INTERMITTENCY AND CORRELATIONS AT LEP AND AT HERA

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A review on recent investigations of local fluctuations and genuine correlations in e^+e^- annihilations at LEP and in e^+p collisions at HERA is given.

1 Introduction

Local fluctuations and genuine correlations of hadrons in high energy interactions provide us with important information about multihadron production mechanism. This information gives detailes which are beyond those obtained from studies of single-particle distributions.

The peculiarity of hadrons to group when produced have been studied a long time using correlation functions, while in the recent decade the interest in correlations have been revived due to a new method of factorial moments and due to the obtained intermittency phenomenon, i.e. self-similarity, or fractality, of hadroproduction.[1,2]

The fractal structure of particle distribution in e^+e^- collisions has been realized many years ago due to a jet evolution picture (see e.g. [2]). However, even if the parton shower is expected to exhibit intermittency, it does not guarantee the effect to appear at hadron level. Recently, different approaches have been proposed to describe experimentally observed fractality by the analytical QCD calculations.[3]

In this talk we review experimental studies of intermittency and correlations at LEP [4,5,6,7,8] and at HERA [9,10], which appeared recently and provide us with further information in addition to discussions of recent reviews.[1,2] From the studies, it is seen that, despite considerable success in understanding different properties of the phenomenon, further investigations are needed.

2 Definitions

2.1 A Tool of Factorial Moments and Cumulants

In order to measure local dynamical fluctiations, a method of normalized factorial moments, F_q, is applied.[1,2] The factorial moment of order q is defined as a function of a phase-space region size δ,

$$F_q(\delta) = \langle n(n-1)\cdots(n-q+1)\rangle / \langle n\rangle^q. \quad (1)$$

Here n is the number of particles in δ-region, and the brackets $\langle\cdots\rangle$ denote averaging over events.

The normalised factorial moments allow us to extract dynamical fluctuations. For uncorrelated particle production mechnism, the moments are independent of δ, $F_q \equiv 1$. Correlations beetween particles lead to increasing factorial moments with decreasing δ (increasing number M of the δ-regions), and if exhibiting a power-law the dependence is called intermittency.

To extract genuine correlations contributing to the fluctuations, one uses the technique of normalised factorial cumulant moments, cumulants.[1,2] The cumulants, K_q, are constructed from the unnormalised factorial moments in a way they vanish whenever particles in q-tuple are statistically independent.

The normalised cumulants are defined as

$$K_q(\delta) = k_q / \langle n\rangle^q. \quad (2)$$

with the Mueller moments k_q,

$$k_1 = \langle n\rangle\,, \quad k_2 = \langle n(n-1)\rangle - \langle n\rangle^2,$$
$$k_3 = \langle n(n-1)(n-2)\rangle - 3\,\langle n(n-1)\rangle\,\langle n\rangle + 2\,\langle n\rangle^3\,, \quad \text{etc.} \quad (3)$$

Normalised cumulants share with the normalised factorial moments their property to measure dynamical component of the underlying particle density.

From Eqs. (2) and (3), one finds the interrelations between normalised factorial moments and cumulants,

$$F_1 = K_1, \quad F_2 = K_2 + 1, \quad F_3 = K_3 + 3K_2 + 1, \text{ etc.} \quad (4)$$

which provide us with the information whether the p-order genuine correlations are important in the q-particle dynamical fluctuation.

2.2 Analytical QCD Predictions

The QCD description of multihadron production is based on partonic picture, i.e. on gluon cascades radiated off the initial parton.[3] In order to describe hadroproduction mechanism, one has to cut off the parton cascade at some scale $Q_0 \leq 1$ GeV, while the following non-perturbative hadronization is considered within the concept of Local Hadron Parton Duality (LPHD), which connects multihadron final states and partons.

The QCD calculations are given in the angular phase space, i.e. in 1-dimensional, 1D, rings (2-dimensional, 2D, cones) around a jet axis with mean opening angle Θ (a direction (Θ,Φ)) and a half width (opening angle) $\delta \equiv \vartheta$.

