TOPOLOGICAL ORIGIN OF INERTIA

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Summary

The purpose of the present article, following "Mach's principle" (the main elements of which have contributed to the foundations of general relativity) is to propose a new non-local interpretation of the inertial interaction. We suggest a novel approach according to which the inertial interaction can be correctly described by the topological field theory proposed by Edward Witten in 1988. In this context, the instantaneous propagation and the infinite range of the inertial interaction can be explained in terms of the topological amplitude associated to a 0 size singular gravitational instanton.

1. INTRODUCTION

The phenomenon of inertia - or "pseudo-force" according to E. Mach [1] - has recently been presented by J.P. Vigier [2] as one of the "unresolved mysteries of modern physics". According to us, this important question, which is well formulated in the context of Mach's principle, cannot be resolved or even understood in the context of conventional field theory.

Here we suggest a novel approach, a direct outcome of the topological field theory proposed by Edward Witten in 1988 [3]. According to this approach, beyond the interpretation proposed by Mach, we consider inertia as purely a *topological* source, linked to the topological charge Q = 1 of the 0 size singular gravitational instanton, identifiable, according [4], to the initial singularity of spacetime in the standard model.

Evaluation of the total inertial (or inertial potential) contribution resulting from the sum of the masses in the universe gives :

$$\mathbf{U}_{\text{inertial total}} = \sum_{univers} \frac{GM}{c^2 r} \approx 1 \qquad (1.)$$

which turns out to be an invariant for each local mass. The topological charge of the singular gravitational instanton, of the form

$$Q = \frac{1}{32\pi^2}\int d^4x\, R_{\mu\nu}\tilde{R}^{\mu\nu} = 1 \qquad (2.)$$

is also an invariant equal to the identity, just as the total inertial contribution. The equality between the inertial mass and the gravitational mass is explained here in terms of the quantisation of the topological charge of the singular gravitational instanton.

We therefore suggest that the topological charge of the singular gravitational instanton of 0 size represents the source of the inertial interaction. As the topological charge, the inertial interaction is equally invariant and propagates itself instantaneously from one point to another in spacetime. Such a property is not explicable within the frame of field theory but may find a solution in topological theory. In this new context, the initial singularity is the source of a topological amplitude corresponding to the charge of the 0 size singular gravitational instanton, i.e. $Q = \int d^4x\, R_{\mu\nu}\tilde{R}^{\mu\nu}$, detectable at the boundary S^3 of the singular gravitational instanton having the topology of the 4-dimensional Euclidean ball B^4. The pseudo-observables of Riemannien spacetime at the origin are here interpreted as co-cycles on instanton moduli space and are associated with γ_i cycles of the B^4 4-manifold (Donaldson application). Considering a point X in B^4, the topological amplitude responsible for the propagation of the instantonic charge takes then the form :

$$\left\langle 0_{S^3} .\ 0_X \right\rangle = \#(S^3, X)$$

The topological amplitude of the theory is given by the pseudo-observables of the left member, while the right member designates the number of intersections of $\gamma_i \subset B^4$. The function $\#(S^3, X)$ is zero if the point X is situated outside the sphere S^3 and unity if X is inside S^3 (i.e. if $X \in B^4$), the case where there exists a topological amplitude.

The present article is organised as follows. In paragraph 2, we briefly recall the context in which the problem of inertia rests (in classical mechanics). Equally we recall the canonical formulation of Mach's principle, which suggests a non-local approach of the inertial interaction. In paragraph 3, we consider " Mach's topological principle", a new formulation of Mach's principle in the context of topological field theory. In paragraph 4, we suggest that propagation of information characterising the Initial Singularity of spacetime can conveniently be described by the topological amplitude associated with a singular gravitational instanton of 0 size. We complete this approach by showing, from another point of view, that if the topological impulse at the origin represents a Dirac shock (distribution with zero support), then this impulse is sent to infinity - i.e. to the sphere S^3, the co-boundary of spacetime E^4 and the ball B^4 representing the singular gravitational instanton identified to the initial singularity. In conclusion, we discuss these diverse results and conjectures in paragraph 6.

2. REMINDER : INERTIA AND MACH'S PRINCIPLE

It has long been known in classical mechanics that the inertial interaction demonstrates itself under the form of an instantaneous reaction to acceleration of any material object :

$$f_i = -m\, a$$

a being the acceleration of the system itself. According the Newtonian point of view, - and, to some extent, in special relativity - , this concerns an inherent property of matter, which does not conflict with the conception of absolute space and time.

On the contrary, according to the general relativity, although independent of the observer, the intrinsic properties of space-time and its geometry are described by a metric inseparable from the distribution of the matter. This relationship has been defined on an axiomatic basis in the "equivalence principle" between inertia and gravitational mass. However, how to explain the *instantaneous* propagation (i.e. non-physical in the strict sense) of the inertial interaction ? How does this interaction propagate itself at infinite speed from one point to another in spacetime ?

The first attempt to provide a global response was qualitatively formulated by E. Mach [1] for whom, on the basis of the relativity of any movement, the source of inertia is the "interconnection" of all matter in the universe. We take from this the generic expression of Mach's principle as initially formulated :

Definition 1.1. (E. Mach) : *the inertial reference point defined by local physics coincides with the reference point in which distant objects are at rest, where it results that the most distant masses distributed in the universe determine the inertial behaviour of local masses.*

Numerous other variants of Mach's principle still exist, developed notably by C. Bras and R.H. Dicke [5], H. Bondi and J. Samuel [6], and D.R. Brill *et al* [7]. However, none of these approaches appears able to explain, in a compatible framework with relativistic constraints, the nature of the inertial interaction as well as its mode of propagation.

We therefore propose here to renew the approach of Mach's principle in the context of topological field theory.

3. TOPOLOGICAL MACH'S PRINCIPLE

We suggest below that the foundation on which Mach's principle rests (the same as the notion of inertia) is not physical, but falls within topological theory, defined by Edward Witten in 1988 [3]. As an example, we consider Foucault's pendulum experiment $\mathcal{G}$, which cannot be explained satisfactorily in either classical or relativistic mechanics. Recalling that the problem presented is that of the fixity of the plane of oscillation of the pendulum $\mathcal{G}$. The " topological Mach's principle" assumes then that the interaction between $\mathcal{G}$ and "global spacetime" **E** is itself of the topological type - which by its very nature explains precisely the character at the same time invariant and global of the system formed by the plane of oscillation of the pendulum and the rest of the universe.

In [4] the existence of a 0 scale limit in pre-spacetime was suggested, the said limit concentrates then the energy density of the "entire universe", in the Mach sense. We begin by shoving that in the context of such an approach, the 0 scale configuration has no *physical* content (there exists no stable physical length inferior to the Planck length) but is a *topological* configuration, corresponding to a singular gravitational instanton of 0 size.

