

# CFT, Holography, and Causal Entropy Bound

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## Abstract

The causal entropy bound (CEB) is confronted with recent explicit entropy calculations in weakly and strongly coupled conformal field theories (CFTs) in arbitrary dimension  $D$ . For CFT's with a large number of fields,  $N$ , the CEB is found to be valid for temperatures not exceeding a value of order  $M_P/N^{\frac{1}{D-2}}$ , in agreement with large  $N$  bounds in generic cut-off theories of gravity, and with the generalized second law. It is also shown that for a large class of models including high-temperature weakly coupled CFT's and strongly coupled CFT's with AdS duals, the CEB, despite the fact that it relates extensive quantities, is equivalent to (a generalization of) a purely holographic entropy bound proposed by E. Verlinde.

# I. INTRODUCTION

Recently, there has been growing interest in, and proliferation of, various kinds of entropy bounds. Much of this interest stems from the idea of holography [1], a bold conjecture that some dynamical systems in  $D$  space-time dimensions can be completely described in terms of degrees of freedom living on their  $(D - 2)$ -dimensional boundary. Maldacena's AdS/CFT correspondence [2] is the prototypical example realizing such a conjecture. A necessary condition for holography is that the number of degrees of freedom of the system does not exceed the area of the  $(D - 2)$ -dimensional hypersurface surrounding it in units of some fundamental area, usually taken to be Planck's. Consequently, the validity of holography hinges upon, although it is by no means guaranteed by, a holographic bound on entropy.

Many systems seem to obey a holographic entropy bound. For instance, limited-gravity systems whose size  $R$  is larger than their gravitational radius  $R > R_g \equiv 2G_N E$  ( $E$  is the total energy of the system), satisfying Bekenstein's bound [3]  $S < S_B$ ,  $S_B \sim ER$  automatically satisfy the holography bound since<sup>1</sup>  $ER = R_g^{D-3} R l_P^{2-D} < (R/l_P)^{D-2} = S_{HOL}$ . The real challenge to holography, therefore, is associated with its application to strong-gravity systems, such as the whole Universe.

Bekenstein himself [4] proposed an extension of his bound to cosmology by identifying the linear size  $R$  appearing in his bound with the particle horizon. For regions much larger than the particle horizon, or after reheating at the end of inflation, the cosmological Bekenstein bound becomes too loose. Instead, it could become too strong if one tried to apply it to sufficiently small regions. Fischler and Susskind (FS) [5] applied holography to cosmology, and proposed that the area of the particle horizon should holographically bound the entropy on the backward-looking light cone. It was soon realized, however, that the FS proposal requires modifications, since violations of it were found to occur in physically reasonable

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<sup>1</sup>We will use units in which  $c = k_B = \hbar = 1$ , define Planck's length by  $l_P^{D-2} = G_N$ , and often ignore numerical factors of order unity.

situations, such as a closed, adiabatically contracting FRW universe. Several attempts were made to mend the FS proposal, which finally resulted in a covariant proposal by Bousso involving entropy on suitably constrained forward and/or backward-looking light-cones [6]. Bousso’s proposal is defined such that it can be applied to more general space-times and not just to cosmological ones.

In parallel, several groups tried to modify the FS proposal by bounding entropy inside space-like regions. This resulted in various proposals [7], [8], [9], [10], all roughly identifying the maximal scale over which holography applies with a scale of about the Hubble radius  $H^{-1}$ . This line of reasoning resulted in the so-called Hubble Entropy Bound (HEB), bounding entropy density by  $Hl_P^{2-D}$ . Eventually, these ideas were synthesized in an improved covariant form through the introduction of a causal bound on entropy in a generic space-like region (CEB) [11].

Interestingly enough, both Bousso’s proposal and CEB appear to follow from the same underlying bound (of the kind first proposed by Flanagan et al. [12]) on a local entropy current. Bousso’s proposal is obtained by projecting along an arbitrary null vector and CEB by projecting on an arbitrary time-like vector. HEB, CEB, and the local bound on entropy current, all scale with the square root of the energy of the system and thus lie around the geometric mean between  $S_B$ , which scales as  $S_B \sim E^1$ , and  $S_{HOL}$ , scaling as  $S_{HOL} \sim E^0$ .