For the normalised cumulants and factorial moments, the power-law,

$$K_q(\Theta,\vartheta) \text{ or } F_q(\Theta,\vartheta) \propto (\Theta/\vartheta)^{(q-1)(D-D_q)}, \tag{5}$$

with fractal or Rényi dimensions D_q is predicted. D is a dimensional factor: $D=1$ for ring regions and 2 for cones.

The QCD expectations for D_q are as follows (see [3] and refs. therein).

- In a fixed-coupling regime ($\alpha_s = \text{const.}(\vartheta)$) of the Double Log Approximation (DLA),

$$D_q \equiv D_q^{(\mathrm{c})} = \gamma_0(Q)\,\frac{q+1}{q}, \quad \gamma_0(Q) = \sqrt{2N_c\alpha_s/\pi}, \quad Q = E\Theta, \tag{6}$$

  for moderately small angular regions, $\vartheta \leq \Theta$.

- In a running-coupling regime of the DLA,

$$\text{(a)}\quad D_q = D_q^{(\mathrm{c})}\left(1+\frac{q^2+1}{4q^2}\cdot\varepsilon\right), \qquad \text{(b)}\quad D_q = 2D_q^{(\mathrm{c})}\left(\frac{1-\sqrt{1-\varepsilon}}{\varepsilon}\right),$$

$$\text{(c)}\quad D_q = 2\gamma_0(Q)\,\frac{q-w(q,\varepsilon)}{\varepsilon(q-1)}, \quad w(q,\varepsilon) = q\sqrt{1-\varepsilon}\left(1-\frac{\ln(1-\varepsilon)}{2q^2}\right). \tag{7}$$

- In the Modified Leading Log Approximation (MLLA), Eq. (7a) remains valid but γ_0 is replaced by an effective one, $\gamma_0^{\mathrm{eff}}(Q) = \gamma_0(Q) + \gamma_0^2(Q)\cdot f(q, N_f, N_c)$.

Here, E is the jet energy, N_c and N_f are the number of flavors and colors, respectively.

A scaling variable,

$$\varepsilon = \frac{\ln(\Theta/\vartheta)}{\ln(E\Theta/\Lambda)}\,. \tag{8}$$

is utilised in the calculations, For the maximum phase-space $\vartheta = \Theta$, $\varepsilon = 0$.

The analytical predictions involve only one adjustable parameter, the QCD scale Λ, while a strong coupling α_s is based on first-order QCD relation,

$$\alpha_s = \frac{\pi\beta^2}{6}\frac{1}{\ln(Q/\Lambda)}, \quad \beta^2 = 12\left(\frac{11}{3}N_c - \frac{2}{3}N_f\right)^{-1}. \tag{9}$$

The QCD calculations are made at asymptotic energies, which corresponds to an infinite number of partons in an event. No energy-momentum conservation is taken into account in the calculations above.

3 Experimental Results

3.1 Spatial Fluctuations and Correltions

At LEP, L3 [5] and OPAL [8] have studied fluctuations and correlations in the $Z^0 \to e^+e^- \to$hadrons process.

L3 measured fluctuaions in rapidity and in the 4-momentum difference ($Q_{ij}^2 = -(p_i - p_j)^2$ of all ij-pairs) using factorial moments for the former variable and the bunching parameters for both variables. The measurements have shown the multifractal character of the local fluctuations as it is expected from the QCD parton shower picture.

A large statistics of more than 4 million events has been used by OPAL to measure local fluctuations and genuine correlations in one-, two- and three-dimensional subspaces of rapidity, azimuthal angle, and transverse momentum (w.r.t. the sphericity axis). As well as L3, OPAL has observed multifractal behavior of factorial moments in one dimension, rapidity and azimuthal angle. Such an intermittency behaviour is more pronounced with increasing dimension which is ascribed to a jet-like structure of the events as predicted.[2]

OPAL has measured factorial cumulants and found the genuine correlations up to the 5th order, being especially large in rapidity vs. azimuthal angle subspace. Using Eqs. (4) reduced to some pth order, $p<q$ (e.g. $F_3^{(2)} = 3K_2 + 1$), OPAL checked the importance of the genuine correlations in the dynamical fluctuations obtained. It was found that genuine correlations of high-order are needed to describe the intermittency effect, see Fig. 1.

