United Nations Educational Scientific and Cultural Organization
and
International Atomic Energy Agency
THE ABDUS SALAM INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

LABORATORY TEST OF PULSAR WINDS BY INTENSE LASER RADIATION

U.A. Mofiz*
School of Science and Technology, Bangladesh Open University, Gazipur-1705, Bangladesh
and
The Abdus Salam International Centre for Theoretical Physics, Trieste, Italy

and

A. Sen†
Institute for Plasma Research, Bhat, Gandhinagar 382 428, India
and
The Abdus Salam International Centre for Theoretical Physics, Trieste, Italy.

Abstract

An ambient field generated in the electron-positron plasma due to fluctuation or other reasons, may be enhanced by intense ultra short electromagnetic radiation and $e^{\mp}$ bunches may be accelerated to high energies. The phenomenon is well matched with the cause of pulsar winds in the polar cap region and it may be accomplished in the laboratory with the ultra short laser pulses.

MIRAMARE – TRIESTE
August 2000

*Regular Asociate of the Abdus Salam ICTP. E-mail: mofizua@bou.bangla.net; amberm@bangla.net

†Senior Associate of the Abdus Salam ICTP. E-mail: abhijit@plasma.ernet.in

Pulsar winds have been mapped for the Crab and Vela pulsars as the flux of relativistic pair plasma. It is argued that polarized $e^{\pm}$ bunches are ejected from near the polar caps at high Lorentz factors [1]. Cascade generation of $e^{\pm}$ which occurs in the pulsar magnetosphere via curvature radiation, is noted [2,3]. Intense electromagnetic radiation propagating in the polar cap region of the pulsar magnetosphere may generate a huge potential difference which causes the pulsar winds. In this paper, we study such a mechanism analytically. We investigate the nonlinear propagation of intense electromagnetic radiation in an electron - positron plasma with an ambient potential due to thermal or other physical reasons. It is found that a short pulse of intense radiation with the group velocity very close to the speed of light may excite large amplitude ambipolar potential, subsequently intense wakefields to accelerate $e^{\pm}$ bunches to high energies. Recent success in the confinement of $e^{\pm}$ plasma in penning traps [4,5] and the possibility of having intense laser radiation [6] may accomplish such an experiment in the laboratory.

The system is described by the cold relativistic two fluids equations. The longitudinal motion is given by the continuity equation

$$\partial_t n_s + \nabla \cdot n_s \mathbf{v_s} = 0, \tag{1}$$

and the z component of equation of motion

$$\partial_t P_{sz} = -\frac{e_s}{c}\partial_t A_z - e_s \partial_z \Phi - mc^2 \partial_z \gamma_s, \tag{2}$$

where, $s = (-, +)$ represents the electron, positron plasma species, m is the rest mass, $\gamma_s = (1 + P_s^2/m^2c^2)^{1/2}$ is the relativistic factor and P_{sz} is the longitudinal momentum. A_z is vector potential of the ambient field and Φ is the scalar potential, for which the Lorentz gauge

$$\frac{1}{c}\partial_t \Phi + div\mathbf{A} = \mathbf{0}, \tag{3}$$

is considered. For one dimensional variations (in z) the symmetry of the problem permits [7,8] elimination of transverse momentum by the exact integration $\mathbf{P}_{s\perp} = -e_s \mathbf{A}_\perp/c$, where $\mathbf{A}_\perp$ is the vector potential of the propagating electromagnetic radiation. We introduce the following dimensionless quantities:
$t \to \omega_p t; z \to k_p z; k_p = \omega_p/c, \omega_p = (\frac{4\pi e^2 n_0}{m})^{1/2}; \mathbf{P} \to \frac{\mathbf{P}}{mc}; \mathbf{A}_\perp \to \frac{e\mathbf{A}_\perp}{mc^2}; \Phi \to \frac{e\Phi}{mc^2}; N_s \to \frac{n_s}{n_0}$.

Then the equations for the potentials are

$$(\partial_z^2 - \partial_t^2)\Phi = N_- - N_+, \tag{4}$$

$$(\partial_z^2 - \partial_t^2)\mathbf{A}_\perp = (\frac{\mathbf{N}_-}{\gamma_-} + \frac{\mathbf{N}_+}{\gamma_+}))\mathbf{A}_\perp, \tag{5}$$

$$\partial_t^2 A_z = -\partial_t(\partial_z \Phi) - (N_- V_- - N_+ V_+), \tag{6}$$

where, $V_s = P_{sz}/\gamma_s$ and $\gamma_s = \sqrt{1 + A_\perp^2 + P_{sz}^2}$.