Recently, E. Verlinde [13] argued that the radiation in a closed, Radiation-Dominated(RD) Universe can be modeled by a CFT, and that its entropy can be evaluated using a generalized Cardy formula, which, in some cases, can be derived using the AdS/CFT correspondence [14]. On the basis of this entropy formula, Verlinde proposed a new, entirely holographic bound on entropy stating that the subextensive component of the entropy (the “Casimir entropy”) of the entire closed universe has to be less than the entropy of a black hole of the same size. The well-known square-root appearing in Cardy’s formula, reminiscent of the square-root occurring in the above-mentioned geometric mean of  $S_B$  and  $S_{HOL}$ , prompted Verlinde to point out a close connection between HEB and his new proposal. But,

in spite of their close similarity, Verlinde's new bound still holds for cases for which HEB appears to be violated.

Subsequently, Kutasov and Larsen (KL) [15] (see also [16]) have shown, by explicit weak-coupling, high-temperature CFT calculations, that Verlinde's generalization of Cardy's formula is not always correct and that, consequently, his proposed bound between two holographic quantities cannot be generally valid.

In this paper we try to shed some (hopefully bright!) light on this rather puzzling state of affairs. In Section II we give a generalization of CEB to arbitrary  $D$  and then, in Section III, we check it against the CFT calculations of Refs. [15], [16]. We find that CEB passes the CFT test provided temperatures are kept below a certain scale  $\Lambda$  which differs from  $M_P$  by a  $D$ -dependent factor scaling as an inverse power of the number of species  $N$  in the CFT. We also present, in Section IV, a modification of Verlinde's bound between holographic quantities which evades the KL criticism and show that the new bound, within the CFT framework, is exactly equivalent to CEB. We finally point out the reasons why the naive HEB is problematic, as pointed out in Ref. [13], while its CEB improvement is not.

## II. CEB IN $D$ DIMENSIONS

As mentioned in the introduction, CEB is an improved, covariant version of HEB, which is applicable, in principle, to any space-like region. Before extending CEB to any dimension  $D$ , let us briefly recall the basic ideas behind its predecessor, HEB. HEB was motivated by the following reasonable assumptions [7] (see also [8–10])

(*i*) entropy is maximized by the largest stable black hole that can fit in a given region of space. This is because the merging of two black holes into a larger one always results in an entropy increase.

(*ii*) the largest stable black hole in a cosmological background is typically of size comparable to that of the Hubble horizon. This assumption is qualitatively supported by previous calculations [20]

In cosmological backgrounds, CEB refines HEB by defining more precisely the “horizon” concept through the identification of a critical (“Jeans”-like) causal connection scale  $R_{CC}$ , above which perturbations are causally disconnected, so that black holes of larger size, very likely, cannot form, and by putting the resulting entropy bound in an explicitly covariant form.

The causal-connection scale  $R_{CC}$  is found by looking at perturbation equations in  $D$  dimensions. For gravitons, in the case of flat universe, one finds [21]

$$R_{CC}^{-2} = \frac{D-2}{2} \text{Max} \left[ \dot{H} + \frac{D}{2} H^2, -\dot{H} + \frac{D-4}{2} H^2 \right]. \quad (1)$$

If  $H \gg \dot{H}$ ,  $R_{CC} \propto H^{-1}$  and one recovers HEB with a  $D$ -dependent prefactor scaling as  $\sqrt{D(D-2)}$ . The above result generalizes to the case of a spatially curved universe in the form [22,23]

$$R_{CC}^{-2} = \frac{D-2}{2} \text{Max} \left[ \dot{H} + \frac{D}{2} H^2 + \frac{D-2}{2} \frac{\kappa}{a^2}, -\dot{H} + \frac{D-4}{2} H^2 + \frac{D-2}{2} \frac{\kappa}{a^2} \right]. \quad (2)$$

A covariant definition of  $R_{CC}$  is obtained by expressing (2) in terms of the “00” components of curvature tensors. One easily finds:

$$R_{CC}^{-2} = \frac{D-2}{2(D-1)} \text{Max} [G_{00} \mp R_{00}] = 4\pi G_N \left[ \frac{1}{D-1} \rho - p, \frac{2D-5}{D-1} \rho + p \right], \quad (3)$$

where, to derive the second equality, we have used Einstein’s equations,  $G_{\mu\nu} = 8\pi G_N T_{\mu\nu}$  and a perfect-fluid form for the energy-momentum tensor.

