Supersymmetric effects in rare semileptonic decays of B and K mesons

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Rare flavour-changing neutral-current transitions of the type $s \rightarrow d \ \ell^+\ell^-(\nu\bar{\nu})$ and $b \rightarrow s \ \ell^+\ell^-(\nu\bar{\nu})$ are analysed in supersymmetric extensions of the Standard Model with generic flavour couplings. It is shown that these processes are particularly sensitive to the left–right mixing of the squarks and that, in the presence of non-universal A terms, they could lead to unambiguous signatures of new physics in exclusive K and B meson decays.

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1 Introduction

Flavour-changing neutral-current (FCNC) processes are one of the most powerful tools in probing the structure of flavour beyond the Standard Model (SM): the strong suppression of these transitions occurring within the SM, which is due to the Glashow–Iliopoulos–Maiani (GIM) mechanism [1] and to the hierarchy of the Cabibbo–Kobayashi–Maskawa (CKM) matrix [2], ensures a large sensitivity to possible non-standard effects, even if these occur at very high energy scales.

In the present talk we focus on a specific class of $\Delta F = 1$ FCNC transitions:

$$s \to d \; \ell^+ \ell^- (\nu \bar{\nu}) \qquad \text{and} \qquad b \to s \; \ell^+ \ell^- (\nu \bar{\nu}) \;. \tag{1}$$

As we shall discuss, these are particularly interesting for the following reasons:

- These transitions have a strong sensitivity to supersymmetric extensions of the SM with flavour non-universal soft-breaking terms. Taking into account all the existing phenomenological constraints, within this type of models it is possible to generate sizeable non-standard effects to the partonic processes in (1).

- The existing experimental constraints on these transitions are rather weak, but in the near future it will be possible to perform stringent tests by means of exclusive rare K and B meson decays.

In the rest of the talk we shall illustrate these two points in more detail. Section 2 is devoted to the analysis of the supersymmetric contributions to $d_j \to d_i \ell^+ \ell^- (\nu \bar{\nu})$ transitions, including a discussion about the indirect bounds obtained by other processes. In Sections 3–5 we analyse how to extract information on these partonic transitions by means of experimental data on $K \to \pi \nu \bar{\nu}$, $K_L \to \pi^0 e^+ e^-$ and exclusive $b \to s \ell^+ \ell^- (\nu \bar{\nu})$ decays, respectively.

2 SUSY contributions to $d_j \to d_i \ell^+ \ell^- (\nu \bar{\nu})$ transitions in models with non-universal soft-breaking terms

The class of supersymmetric extensions of the SM that we shall consider is the so-called unconstrained MSSM (see e.g. [3,4]). This model has the minimal number of new fields necessary to build a consistent SUSY version of the SM, namely all the superpartners of the SM fields plus an extra SUSY Higgs doublet. On the contrary, the assumptions made on the soft-breaking terms are very general. The only condition we shall impose on the flavour structure of the soft-breaking terms is a non-universal linear relation between the trilinear terms (Y_{ij}^A) and the Yukawa couplings (y_k), leading to

$$Y_{ij}^A = \mathcal{O}(y_k M_S) \;, \qquad k = \max(i,j) \;, \tag{2}$$

where M_S denotes a common soft-breaking scale [$M_{ij} = \mathcal{O}(M_S^2)$ for the bilinear terms]. This condition let us to avoid charge- and colour-breaking minima or unbounded directions in the SUSY potential [5]. The proportionality coefficients will be assumed to be $\mathcal{O}(1)$, unless more stringent constraints are imposed by experimental data.

Similarly to the SM, also within this context FCNC amplitudes involving external quark fields turn out to be generated only at the quantum level. Given the large number of new off-diagonal flavour couplings, the simplest way to parametrize the new effect is provided by the so-called mass-insertion approximation [3,4]. This consists of choosing a simple basis for the gauge interactions and, in that basis, to perform a perturbative expansion of the squark mass matrices around their diagonal. Being interested in processes with external down-type quarks, we will employ in the following a squark basis where all quark–squark–gaugino vertices involving down-type quarks are flavour-diagonal. In this basis we then define the following adimensional couplings:

$$\left(\delta_{AB}^{[U,D]}\right)_{ij} = \left(M_{[U,D]}^2\right)_{i_A j_B} \Big/ \langle M_{[U,D]}^2\rangle \ , \tag{3}$$

where A, B denote the helicity (L, R) and i, j the family. These couplings constitute the basic tool to parametrize and classify the new contributions to FCNC amplitudes arising within the unconstrained MSSM.

SUSY contributions to $d_j \rightarrow d_i \ell^+ \ell^- (\nu\bar{\nu})$ transitions can also be divided into three groups according to the diagrams (or the effective operators) that generate it: box and helicity-conserving photon-penguins (generic dimension-6 operators), magnetic penguins (dimension-5 operators) and Z penguins. In each of these classes a different type of delta plays a dominant role.

