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Coherent Exclusive Exponentiation of 2f Processes in e^+e^- Annihilation*

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In the talk we present the Coherent Exclusive Exponentiation (CEEX) which is implemented in the \mathcal{K} CMC event generator for the process $e^+e^- \rightarrow f\bar{f} + n\gamma$, $f = \mu, \tau, d, u, s, c, b$ for center of mass energies from τ lepton threshold to 1TeV, that is for LEP1, LEP2, SLC, future Linear Colliders, b, c, τ -factories etc. We will attempt a short discussion of the theoretical concepts necessary in our approach, in particular the relations between the rigorous calculation of spin amplitudes (perturbation expansion), phase space parametrisation and exponentiation. In CEEX effects due to photon emission from the initial beams and outgoing fermions are calculated in QED up to second-order, including all interference effects. Electroweak corrections are included in first-order, at the amplitude level. The beams can be polarised longitudinally and transversely, and all spin correlations are incorporated in an exact manner. Precision predictions, in particular the photon emission at LEP2 energies, are also shown.

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1 Introduction

At the end of LEP2 operation the total cross section for the process $e^-e^+ \rightarrow f\bar{f} + n\gamma$ has to be calculated with the precision 0.2% – 1%, depending on the event selection. The arbitrary differential distributions have to be calculated with the corresponding precision. Even now, this is not always the case [1] and the calculations are still continuing. In the future, for linear colliders (LC’s), the precision requirement can be even more demanding. These requirements necessitate development of the appropriate calculational schemes for the QED corrections and the construction of new dedicated MC programs. We present here an effort in this direction. Our report is based on refs. [2–4] and the Monte Carlo program is described in ref. [5]. The pedagogical introduction to some concepts necessary in understanding exponentiation can be found e.g. in [6].

Feature	KORALB	KORALZ	KK 4.13	KK 2000+?
QED type	$\mathcal{O}(\alpha)$	EEX	CEEX, EEX	CEEX, EEX
CEEX(ISR+FSR)	none	none	$\{\alpha, \alpha L; \alpha^2 L^2, \alpha^2 L^1\}$	$\{\dots \alpha^2 L^1; \alpha^3 L^3\}$
EEX(ISR+FSR)	none	$\{\alpha, \alpha L, \alpha^2 L^2\}$	$\{\alpha, \alpha L, \alpha^2 L^2, \alpha^3 L^3\}$	$\{\dots \alpha^2 L^2, \alpha^3 L^3\}$
ISR-FSR int.	$\mathcal{O}(\alpha)$	$\mathcal{O}(\alpha)$	$\{\alpha, \alpha L\}_{\text{CEEX}}$	$\{\alpha, \alpha L\}_{\text{CEEX}}$
Exact brems.	1 γ	1, 2coll. γ	1, 2, 3coll. γ	up to 3 γ
Electroweak	No Z-res.	DIZET 6.x	DIZET 6.x	New version?
Beam polar.	long+trans.	longit.	long+trans.	long+trans.
τ polar.	long+trans.	longit.	long+trans.	long+trans.
Hadronization	—	JETSET	JETSET	PYTHIA
τ decay	TAUOLA	TAUOLA	TAUOLA	TAUOLA
Inclusive mode	—	No	Yes	Yes
Beamstrahlung	—	No	Yes	Yes
Beam spread	—	No	Yes	Yes
$\nu\nu$ channel	—	Yes	No	Yes
ee channel	—	No	No	Yes
tt channel	—	No	No	yes?
WW channel	—	No	No	yes?

Table 1: Overview of the KKMC event generator as compared with KORALZ and KORALB.

2 What is precision calculation

New results from high energy particle experiments are obtained as a result of the huge effort of hundreds of experimental physicists over many years. In the cases when

theoretical calculations are needed to interpret the results, it is fair to require, whenever possible, the uncertainty of the calculations to be smaller at least by a factor of 3 than the experimental error. Once the condition is fulfilled, in the final interpretation of experimental data for quantities such as coupling constants, total cross sections or particle masses, the final combined theoretical and experimental uncertainty would not increase more than 10 % with respect to experimental uncertainty alone. This rule of thumb is motivated in cases when the theoretical calculations are possible and require an effort much smaller than that of the experiments.

The crucial requirement of the high precision calculation is however not only that its results agree with the measured data, but also, that the relation of the results with the foundation of the Standard Model field theory is fully controlled. At present, requirements for precision, as defined by experiments, do not exceed the 0.1 % tag. That is why, in general, predictions including complete Standard Model corrections of $\mathcal{O}(\alpha_{QED})$, are sufficient. Only those terms of the higher orders which include enhancement factors such as $\ln \frac{s}{m_f^2}$, $\ln \frac{M_Z}{\Gamma_Z}$, $\frac{m_t}{m_W}$ etc., have to be taken into account.