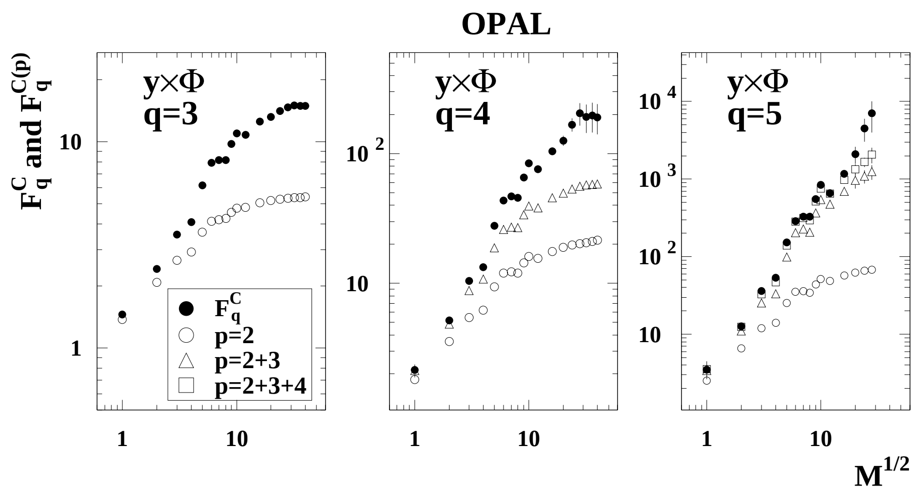


Figure 1. Decomposition of factorial moments F_q into correlation contributions $F_q^{(p)}$ in rapidity vs. azimuthal angle subspace measured by OPAL [8] (see Eq. (4) and text).

In both Collaborations' studies, Monte Carlo (MC) models which have been tuned to reproduce global event-shape distributions and single-particle inclusive spectra in e^+e^- annihilations, were found to reproduce the trend, but not the magnitudes of the measured moments, especially for small δ.

3.2 Angular Fluctuations and Correlations

L3 [4], DELPHI [6,7] and ZEUS [9,10] have measured fluctuations and correlations in angular phase space. While the thrust or sphericity axes are chosen as a jet axis in LEP experiments, the special Breit frame is considered in the HERA experiment to separate hadronic final state from the radiation and to mimic a single e^+e^- reaction hemisphere.

Comparing the measurements to the first-order analytical predictions, Eqs. (6), (7), and (9), it has been found that the DLA and MLLA reasonably well describe the general features of the data (Figs. 2, 3). The measured moments rise approximately linearly for large angles ϑ (small ε) as expected from the parton shower multifractality (cf. Eq. (5)), while level off at smaller angles beleived due to the runing effect of the coupling α_s. The 2D moments rise much more steeply than the 1D one.[7] The factorial moments are found [7] to increase as the energy increases, see Fig. 3. To note is that the ϑ- and D-dependences are analogous, respectively, to multifractality of a parton shower and to the jet structure, obtained in spatial above-discussed analyses.[5,8]

On the other hand, one finds some deviations between the QCD predic-

Figure 2: L3 [4] factorial moments $F_q(z \equiv \varepsilon)/F(0)$ at Λ=0.16 GeV (left) and Λ=0.04 GeV (right) compared to the QCD calculations according to DLA Eqs. (6), (7) and MLLA ($\gamma_0 \to \gamma_0^{\rm eff}$).

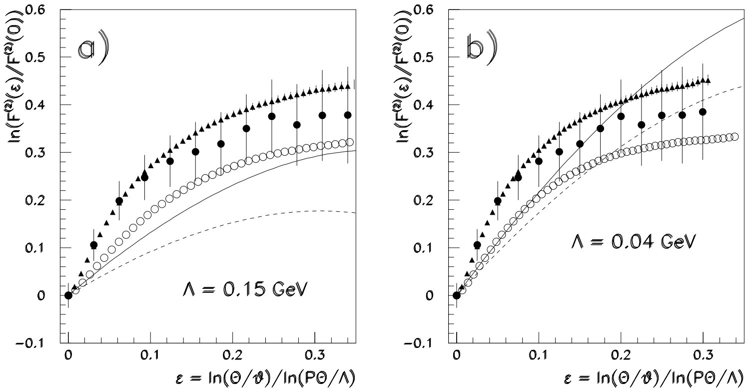


Figure 3. DELPHI factorial moments $F_2(\varepsilon)/F_2(0)$ vs. QCD (Eq.(7c), N_f=3) and MC predictions.[7] Data and the QCD results are shown by, respectively, open circles and dashed lines at $\sqrt{s}$=91 GeV and by full circles and solid lines at $\sqrt{s}$=183 GeV. The full triangles denote the JETSET MC results at 183 GeV. Similar QCD predictions are claimed to be found if one uses Eqs. (7a) or (7b).