Generic dimension-6 operators. Box diagrams with internal chargino or neutralino fields and, in the case of charged leptons, also photon-penguin diagrams with internal gluino, chargino or neutralino fields, can lead to effective FCNC operators of the type

$$\left(\bar{d}_A^i \gamma^\mu d_A^j\right)\left(\bar{l}_B \gamma_\mu l_B\right) \ . \tag{4}$$

Since the external quarks have the same helicity, the potentially leading SUSY contributions to the Wilson coefficients of these operators are generated by helicity-conserving couplings:

$$\frac{\left(\delta_{AA}^{[U,D]}\right)_{ij}}{M_S^2} \ . \tag{5}$$

The dimensional factor in Eq. (5), due to the integration of heavy SUSY degrees of freedom, indicates explicitly that these contributions vanish as $1/M_S^2$ in the limit of a large SUSY-breaking scale.

The helicity-violating couplings δ_{LR}^Q appear in the Wilson coefficients of dimension-6 operators only to second order in the mass expansion, with contributions

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of the type [6]

$$\frac{(\delta^U_{LR})_{i3}(\delta^U_{RL})_{3j}}{M^2_S} \ . \tag{6}$$

Since the left–right mixing is generated by the trilinear terms, then $\delta^Q_{LR} = \mathcal{O}(m_q/M_S)$ and the contribution in (6) vanish as $1/M^4_S$ for large M_S. Thus the effect of helicity-violating couplings is not only disfavoured by the fact that it requires a double insertion, but it is also parametrically suppressed in the limit of a large SUSY-breaking scale. As we shall see below, this is not the case only in a specific type of dimension-6 operators: those generated by Z penguins.

On the other hand, both helicity-conserving and helicity-violating contributions to generic dimension-6 operators turn out to be negligible with respect to the SM ones, once the bounds from $\Delta F = 2$ processes are taken into account [7]. This fact can be understood by a naive dimensional argument in the limit of large M_S [8]. Indeed, considering for simplicity only the case of δ^Q_{AA}, it is easy to show that the SUSY contribution to $\Delta F = 2$ amplitudes –appearing necessarily at the second order in the mass expansion– are of $\mathcal{O}[(\delta^Q_{AA}/M_S)^2]$. Thus the limits on δ^Q_{AA} arising from $\Delta F = 2$ amplitudes scale linearly with M_S and not quadratically, as in the $\Delta F = 1$ case. As a result, SUSY contributions to $d_j \to d_i \ell^+ \ell^- (\nu\bar{\nu})$ transitions generated by box diagrams and helicity-conserving photon-penguins turn out to be extremely suppressed for $M_S \gtrsim 1$ TeV.[1]

Magnetic penguins. The integration of the heavy SUSY degrees of freedom in penguin-like diagrams can also lead to operators with dimension lower than 6, creating an effective FCNC coupling between quarks and SM gauge fields. In the case of the photon field, the unbroken electromagnetic gauge invariance implies that the lowest-dimensional coupling is provided by the so-called magnetic operator

$$\bar{d}^i_{L(R)} \sigma^{\mu\nu} d^j_{R(L)} F_{\mu\nu} \ . \tag{7}$$

Here the potentially leading SUSY contribution is induced by helicity-violating couplings, and in particular by the left–right mixing of down-type squarks, which can appear in gluino-exchange diagrams:

$$\frac{(\delta^D_{LR})_{ij}}{M_S} \ . \tag{8}$$

Since the operator (7) has dimension 5, the explicit dimensional suppression of the left–right mixing contribution is only $1/M_S$. Nonetheless, also in this case the overall SUSY effect decouples as $1/M^2_S$ since $\delta^Q_{LR} = \mathcal{O}(m_q/M_S)$.

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[1]A similar argument holds for SUSY contributions to $d_j \to d_i \bar{q}q$ transitions [8], with the notable exception of $\Delta I = 3/2$ amplitudes [9].

The appearance of a single inverse power of M_S in Eq. (8) has the important consequence that this contribution can naturally evade the $\Delta F = 2$ constraints and compete with the SM term [4,12,8]. This is not the case for contributions generated by helicity-conserving couplings or left–right mixing in the up sector, which appear only beyond the first order in the mass insertion.

In the $b \to s$ case the most significant constraint on possible non-standard effects in the magnetic operator is provided by the inclusive process $B \to X_s\gamma$ (see e.g. [10] for an updated discussion). The recent measurements [11] exclude SUSY contributions substantially larger that the SM one, or imply bounds of $\mathcal{O}(10^{-2})$ on $|(\delta^D_{LR})_{23}|$. Note, however, that the assumption made on the trilinear terms implies

$$\left|(\delta^D_{LR})_{23}\right| \lesssim \frac{m_b}{M_S} \simeq 10^{-2} \left(\frac{500 \text{ GeV}}{M_S}\right) , \tag{9}$$

then the $B \to X_s\gamma$ measurement does not pose a serious fine-tuning constraint about the non-universality of A terms.