Thanks to this, one can define schemes of calculation where QED calculations can be separated from the rest, and dealt with to large degree individually¹. As it was presented in [1,7] this was indeed the solution successful for LEP2 $e^+e^- \rightarrow 2f$ and $e^+e^- \rightarrow 4f$ processes. Exponentiation is a convenient way of dealing with the QED corrections, which are large, and depend on the detection conditions (cuts).

3 What is coherent exclusive exponentiation CEEEX?

The *exponentiation* is generally a method of summing up real and virtual photon contributions to infinite order such that infrared (IR) divergences cancel. The *exclusivity* means that the procedure of exponentiation, that is summing up the infrared (IR) real and virtual contribution, within the standard perturbative scheme of quantum field theory, is done at the level of the fully differential (multiphoton) cross section, or even better, at the level of the scattering matrix element (spin amplitude), *before any phase-space integration over photon momenta is done*. The other popular type of the exponentiation is *inclusive* exponentiation (IEX), which is done at the level of inclusive distributions, structure functions, etc. see discussion in ref. [8]. The classical work of Yennie-Frautschi-Suura [9] (YFS) represents the best example of the exclusive exponentiation and we nickname it as EEX. Finally, why do we use word *coherent*? In CEEEX the essential part of the summation of the IR real and virtual photon contributions is done at the amplitude level. Of course, IR cancellations occur as usual at the probability level, however, the transition from spin amplitudes to dif-

¹Even further separation is possible: emission of additional real fermion pairs can be calculated separately. At the same step, the appropriate virtual corrections have to be included into predictions for $2f$ -processes.

ferential cross sections, and the phase space integration are done entirely numerically! As a consequence of the above *coherent* approach it follows that CEEX is friendly to coherence among Feynman diagrams, narrow resonances, interferences, etc. This is a great practical advantage. In our many previous works which led to the development of the Monte Carlo event generators like YFS2, YFS3, KORALZ, KORALW, YFS3WW, BHLUMI, BHWIDE, see refs. [10–15], we have generally employed EEX, which is closely related to the YFS work [8]. The CEEX is a recent development and is so far used only in the new $\mathcal{K}\mathcal{K}\text{MC}$ program [5].

Let us now show in a very simplified schematic way what is the the main difference between the old EEX/YFS and the CEEX for the fermion pair production the process:

$$e^-(p_1, \lambda_1) + e^+(p_2, \lambda_2) \rightarrow f(q_1, \lambda'_1) + \bar{f}(q_2, \lambda'_2) + \gamma(k_1, \sigma_1) + \dots + \gamma(k_n, \sigma_n). \quad (1)$$

The EEX total cross section is

$$\sigma = \sum_{n=0}^{\infty} \int_{m_\gamma} d\Phi_{n+2} e^{Y(m_\gamma)} D_n(q_1, q_2, k_1, \dots, k_n), \quad (2)$$

where in the $\mathcal{O}(\alpha^1)$ the distributions for $n_\gamma = 0, 1, 2$ are

$$\begin{aligned} D_0 &= \bar{\beta}_0 \\ D_1(k_1) &= \bar{\beta}_0 \tilde{S}(k_1) + \bar{\beta}_1(k_1) \\ D_2(k_1, k_2) &= \bar{\beta}_0 \tilde{S}(k_1) \tilde{S}(k_2) + \bar{\beta}_1(k_1) \tilde{S}(k_2) + \bar{\beta}_1(k_2) \tilde{S}(k_1) \end{aligned} \quad (3)$$

and the real soft factors are defined as usual

$$4\pi \tilde{S}(k) = \sum_{\sigma} |\mathfrak{s}_\sigma(k)|^2 = |\mathfrak{s}_+(k)|^2 + |\mathfrak{s}_-(k)|^2 = -\frac{\alpha}{\pi} \left(\frac{q_1}{kq_1} - \frac{q_2}{kq_2} \right)^2. \quad (4)$$

What is important for our discussion is that the IR-finite building blocks

$$\begin{aligned} \bar{\beta}_0 &= \sum_{\lambda} |\mathcal{M}_\lambda|^2, \\ \bar{\beta}_1(k) &= \sum_{\lambda\sigma} |\mathcal{M}_{\lambda\sigma}^{1\text{-phot}}|^2 - \sum_{\sigma} |\mathfrak{s}_\sigma(k)|^2 \sum_{\lambda} |\mathcal{M}_\lambda^{\text{Born}}|^2 \end{aligned} \quad (5)$$

in the multiphoton distributions are all in terms of $\sum_{spin} |\dots|^2$!! We denoted: $\lambda =$ fermion helicities and $\sigma =$ photon helicity.