0 scale of spacetime and topological theory

Beginning with the results of S. Weinberg [8], one can reasonably consider that spacetime at the Planck scale forms a system globally in thermal equilibrium. From an algebraic point of view, a state of equilibrium is a state on a C^*- quasi-local algebra, generated by a sub-algebra corresponding to the kinematic observables of the sub-system. Starting from the equilibrium state, it has been shown in [4] that pre-spacetime at the Planck level can be seen as subject to the Kubo-Martin-Schwinger (KMS] condition [9]. In the limits of the holomorphic KMS strip, it is therefore natural to consider the direction of the timelike metric as complex, the metric taking on the new form :

$$\eta_{\mu\nu} = \mathrm{diag}(1\ ,\ 1\ ,\ 1\ ,\ e^{i\theta}) \tag{3}$$

The signature of the metric (3), endowed with a supplementary degree of liberty on the g_{44} component, is Lorentzian for $\theta = \pm \pi$ and can become Euclidean for $\theta = 0$. The modular theory of Tomita [10] suggests then the "dualisation" of the signature given by the generalised automorphisms of the algebra **A**, which can be written :

$$\alpha_{\tau_c} = e^{\beta_c H} A\ e^{-\beta_c H} \tag{4}$$

The temporal flow associated to (4) is formally holomorphic in the variable $ß_C = ß_r + iß_i \in \mathbb{C}$. The group of modular automorphisms α_{τ_c} generates two dual flows, of which one part is :

$$\alpha_{\tau_r} = e^{i\beta_i H} A\ e^{-i\beta_i H} \tag{5}$$

corresponding to the algebra of observables and to the Lorentzian flow in real time. On the other hand, the dual current, is :

$$\alpha_{\tau_i} = e^{\beta_r H} A\ e^{-\beta_r H} \tag{6}$$

giving on **A** a semi-group of non-bounded and non-stellar operators. The flow α_{τ_i} of **A** is defined not throughout the whole of **A** but on an ideal $\{\mathfrak{I}\}$ of **A** and coupled to the topological flow in pure $ß = i\ t$ imaginary time. In the proposed model [4], the algebra of observables described by (5) is replaced at scale 0 of spacetime by an "algebra of pseudo-observables", dual to the algebra of Heisenberg α_{τ_r} in the form (6). At the singular $ß = 0$ scale, it is naturally no longer possible to conserve the notion of physical observables; instead, we consider homology cycles in module space of (0 size) gravitational instanton. This latter conclusion remains true, in pure imaginary time, for all real ß > 0. Such an approach allows us to distinguish three different domains in the "cosmological light cone", each of these domains being described by a specific Von Neumann algebra. If we call $M_{0,1} = R \otimes F$ the factor $R_{0,1}$ of type II_∞ corresponding to the singular 0 scale, as all ergodic transformations starting from $M_{0,1}$ (flows associated with 0 scale) are weakly equivalent [11], therefore $M_{0,1}$ is a hyperfinite factor of the Araki-Woods ITPFI type [12]. The factor $M_{0,1}$ is then canonical. More generally, there exist thus three scales (corresponding to the three regions of the of cosmological light cone) :

(i) the *classical* scale (ß > 1_{Planck}), described by the factor M_C of type I_∞;

(ii) the *quantum* superposition scale ($0<ß<\lambda_{\text{Planck}}$) described by the ITPFI of type III_λ , i.e. $R_\lambda = II_\infty \rtimes_\theta \mathbb{Z}$. We write then $M_q = M_{0,1} \rtimes_\theta \mathbb{Z}$;

(iii) the *topological* scale (0 scale associated with ß=0) described by the ITPFI of type II_∞ $M_{0,1}$.

To finish, we note that the algebraic flow of weights associated to the factor $M_{0,1}$ of type II_∞ at 0 scale of spacetime is an invariant of $M_{0,1}$. Thus, it has equally been shown, again in [4], that the initial singularity, out of reach of quantum field theory but well defined by topological theory, can be considered not in terms of divergences of physical fields but in terms of topological field symmetries and associated invariants (such as the first Donaldson invariant [13-14]) :

$$I = \sum_i (-1)^{n_i} \qquad (7)$$

A possible resolution of the Initial Singularity consists then of considering that 0 scale, which cannot be described by the (perturbative) physical theory, should be described by the (non-perturbative) dual theory, of the topological type.

Topological singularity invariant

Starting from Witten [3], one normally defines topological theory as the quantisation of zero, the Lagrangian of the theory being either (**i**) a 0 mode, or (**ii**) a characteristic class $c_n(V)$ of a vectorial bundle $V \xrightarrow{\pi} M$ built on spacetime. A new topological limit of the theory has therefore been defined [4], which is both non-trivial and no longer based on H = 0 but on ß = 0 and hence *independent* of H. The ordinary topological limit of quantum field theory, described by the Witten invariant $Z = Tr(-1)^n$ is given by the limit of the partition function $Z = Tr(-1)^n e^{-ßH}$ for zero (or invariant) values of H. On the contrary, in our case, we choose the 0 mode of the scale (ß = 0). Hence Z becomes (s representing the number of instantons of the theory) :

$$Z_{ß=0} = Tr\,(-1)^s \tag{8}$$

This new invariant, isomorphic to the Witten invariant $Z = Tr(-1)^n$, can be explicitly associated to the initial singularity of the pre-spacetime, reached for the value ß = 0 of the states partition function. One can therefore extend the iso-dimensional monopole / instanton duality demonstrated in [4] by suggesting that such a duality symmetry links the BRST cohomology ring (physical sector of the theory) and the cohomology ring of the instanton module space (topological sector). The BRST cohomology groups [15], having the generic form

$$H^{(g)}_{BRST} = \frac{\ker Q^{(g)}_{BRST}}{\mathrm{im}\, Q^{(g-1)}_{BRST}} \tag{9}$$

we then consider that the topological theory realise the injection of rings :

$$H^{\star}_{BRST} = \otimes^{\Delta U_k}_{g=0} \; H^{g}_{BRST} \xrightarrow{\iota} H^{\star}\left(\mathcal{M}^{(k)}_{\mathrm{mod}}\right) = \otimes^{d_k}_{i=0} H^{(i)}\left(\mathcal{M}^{(k)}_{\mathrm{mod}}\right) \tag{10}$$

which provides an injective path of the physical mode into the topological mode. In terms of observables 0_i and homology cycles $H_i \subset M_{\mathrm{mod}}$ in module space M_{mod} of configurations of the gravitational instanton type $\Im[\phi(x)]$ on the gravitational fields ϕ of the theory, we bring out the equivalence :

$$\langle 0_1 0_2 \;\ldots\; 0_n \rangle = \#(H_1 \cap H_2 \cap \;\ldots\; \cap H_n) \tag{11}$$

where the physical sector of the theory is described by the observables 0_i and the dual sector, of the topological type, by the homology cycles $H_i \subset M_{\mathrm{mod}}$. The oscillation of the signature of the metric between physical and topological sectors is then induced by the divergence $\Delta U_k = \int \partial^\mu j_\mu \, d^4x$ of the ghost flow [16][15] j_μ. When $\Delta U = 0$, as there exists no embedding space for module space, we suggest that the theory is then projected in the Coulomb branch, at the origin of M_{mod}, on a singular instanton of 0 size [17] which we identify to spacetime at 0 scale. The theory is ramified on the purely topological sector H_i, the corresponding signature at this sector being Euclidean (++++).