Concerning the $s \to d$ case, there are no significant constraints on $|(\delta^D_{LR})_{12}|$, whereas a stringent bound on $|\text{Im}(\delta^D_{LR})_{12}|$ can be derived from ε'/ε [12]. The latter is obtained by constraining the SUSY contribution to the chromo-magnetic operator (closely related to the magnetic one) and implies [8]:

$$\left|\text{Im}(\delta^D_{LR})_{12}\right| \leq 4 \times 10^{-5} \left(\frac{M_S}{500 \text{ GeV}}\right) . \tag{10}$$

This limit is more stringent than the upper bound on $|(\delta^D_{LR})_{12}|$ imposed by (2), namely

$$\left|(\delta^D_{LR})_{23}\right| \lesssim \frac{m_s}{M_S} \simeq 2 \times 10^{-4} \left(\frac{500 \text{ GeV}}{M_S}\right) , \tag{11}$$

but it is much higher than the value assumed by $|\text{Im}(\delta^D_{LR})_{12}|$ within the flavour-constrained MSSM [13]. Interestingly, if $\text{Im}(\delta^D_{LR})_{12} = \mathcal{O}(10^{-5})$ it is possible to conceive a scenario where all CP-violating effects observed so far in the kaon sector (ε and ε') are of SUSY origin [14,15]. As we shall discuss in the next sections, this scenario would produce very clear signatures in rare K decays.

Z **penguins.** Thanks to the spontaneous breaking of $SU(2)_L$, in the case of Z penguins the integration of the SUSY degrees of freedom can lead to an effective FCNC operator of dimension 4:

$$\bar{q}^i_L \gamma^\mu d^j_L Z_\mu . \tag{12}$$

This operator generates a dimension-6 structure like the one in Eq. (4) when the heavy Z field is integrated out. In this case, however, the dimensional suppression is provided by $1/M^2_Z$ and there is no explicit trace of M_S. The

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latter is hidden in the dimensionless coupling of the operator (12), denoted by Z_{ji}^L, that requires a double mixing between $SU(2)_L$-singlet and $SU(2)_L$-doublet fields,[2] and thus vanishes as $1/M_S^2$ for large M_S. The potentially dominant contributions to Z_{ji}^L arise from chargino loops, either by a double left-right insertion in the up-squark propagators [6] or by a single insertion together with wino-higgsino mixing [7,18]:

$$Z_{ji}^L \sim \begin{cases} (\delta_{LR}^U)_{j3}(\delta_{RL}^U)_{3i} \\ (m_t/M_S)V_{j3}(\delta_{RL}^U)_{3i} \end{cases} \tag{13}$$

As can be noted, in both cases $Z_{ji}^L = \mathcal{O}(m_t^2/M_S^2)$, where the m_t factor arises from the Yukawa coupling of the third generation. Since the left–right mixing in the up sector provides a subleading contribution to generic dimension-6 operators and, in particular, to $\Delta F = 2$ transitions, the indirect constraints on these effects are rather weak. If $(\delta_{RL}^U)_{3i}$ lies in the window

$$\frac{m_t}{M_W}|V_{3i}| \lesssim \left|(\delta_{LR}^U)_{3i}\right| \lesssim \frac{m_t}{M_S} , \tag{14}$$

then SUSY contributions to Z_{ji}^L turn out to be comparable or even larger than the SM one. On the contrary, contributions to Z_{ji}^L from helicity-conserving couplings or left–right mixing in the down sector are always negligible.

In the $b \to s$ case some phenomenological constraints on $|Z_{sb}^L|$ can be obtained directly from exclusive and inclusive $b \to s \, \ell^+\ell^-(\nu\bar\nu)$ transitions [18,16]. The latter are certainly cleaner from the theoretical point of view; however, their experimental determination is quite difficult. Indeed the most stringent constraint, at present, is the one extracted from $B \to K^*\mu^+\mu^-$ [16], where the experimental upper bound on the non-resonant branching ratio lies only about a factor of 2 above the SM expectation. This constraint implies a bound of $\mathcal{O}(1)$ on $|(\delta_{LR}^U)_{3i}|$, which is still outside the window (14).

Owing to the smallness of V_{td}, the window (14) is much larger in the case of $s \to d$ transitions. Here the most stringent constraints on Z_{ds}^L arise from $K_L \to \mu^+\mu^-$ (on the real part) and ε'/ε (on the imaginary part) [19]. Without entering into a detailed discussion about these bounds, which can be found elsewhere [8], we

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[2]Here and in the following we employ the normalization of Z_{ji}^L in [8,16]:

$$\mathcal{L}_{FC}^Z = \frac{G_F}{\sqrt{2}} \frac{e}{\pi^2} M_Z^2 \frac{\cos\Theta_W}{\sin\Theta_W} Z^\mu Z_{ji}^L \, \bar{q}_L^i \gamma_\mu q_L^j \, + \text{ h.c. } .$$

With this normalization the SM contribution to Z_{ji}^L, evaluated in the 't Hooft–Feynman gauge, is given by $Z_{ji}^L|_{SM} \simeq C_0(x_t)V_{3i}^*V_{3j}$, where V_{ij} denote CKM matrix elements, $x_t = m_t^2/m_W^2$ and the function $C_0(x)$ can be found in [17]. We further stress that the leading $\mathcal{O}(x_t)$ contributions to FCNC Z penguins are gauge-invariant within both SM and MSSM.