The above is to be contrasted with the analogous $\mathcal{O}(\alpha^1)$ case of CEEX

$$\sigma = \sum_{n=0}^{\infty} \int_{m_\gamma} d\Phi_{n+2} \sum_{\lambda, \sigma_1, \dots, \sigma_n} |e^{B(m_\gamma)} \mathcal{M}_{n, \sigma_1, \dots, \sigma_n}^\lambda(k_1, \dots, k_n)|^2, \quad (6)$$

where the differential distributions for $n_\gamma = 0, 1, 2$ photons are the following:

$$\begin{aligned}\mathcal{M}_0^\lambda &= \hat{\beta}_0^\lambda, \quad \lambda = \text{fermion helicities}, \\ \mathcal{M}_{1,\sigma_1}^\lambda(k_1) &= \hat{\beta}_0^\lambda \mathfrak{s}_{\sigma_1}(k_1) + \hat{\beta}_{1,\sigma_1}^\lambda(k_1), \\ \mathcal{M}_{2,\sigma_1,\sigma_2}^\lambda(k_1, k_2) &= \hat{\beta}_0^\lambda \mathfrak{s}_{\sigma_1}(k_1) \mathfrak{s}_{\sigma_2}(k_2) + \hat{\beta}_{1,\sigma_1}^\lambda(k_1) \mathfrak{s}_{\sigma_2}(k_2) + \hat{\beta}_{1,\sigma_2}^\lambda(k_2) \mathfrak{s}_{\sigma_1}(k_1)\end{aligned}\quad (7)$$

and the IR-finite building blocks are

$$\begin{aligned}\hat{\beta}_0^\lambda &= (e^{-B} \mathcal{M}_\lambda^{\text{Born+Virt.}})|_{\mathcal{O}(\alpha^1)}, \\ \hat{\beta}_{1,\sigma}^\lambda(k) &= \mathcal{M}_{1,\sigma}^\lambda(k) - \hat{\beta}_0^\lambda \mathfrak{s}_\sigma(k).\end{aligned}\quad (8)$$

As shown explicitly, this time everything is in terms of \mathcal{M} -spin-amplitudes! This is the basic difference between EEX/YFS and CEEEX. The complete expressions for spin amplitudes with CEEEX exponentiation, for any number of photons, are shown in ref. [2] for the $\mathcal{O}(\alpha^1)$ case and in ref. [4] for the $\mathcal{O}(\alpha^2)$ case.

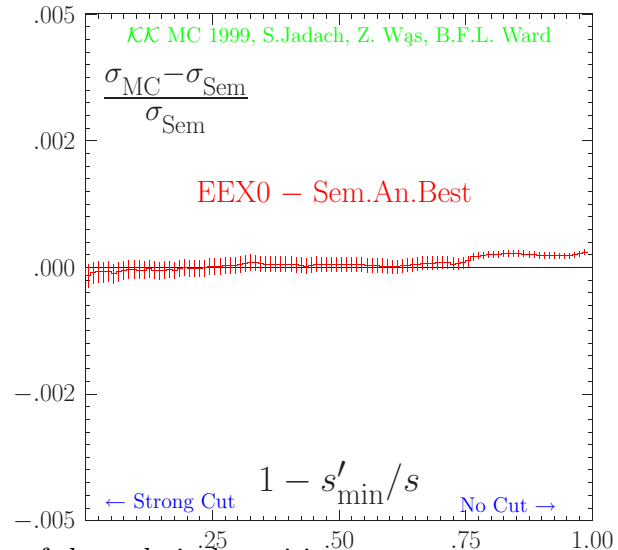


Figure 1: Baseline test of the technical precision.

4 Monte Carlo numerical results

The $\mathcal{O}(\alpha^2)$ CEEEX-style matrix element is implemented in $\mathcal{K}\mathcal{K}\text{MC}$ which simulates the production of muon, tau and quark pairs. Electrons (Bhabha scattering) and neutrino channels are not available. The program includes for the optional use the older, EEX-style matrix element. It is then functionally similar to KORALZ [11] and the older KORALB [16] programs. In Table 1 we provide the complete comparison of the features of $\mathcal{K}\mathcal{K}\text{MC}$ and the older programs.