We assume that all quantities are the functions of new independent variables (ξ, τ), where $\xi = z - v_g t$ and $\tau = t$, $v_g(\to v_g/c)$ is the dimensionless group velocity of the propagating radiation. In these new variables, the continuity and the longitudinal motion are described by the equations

$$\partial_\tau N_s - \partial_\xi[N_s(v_g - V_{sz})] = 0, \tag{7}$$

$$\partial_\tau(P_{sz}+A_{sz}) = \partial_\xi(v_g A_{sz} - \Phi_s - \gamma_s + v_g P_{sz}), \tag{8}$$

where $A_{-z} = -A_z, A_{+z} = A_z$ and $\Phi_- = -\Phi, \Phi_+ = \Phi$ respectively.

We consider an ultra short electromagnetic (laser) radiation propagating through the plasma. In this case τ may be considered much smaller than the characteristic time of changing the envelope of $A_\perp$. Thus a quasistatic approximation may be taken into account [9]. In this approximation, the continuity equation gives

$$N_- = \frac{v_g}{v_g - V_-}; N_+ = \frac{v_g}{v_g - V_+}, \tag{9}$$

and the equation for longitudinal motion yields an integral of motion

$$\gamma_s + \hat{\Phi}_s - v_g P_{sz} = 1, \tag{10}$$

where $\hat{\Phi}_s = \Phi_s - v_g A_{sz}$. Since $P_{sz} = \gamma_s V_{sz}$, from the above integral of motion, we find

$$V_- = \frac{1}{v_g\gamma_-}(\hat{\Phi}_- + \gamma_- - 1), \tag{11}$$

$$V_+ = \frac{1}{v_g\gamma_+}(\hat{\Phi}_+ + \gamma_+ - 1), \tag{12}$$

Furthermore, the equation for A_z [Eq.(6)] yields the relation

$$A_z = A_{0z} + \Phi, \tag{13}$$

where A_{0z} is the potential of the ambient field.

Using the definition of γ_s and eliminating P_{sz} from the integral of motion, γ_s may be expressed in terms of potentials, explicitly

$$\gamma_- = \frac{1}{1-v_g^2}\{(1-\hat{\Phi}_-) - v_g\sqrt{(1-\hat{\Phi}_-)^2 - (1-v_g^2)(1+A_\perp)}\}, \tag{14}$$

$$\gamma_+ = \frac{1}{1-v_g^2}\{(1-\hat{\Phi}_+) - v_g\sqrt{(1-\hat{\Phi}_+)^2 - (1-v_g^2)(1+A_\perp)}\}, \tag{15}$$

with

$$\hat{\Phi}_- = -(1-v_g^2)\Phi + v_g A_{0z}, \hat{\Phi}_+ = (1-v_g^2)\Phi - v_g A_{0z}, \tag{16}$$

respectively.

Now, we can write the coupled set of field equations which describes the 1D nonlinear interaction of intense electromagnetic field with the ambient potential and the $e^\pm$ plasma in the quasistatic approximation:

$$[(1-v_g^2)\frac{\partial^2}{\partial\xi^2} + 2v_g\frac{\partial^2}{\partial\xi\partial\tau} - \frac{\partial^2}{\partial\tau^2}]A_\perp = v_g^2[\frac{1}{\sqrt{(1-\hat{\Phi}_-)^2 - (1-v_g^2)(1+A_\perp^2)}} + \frac{1}{\sqrt{(1-\hat{\Phi}_+)^2 - (1-v_g^2)(1+A_\perp^2)}}]A_\perp, \tag{17}$$

$$\partial^2{}_\xi\Phi = \frac{v_g^2}{1-v_g^2}[\frac{1-\hat{\Phi}_-}{\sqrt{(1-\hat{\Phi}_-)^2-(1-v_g^2)(1+A_\perp^2)}} - \frac{1-\hat{\Phi}_+}{\sqrt{(1-\hat{\Phi}_+)^2-(1-v_g^2)(1+A_\perp^2)}}]. \quad (18)$$

We are considering a very short pulse of radiation. Therefore, the modulation of pump radiation in this ultra short time may be ignored. We suppose that

$$A_\perp = A_{\perp 0} + \Delta A_\perp, \quad (19)$$

where, $|\Delta A_\perp| \ll |A_{\perp 0}|$.

