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Mass and Transverse Mass Effects on the Hadron Emitter Size

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Abstract

We investigate the dependence of the longitudinal emitter dimension $r_{||}$ of identical bosons, produced in the hadronic Z^0 decays, on their transverse mass m_T obtained from 2-dimensional Bose-Einstein correlations (BEC) analyses. We show that this dependence is well described by the expression $r_{||} = c\sqrt{\hbar\Delta t}/\sqrt{m_T}$, deduced from the uncertainty relations, setting Δt to be a constant of the order of 10^{-24} sec. This equation is essentially identical to the one previously applied to the 1-dimensional BEC results for the emitter radius dependence on the boson mass itself. It is further shown that a very similar behaviour exists also for the dependence of the interatomic separation in Bose condensates on their atomic masses when they are at the same very low temperature.

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1 Introduction

Bose-Einstein Correlations (BEC) of identical bosons, produced in multihadron final states of high energy particle interactions, have been analysed for some 40 years [1]. Many BEC analyses utilised pairs of identical charged pions produced in multihadron final states where the emitter has often been assumed to be a sphere with a Gaussian distribution. The experimental results have been then subjected to a Coulomb correction to account for the repulsive force between the equally charged particles. The kinematic variable frequently used, and still in use today, is defined by

$$Q = \sqrt{-(q_1 - q_2)^2} \ ,$$

where q_1 and q_2 are the four momenta of the two identical hadrons. In the limit of $Q \to 0$ the two identical bosons are occupying the same lowest energy ground state defined in their centre of mass system. To observe the effect of the BEC the experimental Q distribution is divided by a corresponding reference sample distribution which is selected so as to be, as much as possible, identical to the data sample in all its features but void of Bose-Einstein statistics effects. The distribution of this ratio is in general described by the expression

$$C_2(Q) = 1 + \lambda e^{-Q^2 r^2} \ , \tag{1}$$

where r measures the average distance between the two boson when they are predominantly in an s-wave. This r value, extracted from $C_2(Q)$ as Q approaches zero, is taken to represent the dimension of the hadron emitter. The factor λ in Eq. 1, which can vary between $0 \leq \lambda \leq 1$, is the strength of the effect which depends on the chaoticity of the emitter and on the purity of the measured data sample. In e^+e^- annihilations r was found to be within the range of 0.7 to 1.0 fm (see e.g. Ref. [2]), essentially independent of the centre of mass energy $\sqrt{s_{ee}}$.

In recent years the BEC analyses in e^+e^- annihilations have been extended in several directions. Among them, the search for the so called higher order BEC namely, of three or more genuine identical hadrons correlations; the search for deviation from an ideal spherical emitter and to studies aimed to determine whether the correlation dimension is a function of the hadron mass.

In Ref. [3] it was first pointed out that in $e^+e^- \to Z^0 \to hadrons$ the measured dimension r values are a decreasing function of the hadron mass m (see Fig. 1). This observation was first deduced from the measured r values obtained from BEC analyses of identical $\pi^\pm\pi^\pm$ and $K^\pm K^\pm$ boson-pairs. That indeed $dr(m)/dm < 0$ was significantly strengthened by the recent emitter size measurements [4] of the $\Lambda\Lambda$ and $\bar{\Lambda}\bar{\Lambda}$ pairs. These last measurements utilised a method proposed in Ref. [5] where the mixture of S=0 and S=1 spin states can be determined for the hyperon-pair as a function of their centre of mass (CM) energy. The onset of the Pauli exclusion principle, as the CM kinetic energy decreases to zero and the s-wave of the systems dominates, determines the $r(m_\Lambda)$ value which was found to be of the order of 0.15 fm.