From this viewpoint, the image of 0 symmetry, described by the non-broken gauge group of type SU(2) ⊗ SU(2), is given by the first Donaldson invariant [13][14], associated with the existence of a "topological amplitude" characterising the theory. When the dimension dim$\mathcal{M}$ of instanton module space is non-zero, the Donaldson invariants are given by the correlation function of the theory :

$$Z(\gamma_1 \ldots \gamma_r) = \int DX \, e^{-S} \prod_{i=1}^{r} \int_{\gamma i} W_{k_1} = \left\langle \prod_{i=1}^{r} \int_{\gamma_i} W_{k_i} \right\rangle \quad (\mathrm{Dim}\ \mathcal{M}_\mathrm{K} \neq 0) \tag{12}$$

Thus, our most surprising formal result is that at scale ß = 0 associated with the high temperature limit, instanton module space being zero at this limit, the partition function, given by

$$\underset{ß=0}{Z} = \mathrm{Tr}(-1)^S \; e^{-\beta H} \tag{13}$$

must give again the first Donaldson invariant

$$\mathrm{I} = \sum_i (-1)^{n_i} , \tag{14}$$

a non-polynomial topological invariant, reduced to an integer for dim $\mathcal{M}_\mathrm{K} = 0$ [14]. This limit

$$\underset{ß=0}{Z} = \mathrm{Tr}(-1)^S \tag{15}$$

of the partition function (13) corresponds to a generalised symmetry of all possible states of the metric, all instantonic states of $g_{\mu\nu}$, given by the topological charge of the singular gravitational instanton, being equivalent at 0 scale. We call "0 symmetry" the generalised symmetry characterising the singular 0 scale. The above approach establishing, in the context of a model σ, the coupling at the Planck scale between 3-dimensional Euclidean gravity and a

2-dimensional "target space" (scalar sector), this provides a qualitative image of the initial spacetime singularity as a conic orbifold (or conifold) G such that $\Gamma_i = \frac{R^2}{Z_n}$.

In effect, we suggest as a geometric model of the instanton the ball B^4 bounded by the sphere S^3. The propagation of the solution depends then on the support of the gravitational instanton : in the region of 0 limit, there exists an accumulation of topological charge above the singular point S_0 such that the topological charge density $RR^* \to \infty$; in the dual situation, corresponding to the fundamental state, the support of the instanton is extended to infinity and $RR^* \to 0$. The transition of 0 at infinity is then described by the conformal transformations of the sphere.

From this point of view, the Machian interaction established between the pendulum $\mathcal{J}$ and the global universe is described exactly by the interaction between $\mathcal{J}$ and the 0 size instanton - almost like a renormalisation transformation. We therefore propose reformulating Mach's principle in the context of topological field theory :

3.1 Topological Mach's principle : *the topological amplitudes associated with propagation of the topological charge of the 0 size singular gravitational instanton, the solution of the Initial Singularity of spacetime, determines the inertial behaviour of local masses.*

In the following paragraph, we put forward a number of natural arguments to establish Mach's topological principle. In particular, we consider that the propagation of information characterising the Initial Singularity of spacetime can conveniently be described by the topological amplitude associated with the 0 size singular gravitational instanton.

4. 0 SIZE SINGULAR INSTANTON AND TOPOLOGICAL INTERACTION

The principal argument demonstrating the existence of an interaction between $\mathcal{J}$ and the charge Q resides in the fact that the topological charge of the instanton is entirely determined by the asymptotic behaviour of the gauge field (A_μ in the Yang et Mills case and $g_{\mu\nu}$ in N = 2 supergravity). The field F, non-zero in $E^4 = \mathbb{R}^4$ instanton space, cancels itself out at the boundary $\partial E^4 = S^3$, the three-dimensional sphere or the gauge potential becoming a pure gauge. Hence, we suggest that $\partial E^4 = S^3$ represents the curved and compact three-dimensional physical space $\mathbb{R}^3$ in which the rotation plane $\mathcal{Y}$ is inscribed. We draw from this a relation of topological type between the topological charge defined by ∂E^4 and the fixed orientation of the oscillation plane $\prod^2$ of $\mathcal{Y}$, equally defined by $\partial E^4 = S^3$.

Proposition 4.1. *The topological charge **Q** of the Yang and Mills instanton is entirely determined by the asymptotic behaviour of the gauge field A_μ, at the boundary ∂E^4 represented by the three-dimensional sphere S^3 where the gauge potential becomes a pure gauge. $\partial E^4 = S^3$ represents the curved and compact three-dimensional physical space $\mathbb{R}^3$ in which the plane of rotation $\prod^2$ of $\mathcal{Y}$* ***is inscribed.***

Demonstration 4.2. The Lagrangian of the Yang and Mills theory in Euclidean spacetime has the form :

$$\mathbf{L} = \frac{1}{4g^2}\int d^4x F^{\alpha}_{uv} F^{\alpha uv} \tag{16}$$

with f^{abc} being the structure constants of the gauge group SU(2) :

$$F^a_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + f^{abc} A^b_\mu A^c_\nu \tag{17}$$

The classical Yang and Mills action can then be written :

$$S = \frac{1}{8g^2}\int d^4x \left\{ (F^a_{\mu\nu} \pm \tilde{F}^{a\mu\nu})^2 \mp \frac{8\pi^2}{g^2} Q \right\} \tag{18}$$

with $\tilde{F}^{a\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\alpha\beta} F^a_{\mu\nu}$, Q being the topological charge (or Pontryagin index) of the instanton:

$$Q = \frac{1}{32\pi^2}\int d^4x\, F^a_{\mu\nu} \tilde{F}^{a\mu\nu} \tag{19}$$

As the action of the configuration must be finite, on spheres at infinity of radius r → ∞, the field $F^a_{\mu\nu}$ must fall to 0 more rapidly than r**-3**:

$$F^a_{\mu\nu}(|x| = \mathrm{r}) \to 0 \tag{20}$$

such that A^a_μ must be a pure gauge at infinity (r → ∞):

$$A^a_\mu T^a \to iU(x)\partial_\mu U^{-1}(x) \tag{21}$$

T^a being the generators of the gauge group, i.e. in the case of the fundamental representation with Pauli matrices σ^a , $T^a = \frac{1}{2}\sigma^a$ for SU(2). The element U ∈ SU(2). Considering the charge Q, it is possible to represent it by an integral over the total derivative :

$$\mathrm{Q} = \frac{1}{32\pi^2}\int d^4x\, \partial_\mu K_\mu \tag{22}$$

that is to say, according the Gauss theorem :

$$\mathrm{Q} = \frac{1}{32\pi^2}\int_{|x|=r} K_\mu dS_\mu \tag{23}$$

where the vector K_μ represents the Chern-Simons current :

$$K_\mu = 2\varepsilon_{\mu\nu\alpha\beta}(A^a_\nu \partial_\alpha A^a_\beta + \frac{1}{3} f^{abc} A^a_\nu A^b_\alpha A^c_\beta) \tag{24}$$

