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ON THE LANGMUIR OSCILLATION AND LANDAU DAMPING IN QUARK GLUON PLASMA

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Abstract

On the basis of semiclassical kinetic equations for quark-gluon plasma (QGP) and Yang-Mills equation in covariant gauge, Langmuir oscillation and linear Landau damping is investigated. It is found that plasma eigen modes are directly related with the wave number and it is highly coupled with the thermal part of QGP. The linear Landau damping also exists in QGP, which shows that plasma modes heavily damp for $| k | \to 0$.

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I. INTRODUCTION

Recently, there has been much interest in theoretical and experimental study of quark-gluon plasma (QGP) [1-3]. Because of asymptotic freedom of QCD, quarks are liberated at suffieciently high temperature and densities. There is a phase transition which occurs at temperatures of several hundred MeV. The thermodynamic properties of a quark-gluon plasma were considered in detail in Ref.[4], in which fairly good expressions were obtained for its free energy. Kinetic properties of the quark-gluon plasma and its collective excitations are investigated in Ref. [5]. Kalashnikov and Klimov [6] investigated the properties of the polarization operator calculated in QCD at finite temperatures and densities in the "one-loop" approximation and the spectrum of the elementary excitations in such a system is found explicitly.

Most of the theoretical analyses of the dynamics and signatures of QGP plasma phase have relied on the assumption of local thermal and chemical equilibrium. These assumptions have promoted the construction of a kinetic frame work for plasmas with non-abelian interactions, which allow discussions of nonequilibrium phenomenon with the approach to equilibrium.

Heinz and Siemens [7] carried out an analyses of colored collective modes in a QGP on the basis of 'quark-gluon transport theory' near equilibrium. They found that two optical modes (one longitudinal and one transverse) exist starting for k=0 at the plasma frequency, while there is no acoustic mode starting at $\omega = 0$. An important conclusion is done here, that linear Landau damping is absent in QGP due to the contribution of massless gluon in the collective modes. Further, Markov and Markova [8] developed the theory of nonlinear Landau damping on the basis of hard thermal loop approximation. Linear Landau damping is abandoned on the basis of an earlier paper [7].

In this paper, we re-examine the Linear Landau damping in QGP on the basis of semi-classical kinetic equations and Yang-Mills [SU(3)] equation in a covariant gauge. In Sec.II. we formulate the linearized system of equations for quark-gluon plasma with thermal and chemical equilibrium. The regular distribution functions are bosonic for quarks and anti quarks, while it is fermionic for the gluons with the global equilibrium in QGP. In Sec.III. we do the Fourier transformation of the linearized system and a generalized expression for the conductivity (polarization) tensor is obtained. The Langmuir oscillation and linear Landau damping is studied in detail. It is found that the eigen modes in QGP is strongly related to the wave number and also to the thermal part of the system. Linear Landau damping, depending stongly on the wave number and temperature, also exists in QGP. For smaller $\mid k \mid\rightarrow 0$, we find that $\omega_p \sim \mid k \mid^{\frac{1}{3}},, \sim \frac{1}{|k|}$.

II. EQUATIONS FOR DISTRIBUTION FUNCTIONS AND GAUGE FIELDS

We consider an ultrarelativistic quark-gluon plasma in, or, close to thermal equilibrium, at a temperature T. We use the natural units, $c = k_B = 1$ and the metric $g^{\mu,\nu} = diag(1, -1, -1, -1)$. We consider a $SU(N_c)$ gauge theory with N_f flavors of quarks. The color indices $a, b, ..$ run from 1 to $N_c^2 - 1$. The generators of the gauge group are denoted by t^a and T^a, respectively, for the fundamental and the adjoint representations, and are normalized such that $Tr(t^a t^b) = \frac{1}{2}\delta_{ab}$ and $Tr(T^a T^b) = N_c \delta^{ab}$. It follows that $(T^a)^{bc} = -if^{abc}$, and $t^a t^a = C_f$, where $C_f = (N_c - 1)/(2N_c)$ is the Casimir of the fundamental representation and f^{abc} are the structure constants of the group: $[t^a, t^b] = if^{abc}t^c$. Furthermore, D_μ and $\hat{D}_\mu$ are the covariant derivatives which act as $D_\mu = \partial_\mu - ig[A_\mu(x), \cdot]$, $\hat{D}_\mu = \partial_\mu - ig[\hat{A}_\mu(x), \cdot]$, $[,]$ denotes the commutator, $\{,\}$ denotes the anticommutator, and $A_\mu = A^a_\mu t^a$ for the fundamental representation and $\hat{A}_\mu = A^a_\mu T^a$ for the adjoint representation. The field $F_{\mu\nu} = F^a_{\mu\nu} t^a$ with