Whereas the experimental findings that $r(m_\pi)$ is somewhat larger than $r(m_K)$, but still equal within errors, may still be consistent with the string fragmentation model, although in its basic form it expects $r(m)$ to increase with m [6], the much smaller value obtained for $r(m_\Lambda)$ poses a challenge to the model [7]. At the same time however, it was shown in Ref. [3] that by applying the Heisenberg uncertainty relations, one can derive an expression for $r(m)$ which decreases as

m increases, namely:

$$r(m) \;=\; \frac{c\sqrt{\hbar\Delta t}}{\sqrt{m}}\;. \qquad (2)$$

Taking for Δt the value 10^{-24} sec to represent the time scale of the strong interactions sector, independent of the hadron mass, one obtains the continuous thin line in Fig. 1 which follows rather well the trend of the r values measured in the LEP1 data. A fit of Eq. 2 to the data yields for Δt the value $(1.2 \pm 0.3) \times 10^{-24}$ sec. The continuous thick line in Fig. 1, which is almost identical to the one obtained from the uncertainty relations, was derived from the virial theorem assuming Local Parton Hadron Duality [8] using a general QCD potential [9].

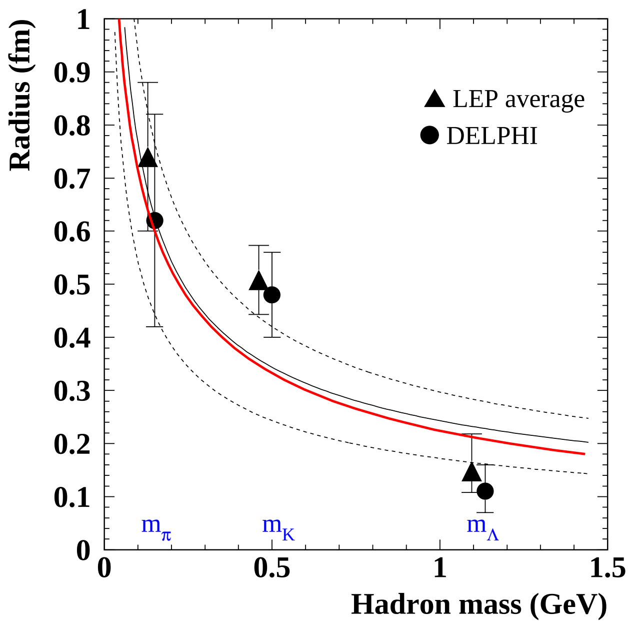

Figure 1: The measured emitter radius $r(m)$ as a function of the hadron mass determined from BEC analyses using hadronic Z^0 decay events at LEP1 (taken from Ref. [2]). The continuous thin line is the prediction from the Heisenberg uncertainty relations setting $\Delta t = 10^{-24}$ sec, the upper and lower dashed lines correspond respectively to the Δt values of 1.5×10^{-24} sec and 0.5×10^{-24} sec. The continuous thick line is derived from the virial theorem assuming Local Parton Hadron Duality and using a general QCD potential.

The effective range of the two-pion sources was also estimated in 2-dimensional BEC analyses, in heavy-ion collisions [10] and in the hadronic Z^0 decays [11], as a function of the transverse mass m_T of the pion-pair, defined as

$$m_T \;=\; 0.5 \times \left(\sqrt{m^2 \;+\; p_{1,T}^2} \;+\; \sqrt{m^2 \;+\; p_{2,T}^2}\;\right)\;. \qquad (3)$$

Here $p_{1,T}$ and $p_{2,T}$ are the transverse momentum of the two identical bosons in the longitudinal centre of mass system (LCMS) [12]. As can be seen e.g. in Fig. 2, the results of these studies show also a decrease of the longitudinal range r_z ($\equiv r_{||}$) of the $\pi\pi$ system as m_T increases. Moreover, the behaviour of $r_z^{\pi}(m_T)$ follows very closely the dependence of $r(m)$ which is a function of the hadron mass itself.

These findings pose the obvious question why the values of $r(m)$ and $r_z^\pi(m_T)$ essentially coincide when $m = m_T(\pi)$. Also of interest is the question whether both or one of the two quantities, m and m_T, are the basic variables on which the hadron emitter dimension depends on and thus should play an integral part in any model describing multi-hadron production.

In the quest to understand the interrelation between $r(m)$ and $r_z^\pi(m_T)$ we explore in Section 2 the possibility that the dependence of r_z on m_T can also be described in terms of the Heisenberg uncertainty relations. Next in Section 3 we turn to another phenomenon related to the Bose statistics namely the Bose-Einstein Condensation. Here we show that at a fixed very low temperature the dependence of the interatomic atomic separation on the mass of the condensates atoms is proportional to $1/\sqrt{m_{atom}}$. Finally a summary is presented in Section 4.