From equ. (24) we conclude that the topological charge Q is thus entirely determined by the asymptotic behaviour of the field A^a_μ. As established in [18], Q depends solely on the global properties of the function $A^a_\mu(|x| = r)$. Indeed, at infinity we have :

$$F \xrightarrow{|x|\to\infty} 0$$

but this is not (necessarily) the case for the gauge potential A_μ^a which becomes a pure gauge :

$$A(x) \xrightarrow{|x| \to \infty} U(x) \partial_\mu U^{-1}(x) \tag{25}$$

the void of the theory being non-trivial. The gauge elements $U(x) \in \mathrm{SU}(2)$, $x \in S^3$ are such that :

$$\mathrm{U} = A + i\vec{\sigma}\,\vec{B},\ A^2 + \vec{B}^2 = 1 \tag{26}$$

and $U(x)$ represents :

$$U:\ S^3 \to \mathrm{SU}(2) \cong S^3 \tag{27}$$

where we find the applications of the sphere S^3 representing the compact physical space E^3, boundary of the space E^4, on the isotopic space of SU(2), equally isomorphic to S^3. We draw the identification of S^3, boundary of the 4-dimensional instanton solution to physical space, of the double embedding of SU(2) in SL(2, C) - the universal convening of the Lorentz group - and in SU(2) ⊗ SU(2), the convening of SO(4). As SU(2) → S^3, we therefore propose to interpret S^3 both (**i**) as the 3-dimensional boundary of the 4-dimensional instanton Euclidean solution B^4 and (**ii**) as the 3-dimensional boundary of spacetime. From this identification, the application $S^3 \to S^3$ is designated by $\pi_3(S^3)$ and is such that :

$$\pi_3(S^3) = \mathbb{Z} \tag{28}$$

such that the applications $S^3 \to S^3$ are classified according to the homotopy classes characterised by integers, in our case n = 1. Thus, the 2-dimensional plane of oscillation $\mathbf{\Pi^2}$ of Foucault's pendulum is immersed in the 3-dimensional physical space corresponding precisely to S^3. As the topological charge **Q** of the instanton is determined by the behaviour of the gauge field on S^3, it follows from this that $\mathbf{\Pi^2}$ is determined by **Q**, as required. □

We have demonstrated the relation between **Q** and Π^2 in the case of Yang and Mills instantons. One can extend this result to supergravity, insofar as, in the context of non-linear curvature theories, the field R is, like F, asymptotically free. The action of the gravitational instanton becomes :

$$S = = \frac{1}{g^2(\rho)} \int d^4x\, R_{\mu\nu}\, R^{\mu\nu} + \vartheta \int d^4x \,\mathrm{Tr}\; R_{\mu\nu} \tilde{R}^{\mu\nu} \tag{29}$$

the topological term being :

$$Q = \vartheta \int d^4x \,\mathrm{Tr}\; R_{\mu\nu} \tilde{R}^{\mu\nu} \tag{30}$$

In this case, the surface term K, associated with the Chern-Simons topological flow, becomes :

$$\begin{aligned} Q_3^0(\Gamma, R) &\sim \mathrm{tr}\left(\Gamma \mathrm{d}\Gamma + \frac{2}{3}\Gamma^3\right) \\ Q_3^0(\omega, R) &\sim \mathrm{tr}\left(\omega \mathrm{d}\omega + \frac{2}{3}\omega^3\right) \end{aligned} \tag{31}$$

Γ representing the Christophel symbols, ω the spinorial connections and R the curvature of the 4-manifold. In putting R = dΓ + Γ^2 or R = dω + ω^2, we therefore find again the result of the first part of (4.1.) in supergravity. Concerning the Chern-Simons form, we recall that Witten has shown [19] that general relativity in dimension (2+1) is equivalent to Chern-Simons topological theory [20] in dimension 3. The Chern-Simons Lagrangian is :

$$h(A) = \int Tr(A \wedge dA) + \frac{2}{3} A \wedge A \wedge A) \tag{32}$$

Hence, the action of the theory of gravity in dimension (2+1) has the form :

$$\begin{aligned} L &= \varepsilon^{abc} \varepsilon^{ijk} e_i^a R_{jk}^{bc}(\omega) \\ R_{jk}^{bc} &= \partial_j \omega_k^{bc} + \omega_j^{be} \omega_k^{ec} - (j \leftrightarrow k) \end{aligned} \tag{33}$$

Analysis of the two Lagrangians (32) and (33) shows the equivalence between gravity (2+1) and the Chern-Simon theory at the limit of 0 scale associated with 0 energy modes. In dimension 4, we find then that at the limit of 0 scale, Lorentzian (3+1) field theory must be replaced by Donaldson topological theory [21] (4,0).

One can therefore consider that the propagation of the initial singularity is induced by the existence of a topological amplitude – the charge of the 0 size singular gravitational instanton, i.e. $Q = \int d^4x \, R_{\mu\nu}\tilde{R}^{\mu\nu}$, - detectable at the boundary S^3 of the singular gravitational instanton provided with the topology of the B^4 Euclidean ball of dimension D=4. The pseudo-observables are here interpreted as co-cycles on instanton module space and are associated with cycles γ_i of the B^4 4-manifold (Donaldson application). Considering a point X of B^4, the topological amplitude responsible for the propagation of the instantonic charge takes then the form:

$$\left\langle 0_{S^3} . \ 0_X \right\rangle = \#(S^3, X)$$

The topological amplitude of the theory is given by the pseudo-observables of the left member, while the right member designates the number of intersections of $\gamma_i \subset B^4$. The function $\#(S^3, X)$ is zero if the point X is situated outside the sphere S^3 and equals 1 if X is inside S^3 (i.e. if $X \in B^4$), the case where there exists a topological amplitude.

Now, we return to the inertial interaction. In the following proposition, we suggest that the source of inertia is not, in the strict sense, physical but has a *topological* content, linked to the topological charge Q=1 of the singular instanton.