The Bekenstein-Hawking entropy of a Schwarzschild black hole of radius  $R_{BH}$  in  $D$  dimensions is given by  $S = \mathcal{A}/4l_P^{D-2}$ . The generalization of  $S_{CEB}$  for a region of proper volume  $V$  is therefore

$$S_{CEB} = \beta n_H S^{BH} = \beta \frac{V}{V(R_{CC})} \frac{\mathcal{A}}{4l_P^{D-2}} \quad (4)$$

where  $n_H \equiv \frac{V}{V(R_{CC})}$  is the number of causally connected regions in the volume considered,  $V(x)$  denotes the volume of a region of size  $x$ , and  $\beta$  is a fudge factor reflecting current uncertainty on the actual limiting size for black-hole stability. For a spherical volume in flat

space we have  $V(x) = \Omega_{D-2} x^{D-1} / (D-1)$ , with  $\Omega_{D-2} = 2\pi^{(D-1)/2} / \Gamma\left(\frac{D-1}{2}\right)$ , but in general the result is different and depends on the spatial-curvature radius.

Following Ref. [11], the expression for  $S_{\text{CEB}}$  in  $D$  dimensions can be rewritten in the explicitly covariant form

$$S_{\text{CEB}} = B l_P^{-(D-2)} \int_{\sigma < 0} d^D x \sqrt{-g} \delta(\tau) \sqrt{\text{Max}_{\pm} [(G_{\mu\nu} \pm R_{\mu\nu}) \partial^\mu \tau \partial^\nu \tau]} =$$

$$B (8\pi)^{1/2} l_P^{-D/2+1} \int_{\sigma < 0} d^4 x \sqrt{-g} \delta(\tau) \sqrt{\text{Max}_{\pm} \left[ (T_{\mu\nu} \pm T_{\mu\nu} \mp \frac{1}{2} g_{\mu\nu} T) \partial^\mu \tau \partial^\nu \tau \right]}, \quad (5)$$

where  $\sigma < 0$  defines the spatial region inside the  $\tau = 0$  hypersurface whose entropy we are discussing, and  $T$  is the trace of the energy-momentum tensor.

The prefactor  $B$  can be fixed by comparing eqns. (4) and (5). In fact, consider the expression (4) in the limit  $R_{CC} \ll a$ , where  $a$  is the radius of the Universe: in this case, over a region of size  $R_{CC}$  we may neglect spatial curvature and write  $V(R_{CC}) = \Omega_{D-2} R_{CC}^{D-1} / (D-1)$ , and the area of the black hole horizon as  $\mathcal{A} = \Omega_{D-2} R_{BH}^{D-2}$ , thus giving (apart for negligible terms of order  $(R_{CC}/a)^2$ )

$$S_{\text{CEB}} = \beta \frac{D-1}{4} V R_{CC}^{-1} l_P^{-(D-2)} = B \sqrt{\frac{2(D-1)}{D-2}} V R_{CC}^{-1} l_P^{-(D-2)}. \quad (6)$$

This fixes  $B = \sqrt{\frac{(D-1)(D-2)}{32}} \beta$ .

Since (5) applies to any space-like region, it can be rewritten in a local rather than integrated form by introducing an entropy current  $s_\mu$  such that  $S = \int d^D x \sqrt{-g} \delta(\tau) s_\mu \partial^\mu \tau$ . Then (5) becomes equivalent to (with  $\lambda^\mu$  a arbitrary time-like vector):

$$s_\mu \lambda^\mu \leq l_P^{-D/2+1} (8\pi)^{1/2} B \sqrt{\text{Max}_{\pm} \left[ (T_{\mu\nu} \pm T_{\mu\nu} \mp \frac{1}{2} g_{\mu\nu} T) \lambda^\mu \lambda^\nu \right]}. \quad (7)$$

In the limit of a light-like vector  $\lambda$  we get one of the conditions proposed by Flanagan et al. [12] in order to recover Bousso's proposal. Their bound corresponds (in  $D = 4$ ) to  $B = \frac{1}{4\pi}$  and could be used to fix  $\beta$  (assuming that it is  $D$ -independent).