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc} A^a_\mu A^a_\nu, \tag{1}$$

obeys the Yang-Mills (YM) equation in a covariant gauge

$$\partial_\mu F^{\mu\nu}(x) - ig[A_\mu(x), F^{\mu\nu}(x)] - \xi^{-1}\partial^\nu \partial^\mu A_\mu(x) = -J^\nu(x), \tag{2}$$

where, ξ is a gauge parameter and g is the dimensionless coupling parameter. J^ν is the color current

$$J^\nu = gt^a \int d^4 p p^\nu [Trt^a(f_q - f_{\bar{q}}) + Tr(T^a f_g)]. \tag{3}$$

We are neglecting the spin effects. Thus the distribution functions of quarks f_q, antiquarks $f_{\bar{q}}$, and gluons f_g satisfy the semiclassical kinetic equations

$$\begin{aligned} p^\mu D_\mu f_{q,\bar{q}} \pm \frac{1}{2} g p^\mu \{F_{\mu\nu}, \frac{\partial f_{q,\bar{q}}}{\partial p_\nu}\} = 0, \\ p^\mu \hat{D}_\mu f_g + \frac{1}{2} g p^\mu \{\hat{F}_{\mu\nu}, \frac{\partial f_g}{\partial p_\nu}\} = 0, \end{aligned} \tag{4}$$

where, the upper sign refers to quarks and the lower one to antiquarks, and $\hat{F}_{\mu\nu} = F^a_{\mu\nu} T^a$.

Initially, we are interested with the linear response in the QGP plasma. Therefore, we decompose the distriburion functions into two parts, namely, regular and random (turbulent) ones

$$f_s = f^R_s + f^T_s, \qquad s = q, \bar{q}, g, \tag{5}$$

so that

$$< f_s >= f^R_s, < f^T_s >= 0, \tag{6}$$

where angular brackets $< . >$ indicate a statistical ensamble of averaging. Further we set

$$A_\mu = A^R_\mu + A^T_\mu, < A^T_\mu >= 0, \tag{7}$$

by definition. For simplicity, the regular part of the field A^R_μ is considered equal to zero. Similarly,

$$J_\mu = J^R_\mu + J^T_\mu, < J_\mu >= J^R_\mu, < J^T_\mu >= 0. \tag{8}$$

Let

$$\begin{aligned} f^T_s &= f^{T(1)}_s + f^{T(2)}_s + ... \\ J^T_\mu &= J^{T(1)}_\mu + J^{T(2)}_\mu + ... \end{aligned} \tag{9}$$

Substituting Eqs. (5) - (9) into Eqs.(2) - (4), and collecting only the first order of perturbations, we obtain the following linearized system of equations for the QGP plasma:

$$\partial_\mu (F^{T\mu\nu})_L - \xi^{-1}\partial^\nu\partial^\mu A^T_\mu = -J^{T(1)\nu} = -gt^a \int d^4 p p^\nu [Tr t^a (f_q^{T(1)} - f_{\bar{q}}^{T(1)}) + Tr(T^a f_g^{T(1)})]. \quad (10)$$

$$p^\mu \partial_\mu f^{T(1)}_{q,\bar{q}} = \mp\frac{1}{2} g \frac{1}{2} g p^\mu \{(F^T_{\mu\nu})_L, \frac{\partial f^R_{q,\bar{q}}}{\partial p_\nu}\} = 0, \quad (11)$$

$$p^\mu \hat{D}_\mu f^{T(1)}_g = -\frac{1}{2} g p^\mu \{(\hat{F}^T_{\mu\nu})_L, \frac{\partial f^R_g}{\partial p_\nu}\} = 0, \quad (12)$$