Conjecture 4.3. *The inertial interaction is a topological interaction, of which the source is the topological charge of the 0 size singular gravitational instanton.*

Elements of demonstration 4.4. We have shown that the total inertial force likely to contribute in the case where the entire Universe would be submitted to an acceleration ***a*** in respect to a given object, could be obtained by summing the forces F'_{12} on masses other than *m* itself :

$$F_{inertielle} = ma \sum \frac{GM}{c^2 r} \qquad (34)$$

If M represents the totality of the mass of the Universe, a good approximation of $\sum \frac{GM}{c^2 r}$ gives us :

$$F_{totale} = \sum_{univers} \frac{GM}{c^2 r} \approx 1 \qquad (35)$$

The total inertial interaction F_{totale} is therefore of the order of unity and represents a topological invariant - i.e. invariant of gravitational gauge and scale invariant -. In all points of space, the contribution of F_{totale} of is identical. Hence, we suggest that the total inertia F_{totale} has its origin in the topological charge Q :

$$Q = \frac{1}{32\pi^2} \int d^4 x \, R_{\mu\nu} \tilde{R}^{\mu\nu} = 1 \qquad (36)$$

Indeed, we have seen that the topological charge Q of the instanton is entirely determined by the asymptotic behaviour of the gauge field at the boundary ∂E^4 represented by the three-dimensional sphere S^3 where the gauge potential becomes a pure gauge. We show then that at the 0 scale limit, F_{totale} is isomorphic to Q :

$$F_{totale} \equiv Q$$

At the 0 scale limit, $\sum_{univers} \frac{GM}{c^2 r}$ is identifiable to the Schwartzshild radius l_S of spacetime quotiented by the radius $r \rightarrow \varepsilon$ of the configuration in region of 0 scale, i.e. :

$$\sum_{univers} \frac{GM}{c^2 r} = \sum_{univers} \frac{l_S}{r} \qquad (37)$$

We therefore draw from (37) that $\frac{GM}{c^2 r}$ represents the Gaussian curvature of the manifold, which gives :

$$\frac{GM}{c^2 r} = K_{r\varphi} \qquad (38)$$

$K_{r\varphi}$ being the Gaussian curvature of the geodesic surface (r, φ). Hence, at the Planck scale, we must extend Einstein's gravitational term R towards an asymptotically free non-linear theory. Our approach is that the presence of terms in R^2 in the Lagrangian is the result of a superposition of two scales of quantum gravity : (**i**) the gravitational scale - i.e. macroscopic scale - characterised by the presence of an interaction in R and (**ii**) the quantum scale - i.e. microscopic scale - characterised by the presence of dual gravity terms, in R* = iR. The curvature term $K_{r\varphi}$ takes then the new form in supergravity :

$K_{r\varphi}$ **(Planck)** $\rightarrow R_{\mu\nu}R^{\mu\nu}$

such that $\sum_{univers} \frac{l_S}{r}$ is identifiable to

$$\Theta = \int d^4x \; R_{\mu\nu}R^{\mu\nu} \qquad (39)$$

As the theory is auto-dual, one has therefore R = R^*, and Θ becomes :

$$\Theta = \int d^4x \; R_{\mu\nu}\tilde{R}^{\mu\nu} \qquad (40)$$

which, by construction, is isomorphic to the topological charge Q of the gravitational instanton, as required. □

From the above demonstration, we can therefore conclude that the inertial interaction admits the topological charge of the 0 size gravitational instanton as source. The topological nature of the inertial interaction explains then (i) its global invariance properties and (ii) its instantaneous propagation between two points in spacetime.

We complete the conjecture (2.1.6) in specifying why the plane of oscillation **∏2** has a static character.

Corollary 4.5. *The static character of the plane of oscillation* ***∏2*** *of the pendulum* $\mathcal{J}$ *is induced by the structure of topological invariant of* ***Q*** *:*

Elements of demonstration 4.6. Instanton theory has established that Q is a topological invariant. In our case, Q = 1. The resulting vacuum topology θ - and, as a consequence, that of physical space S^3, boundary of E^4, in which $\prod^2$ is embeded - is equally invariant. Indeed, the invariance of the topological charge implies the invariance of the topological structure of S^3, i.e. of the three-dimensional physical space in the course of temporal evolution. It results from this that $\prod^2 \subset S^3$ is itself invariant - i.e. static -. Furthermore, the static character of $\prod^2$ implies that the underlying symmetry is Euclidean and can be described by the action of the group :

$$G_S = SU(2) \otimes SU(2) \tag{41}$$

G_S being, precisely, the symmetry group describing the instanton configuration characterised by the topological charge Q, as required. □

We now show that a good model of the propagation of the topological interaction $\mathbf{Int}_{\text{top}}$ can be given by the conformal transformations Conf (S^3) of the sphere S^3.

Conformal group of S^3

We propose to describe the transformations Conf (S^3) by the Möbius group [22], defined from the inversion of S^3. Recalling that the pole inversion c, of strength a $\in \mathbb{R}^*$ is the application $i = i_{c,\alpha} : X \setminus c \to X \setminus$ where c is defined, in X vectorialised in c, by $i(x) = \frac{\alpha}{\|x\|^2}.x$. From this :

Definition 4.8. *By inversion of S^3 is meant all restriction to S^3 of an inversion of R^4 globally conserving S^3. The Möbius group Möb (3) of S^3 is the sub-group of the S^3 bijection group engendered by inversions of S^3.*

At this point, we establish a link between the application defining the topological charge of the instanton and the similitudes of $\mathbb{R}^4$. We recall first that the group GL(n k) operates on M(n, k) by simillitude :

$$(P, M) \mapsto P.M = P.M.P^{-1} \tag{42}$$

The equivariant canonical bijection associated to the matrix M is then a diffeomorphism, representing the *similitude class* of M :

$$\varphi(M) : GL(n, \mathbb{R}) / Z(M) \rightarrow O(M) \tag{43}$$

which leads us to the important definition, taken from the reduction of endomorphisms [22] :

Definition 4.9. *The application $\psi : M(n, \mathbb{R}) \rightarrow O(M)$ describing the similitude class of M is given by :*

$$X \mapsto \exp(X).M.\exp(-X) \tag{44}$$

From this definition, we show in the two following propositions that the application defining the topological charge of the instanton belongs to the conformal group of S^3.