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simply note that: i) the sizeable uncertainties due to non-perturbative effects in both $K_L \to \mu^+\mu^-$ and ε'/ε do not allow us to extract precise constraints; ii) taking into account these uncertainties, the present bounds on Z_{ds}^L are within the window (14) and allow for $\mathcal{O}(1)$ deviations from the SM at the amplitude level.

Summarizing, we can say that only the flavour-violating left–right mixing among the squarks can naturally lead to large effects in the transitions (1). In the $s \to d$ case this can happen either via magnetic penguins [ruled by $(\delta_{LR}^D)_{12}$] or via Z penguins [ruled by $(\delta_{LR}^U)_{13}$ and $(\delta_{LR}^U)_{23}$], whereas $b \to s$ magnetic penguins are strongly constrained by $B \to X_s\gamma$. Moreover, we have seen that under the assumption (2) the present constraints about the non-universality of the trilinear terms are all rather weak, both for up- and down-type squarks. We believe that this observation strengthens the interest in searching for sizeable non-standard effects in the transitions (1).

3 $K \to \pi\nu\bar\nu$

These decays are considered the golden modes for a precise measurement of the $s \to d\nu\bar\nu$ transition. Within the SM, separating the contributions to the $s \to d\nu\bar\nu$ amplitude according to the intermediate up-type quark running inside the loop, one can write

$$\mathcal{A}(s \to d\nu\bar\nu)_{\rm SM} = \sum_{q=u,c,t} V_{qs}^* V_{qd} \mathcal{A}_q \sim \begin{cases} \mathcal{O}(\lambda^5 m_t^2) + i\mathcal{O}(\lambda^5 m_t^2) & (q=t) \\ \mathcal{O}(\lambda m_c^2) + i\mathcal{O}(\lambda^5 m_c^2) & (q=c) \\ \mathcal{O}(\lambda \Lambda_{QCD}^2) & (q=u) \end{cases} \qquad (15)$$

The hierarchy of the CKM matrix elements would favour up- and charm-quark contributions; however, the hard GIM mechanism of the parton-level calculation implies $\mathcal{A}_q \sim m_q^2/M_W^2$, leading to a completely different scenario. As shown on the r.h.s. of Eq. (15), where we have employed the standard phase convention ($\mathrm{Im}V_{us} = \mathrm{Im}V_{ud} = 0$) and expanded the CKM matrix in powers of the Cabibbo angle ($\lambda = 0.22$) [20], the top-quark contribution dominates both real and imaginary parts. This structure implies that $\mathcal{A}(s \to d\nu\bar\nu)_{\rm SM}$ is dominated by short-distance dynamics and therefore calculable with high precision in perturbation theory.

The leading short-distance contributions to $\mathcal{A}(s \to d\nu\bar\nu)$, both within the SM and within its SUSY extension discussed before, can be described by means of a single effective dimension-6 operator:

$$Q_L^\nu = (\bar{s}_L \gamma^\mu d_L)(\bar\nu_L \gamma_\mu \nu_L) \ , \qquad (16)$$

whose Wilson coefficient has been calculated at the next-to-leading order within the SM [21] (see also [22,23]). The simple structure of Q_L^ν has two major advantages:

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- the relation between partonic and hadronic amplitudes is very accurate, since the hadronic matrix elements of the $\bar{s}\gamma^\mu d$ current between a kaon and a pion are related by isospin symmetry to those entering K_{l3} decays, which are experimentally well known;

- the lepton pair is produced in a state of definite CP and angular momentum, implying that the leading contribution to $K_L \to \pi^0 \nu\bar{\nu}$ is CP-violating.

The dominant theoretical error in estimating $\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})_{\rm SM}$ is due to the uncertainty of the QCD corrections to the charm contribution (see [23] for an updated discussion), which can be translated into a 5% error in the determination of $|V_{td}|$ from $\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})$ [3]. Genuine long-distance effects associated to the up quark have been shown to be substantially smaller [25].