where,

$$(F^T_{\mu\nu})_L = \partial_\mu A^T_\nu - \partial_\nu A^T_\mu = (\partial_\mu A^{Ta}_\nu - \partial_\nu A^{Ta}_\mu) t^a \equiv (F^{Ta}_{\mu\nu}) t^a. \quad (13)$$

$$(\hat{F}^T_{\mu\nu})_L = \partial_\mu \hat{A}^T_\nu - \partial_\nu \hat{A}^T_\mu = (\partial_\mu \hat{A}^{Ta}_\nu - \partial_\nu \hat{A}^{Ta}_\mu) T^a \equiv (\hat{F}^{Ta}_{\mu\nu}) T^a. \quad (14)$$

We suppose that the characteristic time of relaxation of oscillations is small as compared to the time of relaxation of the f^R_s. Therefore, we neglect the variation of the regular part of the distribution functions in space and time, assuming that these functions are specified and describe the global equilibrium in QGP [8]

$$f^R_{q,\bar{q}} \equiv f^0_{q,\bar{q}} = 2\frac{2N_f \theta(p_0)}{(2\pi)^3}\delta(p^2)\frac{1}{e^{(pu\mp\mu)/T}+1},$$
$$f^R_g \equiv f^0_g = 2\frac{2\theta(p_0)}{(2\pi)^3}\delta(p^2)\frac{1}{e^{(pu)/T}-1}. \quad (15)$$

III. LANGMUIR OSCILLATION AND LANDAU DAMPING

Taking the Fourier transformation of the linearized equations (11) and (12), we find

$$f^{(1)}_{q,\bar{q}}(k,p) = \mp \frac{g\chi^{\nu\lambda}}{p\cdot k + i\epsilon p_0} A_\nu(k) \frac{\partial f^{(0)}_{q,\bar{q}}}{\partial p^\lambda}, \quad (16)$$

$$f^{(1)}_g(k,p) = -\frac{g\chi^{\nu\lambda}}{p\cdot k + i\epsilon p_0} \hat{A}_\nu(k) \frac{\partial f^{(0)}_g}{\partial p^\lambda}, \quad (17)$$

where,

$$\chi^{\nu\lambda} \equiv (p\cdot k g^{\nu\lambda} - p^\nu k^\lambda). \quad (18)$$

Thus, the Fourier transformation of equation (10) takes the form

$$k_\mu (k^\mu A^{\nu'} - k^{\nu'} A^\mu) - \xi^{-1} k^{\nu'} k^\mu A_\mu = -J^{\nu'}(k)$$
$$= g^2 \int d^4 p \frac{p^{\nu'}\chi^{\nu\lambda}}{p\cdot k + i\epsilon p_0}\{A_\nu(k)(\frac{\partial f^{(0)}_q}{\partial p^\lambda} + \frac{\partial f^{(0)}_{\bar{q}}}{\partial p^\lambda}) + \hat{A}_\nu(k)\frac{\partial f^{(0)}_g}{\partial p^\lambda}\}. \quad (19)$$

We may write

$$J^{(1)\nu'}(k) = g^2 \int d^4p \frac{p^{\nu'}}{p\cdot k + i\epsilon p_0}(p^\nu k \cdot \frac{\partial}{\partial p} - p\cdot k \frac{\partial}{\partial p^\nu})[f_q^{(0)} + f_{\bar{q}}^{(0)} + N_c f_g^{(0)}]A_\nu(k). \tag{20}$$

In the above, for simplicity, we omit the suffix " T " for the gauge field.

Now, we can write the linear current as

$$J^{\nu'}(k) = \Pi^{\nu'\nu} A_\nu(k), \tag{21}$$

where,

$$\Pi^{\nu'\nu} = g^2 \int d^4p \frac{p^{\nu'}}{p\cdot k + i\epsilon p_0}(p^\nu k \cdot \frac{\partial}{\partial p} - p\cdot k \frac{\partial}{\partial p^\nu}) N_{eq}, \tag{22}$$

with $N_{eq} = \frac{1}{2}(f_q^{(0)} + f_{\bar{q}}^{(0)}) + N_c f_g^{(0)}$, is the conductivity (polarization) tensor of QGP.