Proposition 4.10. *For all similitude $h \in \mathrm{Sim}(\mathbb{R}^3)$, the application defining the topological charge of the instanton, i.e. $f : S^3 \rightarrow S^3$, defined by $f(n) = n$ and $f = g^{-1} \circ h \circ g$ on $S^3 \setminus n$ belongs to Möb (3)).*

Demonstration 4.11. Let n be the north pole and s the south pole of the sphere S^3. We can then show that $g^{-1} \circ h \circ g$, conveniently extended to the whole of S^3, is in Möb (3) if h is an inversion or a hyperplane symmetry. Yet, the h engender the group of similitudes Sim ($\mathbb{R}^3$). Indeed, considering $\vec{f}$, it has been shown [25] that if the kernel

$$\mathrm{Ker}\,(\vec{f} - \mathrm{Id}_{\vec{X}}) = \{0\} \tag{45}$$

then f admits a unique fixed point corresponding to its centre. Moreover, $f \in \mathrm{Sim}(X) \setminus \mathrm{Is}(X)$, there exists a unique $\omega \in X$ such that $f(\omega) = \omega$. The point ω is the centre of the similitude f and one can write :

$$f = h \circ g = g \circ h, \quad h \in H_{\omega,\mu} \ \text{ et } \ g \in \mathrm{Is}_{\omega}(X) \tag{46}$$

The above assumes convenient extensions, the most immediate consisting of attaching a point at infinity on $\mathbb{R}^3$ and extending g to $S^3 \rightarrow \mathbb{R}^3$ by g(n) = ∞. Thus, following [22], their exists in Möb (3) the applications

$$f_\lambda = g^{-1} \circ H_{0,\lambda} \circ g \quad \mathbf{a} \; e^{-\lambda} \circ H_{0,\lambda} \circ e^{\lambda} \tag{47}$$

associated with vectorial homothetias of $\mathbb{R}^3$. If $\lambda > 1$, the application f_λ admits the north pole as an attractor and the south pole as a repeller, i.e. the iterates f_λ^n (n ∈ ℕ) making all points of $S^3 \setminus$ s converge towards n. The only point of S^3 out of the attraction of n is the south pole s. □

We show now that Möb(3) is the conformal group Conf (S^3) of S^3. Letting Conf (S^3) describe the scale invariance (i.e. the conformal invariance) of the sphere identified here (following the inclusion $S^3 \subset SL(2, C)$) to physical space $\mathbb{R}^3$ compactification.

Proposition 4.12. *Let Möb*$^{\pm}$ *(3) = Conf*$^{\pm}$ *(S^3). ∀ the radius r → 0 of S^3 engendering* $S^3_{r\rightarrow 0}$, *and ∀ f ∈ Möb(3), then* $S^3_{r\rightarrow 0}$ *belongs to the bundle f (S^3) of spheres S^3. Reciprocally, a bijection of S^3 verifying this property necessarily belongs to Möb (3) . The group Möb (3) presents a natural isomorphism with PO(α) of the quadric of equation*

$$q = -\sum_{i=1}^{4} x^2 + x_5^2 \, .$$

Demonstration 4.13. Let $i_{c,\,\alpha}$ be an inversion of X of dimension n. Let its derivative be $i'(x)$ composed of the vectorial hyperplane symmetry $x^{\perp}$ and the homothetia of ratio $\alpha / \|x\|^2$. We show then that $i'(x)$ in all $x \in X \setminus$ c is a direct similitude for $\alpha^n < 0$ and an indirect similitude for $\alpha^n > 0$. In fact, $i'(x)$ conserves the right and right-oriented angles. Since the composite of two conformal applications is conformal, then Möb(3)⊂Conf(S^3). Reciprocally, as Möb(3) ⊂ Conf (S^3) is transitive on S^3, then f ∈ Conf (S^3) leaves the north pole n fixed. According to the stereographic projection g of n, for f(n) = n, we obtain :

$$g \circ f \circ g^{-1} \in \text{Conf}\,(\mathbb{R}^3) \tag{48}$$

g and f being conformal. Applying Liouville's similitudes theorem, we have

$$g \circ f \circ g^{-1} \in \mathrm{Sim}(\mathbb{R}^3) \Rightarrow f \in \text{Möb}(3) \tag{49}$$

It follows from the inversion properties [25] that $f(\sigma)$ conserves the structure of the sphere S^3 when the radius $r \to 0$. Reciprocally, on putting $f(n) = n$, $g \circ f \circ g^{-1}$ transforms the $\mathbb{R}^3$ (half) lines into (half) lines, such that $S^3_{r\to 0}$ belongs to the bundle $f(S^3)$ of spheres S^3.

Finally, it has been established in [25] that $\text{Möb}(3) = \left\{ \Sigma \circ (f| \operatorname{im}(\alpha)) \circ \Sigma^{-1} : f \in \mathrm{PO}(\alpha) \right\}$, i.e. the Möbius group of S^3 corresponds to the restriction of the group $\mathrm{PO}(\alpha)$ on $\operatorname{im}(\alpha)$. □

We now conjecture that the plane of oscillation of $\mathcal{I}$ conserves the initial singularity $\mathbb{S}$ for inertial reference point, whatever the orientation of this plane in physical space $\mathbb{R}^3$

Conjecture 4.14. *Whatever the orientation in physical space $\mathbb{R}^3$ of the plane of oscillation Π^2 of the pendulum $\mathcal{I}$, this 2-dimensional plane necessarily intersects the initial singularity $\mathbb{S}$, i.e. Π^2* ***is always aligned on*** *$\mathbb{S}$.*

Elements of demonstration 4.15. We have established the identification between physical space $\mathbb{R}^3$ compactification and S^3, boundary of spacetime and equally boundary of the singular gravitational instanton solution. Each orientation of the plane of oscillation Π^2 corresponds therefore to an orientation in S^3. We have also established that S^3 can be identified to physical space :

$$S^3 \leftrightarrow \mathbb{R}^3$$

such that the three-dimensional information coming from physical 3-geometry is concentrated on the 3-surface S^3 but is not detectable in the interior of the sphere. Thus, the conformal invariance of S^3 implies that the temporal direction x^4 is necessarily orthogonal to the tangent space in a point of S^3. Putting this point as the south pole **s** of S^3, we have shown above that there exists in Möb (3) the applications

$$f_\lambda = g^{-1} \circ H_{0,\lambda} \circ g \;\; \mathbf{a} \;\; e^{-\lambda} \circ H_{0,\lambda} \circ e^{\lambda} \tag{50}$$

associated to the vectorial homothetias of $\mathbb{R}^3$. If $\lambda > 1$, the application f_λ admits the north pole as attractor and the south pole as repeller, i.e. the iterations f_λ^n ($\mathbf{n} \in \mathbb{N}$) make all points in $S^3 \setminus$ s converge towards **n**. The only point in S^3 escaping the attraction of **n** is the south pole **s**. The north pole **n** of the 3-sphere is therefore the fixed point of the conformal transformation Conf(S^3) and the temporal direction x^4, orthogonal to the tangential plane of the sphere in **s**, intersects necessarily the centre O of S^3 as well as the north pole **n**. Since, by construction, the plane of oscillation $\prod^2$ contains x^4 , then :

$$x^4 \subset \prod^2$$

it follows that the plane of oscillation $\prod^2$ is orthogonal to the tangential plane of the sphere at the south pole **s** and cuts therefore necessarily the centre O of S^3 as well as the north pole **n**, the singular attractor point of S^3. □

The above conjecture suggests therefore that the symmetry of rotation in $\mathbb{R}^3$ (manifested by the plane of oscillation of Foucault's pendulum) is explicitly linked to the symmetry of the 0 instanton configuration, SU(2) $\approx$ S^3 being a sub-group of both SU(2) $\otimes$ SU(2) and SL(2, C). Once the identification Initial Singularity / instanton 0 is admitted, the above approach allows us reasonably to consider that, whatever the orientation of the plane of the pendulum in physical space, this plane remains necessarily aligned with the singular origin of spacetime, identified here to the singular origin **n** of the sphere S^3, **n** being the north pole of the 3-sphère, the unique fixed point of the system. Indeed. the different possible orientations in physical space of the plane of oscillation of the pendulum are given by all the possible orthogonal directions to the tangent plane at S^3. We obtain the different orientations of $\prod^2$ in $\mathbb{R}^3$ by making the south pole **s** "turn" on the 3-surface S^3, this rotation conserving the alignment between **s**, O and **n** in the same plane $\prod^2$.