The case of $K_L \to \pi^0\nu\bar{\nu}$ is even cleaner from the theoretical point of view [26]. Indeed, because of the CP structure, only the imaginary parts in (15) –where the charm contribution is absolutely negligible– contribute to $\mathcal{A}(K_2 \to \pi^0\nu\bar{\nu})_{\rm SM}$. Thus the dominant direct-CP-violating component of $\mathcal{A}(K_L \to \pi^0\nu\bar{\nu})_{\rm SM}$ is completely saturated by the top contribution, where the QCD uncertainties are very small (around 1%). Intermediate and long-distance effects in this process are confined to the indirect-CP-violating contribution [27] and to the CP-conserving one [28] which are both extremely small. Taking into account also the isospin-breaking corrections to the hadronic matrix element [29], one can write an expression for $\mathcal{B}(K_L \to \pi^0\nu\bar{\nu})_{\rm SM}$ in terms of short-distance parameters with a theoretical error below 3% [23,27]:

$$\mathcal{B}(K_L \to \pi^0\nu\bar{\nu})_{\rm SM} = 4.16 \times 10^{-10} \left[\frac{\overline{m}_t(m_t)}{167 \text{ GeV}}\right]^{2.3} \left[\frac{{\rm Im}\lambda_t}{\lambda^5}\right]^2, \qquad (17)$$

where $\lambda_t = V_{ts}^* V_{td}$.

Taking into account all the indirect constraints on ${\rm Im}(V_{ts}^* V_{td})$ [30], the present range of SM predictions for the two $K \to \pi\nu\bar{\nu}$ branching ratios is reported in the second column of Table 1. In the following three columns, we show the upper bounds obtained within three SUSY scenarios with non-trivial $(\delta^U_{LR})_{i3}$ and $(\delta^U_{LR})_{12}$. In all cases the SUSY flavour-mixing terms, as well as CKM matrix elements, have been constrained taking into account the measurement of ε, ε', $K_L \to \mu^+\mu^-$ and the respective theoretical uncertainties [8]. As can be noticed, the two neutrino modes could provide sizeable unambiguous signatures of SUSY, but only in the presence of a large left–right mixing in the up sector. Interestingly, the present measurement of $\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})$ [31] is very close to putting serious constraints (or to providing some evidence...) on this scenario.

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[3]Very recently also the subleading effect of $\mathcal{O}(m_K^2/m_c^2)$ induced by dimension-8 operators has been estimated [24]. This effect is not calculable precisely, but it is likely to be smaller than (or at most as large as) the uncertainty in the QCD corrections to the leading term [24].

Observable	SM	SUSY scenarios			exp. data
		A	B	C	
$10^{10} \times \mathcal{B}(K^+ \to \pi^+ \nu \bar{\nu})$	0.71 ± 0.12	$\leq \mathcal{B}_{\mathrm{SM}}$	≤ 2.1	≤ 2.7	$1.5^{+3.5}_{-1.3}$ [31]
$10^{10} \times \mathcal{B}(K_L \to \pi^0 \nu \bar{\nu})$	0.22 ± 0.05	$\leq \mathcal{B}_{\mathrm{SM}}$	≤ 1.7	≤ 4.0	$< 5.9 \times 10^3$ [32]
$10^{11} \times \mathcal{B}(K_L \to \pi^0 e^+ e^-)_{\mathrm{dir}}$	0.35 ± 0.07	≤ 2.0	≤ 3.0	≤ 10	< 58 [33]

Table 1: SM expectations, experimental data and upper bounds within different SUSY scenarios for the branching ratios of the rare decays $K_L \to \pi^0 \nu \bar{\nu}$, $K_L \to \pi^0 e^+ e^-$ and $K^+ \to \pi^+ \nu \bar{\nu}$. The three SUSY scenarios correspond to [8]: A) $(\delta_{LR}^U)_{i3} = 0$, $(\delta_{LR}^U)_{12} \neq 0$, $0 \leq \mathrm{Im}(\lambda_t) \leq \mathrm{Im}(\lambda_t)_{\mathrm{SM}}$; B) $(\delta_{LR}^U)_{12} = 0$, $(\delta_{LR}^U)_{i3} \neq 0$, $0 \leq \mathrm{Im}(\lambda_t) \leq \mathrm{Im}(\lambda_t)_{\mathrm{SM}}$; C) $(\delta_{LR}^U)_{12} \neq 0$, $(\delta_{LR}^U)_{i3} \neq 0$, $|\mathrm{Im}(\lambda_t)| \leq 1.73 \times 10^{-4}$.

4 $K_L \to \pi^0 e^+ e^-$

Similarly to $K \to \pi \nu \bar{\nu}$ decays, also the short-distance contributions to $K \to \pi \ell^+ \ell^-$ transitions are calculable with high accuracy. Long-distance contributions to the latter, however, are much larger owing to the presence of electromagnetic interactions. Only in few cases (mainly in CP-violating observables) are long-distance contributions suppressed and is it possible to extract the interesting short-distance information.