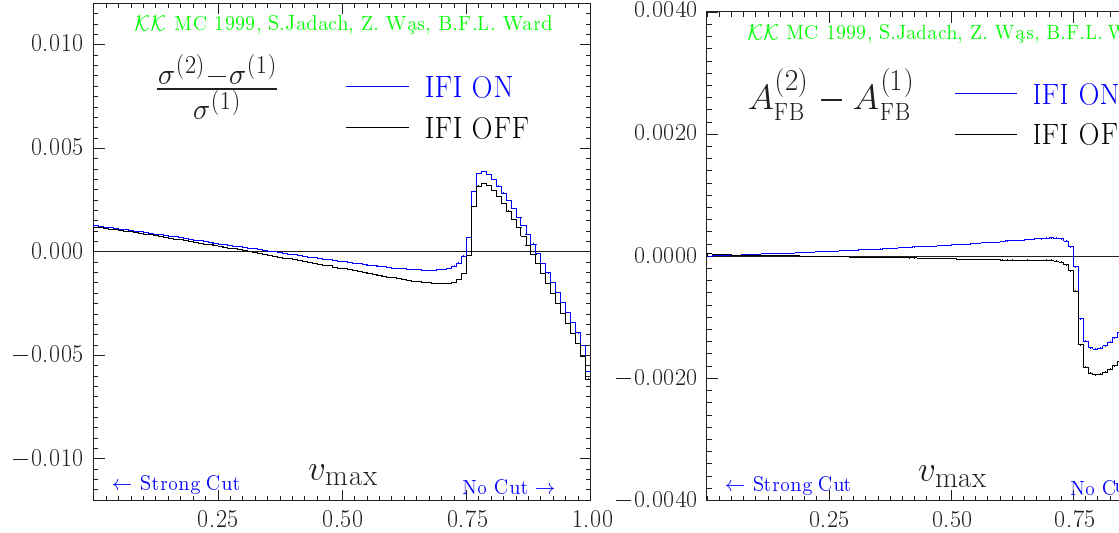


Figure 2: Test of the technical precision of $\mathcal{K}\mathcal{K}\text{MC}$.

4.1 Technical precision

For the new MC program of the high complexity like $\mathcal{K}\mathcal{K}\text{MC}$ it is important to check very precisely the overall normalisation. This is the cornerstone of the evaluation of the *technical precision* of the program, especially for $\mathcal{K}\mathcal{K}\text{MC}$ which is aimed at the end of testing at the total precision of 0.1%. In Fig. 1 we present the comparison of the $\mathcal{K}\mathcal{K}\text{MC}$ with simple semi-analytical integration for the total cross section, as a function of the minimum mass $\sqrt{s'_{\min}}$ of the final muon pair. It is done for muon-pair final state at $\sqrt{s} = 200\text{GeV}$. For $\sqrt{s'_{\min}} \rightarrow \sqrt{s}$, when emission of hard photons is suppressed, there is an agreement $< 0.02\%$ between $\mathcal{K}\mathcal{K}\text{MC}$ and the analytical calculation. For $\sqrt{s'_{\min}} < M_Z$ the on-shell Z-boson production due to emission of the hard initial state radiation (ISR), the so called Z radiative return (ZRR), is allowed kinematically. Even in this case (more sensitive to higher orders) the agreement $< 0.02\%$ is reached. For the above exercise we used the simplified $\mathcal{O}(\alpha^0)$ CEEX matrix element, because in this case the precise phase-space analytical integration is relatively easy.

4.2 Physical precision

The equally important component of the overall error is the physical error which we estimate conservatively as the half of the difference $\mathcal{O}(\alpha^2) - \mathcal{O}(\alpha^1)$. In Fig. 2 we show the corresponding result for the total cross section and charge asymmetry for $\sqrt{s} = 189\text{GeV}$ as a function of the cut on energies of all photons ($s'_{\min} < s$ limits the total photon energy.) We obtain in this way the estimate 0.2% for the physical

precision of the total cross section and 0.1% for the charge asymmetry. Both plots in Fig. 2 show as expected a strong variation at the position of the ZRR. The quoted precision is good enough for the LEP2 combined data.