We draw from the above that whatever the orientation, the plane of oscillation of Foucault's pendulum is necessarily aligned with the initial singularity marking the origin of physical space

S^3, that of Euclidean space E^4 (described by the family of instantons I_β of whatever radius ß) and, finally, that of Lorentzian spacetime M^4.

The static character of $\prod^2$ comes therefore *in fine* from the fact that the direction x^4 represents equally the fourth direction (in ß = *i* t imaginary time) of the Euclidean configuration of the type instanton E^4 bounded by S^3, such that at each point **s** of S^3, $\prod^2$ cuts O and the origin of E^4 represented by the north pole **n**. We suggest then that this interpretation of x^4 as imaginary time explains in a non-trivial way the nature of the inertial force as well as its instantaneous propagation from one point to another in spacetime.

5. DIRAC IMPULSE AND INITIAL SINGULARITY

We complete this paper by suggesting a complementary argument concerning the propagation of topological type of a "causal information" from the singular point S, the origin of the system, to the boundary of spacetime. In the following, we consider that the topological impulse at the origin represents a Dirac shock and is sent to infinity - i.e. to S^3, boundary of spacetime.

In effect, the initial singularity can be interpreted as a causal signal giving rise to, at instant 0, a shock at the origin of the Dirac impulse type [23]. The shock at the origin, or Imp (t), distributed at the 0 scale of spacetime, must satisfy :

$$\text{(i)}\quad \forall t \in R,\ f(t) \geq 0$$

$$\text{(ii)}\quad \mathrm{Imp}(t) = \begin{cases} 0 & \text{si} \ \ t \neq 0 \\ \infty & \text{si} \ \ t = 0 \end{cases} \qquad (51)$$

$$\text{(iii)}\quad \int_R \mathrm{Imp}(t)\,dt = 1$$

The unity impulse at the singular origin $\mathbb{S}$ can then be considered as an ideal signal, of causal type.

Proposition 5.1. *The initial singularity, distribution of zero support, can be interpreted as a Dirac impulse. It follows from this that the Fourier transform is a function that can be extended in the complex plane under the form of a holomorphic entire function, or bilateral Laplace transform.*

Demonstration 5.2. It has been shown in [4] that the initial singularity can be identified as a 0 size singular gravitational instanton, configuration built by E. Witten in [3]. The support of all associated distributions δ is therefore reduced to the singular point $\mathbb{S}$. The function δ associated to the curvature is therefore a Dirac distribution, such that, as established in [23], its Fourier transform is holomorphic intire and given by :

$$f\,[\,\delta] = \left\langle \delta(x)\ ,\ e^{-\,2i\pi\nu\beta} \right\rangle = [e^{-2i\pi\nu\beta}]_{\beta\,=\,0} \tag{52}$$

or, when the scale ß (or the time t) of the theory is zero :

$$f[\delta] = 1 \tag{53}$$

becomes a real and even distribution which, insofar as $f\,[1] = \delta$, must satisfy :

$$f[\delta] = \overline{f}\,[\,\delta\,] = 1 \tag{54}$$

The holomorphic function resulting from the Fourier transform of the δ function of zero support can equally be written in the form of a bilateral Laplace transform f(ß) :

$$f(ß_C) = = \int f(H) e^{-ß_c H} dH \tag{55}$$

where ß is a complex variable. By decomposing ß into real and imaginary parts, i.e. $ß_C = ß_r + iß_i$, we observe that, for $ß_r = 0$:

$$f\,(iß_i\,) = \int f(H) e^{-ißH} dH \tag{56}$$

which, up to the change of variable, is the Fourier transform of f(H). For a fixed $ß_r$, we have:

$$f\,(ß_r + iß_i\,) = \int e^{-\,ßH} f(H)\ e^{-ißH} dH \tag{57}$$

which is the FT of f(H) $e^{-\,ßH}$. Deriving under the sum the expression for $f(ß_C)$:

$$d/dß_C\ f(ß_C) = \int - ß_c f(H)\ e^{-\,ß_c H} dH \tag{58}$$

or, in general :

$$d^m/d(ß_c)^m\, f(ß_c) = = \int -(H)^m f(H) e^{-ß_c H} dH \qquad (59)$$

and the summability abscissae of $-(H)^m f(H)$ are the same as those of $f(H)$, such that $f(ß_c)$ is indefinitely derivable for all values of $ß_c$ where $f(ß_c)$ exists. $f\,(ß_r + iß_i\,)$ is therefore holomorphic in all the band of summability, i.e. for all values situated to the right of 0 in the complex plane formed by $ß_r > 0$ and $ß_i > 0$. The function $ß_c$ is therefore analytic in this domain of the complex plane. As the Dirac impulse at the origin has for support the point $\mathbb{S}$, its FT describes therefore an impulsional response non-decreasing at infinity. □

To understand the necessarily non-compact character of the impulsional response giving the evolution of the system, we complete the above proposition by the following corollary :

Corollary 5.3. *The Fourier transform of the singular distribution δ_S of punctual support describing the impulsional response of the spacetime system cannot be of compact support.*

Demonstration 5.4. Considering the Dirac impulse $\delta_S \in E(\mathbb{R})$ and the function $f = \hat{\delta}_s$.

f is analytic on $\mathbb{R}$, insofar as it is the trace of a holomorphic function on $\mathbb{C}$. We suppose that f is of compact support, hence f cancels itself out on an non-empty open set of $\mathbb{R}$. Thus, if a general analytical function on $\mathbb{R}$ is zero on an non-empty open set, then it is identically zero. It follows from this that f cannot be of compact support. □

The above results suggest in fact that the interaction here considered, (not physical but purely topological), is ergodic. As the Dirac impulse at the origin has for support the point $\mathbb{S}$, its FT describes therefore an impulsional response non-decreasing at infinity. □ Indeed, (i) the behaviour of $\prod^2$ is scale invariant and (ii) the 0 size singular gravitational instanton characterising, according to [4], the initial singularity, represents a critical point S_0 in the system formed by the pre-spacetime manifold and $\mathcal{G}$, such that the correlation length of the system $\xi \to \infty$. From this viewpoint, the interaction **Int** $_{top}$ is subject to the action of a renormalisation group G_n assuring the scale invariance of the system.