The single-photon exchange amplitude, dominated by long-distance dynamics, provides the largest contribution to the CP-allowed transitions $K^+ \to \pi^+ \ell^+ \ell^-$ and $K_S \to \pi^0 \ell^+ \ell^-$. The former has been observed, both in the electron and in the muon mode, whereas only an upper bound of 1.6×10^{-7} exists on $\mathcal{B}(K_S \to \pi^0 e^+ e^-)$ [34]. On the contrary, the long-distance part of the single-photon exchange amplitude is forbidden by CP invariance in the $K_L \to \pi^0 \ell^+ \ell^-$ channels, which are much more interesting from the short-distance point of view (especially the electron mode).

In $K_L \to \pi^0 e^+ e^-$ we can distinguish three independent (and comparable) contributions: direct-CP-violating, indirect-CP-violating and CP-conserving.

The direct-CP-violating part of the $K_L \to \pi^0 e^+ e^-$ amplitude is very similar to the $K_L \to \pi^0 \nu \bar{\nu}$ one, but for the fact that it receives an additional short-distance contribution from the photon penguin. Within the SM, this theoretically clean part of the amplitude leads to [35]

$$\mathcal{B}(K_L \to \pi^0 e^+ e^-)^{\mathrm{SM}}_{\mathrm{dir}} = 6.5 \times 10^{-11} \left[\frac{\overline{m}_t(m_t)}{167 \text{ GeV}} \right]^2 \left[\frac{\mathrm{Im}\lambda_t}{\lambda^5} \right]^2 . \tag{18}$$

The present range of variation, together with SUSY upper bounds, is reported in the last line of Table 1. Being sensitive also to the photon penguin, the $K_L \to \pi^0 e^+ e^-$ amplitude could be substantially modified also in the presence of non-trivial SUSY phases in the down sector. In particular, within the interesting scenario where all CP-violating effects observed in the kaon sector were due to $\mathrm{Im}(\delta_{LR}^D)_{12} = \mathcal{O}(10^{-5})$,

$\mathcal{B}(K_L \to \pi^0 e^+ e^-)^{\mathrm{SM}}_{\mathrm{dir}}$ would be close to its SM value, whereas $\mathcal{B}(K_L \to \pi^0 \nu \bar{\nu})$ would be vanishingly small.

In principle the direct-CP-violating part of the $K_L \to \pi^0 e^+ e^-$ amplitude could be experimentally isolated from the other two contributions, especially if it were large. In order to achieve this goal it would be necessary to measure $\mathcal{B}(K_S \to \pi^0 e^+ e^-)$ or to put a stringent bound on it. The two CP-violating components of the $K_L \to \pi^0 e^+ e^-$ amplitude will in general interfere, and the indirect-CP-violating one alone would lead to

$$\mathcal{B}(K_L \to \pi^0 e^+ e^-)_{\mathrm{CPV-ind}} = 3 \times 10^{-3} \, \mathcal{B}(K_S \to \pi^0 e^+ e^-) \,. \tag{19}$$

Since the relative phase of the two CP-violating amplitudes is known, once $\mathcal{B}(K_S \to \pi^0 e^+ e^-)$ will be measured, it will be possible to determine the interference between direct and indirect CP-violating components of $\mathcal{B}(K_L \to \pi^0 e^+ e^-)_{\mathrm{CP}}$ up to a sign ambiguity.

The CP-conserving contribution to $K_L \to \pi^0 e^+ e^-$, generated by a two-photon intermediate state, does not interfere with the CP-violating ones and is expected to be in the 10^{-12} range. The relative weight of this contribution can be further constrained by appropriate kinematical cuts; it should therefore not represent a problem if $\mathcal{B}(K_L \to \pi^0 e^+ e^-)$ will be found in the 10^{-11} range.

5 Exclusive $b \to s\ell^+\ell^-(\nu\bar{\nu})$ decays

The starting point for the analysis of $b \to s\ell^+\ell^-(\nu\bar{\nu})$ transitions, both within the SM and the SUSY scenario discussed in Section 2, is the following effective Hamiltonian:

$$\mathcal{H}_{\mathrm{eff}} = -\frac{G_F}{\sqrt{2}} V_{ts}^* V_{tb} \left(\sum_{i=1}^{10} [C_i Q_i + C_i' Q_i'] + C_L^\nu Q_L^\nu + C_R^\nu Q_R^\nu \right) + \mathrm{h.c.} \,. \tag{20}$$

Here Q_i denotes the Standard Model basis of operators relevant to $b \to s\ell^+\ell^-$ [17] and Q_i' their helicity flipped counterparts. In particular, we recall that $Q_i \sim (\bar{s}\gamma_\mu b)(\bar{c}\gamma^\mu c)$, for $i = 1\ldots6$, $Q_7 \sim m_b \bar{s}_L(\sigma \cdot F)b_R$, $Q_8 \sim m_b \bar{s}_L(\sigma \cdot G)b_R$, $Q_9 \sim (\bar{s}_L \gamma_\mu b_L)(\bar{\ell}\gamma^\mu \ell)$, $Q_{10} \sim (\bar{s}_L \gamma_\mu b_L)(\bar{\ell}\gamma^\mu \gamma_5 \ell)$ and $Q_{L(R)}^\nu \sim (\bar{s}_{L(R)} \gamma_\mu b_{L(R)})(\bar{\nu}_L \gamma^\mu \nu_L)$. The operators that have a non-vanishing matrix element already at the tree level and thus play the dominant role in $b \to s\ell^+\ell^-$ are Q_7, Q_9, Q_{10} and their helicity flipped counterparts. On the other hand, only $Q_{L(R)}^\nu$ have a non-vanishing matrix element in $b \to s\nu\bar{\nu}$.