2 The longitudinal dimension dependence on m_T

Most of the BEC analyses were carried out under the assumption that the emitter size is a Gaussian sphere. The possibility that the space-time extend of the particle emission region deviates from a sphere and in fact is characterised by more than one dimension has been recently proposed [13]. In particular the Lund group developed for the BEC a model based on a quantum mechanical interpretation of the string fragmentation probability [14]. In this model the correlation length in the longitudinal string direction should be larger than the corresponding range in the transverse direction.

The experimental analyses, most of which were carried out with identical charged pions, have utilised the longitudinal centre of mass system. This coordinate system is defined for each pair of identical pions as the system in which the sum of the pion-pair momenta $\vec{p}_1 + \vec{p}_2$, referred to as the 'out' axis, is perpendicular to the 'thrust' (or jet) direction defined as the z-axis. The momentum difference of the pion-pair $\vec{Q}$ is then resolved into the longitudinal direction $Q_z \equiv Q_{||}$ parallel to the thrust axis, Q_{out} is collinear with the pair momentum sum and the third axis Q_{side}, is perpendicular to Q_z and Q_{out}. In this system the projections of the total momentum of the particle-pair onto the longitudinal and side directions are equal to zero. In particular $p_{1,z} = -p_{2,z}$, where the index 1 and 2 refer to the first and second pion, so that $Q_z = p_{1,z} - p_{2,z} = 2p_{1,z} = 2p_z$. The difference in the emission time of the pions couples to the energy difference between the particles only in the Q_{out} direction. In a 2-dimensional analysis one defines the transverse component of Q by the relation

$$Q_T^2 = Q_{out}^2 + Q_{side}^2 .$$

Thus the correlation function, which is fitted to the data, is of the form

$$C_2(Q_z, Q_T) = 1 + \lambda e^{-(r_z^2 Q_z^2 + r_T^2 Q_T^2)} , \tag{4}$$

where r_z, estimated from Eq. 4 as Q_z approaches zero, is the longitudinal geometrical radius and r_T is a mixture of the transverse radius and the emission time. The experimental findings, both in heavy ion collisions [15] and in e^+e^- annihilations [16], verified the theoretical expectations that r_T/r_z is significantly smaller than one.

In the frame work of the azimuthally symmetric sources of pair of identical bosons [17] another variable of the emission function is considered namely, the transverse mass m_T defined by Eq.

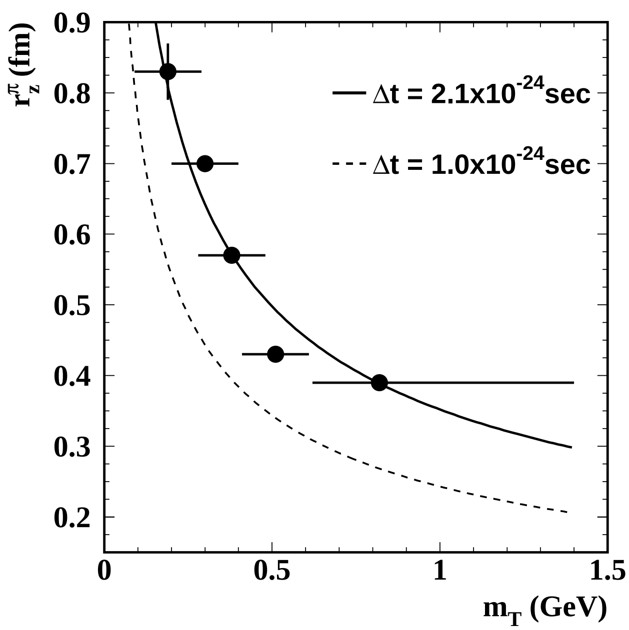

Figure 2: Preliminary results of DELPHI [18] for the dependence of the longitudinal emitter dimension r_z^π on the transverse mass m_T in hadronic Z^0 decays. The data is compared with the expression for r_z given in Eq. 13 setting Δt to the best fitted value of 2.1 $\times 10^{-24}$ sec (continuous line) and the expectation for 1.0 $\times 10^{-24}$ sec (dashed line).