We are interested in the study of oscillation and Landau damping of Langmuir mode in QGP. The term Π^{00} represents such oscillation, thus we study Π^{00} in detail.

$$\Pi^{00} = g^2 \int d^4p \frac{p^0}{p\cdot k + i\epsilon p_0}(p^0 \vec{k} \cdot \frac{\partial}{\partial \vec{p}} - \vec{p}\cdot \vec{k} \frac{\partial}{\partial p^0}) N_{eq} \tag{23}$$

For an easy calculation of the integral, we begin with the one dimensional case of the problem. Therefore, we consider $\vec{p} \parallel \vec{k} \parallel \vec{u}$. Then

$$\Pi^{00} = -\frac{g^2}{T}\frac{2N_f}{(2\pi)^3} 4\pi \int_0^\infty dp \frac{p^0 ku + u^0 kp}{p^0k^0 - kp + i\epsilon p^0} F(p). \tag{24}$$

where,

$$F(p) = \Sigma_s \alpha_s \frac{e^{(p^0u^0 - pu - \mu_s)/T}}{(e^{(p^0u^0 - pu - \mu_s)/T} + 1)^2}, \tag{25}$$

with $\alpha_q = \alpha_{\bar{q}} = 1, \alpha_g = 2N_c, \mu_q = \mu, \mu_{\bar{q}} = -q, \mu_g = 0$, represents a combined distribution of quarks, antiquarks and gluons.

The principal value of the integral related with the combined distribution function $F(p)$ is given by the integral type

$$\int_0^\infty dp \frac{p^0 ku + u^0 kp}{p^0k^0 - kp + i\epsilon p^0} \frac{e^{(p^0u^0 - pu - \mu)/T}}{(e^{(p^0u^0 - pu - \mu)/T} + 1)^2}, \tag{26}$$

which is very much complicated to yield any physical results. Therefore, we chose that

$$F(p) \approx G(p) = \alpha e^{-\beta(p-W)^2}, \tag{27}$$

where α, β, W are some chosen parameters as function of free parameters (μ, T, u, u^0). The function $F(P)$ and $G(p)$ may be close to identical with the proper selection of the parameters satisfying at least the following conditions:

i) $F(0) = G(0)$, *ii)* $F'(W) = G'(W)$, *iii)* $F(\infty) = G(\infty)$.

We plot the functions $F(p)$ and $G(p)$ with the fixed values of parameters $u^0 = P^0 = T = u = 1, \mu = .01, N_c = 8$, with the chosen parameters $\alpha = 4, \beta = .1, W = .8$. These functions are shown in Fig.1a and Fig.1b, respectively. The figures validate our approximation to some extent.