Rate and CP asymmetry in $B \to X_s\gamma$ already provide serious constraints on possible deviations from the SM in C_7 and C_7' [10], and these bounds will soon improve with new data on $B \to X_s\gamma$. However, as we have discussed in Section 2, even if no new-physics effects are found in the magnetic operator, one could still expect sizeable SUSY contributions mediated by the Z penguin. In the following we shall concentrate only on the latter type of effects. Under this assumption, a rather simplified scenario

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emerges, where $C_R^\nu = C_i' = 0$ and only C_{10} and C_L^ν are substantially modified from their SM value [16].

Moreover, even though inclusive measurements are certainly more suitable for precise determinations of short-distance parameters, here we shall discuss only exclusive decays, which have a clear advantage from the experimental point of view. Within the SM the following exclusive branching ratios are expected, compared here with the current experimental limits:

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\begin{array}{rcll}
\mathcal{B}(B \to K \nu \bar{\nu}) & \approx & 4 \times 10^{-6} & (< 7.7 \times 10^{-4} \quad [37]) \\
\mathcal{B}(B \to K^* \nu \bar{\nu}) & \approx & 1.3 \times 10^{-5} & (< 7.7 \times 10^{-4} \quad [37]) \\
\mathcal{B}(B \to K \mu^+ \mu^-)^{\text{n.r.}} & \approx & 6 \times 10^{-7} & (< 5.2 \times 10^{-6} \quad [38]) \\
\mathcal{B}(B \to K^* \mu^+ \mu^-)^{\text{n.r.}} & \approx & 2 \times 10^{-6} & (< 4 \times 10^{-6} \quad [38]) \\
\mathcal{B}(B_s \to \mu^+ \mu^-) & \approx & 3 \times 10^{-9} & (< 2.6 \times 10^{-6} \quad [39])
\end{array}
\tag{21}
$$

The corresponding hadronic uncertainties are typically around $\pm 30\%$ (see e.g. [36] for an updated discussion). As already mentioned, the channel that sets the strongest constraint on the FCNC Z penguin is $B \to K^* \mu^+ \mu^-$. In the optimistic case where Z_{bs}^L is close to saturating this bound, we would be able to detect the presence of non-standard dynamics already by observing sizeable rate enhancements in the above listed branching ratios. In processes such as $B \to K^* \ell^+ \ell^-$ and $B \to K \ell^+ \ell^-$, where the standard photon-penguin diagrams provide a large contribution, the enhancement could be at most a factor of 2. On the other hand, in processes such as $B \to K^* \nu \bar{\nu}$, $B \to K \nu \bar{\nu}$ and $B_s \to \mu^+ \mu^-$, where the photon-exchange amplitude is forbidden, the maximal enhancement could reach a factor of 10 [16].

5.1 Forward-backward asymmetry in $B \to K^* \mu^+ \mu^-$

If SUSY effects were not large enough to produce sizeable deviations in the magnitude of the $b \to Z^* s$ transition, as expected unless $|(\delta_{LR}^U)_{32}|$ were very close to the upper bound in Eq. (14), it would be hard to detect them from exclusive rate measurements. A more interesting observable in this respect is provided by the forward–backward (FB) asymmetry of the emitted leptons. In the $\bar{B} \to \bar{K}^* \mu^+ \mu^-$ case this is defined as

$$
\mathcal{A}_{\text{FB}}^{(\bar{B})}(s) = \frac{1}{d\Gamma(\bar{B} \to \bar{K}^* \mu^+ \mu^-)/ds} \int_{-1}^{1} d\cos\theta
$$
$$
\frac{d^2 \Gamma(\bar{B} \to \bar{K}^* \mu^+ \mu^-)}{ds \; d\cos\theta} \text{sgn}(\cos\theta) \;,
\tag{22}
$$