Specializing now to the case of a RD universe, for which  $\rho = (D-1)p$ , the 00 equation for the scale factor becomes

$$H^2 + \frac{\kappa}{a^2} = \frac{16\pi G_N}{(D-1)(D-2)}\rho = \frac{16\pi G_N}{(D-1)(D-2)}\rho_0 R_0^D a^{-D}, \quad \kappa = \pm 1, 0, \quad (8)$$

and, in terms of the conveniently rescaled conformal time  $\eta$ , defined by  $a(\eta)d\eta = (D-2)dt$ , the solutions can be put in the simple form

$$a(\eta) = A^{\frac{1}{D-2}} \begin{cases} [\sin(\eta/2)]^\alpha & \kappa = 1 \\ (\eta/2)^\alpha & \kappa = 0 \\ [\sinh(\eta/2)]^\alpha & \kappa = -1 \end{cases}, \quad A = \frac{16\pi G_N \rho_0 R_0^D}{(D-1)(D-2)}, \quad \alpha = \frac{2}{D-2}. \quad (9)$$

As can be seen, the qualitative behavior of solutions does not depend strongly on  $D$ . In a (closed, open or flat) RD universe one always has  $R_{00} = G_{00}$ , therefore  $R_{CC}^{-2} = \frac{D-2}{2} \left( -\dot{H} + \frac{D-4}{2}H^2 + \frac{D-2}{2}\frac{\kappa}{a^2} \right)$ . The behaviour of  $S_{CEB}$  is easily derived from the explicit solution for the scale factor and  $R_{CC}$ . In the case  $D=4$  it is shown in Fig. 1.

### III. CEB VS. CFT

E. Verlinde proposed [13] that a radiation-dominated closed Universe in  $D$  space-time dimensions can be modeled by a  $D$ -dimensional CFT, and that its entropy is given by a generalization of Cardy's formula (we will denote it by  $S_{CV}$  for Cardy-Verlinde):

$$S = S_{CV} \equiv \frac{2\pi R}{D-1} \sqrt{2E_C E_E}, \quad (10)$$

where  $R = a$  is the radius of the finite closed universe, and  $E_E$  and  $E_C$  are the extensive and sub-extensive components of the energy. The sub-extensive (Casimir) component,  $E_C$ , is conveniently normalized by

$$E_C = (D-1)(E - TS + pV) = DE - (D-1)TS \sim V/R^2, \quad (11)$$

so that the total energy  $E$  is given by  $E = E_E + \frac{1}{2}E_C$ , and  $E_E$  is purely extensive.

Verlinde motivates his proposal from the AdS/CFT correspondence and provides an example, taken from [14], of strongly coupled CFT's which have AdS duals and satisfy (10). Indeed, for such systems,

$$S = \frac{c}{12} \frac{V}{L^{D-1}} \quad (12)$$

$$E = \frac{c}{12} \frac{D-1}{4\pi L} \left(1 + \frac{L^2}{R^2}\right) \frac{V}{L^{D-1}} \quad (13)$$

$$T = \frac{1}{4\pi L} \left(D + (D-2) \frac{L^2}{R^2}\right), \quad (14)$$

where  $c$  is the central charge of the CFT. The validity of the CV formula can be explicitly verified.

Next Verlinde proposes a new, purely holographic, entropy bound stating that the entropy associated with  $E_C$ ,  $S_C = 2\pi R E_C / (D-1)$ , must be bounded by the entropy of a black-hole filling the whole Universe,  $S_{BH} = (D-2) \frac{V}{4l_P^{D-2} R}$ ,

$$S_C < S_{BH}. \quad (15)$$

This bound is indeed satisfied in the specific cases he considers. We shall come back to Verlinde's bound in Section IV.

Kutasov and Larsen [15] pointed out that, in general, the CV formula (10) is not valid in weakly coupled CFTs. Instead, the free energy  $F$ , the entropy  $S$ , the total energy  $E$ , and the Casimir energy  $E_C$  can be expanded at weak coupling and large  $x \equiv 2\pi R T$ ,

$$-FR = f(x) = \sum_{n \geq 0} a_{D-2n} x^{D-2n} + \dots \quad (16)$$

$$S = 2\pi f'(x), \quad (17)$$

$$ER = (x\partial_x - 1)f(x), \quad (18)$$

$$E_C R = \sum_{n \geq 1} -2na_{D-2n} x^{D-2n} + \dots \quad (19)$$

where the dots represent non-perturbative contributions. It is clear that, unless some special relation holds between  $a_D$  and  $a_{D-2}$ , the CV formula (10) cannot be generally valid.