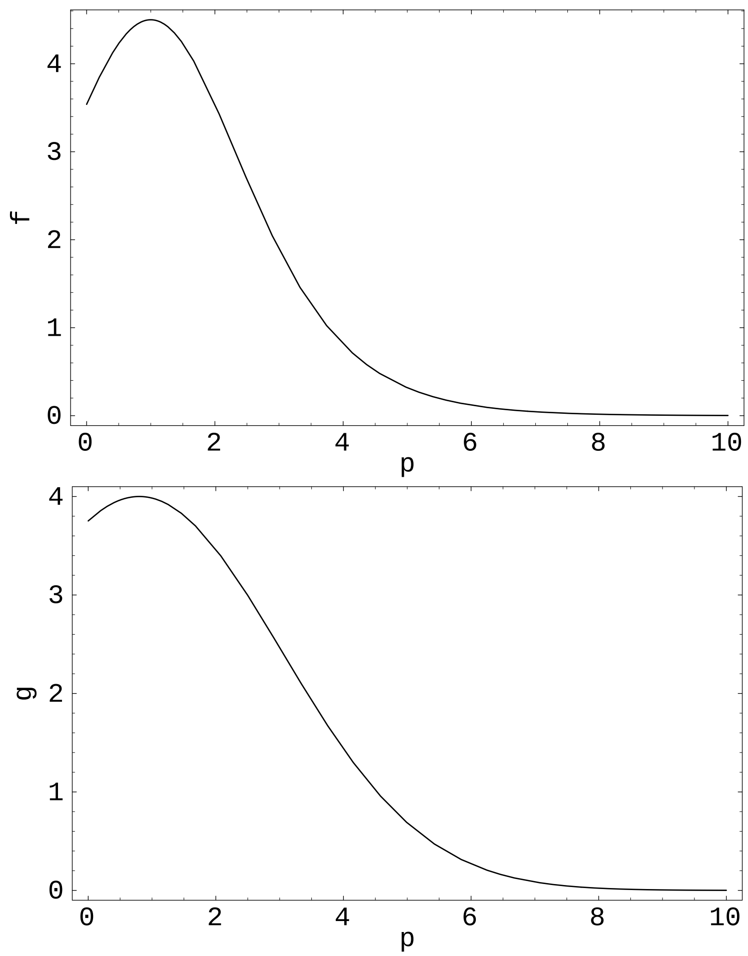

FIG. 1. Dependence of functions F(p) and G(p) for the fixed values free parameters

Thus, the integral equation (24) may be written approximately as

$$\Pi^{00} \approx -\frac{g^2}{T}\frac{8\pi N_f}{(2\pi)^3}\alpha\int_0^\infty dp\frac{p^0ku+u^0kp}{p^0k^0-kp+i\epsilon p^0}e^{-\beta(p-W)^2}. \tag{28}$$

Let

$$I = \int_0^\infty dp \frac{p^0 ku + u^0 kp}{p^0 k^0 - kp + i\epsilon p^0} e^{-\beta(p-W)^2}$$

$$= -p^0 u[\oint \frac{G_1(p)}{p - p_{res}} dp + i\pi G_1(p = p_{res})] - u^0 [\oint \frac{G_2(p)}{p - p_{res}} dp + i\pi G_2(p = p_{res}), \quad (29)$$

where $G_1(p) = e^{-\beta(p-W)^2}, G_2(p) = pe^{-\beta(p-W)^2}, p_{res} = \frac{p^0 k^0_{res}}{k}$, respectively.

Now, expanding

$$\frac{1}{p - p_{res}} = -\frac{1}{p_{res}} \{1 + (\frac{p}{p_{res}}) + (\frac{p}{p_{res}})^2 + ...\}, \quad (30)$$

and keeping up to $2nd$ order in $(\frac{p}{p_{res}})$, from the principal and residual values of the integral, we find

$$\Pi^{00} = \frac{g^2}{T} \frac{8\pi N_f}{(2\pi)^3} \alpha [\frac{p^0 u}{p_r} + (\frac{p^0 u}{p_r^2} + \frac{u^0}{p_r})(\frac{e^{-\beta p_0^2}}{2\beta} + W) + (\frac{p^0 u}{p_r^3} + \frac{u^0}{p_r^2})\{\frac{1}{\beta}(W - \frac{p_0}{2})e^{-\beta p_0^2} + \frac{1+2\beta}{2\beta} W^2\}$$
$$-i\pi\{(p^0 u + p_r u^0) e^{-\beta(p_r - W)^2}\}], \quad (31)$$

From equation (19) we find the dielectric tensor for the Langmuir oscillation

$$\varepsilon^l = (1 - \xi^{-1}) k_0^2 + \Pi^{00}. \quad (32)$$

To extract Langmuir oscillation and damping of longitudinal spectrum, we split $k^0 = k_r^0 + ik_i^0$. In fact, $k_r^0 = \omega_r$ and $k_i^0 = \omega_i$.

Thus the dielectric tensor splits into two parts

$$\varepsilon^l = \varepsilon_r^l + i\varepsilon_i^l, \quad (33)$$

where

$$\varepsilon_r^l(k, k_r^0) = 0, \quad (34)$$

determines the Langmuir spectrum and

$$k_i^0 = -\frac{\varepsilon_i^l(k, k_0^0)}{\frac{\partial \varepsilon_r^l(k, k_r^0)}{\partial k^0}|_{k^0 = k_r^0}}, \quad (35)$$

gives the Landau damping. Accordingly, from equations (28) and (29), we find that

$$\varepsilon_r^l(\omega_r, k) = \omega_r^2 - \frac{\bar{A}k}{\omega_r} - \frac{\bar{B}k^2}{\omega_r^2} - \frac{\bar{C}k^3}{\omega_r^3}, \quad (36)$$

and

$$\varepsilon_i^l(\omega_r, k) = \frac{g^2}{T} \frac{8\pi^2 N_f}{(2\pi)^3} \alpha (1 + \xi^{-1})^{-1} p^0 (u + \frac{\omega_r}{k}) e^{-\beta(\frac{p^0 \omega_r}{k} - W)^2}. \quad (37)$$