where $s = m_{\mu^+ \mu^-}^2 / m_B^2$ and θ is the angle between the momenta of μ^+ and $\bar{B}$ in the dilepton centre-of-mass frame. Assuming that the leptonic current has only a vector (V) or axial-vector (A) structure, then the FB asymmetry provides a direct measure
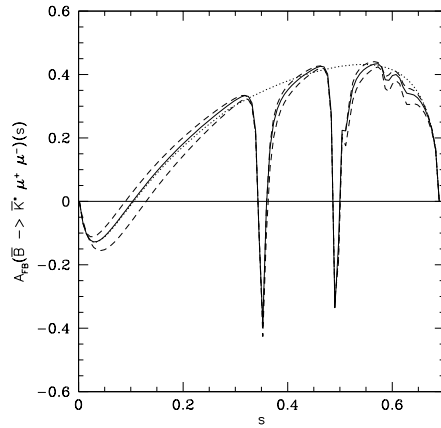
Figure 1: *FB asymmetry of $\bar{B} \to \bar{K}^*\mu^+\mu^-$ within the SM. The solid (dotted) curves have been obtained employing the Krueger–Sehgal [42] approach (using the perturbative end-point effective Hamiltonian [16]). The dashed lines show the effect of varying the renormalization scale of the Wilson coefficients between $m_b/2$ and $2m_b$, within the Krueger–Sehgal approach.*

of the *A-V* interference. Since the vector current is largely dominated by the photon-exchange amplitude and the axial one is very sensitive to the Z exchange, $\mathcal{A}_{FB}^{(\bar{B})}$ and $\mathcal{A}_{FB}^{(B)}$ provide an excellent tool to probe the $Z\bar{b}s$ vertex. Indeed $\mathcal{A}_{FB}^{(\bar{B})}(s)$ turns out to be proportional to[4]

$$\mathrm{Re}\left\{ C_{10}^* \left[s\, C_9^{\mathrm{eff}}(s) + \alpha_+(s)\frac{m_b C_7}{m_B} \right] \right\}, \tag{23}$$

where $\alpha_+(s)$ is an appropriate ratio of hadronic form factors [16,40]. The overall factor ruling the magnitude of $\mathcal{A}_{FB}^{(\bar{B})}(s)$ is affected by sizeable theoretical uncertainties. Nonetheless there are at least three features of this observable that provide a clear short-distance information:

i) Within the SM $\mathcal{A}_{FB}^{(\bar{B})}(s)$ has a zero in the low-s region ($s_0|_{\mathrm{SM}} \sim 0.1$) [40]. The exact position of s_0 is not free from hadronic uncertainties at the 10% level; nonetheless, the existence of the zero itself is a clear test of the relative sign between C_7 and C_9. The position of s_0 is essentially unaffected by possible new-physics effects in the $Z\bar{b}s$ vertex.

ii) The sign of $\mathcal{A}_{FB}^{(\bar{B})}(s)$ around the zero is fixed unambiguously in terms of the relative sign of C_{10} and C_9 [16]: within the SM one expects $\mathcal{A}_{FB}^{(\bar{B})}(s) > 0$ for $s > s_0$, as in Fig. 1. This prediction is based on a model-independent relation between the form factors [41]. Interestingly, the sign of C_{10} could change in the presence of a

---

[4]To simplify the notation we have introduced the parameter $C_9^{\mathrm{eff}}(s)$, which is not a Wilson coefficient but can be identified with C_9 at the leading-log level [16].

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non-standard $Z\bar{b}s$ vertex, leading to a striking signal of new physics in $\mathcal{A}_{\mathrm{FB}}^{(\bar{B})}(s)$, even if the rate of $\bar{B} \to \bar{K}^* \mu^+ \mu^-$ were close to its SM value.

iii) In the limit of CP conservation, one expects $\mathcal{A}_{\mathrm{FB}}^{(\bar{B})}(s) = -\mathcal{A}_{\mathrm{FB}}^{(B)}(s)$. This holds at the per-mille level within the SM, where C_{10} has a negligible CP-violating phase, but again it could be different in the presence of new physics in the $Z\bar{b}s$ vertex. In this case the ratio $[\mathcal{A}_{\mathrm{FB}}^{(\bar{B})}(s) + \mathcal{A}_{\mathrm{FB}}^{(B)}(s)]/[\mathcal{A}_{\mathrm{FB}}^{(\bar{B})}(s) - \mathcal{A}_{\mathrm{FB}}^{(B)}(s)]$ could be different from zero, for s above the charm threshold, even reaching the 10% level in the SUSY scenario of Section 2 [16].

6   Conclusions

Rare FCNC transitions of the type $d_j \to d_i \, \ell^+ \ell^- (\nu \bar{\nu})$ are very sensitive to simultaneous violations of $SU(2)_L$ and flavour symmetries. Within generic supersymmetric extensions of the SM, these processes could be substantially modified in the presence of non-diagonal trilinear soft-breaking terms. At present this possibility is still open for both $b \to s$ and $s \to d$ transitions, but it has more chances to be realized in the $s \to d$ case [43]. The future measurements of $\mathcal{B}(K^+ \to \pi^+ \nu \bar{\nu})$, $\mathcal{B}(K_L \to \pi^0 \nu \bar{\nu})$, $\mathcal{B}(K_L \to \pi^0 e^+ e^-)$ and $\mathcal{A}_{\mathrm{FB}}[B(\bar{B}) \to \bar{K}^* \mu^+ \mu^-]$ will provide very useful insights in this scenario.

Acknowledgements

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