Thus Eq.(17) takes the form

$$[(1-v_g^2)\frac{\partial^2}{\partial\xi^2} + 2v_g\frac{\partial^2}{\partial\xi\partial\tau} - \frac{\partial^2}{\partial\tau^2}]\Delta A_\perp = \frac{v_g^2}{1-v_g^2}[\frac{1}{\sqrt{(1-\hat{\Phi}_-)^2-(1-v_g^2)(1+A_{\perp 0}^2)}} + \frac{1}{\sqrt{(1-\hat{\Phi}_+)^2-(1-v_g^2)(1+A_{\perp 0}^2)}}]\Delta A_\perp. \quad (20)$$

For larger Φ, it is seen that the right-hand side of equation (20) is small, and thus ΔA changes periodically a little in the ultra short time of interaction, and it is confirmed in a later calculation. Therefore Eq.(18) is decoupled and it may be written as

$$\partial^2{}_\xi\Phi = \frac{v_g^2}{1-v_g^2}[\frac{1-\hat{\Phi}_-}{\sqrt{(1-\hat{\Phi}_-)^2-(1-v_g^2)(1+A_{\perp 0}^2)}} - \frac{1-\hat{\Phi}_+}{\sqrt{(1-\hat{\Phi}_+)^2-(1-v_g^2)(1+A_{\perp 0}^2)}}], \quad (21)$$

where, $\hat{\Phi}_-$ and $\hat{\Phi}_+$ are defined by the equation (16).

It is seen that initially for $\Phi = 0$ and $A_{0z} = 0$, the right-hand side (rhs) of equation (21) is zero. But for $A_{0z} \neq 0$ (although $\Phi = 0$), it is not zero. Thus we see that A_{0z} is one of the determining factors for the generation of intense ambipolar field by electromagnetic radiation. Therefore, we numerically solve equation (21) with the boundary conditions:

$\Phi(0) = 0, \Phi'(0) = 0$, for the values of $A_{\perp 0} = 1.4, v_g = .9999, A_{0z} = .001$. The solution is shown in Fig.1

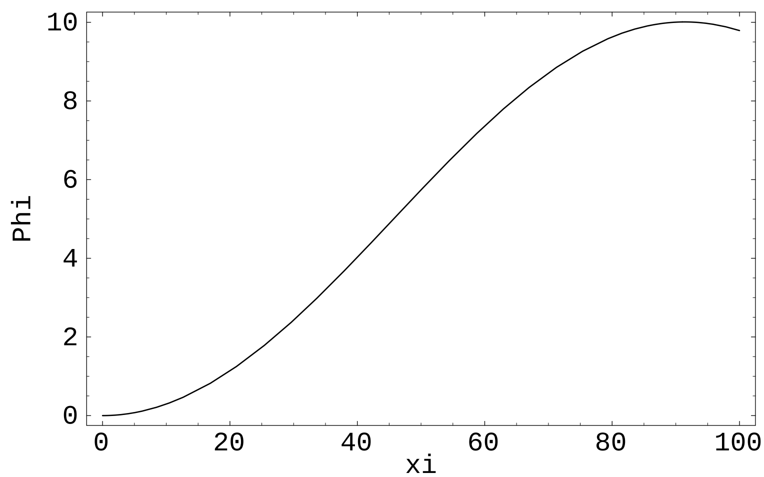

FIG. 1. Generation of intense ambipolar potential in the pulsar environment. Φ is the generated ambipolar potential, $A_{\perp 0} = 1.4$ is the pump radiation, $v_g = .9999$ is the group velocity and $A_{0z} = .001$ is the ambient potential.

As we are considering the unmodulated pump radiation during the time $\tau = \xi / v_g$, the derivative $\frac{\partial}{\partial \xi}$ in Eq.(20) may be put to zero. Therefore, we solve equation (20) along with equation (21) for the short time of interaction. The periodic evolution of pump radiation is shown in Fig.2.