6. DISCUSSION

Here, taking the example of Foucault's experiment, we have suggested a new solution, aimed at resolving two open problems :

(**i**) the problem of the invariance of the inertial interaction at all points in spacetime ;

(**ii**) the problem of the "instantaneous propagation" of inertia from one point to another in spacetime - i.e. the Machian principle according to which the inertial reference frame defined by local physics coincides with the reference frame in which distant objects are at rest. It follows that the masses distributed most distantly in the universe determine the inertial behaviour of local masses.

Our approach is that the problem of inertia, well placed in the context of Mach's principle, cannot be resolved by ordinary field theory. Indeed, it has been suggested in [4] that under the Planck scale, quantum field theory must be analytically continued by topological field theory. We can thus establish that at 0 scale, the associated supergravity is exclusively a matter for topological theory, describing a spacetime provided with a positively defined Euclidean metric. In such a context, the initial singularity can be interpreted as a 0 size singular gravitational instanton marking the origin of the semi-Kählérienne manifold corresponding to pre-spacetime between 0 scale and the Planck scale.

Our conclusion is then that at 0 scale, the topological charge Q = 1 of the 0 size instanton represents the source of the global topological inertial interaction, which can be tested experimentally by the fixity of the plane of oscillation of Foucault's pendulum or by the Thirring- Lense effect.

In a subsequent paper, we will consider that this result can be reinforced by the hypothesis of a correlation between the singular scale (t_0, x_0) and the macroscopic scale (t, x). We begin from the observation of the cosmological radiation at 2.7 ° K and draw from this the existence of a thermal Green function - or Euclidean Green function G_E - describing the correlation between the 0 scale (ß = 0) and the macroscopic scale of spacetime. Such an approach suggests the topological nature of the interaction between 0 and macroscopic scales. Indeed, the correlation described by

$$G_E(t_0,x_0\ ;\ t,x) = \int \langle \phi(t_0,x_0)\phi(t,x)\rangle e^{-\xi} d\phi \qquad (60)$$

is such that all points P of spacetime are correlated - by an Euclidean path - to the singular point O. Insofar as the path between O and P is Euclidean - which is the case since the spacetime system, considered at the non-zero temperature T = 2.7° K is the concern of statistical mechanics - the interaction between O and P depends only on the conditions at the boundary and is thus purely topological. Following this, we specify in this paper to come the notion of "Euclidean propagation" of inertia, according to the flow of weights of the algebra of states describing spacetime in the region of the initial singularity. Beginning with the Von Neumann algebra $\sum$ describing the singular state corresponding to the 0 size gravitational instanton, we conjecture that the sole result from the algebra $\sum$ implies the existence of a "pseudo-dynamic" associated with $\sum$ and characterised by the flow of weights of $\sum$. Such a flow assures the propagation of the topological charge Q of the 0 instanton. In agreement with the results of Connes [24], the homomorphism defining the canonical dynamic δ is such that δ : $\mathbb{R} \rightarrow \mathrm{Out} \sum = \dfrac{\mathrm{Aut}\,\Sigma}{\mathrm{Int}\,\Sigma}$, this invariant having an intrinsic description in terms of flow of weights of $\sum$. In other words, the sole result of the algebra $\sum$ implies the existence of a dynamic associated with $\sum$ and characterised by the flow of weights of $\sum$. We suggest then that this dynamic is based on the semi-group of automorphisms :

$$\alpha_{\tau}(M) = e^{-ß\, \mathbb{D}^2}\, M\, e^{ß\, \mathbb{D}^2} \qquad (61)$$

corresponding to the evolution in imaginary time *i* t of the state M - i.e. to the expansion of the space of states $\mathbb{E}$. This expansion of $\mathbb{E}$ is indexed by increasing values of ß, the radius of $\mathbb{E}$. As (61) describes the flow of weights of the system and that this flow is ergodic, ß is necessarily increasing in the interval $[0, \infty]$.

We suggest then that a test of the topological nature of the interaction existing between the 0 size singular gravitational instanton and local systems is provided by the fixity of the plane of oscillation of Foucault's pendulum. We have shown that Möb(3) is the conformal group Conf (S^3) of S^3. Conf (S^3) describes the scale invariance (i.e. conformal invariance) of the sphere identified here, following the inclusion $S^3 \subset SL(2, C)$, to physical space $\mathbb{R}^3$ compactification. We have then suggested that the flow of weights of the algebra M giving the modular flow $\alpha_t(M)$ on S^3 belongs to the class of similitude S^3.

Finally, we note that an interesting consequence of the above approach is that it allows us to establish an explicit relation between the automorphism semi-group of the algebra of states A and the renormalisation semi-group of the theory. Introduced by Wilson [25] and Kadanoff *et al.* [26], the renormalisation programme - in particular the renormalisation group - allows us to encompass in a unique formalism the different scales of the theory. Experience shows that the behaviour of Foucault's pendulum, notably the fixity of the plane of oscillation, is scale invariant. Everything occurs therefore as if the dynamic of $\mathcal{J}$ were subject to the action of a renormalisation group $G_{\mathbf{R}}$, the group whose structure we define below. The calculation of the correlation length ξ between two localised variables at different points takes then the form, considering the variables ψ_n :

$$\langle \psi_n, \psi_m \rangle - \langle \psi_n \rangle \langle \psi_m \rangle = e^{-ß/ß_0} \tag{62}$$

where the distance ß depends on the number of points on the lattice between n et m. At 0 scale, the correlation length becomes, considering the coupling g_0 :

$$ß_0(g_0) \to \infty \tag{63}$$

The correlation length is infinite, such that at 0 scale, there exists an instantaneous interaction between the point S_0 representing the initial singularity of spacetime and the boundary at infinity of the 4-manifold, representing the 3-dimensional physical space. We have shown that the renormalisation group acting on $\mathcal{J}$ is isomorphic to the automorphism semi-group of the state algebra of the theory at 0 scale.

From the above, we can conclude :

(**i**) the identitfication between the flow of weights of the state algebra and the topological flow responsible for the propagation of the topological charge Q from 0 scale to ß scale characterising the radius of the large scale physical space ;
(**ii**) the explanation according to which, whatever the orientation in physical space $\mathbb{R}^3$ of the 2-dimensional plane of oscillation Π^2 of the pendulum $\mathcal{J}$, this plane necessarily intersects the initial singularity, i.e. Π^2 is always aligned on $\mathbb{S}$.

We therefore suggest, to conclude, that the interpretation of x^4 as a direction in imaginary time explains in a non-trivial manner the nature of the inertial force as well as its instantaneous propagation from one point to another in spacetime.

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