We can explicitly check under which conditions the entropy of weakly coupled CFT's obeys CEB,  $S < S_{CEB} = 4B\sqrt{\pi}\sqrt{EV}l_P^{-(D-2)/2}$ . In the limit  $TR \gg 1$  we find



$$\frac{S^2}{S_{\text{CEB}}^2} = \frac{\pi a_D D^2}{4B^2(D-1)\Omega_{D-1}} (2\pi l_P T)^{D-2}. \quad (20)$$

Thus, CEB is obeyed provided that

$$\left(\frac{T}{M_P}\right)^{D-2} < \frac{K(D)}{a_D}, \quad (21)$$

where  $K(D)$  is a  $D$ -dependent (but CFT independent) constant. We conclude that CEB is obeyed as long as temperatures are below  $M_P$  by a factor  $a_D^{-\frac{1}{D-2}}$ . Since  $a_D$  is proportional to the number  $N$  of CFT-matter species, we obtain a bound on temperature which, in Planck units, scales as  $N^{-\frac{1}{D-2}}$ . We can also explicitly check under which conditions strongly coupled CFT's possessing AdS duals as considered by Verlinde obey CEB. In this case, in the limit  $R/L \sim TR \gg 1$  we find

$$\frac{S^2}{S_{\text{CEB}}^2} = \frac{1}{4(D-1)B^2} \frac{c}{12} \left(\frac{l_P}{L}\right)^{D-2} \quad (22)$$

and thus CEB is obeyed for

$$\frac{1}{4(D-1)B^2} \frac{c}{12} \left(\frac{4\pi T}{DM_P}\right)^{(D-2)} < 1. \quad (23)$$

Since the central charge  $c$  is proportional to the number of CFT fields  $N$ , we obtain a bound on temperature which, in Planck units, scales as  $N^{-\frac{1}{D-2}}$ , exactly as previously obtained for the weakly coupled case.

For the case  $ER \sim a_D$  (which corresponds to  $RT \sim 1$ ) KL have argued that the bound proposed by Verlinde has further problems, which they attributed to its relation with Bekenstein's bound. We notice that CEB is fine also in this case, its validity guaranteed by a condition similar to eq. (21).

Finally, we would like to show that CEB holds also when  $ER \sim 1$ . In this case  $S_{CEB} \simeq 4B\sqrt{\pi}\sqrt{V/R}l_P^{-(D-2)/2}$  scales as  $\left(\frac{R}{l_P}\right)^{\frac{D-2}{2}}$ . As noted by KL the appropriate setup for calculating the entropy is the microcanonical ensemble with the result  $S \sim \log a_D \sim \log N$ ; thus  $S < S_{CEB}$  is guaranteed for a macroscopic Universe as long as

$$\left(\frac{R}{l_P}\right)^{\frac{D-2}{2}} > \log N. \quad (24)$$

In a quantum theory of gravity we expect the UV cut-off  $\Lambda$  to be finite and to represent an upper bound on  $T$  (Cf. the example of superstring theory and its Hagedorn temperature) and a lower bound on  $R$  (Cf. the minimal compactification radius). Thus conditions (21), (23) for the validity of CEB are satisfied as long as  $\left(\frac{\Lambda}{M_{Pl}}\right)^{D-2} < 1/N$ . A bound of the same form was previously proposed in [4] and [17], and independent arguments in support of bounds of this sort have also been recently put forward [18], [19].

#### IV. CEB AND A NEW PURELY HOLOGRAPHIC BOUND

The entropy of all CFT's that we have considered so far could be expressed in terms of the superextensive entropy  $S_B \equiv 2\pi RE/(D-1)$ , and a subextensive entropy  $S_{SUB}$

$$S_{CFT} = \sqrt{2S_B S_{SUB}}. \quad (25)$$

Since  $S_B$  is super-extensive and  $S_{CFT}$  is extensive, eq.(25) can be taken as a definition for sub-extensive entropy  $S_{SUB}$  scaling as  $V/R^2$ .