3. Preliminary results of DELPHI [18] concerning the dependence of $r_z^\pi(m_T)$ on m_T, measured for identical charged pion pairs present in the hadronic Z^0 decays, is shown in Fig. 2. As can be seen, $r_z^\pi(m_T)$ decreases with m_T in a very similar way to the decrease of $r(m)$ as the mass m increases. In fact the continuous line in the figure, which is drawn according to Eq. 2 replacing $r(m)$ by $r_z^\pi(m_T)$ and m by m_T, describes well the r_z measurements using for Δt the value 2.1×10^{-24} sec. That this is the case is not surprising once one realises that Δr_z, the longitudinal distance and Δp_z, the difference in the longitudinal momentum of the two hadrons in the LCMS, are conjugate observables which obey the uncertainty principle

$$\Delta p_z \Delta r_z \;=\; \hbar c \;, \tag{5}$$

Here Δp_z is measured in GeV, $\Delta r_z \;\equiv\; r_z$ is given in fermi units and $\hbar c \;=\; 0.197$ GeV fm. In the LCMS one has

$$\Delta p_z r_z \;=\; 2\mu v_z r_z \;=\; p_z r_z \;=\; \hbar c$$

where $\mu \;=\; m/2$ is the reduced mass of the two identical hadrons of mass m and the longitudinal velocity v_z of these hadrons. Thus

$$r_z \;=\; \frac{\hbar c}{p_z} \;. \tag{6}$$

Simultaneously we also utilise the uncertainty relation expressed in terms of energy and time

$$\Delta E \Delta t \;=\; \hbar \;, \tag{7}$$

where the energy is given in GeV and Δt in seconds. In as much that the total energy E of the two-hadron system is determined essentially only by their mass and their kinetic energy, i.e. the potential energy can be neglected, and since in the LCMS $|p_{1,z}| = |p_{2,z}|$, one has

$$E \;=\; \sum_{i=1}^{2}\sqrt{m^2+p_{i,x}^2+p_{i,y}^2+p_{i,z}^2} \;=\; \sum_{i=1}^{2}\sqrt{m_{i,T}^2+p_z^2}\;, \tag{8}$$

where $m_{1,T}$ and $m_{2,T}$ are the transverse mass of the first and second hadron. As Q_z decreases the longitudinal momentum p_z vanishes so that we can, once $0 \leq p_z^2 < m_{i,T}^2$, expand the hadron energy E in terms of $p_z^2/m_{i,T}^2$ and retain only the two first terms,

$$E \;=\; \sum_{i=1}^{2} m_{i,T}\sqrt{1 \;+\; \frac{p_z^2}{m_{i,T}^2}} \;\approx\; \sum_{i=1}^{2} m_{i,T} + \sum_{i=1}^{2}\frac{p_z^2}{2m_{i,T}}\;. \tag{9}$$

Next we order the identical bosons so that $m_{1,T} \geq m_{2,T}$ and define

$$\delta m_T \;=\; \frac{m_{1,T}-m_{2,T}}{2} \;\geq 0\;,$$

while

$$m_T \;=\; \frac{m_{1,T}+m_{2,T}}{2}\;.$$

Inserting these relations into Eq. 9 one gets, after few algebraic steps, that

$$E \;=\; 2m_T \;+\; \frac{m_T p_z^2}{m_T^2-(\delta m_T)^2}\;. \tag{10}$$

As m_T^2 is larger than $(\delta m_T)^2$, we finally get

$$E \;\approx\; 2m_T \;+\; \frac{p_z^2}{m_T}\;. \tag{11}$$

Since $2m_T$ is not a function of p_z it may be considered to stay fixed as $Q_z \to 0$, so that one has