tions and the data. The analytical calculations are very sensitive to the QCD parameters, Λ and N_f, and are not able to describe simultaneously the factorial moments at all orders and at different dimensions. A better agreement between the factorial moments calculated and the data is obtained for Λ=0.04 GeV than that is found at the expected larger value, Λ=0.16 GeV, Figs. 2 and 3. However, at small Λ, the theory overestimates the data for large ε. The measured cumulants are far from the predictions, and the reduction of the Λ-value was not found to sensibly improve the situation, see Fig. 4.

Likely reasons for the failure of the QCD calculations seem to be their asymptotic character and lack of energy-momentum conservation, as it was mentioned in Sec. 2.2. However, DELPHI has analysed some high-energy events (Fig. 3) and no improvement of agreement at small ε-values was found. Inclusion of energy-conservation terms was obtained to lead to even larger discrepancies.[7]

ZEUS [9] has compared their results on 2-particle angular correlation functions with the DELPHI analysis [6] to check energy-dependence of the correlations, see Fig. 5. According to DELPHI finding, it is steeper rise of the 2-particle correlation functions at $\sqrt{s}$=183 GeV than at $\sqrt{s}$=91 GeV. No such dependence is confirmed by Fig. 5, although ZEUS data is taken at higher energy than that of DELPHI. All the possible checks (of experimental and calculation procedures) kept the results unchanged. It would be interesting to carry out an analysis to understand whether the expected universality of

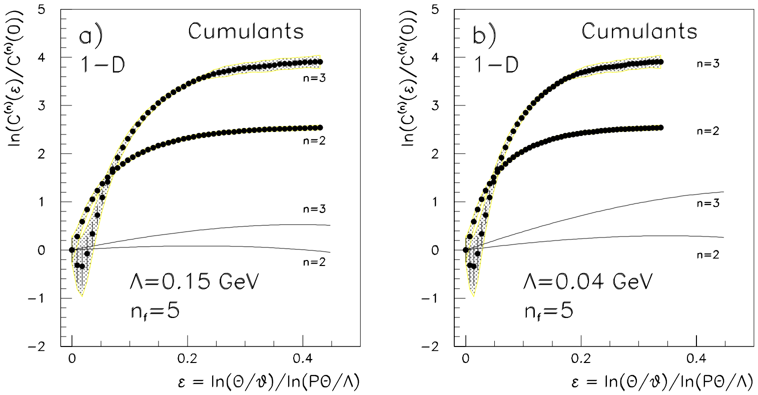
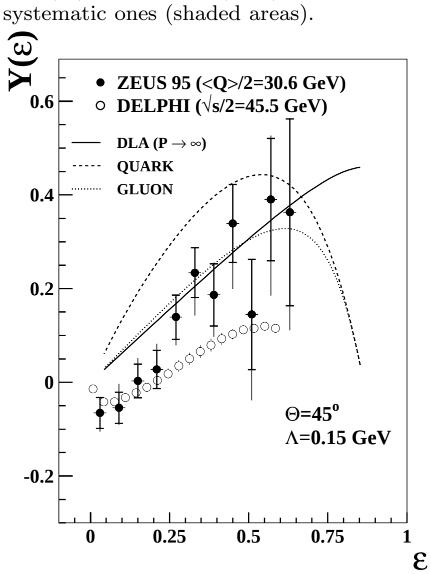


Figure 4. DELPHI cumulants (circles), calculated via Eq. (4), are compared to the QCD Eq. (7c) calculations (lines).[7] The statistical errors (error bars) are shown along with the systematic ones (shaded areas).