Our claim is then simply that in this context, CEB is equivalent to the following holographic bound:

$$S_{SUB} < \beta^2 \frac{(D-2)(D-1)^2}{8} \frac{V}{Rl_P^{D-2}}, \quad (26)$$

which replaces Verlinde's bound for any CFT. Recall that  $\beta$  is a numerical factor introduced in the definition of CEB (4). The proof of our claim should be obvious by now, by writing  $S_{CEB}$  as

$$\begin{aligned} S_{CEB} &= B \sqrt{16\pi \frac{EV}{l_P^{D-2}}} \\ &= \beta(D-1) \sqrt{\frac{D-2}{8}} \sqrt{2S_B \frac{V}{Rl_P^{D-2}}}. \end{aligned} \quad (27)$$

For systems obeying the CV formula,  $S_{SUB} = S_C(1 - S_C/2S_B)$  so CEB coincides, up to a multiplicative factor, with the holographic entropy bound proposed by Verlinde (neglecting terms of order  $E_C/E$ ). In this case the equivalence of the two bounds, which can also be

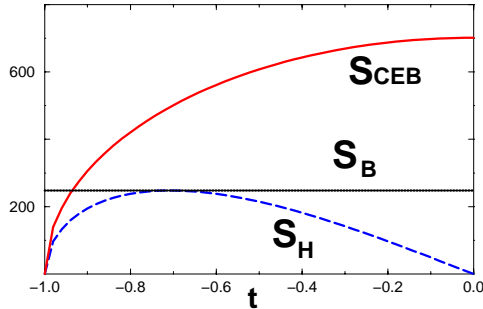


FIG. 1.  $S_{CEB}$  compared with  $S_H$  and  $S_B$  in the expanding phase of a closed  $D = 4$ , RD Universe. Here we set  $\beta = \frac{D-2}{D-1}$ .

visualized in  $D = 4$  using the diagrammatic representation of Verlinde (in this context  $S_{CEB}$  is proportional to the cord subtended by  $\eta$ ), can be checked explicitly by looking at their evolution in a RD closed Universe if, following Verlinde, we write (15) as a combination of Bekenstein's and Hubble Entropy Bound, according to the value of the parameter  $HR$ :

$$\left. \begin{array}{l} S < S_B \\ S < S_H \end{array} \right\} \text{ for } \left\{ \begin{array}{l} HR < 1 \\ HR > 1 \end{array} \right. , \quad (28)$$

where  $S_H = (D - 2)\frac{HV}{4G_N}$  (here we set  $\beta = \frac{D-2}{D-1}$ , in such a way that the normalization is the same of Verlinde). As can be seen in Figure 1, CEB and bound (28) are parametrically equivalent throughout the whole evolution of the Universe.

Consider instead HEB; as noted in [13],  $S_H$  can be expressed as

$$S_H = \sqrt{S_{BH}(2S_B - S_{BH})}. \quad (29)$$

Clearly, CEB and HEB are of the same order of magnitude as long as  $S_{BH} < 2S_B$ . However, while in the regime we have considered  $S_C < 2S_B$  (assuring the validity of CEB),  $S_{BH} < 2S_B$  is not always true. When this happens (e.g. at the turning point), it is possible to violate HEB while respecting CEB. In retrospect we could have expected problems with HEB since it makes a non-covariant split between intrinsic and extrinsic curvature and uses just the latter for the bound. By contrast, CEB uses the full covariant curvature tensors which, through the Einstein equations, can be directly related to the energy-momentum tensor of the matter fields.

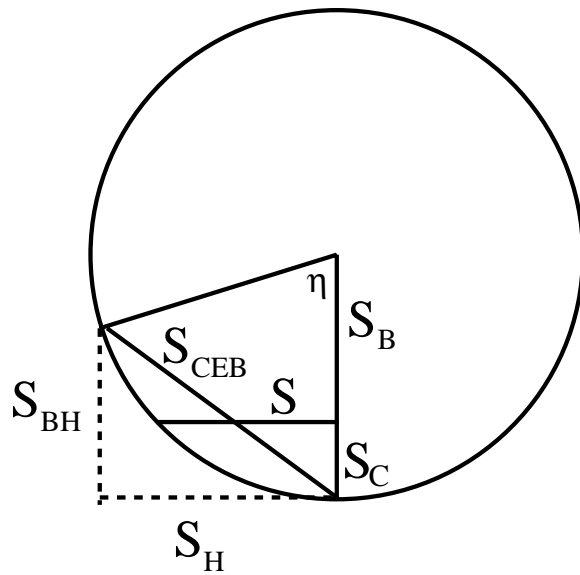


FIG. 2. Entropies in a closed RD Universe with  $D = 4$ .

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