$$\Delta E\Delta t \;=\; \frac{p_z^2}{m_T}\Delta t \;=\; \hbar\;. \tag{12}$$

Combining Eqs. 6 and 12 one obtains

$$r_z(m_T) \;\approx\; \frac{c\sqrt{\hbar\Delta t}}{\sqrt{m_T}}\;. \tag{13}$$

This last equation is identical to the one derived in [3] for the dependence of emitter dimension on the boson mass when $r(m)$ and m are replaced by $r_z(m_T)$ and m_T. A fit of Eq. 13 to the data shown in Fig. 2 yields $\Delta t \;=\; (2.1\pm0.4)\times10^{-24}$ sec so that $r_z^{\pi}(m_T) \;=\; 0.354/\sqrt{m_T(GeV)}$ fm. This value is compatible with the value of $\Delta t \;=\; (1.2\pm0.3)\times10^{-24}$ sec obtained in [3] for $r(m)$ when one also takes into account the relatively wide spread of the 1-dimensional $\pi\pi$ BEC analyses results for $r(m)$ obtained by the four LEP1 experiments (see e.g. Ref. [3]). In heavy-ion collisions of S + Pb, at an energy of 200 GeV per nucleon, the longitudinal range $r_z^{\pi}(m_T)$ was also observed to be inversely proportional to the square root of m_T [17] namely, $r_z^{\pi}(m_T) \;\approx\; 2/\sqrt{m_T(GeV)}$ fm. The ratio between the proportionality factor of 2.0 and 0.354 may well be accounted for by the difference in the extend of the heavy ion target as compared to that of the e^+e^- annihilation leading to hadronic Z^0 decays.

The dependence of the transverse dimension, r_T^{π}, on the two-pion transverse mass m_T has also been measured in the hadronic Z^0 decays [19]. Here again the transverse range was found to decrease as m_T increases. However unlike r_z which is a geometrical quantity, r_T is a mixture of the transverse radius and the emission time so that an application of the uncertainty relations is not straightforward.

3 Interatomic separation in Bose Condensates

When bosonic atoms are cooled down, below a critical temperature T_B, the atomic wave-packets overlap and the equal identity of the particles becomes significant. At this temperature, these atoms undergo a quantum mechanical phase transition and form a Bose-Einstein (BE) condensate, a coherent cloud of atoms all occupying the same quantum mechanical state. This phenomenon, first predicted by A. Einstein in 1924/5, is a consequence of quantum statistics [20]. Detailed theoretical aspects of the Bose-Einstein condensation can be found in Ref. [21] and its up to date experimental situation is described in Ref. [22]. Concise summaries, aimed in particular to the non-expert, both of the experimental situation and the theoretical background, can be found in Refs. [23, 24]. To form Bose condensates one cools down, below the critical temperature T_B, extremely dilute gases so that the formation time of molecules and clusters in three-body collisions is slowed down to seconds or even minutes to prevent the creation of more familiar transitions into liquid or even solid states.

The existence of BE Condensation was first demonstrated in 1995 by three groups [25] in cooling down rubidium, sodium and lithium. Typical temperatures where BE condensates occur are in the range of 500 nK to 2 μK with atom densities between 10^{14} and 10^{15} cm^{-3}. The largest sodium condensate has about 20 million atoms whereas hydrogen condensate can reach even one billion atoms.

Let us consider a dilute homogeneous ideal gas of N identical bosonic atoms of spin zero, confined in a volume V. These atoms occupy energy levels ϵ, handled here as a continuous variable, which are distributed according to the Bose-Einstein statistics. We further set the ground state to be $\epsilon_0 = 0$. If N_0 is the number of atoms in this ground state and N_{ex} is the number of atoms in the excited states then $N = N_0 + N_{ex}$. For a homogeneous ideal gas of identical bosonic atoms it can be shown [26] that at a low temperature T one has

$$N_{ex} = 2.612\, V \left(\frac{2\pi mkT}{h^2}\right)^{3/2} , \tag{14}$$

where V is the volume occupied by the atoms of mass m. Since T_B is defined as the temperature where almost all bosons are still in excited states, we can, to a good approximation, equate N with N_{ex}. That is

$$N = 2.612\, V \left(\frac{2\pi mkT_B}{h^2}\right)^{3/2} , \tag{15}$$

For $T < T_B$ one obtains from Eqs. 14 and 15 that the number of atoms N_0 which are in the condensate state is,

$$N_0 = N - N_{ex} = N\left[1 - \left(\frac{T}{T_B}\right)^{3/2}\right] , \tag{16}$$

where T is the temperature of the atoms lying at the excited energy states above the condensate energy level $\epsilon_0 = 0$. The atomic density of the Bose gas at very low temperatures, $T/T_B << 1$ where $N \approx N_0$, is then given by