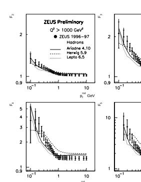


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Figure 5 (left): ZEUS 2-particle correlation function, compared to the runing α_s QCD calculations (solid line), to results of DELPHI [6], and to predictions for 30.6 GeV quark and gluon jets (dashed and dotted lines, repectively).[9]

Figure 6 (right): ZEUS [10] factorial moments $F_q(p_t^{\rm cut})$, compared to MC models. The statistical and total errors are shown by the inner and outer bars, respectively.

the 2-particle inclusive density violates. To note is that QCD calculations for gluon jet better describe the data[9] than those of quark jet, as shown in Fig. 5. This could be connected with the approximation used (DLA instead of, e.g., MLLA) in calculating the ratio of the mean multiplicity in gluon and quark jets. Nevertheless, it is seen that both predictions disagree with the data for small ϑ values (large ε).

The better agreement between the data and the calculations is found when one compares the measurements with MC simulations. At the Z^0 peak, as well as at high energies, the simulations well reproduce the data. This is presumably due to the fact that MC models take into account the energy-momentum conservation and are tuned to the data global variables. On the other hand, there is still difference in choicing the cut-off parameter Q_0, which "terminates" parton cascade: Q_0 is about 0.3-0.6 GeV in MC models used while due to the LPHD it is expected to be of $\simeq$0.25 GeV. However, although parton level MC studies by L3 indicate some disagreement with the LPHD assumption, DELPHI investigation tells us that even a possible violation of the LPHD seem ulikely to be a reason of discrepancies between hadron and parton levels.

3.3 Fluctuations, Correlations and QCD Coherence

Recently, ZEUS [10] has studied correlations in momentum-restricted regions in view of recent QCD+LPHD calculations[11]. The normalised factorial moments of the multiplicity distributions are theoretically expected to behave as

$$F_q(p_t^{\rm cut}) \simeq 1 + \frac{q(q-1)}{6}\frac{\ln(p_t^{\rm cut}/Q_0)}{\ln(E/Q_0)}\,, \qquad F_q(p^{\rm cut}) \simeq {\rm const} > 1, \qquad (10)$$

when particles are restricted (cylindrically) in either the transverse momentum $p_t < p_t^{\rm cut}$ or (spherically) in absolute momentum $|\,\vec{p}\,| < p^{\rm cut}$.

The predictions of Eq. (10) are the followings. There are correlations between partons because the factorial moments exceed unity. The correlations vanish for small $p_t^{\rm cut}$, $F_q \to 1$ as $p_t^{\rm cut} \to Q_0$, due to angular ordering of the partons in the jet (QCD coherence). This leads to the Poissonian (independent) emission. However, soft gluons with spherically limited momenta ($|\,\vec{p}\,| < p^{\rm cut}$) obey the non-Poissonian distribution even for small $p^{\rm cut}$.

Fig. 6 of ZEUS shows a first evident disagreement between the LPHD hypothesis and the measurements. The data factorial moments represent a strongly increasing function of p_t at $p_t < 1$ GeV which disagrees with what the theory predicts, Eq. (10). A similar behaviour is found for the factorial moments in $p^{\rm cut}$, while the errors are substantial to conclude. MC models show good agreement with the data.

It would be interesting to find out to whether this observation in e^+p collision agree with that in e^+e^- annihilation, where one does not need to transform to a specific frame, a probable reason of the above-mentioned disagreement in 2-particle correlation functions.

4 Conclusions

A review on recent results from the investigations of intermittency and genuine correlations by DELPHI, L3 and OPAL in e^+e^- and by ZEUS in e^+p collisions is given. The tool of factorial multiplicity and cumulant moments has been applied. The studies show an existence of strong correlations between produced hadrons. The analytical QCD calculations well describes the scaling behaviour of the measured moments, while some discrepancies are found, especially for genuine correlations. Monte Carlo models reasonably well reproduce the trend in the data, although some disagreement in magnitudes is seen. It is observed by ZEUS a violation of LPHD predictions for momentum-limited factorial moments. The universality between e^+e^- and e^+p collisions is analysed. The observations reviewed give clear evidence of futher efforts for studying the subject.

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