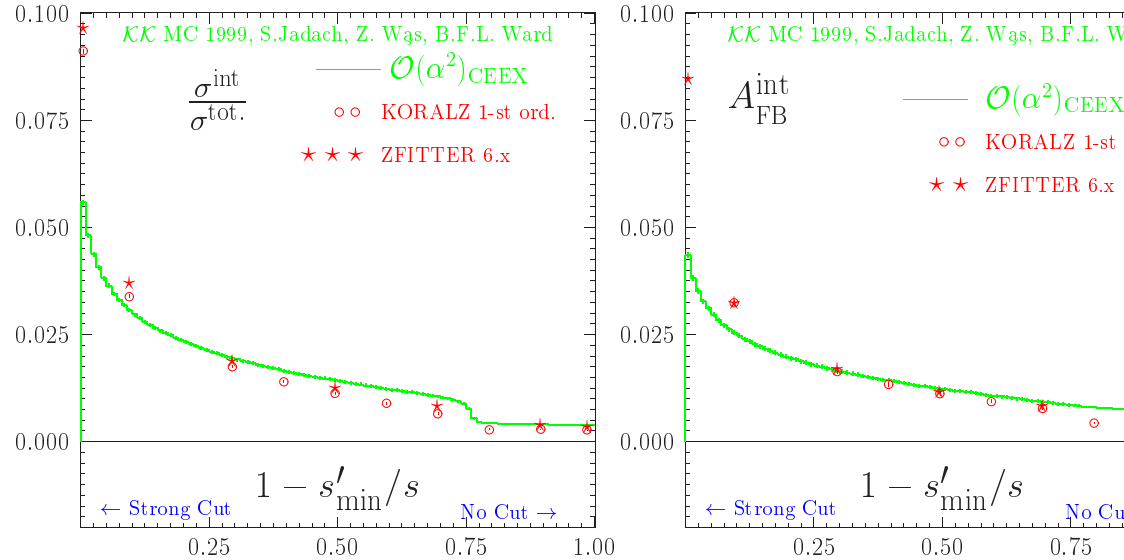


Figure 3: The effect of the initial-final state QED interference in total cross-section and charge asymmetry.

4.3 Initial-final state QED interference

One important benefit from CEEEX with respect to the older EEX is the inclusion of the Initial-Final state QED Interference (IFI). The effect of the IFI is comparable with the precision of the LEP2 combined data and should be under good control. Results of our analysis of the size of the IFI at LEP2 energies ($\sqrt{s} = 189\text{GeV}$) are shown in Fig. 3. In this figure we compare the CEEEX result of $\mathcal{K}\mathcal{K}\text{MC}$ first of all with the result of KORALZ which is run in the $\mathcal{O}(\alpha^1)$ mode without exponentiation (The IFI is neglected for KORALZ with the EEX matrix element.) The $\mathcal{O}(\alpha^1)$ IFI contribution from KORALZ was extensively cross-checked in the past with the dedicated semi-analytical calculations [17]; it is therefore a good reference and starting point. As we see the IFI contribution of CEEEX differs slightly from the pure $\mathcal{O}(\alpha^1)$ result. It is related to exponentiation which makes the angular dependence (in the muon scattering angle) of the IFI contribution less sharp and it is also due to the convolution of the IFI with the $\mathcal{O}(\alpha^2)$ ISR. The expected modification of the interference correction due to higher orders is about 20% for the cross section and asymmetry, if the ZRR is excluded, (the size of the ISR correction in the cross section) and it is indeed of this size. Apparently, this principle works also in the case of ZRR included, remembering

that in this case the ISR correction is 100% or more. However, we feel that this case requires further study. We have also included results of the semianalytical program ZFITTER [18] in our plots². They agree well with the $\mathcal{O}(\alpha^1)$ IFI of KORALZ. This is expected because they are without exponentiation.

5 Outlook and summary

The most important new features in the present CEEEX are the ISR-FSR interference, the second-order subleading corrections, the exact matrix element for two hard photons, and the full density matrix treatment for the spin states of initial and final state fermions³. This makes CEEEX already a unique source of SM predictions for the LEP2 physics program and for the LC physics program. Note that for these the electroweak correction library has to be reexamined at LC energies. The most important omission in the present version is the lack of neutrino and electron channels. Let us stress that the present program is an excellent starting platform for the construction of the second-order Bhabha MC generator based on CEEEX exponentiation. We hope to be able to include the Bhabha and neutrino channels soon, possibly in the next version⁴. The other important directions for the development are the inclusion of the exact matrix element for three hard photons, together with virtual corrections up to $\mathcal{O}(\alpha^3 L^3)$ and the emission of the light fermion pairs. The inclusion of the W^+W^- and $t\bar{t}$ final states is still in a farther perspective.

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²We would like to thank D. Bardin for providing us results from ZFITTER.

³The recent presentation of the τ lepton decay library TAUOLA can be found in ref [19].

⁴At the time of the completion of the conference contribution, the program version including the neutrino channel can be obtained upon individual request only. It is still at the stage of the pre-release tests.

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