$$\rho = \frac{N}{V} = 2.612 \left(\frac{2\pi mkT}{h^2}\right)^{3/2} , \tag{17}$$

where k is the Boltzmann constant and ρ, the atomic density, has the dimension of L^{-3}. From this follows that $\rho^{-1/3}$ is the average interatomic separation in the Bose condensate. At the same time the thermal de Broglie wave length is equal to

$$\lambda_{dB} = \left(\frac{h^2}{2\pi mkT}\right)^{1/2} . \tag{18}$$

Combining Eqs. 17 and 18 one has for the state of a Bose condensate the relation

$$\rho\lambda_{dB}^3 \approx 2.612 . \tag{19}$$

Thus the average interatomic distance in a Bose condensate, d_{BE}, is equal to

$$d_{BE} \equiv \rho^{-1/3} \approx \lambda_{dB}/1.378 . \tag{20}$$

Next we consider two different bosonic gases, having atoms with masses m_1 and m_2, which are cooled down to the same very low temperature T_0, below the critical temperature T_B of each of them. In this case we will produce two Bose condensates with interatomic distances

$$d_{BE}(m_i) \approx \frac{\sqrt{2\pi}}{1.378}\left(\frac{\hbar^2}{m_i kT_0}\right)^{1/2} \quad ; \quad i = 1, 2 . \tag{21}$$

From this follows that when two condensates are at the same fixed temperature T_0 one has

$$\frac{d_{BE}(m_1)}{d_{BE}(m_2)} = \sqrt{\frac{m_2}{m_1}} , \tag{22}$$

which is also the expectation of Eq. 2 for the dimension dependence on the mass of the hadron produced in high energy reactions provided Δt is fixed. Finally it is interesting to note that in as much that one is justified to replace in Eq. 21, at a very low temperature, kT_0 by ΔE and use the uncertainty relation $\Delta E = \hbar/\Delta t$ one derives the expression for $r(m)$ as given by Eq. 2 multiplied by the factor $\sqrt{2\pi}/1.378$.

In relating the condensates with the production of hadrons in high energy reactions one should however keep in mind that the interatomic separation proportionality to $1/\sqrt{m}$ does not necessarily imply that this should also be so for hadrons produced in high energy reactions. Common to both systems is their bosonic nature which allow all hadrons (atoms) to occupy the same lowest energy state. In addition the condensates are taken to be in a thermal equilibrium state. Among the various models proposed for the production of hadrons in high energy e^+e^- and Nucleon-Nucleon reactions some attempts have also been made to explore the application of a statistical thermal-like model [27]. However, whether this approach will eventually prevail is at present questionable. Finally the condensates are taken to be in a coherent state. In the case of the of hadrons one is able to measure r via BEC only if the chaoticity factor λ in Eq. 1 is different from zero i.e., only if the source is not 100% coherent. However so far there is no evidence for a dependence of r on λ apart from that introduced by the correlated errors between Δr and $\Delta\lambda$ produced by fitting Eq. 1 to the data.

4 Summary

It is shown that the longitudinal range of the two-pion emitter size obtained from 2-dimensional BEC analyses of the hadronic Z^0 decays as a function of the transverse mass, is well described by the expectation of the Heisenberg uncertainty relations using a constant value for Δt of the order of 10^{-24} seconds. As a consequence it is not surprising that $r_z^\pi(m_T)$ has essentially the same behaviour as the one observed experimentally for $r(m)$ which is a function of the hadron mass itself. In both cases the range, in fm, is equal to $(0.2-0.4)/\sqrt{m_T}$. This kind of dependence on the transverse mass is also seen in heavy-ion collisions where it was found that $r_z^\pi \approx 2/\sqrt{m_T}$.

It is interesting to note that the interatomic separation of atoms in different Bose condensates, having the same fixed temperature, is proportional to $1/\sqrt{m}$, where here m is the atom mass. This behaviour is the same as found for the range between identical hadron-pair produced in high energy reactions when the time scale Δt is fixed. This similarity can be traced back to the close connection between the de Broglie wave length, applied to the bosonic atoms in condensates, and the Heisenberg uncertainty relations, used here to connect in high energy reactions the range between identical hadrons to their mass.

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