Then the equation

$$\varepsilon_r^l(\omega_r, k) = 0, \quad (38)$$

yields

$$\omega_r^2 = \frac{\bar{A}k}{\omega_r} + \frac{\bar{B}k^2}{\omega_r^2} + \frac{\bar{C}k^3}{\omega_r^3}, \tag{39}$$

where

$$\bar{A} = \frac{g^2}{T}\frac{8\pi N_f}{(2\pi)^3}\alpha(1+\xi^{-1})^{-1}[u + \frac{u^0}{p^0}(\frac{e^{-\beta p_0^2}}{2\beta} + W)],$$

$$\bar{B} = \frac{g^2}{T}\frac{8\pi N_f}{(2\pi)^3}\alpha(1+\xi^{-1})^{-1}[\frac{u}{p_0}(\frac{e^{-\beta p_0^2}}{2\beta} + W) + \frac{u^0}{p_0^2}\{\frac{1}{\beta}(W - \frac{p_0}{2})e^{-\beta p_0^2} + \frac{1+2\beta}{2\beta}W^2\}],$$

$$\bar{C} = \frac{g^2}{T}\frac{8\pi N_f}{(2\pi)^3}\alpha(1+\xi^{-1})^{-1}[\frac{u^0}{p_0^3}\{\frac{1}{\beta}(W - \frac{p_0}{2})e^{-\beta p_0^2} + \frac{1+2\beta}{2\beta}W^2\}]. \tag{40}$$

Equation (35) determines the Landau damping, which is given by

$$, = \omega_i = -\frac{\frac{g^2}{T}\frac{8\pi^2 N_f}{(2\pi)^3}\alpha(1+\xi^{-1})^{-1}p^0(u + \frac{\omega_r}{k})e^{-\beta(\frac{p^0\omega_r}{k} - W)^2}}{2\omega_r + \frac{\bar{A}k}{\omega_r^2} + \frac{\bar{B}k^2}{\omega_r^3} + \frac{\bar{C}k^3}{\omega_r^4}}. \tag{41}$$

For $k \to 0$, we find

$$\omega_r \sim \{\frac{g^2}{T}\frac{8\pi N_f}{(2\pi)^3}\alpha(1+\xi^{-1})^{-1}[u + \frac{u^0}{p^0}(\frac{e^{-\beta p_0^2}}{2\beta} + W)]k\}^{\frac{1}{3}}, \tag{42}$$

and

$$, \sim -\frac{\frac{g^2}{T}\frac{8\pi^2 N_f}{(2\pi)^3}\alpha(1+\xi^{-1})^{-1}p^0(u + \frac{\omega_r}{k})e^{-\beta(\frac{p^0\omega_r}{k} - W)^2}}{3\omega_r} \tag{43}$$

For $u = 0$ (thermal bath)

$$\omega_r \sim\mid k \mid^{\frac{1}{3}}, \qquad , \sim \frac{1}{\mid k \mid}. \tag{44}$$

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REFERENCES

[1] J. I. Kapusta, S. M. H. Wong, Phys. Rev. **D62**, 037301 (2000).

[2] Quark-Gluon Plasma 2, Edited by R. C. Hwa, World Scientific, 1995,

[3] Quark-Gluon Plasma, Edited by B. Sinha, S. Pal ans S. Raha, Springer-Verlag, 1990.

[4] J. I. Kapusta, Nucl. Phys. **B148**, 461 (1979).

[5] J. P. Blaizot and J.-Y. Ollitraut, Phys. Rev. **D48**, 1390 (1993).

[6] O. K. Kalashnikov and V. V. klimov, Sov. J. Nucl. Phys. **31**, 699 (1981). V. V. Klimov, Sov. Phys. JETP **55**, 199 (1982).

[7] U. Heinz and P. J. Siemens, Phys. Lett, **158B**, 11 (1985).

[8] Yu. A. Markov and M. A. Markova, ph/9902397 v2 27 Apr 1999.