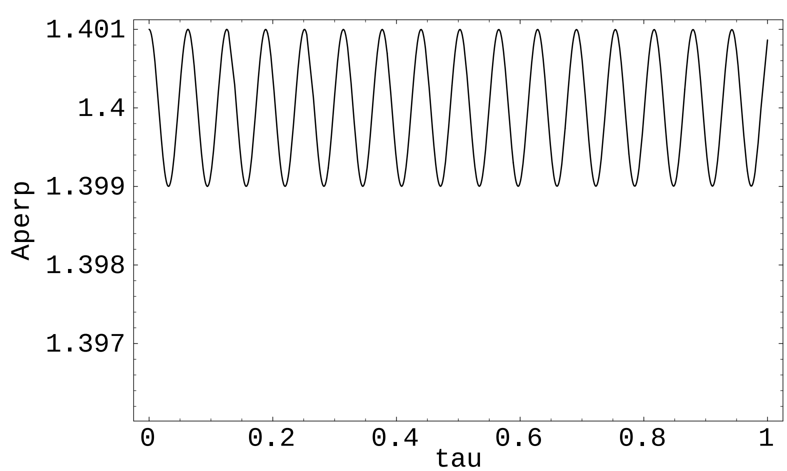

FIG. 2. Pump wave during the short time of interaction. $A_{\perp 0} = 1.4, v_g = .9999, A_{0z} = .001$.

The wake field generated during the interaction may also be calculated using the relation $\mathbf{E} = -\frac{\mathbf{d\Phi}}{\mathbf{d\xi}}$. It is calculated and it is shown in Fig.3. The field is found to be maximum at the mid value of the generated Φ. It means the maximum wake field is generated at half of the time taken to create the saturated ambipolar potential.

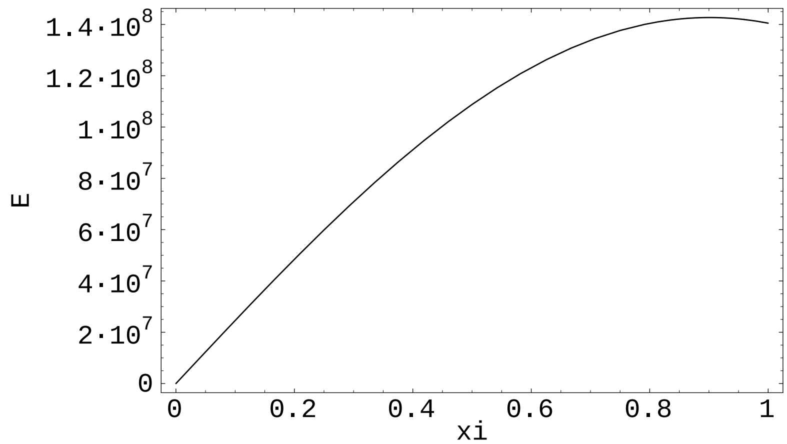

FIG. 3. Wake field generation during the ultra short interaction. $A_{\perp 0} = 1.4, v_g = .9999, A_{0z} = .001$.

Now, we are interested to find some analytical interpretation of the numerical results. We see that equation (21) admits an integral of motion, which is given by

$$(\frac{d\Phi}{d\xi})^2 + V(\Phi) = 0, \tag{22}$$

where,

$$V(\Phi) = 2v_g\gamma_g^4[\sqrt{(1+v_gA_{0z})^2 - \frac{\gamma_{0\perp}^2}{\gamma_g^2}} + \sqrt{(1-v_gA_{0z})^2 - \frac{\gamma_{0\perp}^2}{\gamma_g^2}} - \sqrt{(1-\frac{\Phi}{\gamma_g^2}+v_gA_{0z})^2 - \frac{\gamma_{0\perp}^2}{\gamma_g^2}} - \sqrt{(1+\frac{\Phi}{\gamma_g^2}-v_gA_{0z})^2 - \frac{\gamma_{0\perp}^2}{\gamma_g^2}}], \tag{23}$$

with $\gamma_g = \frac{1}{\sqrt{1-v_g^2}}$ and $\gamma_{0\perp} = \sqrt{1+A_{\perp 0}^2}$.

We plot $V(\Phi)$ as a function of Φ, which is shown in Fig. 4.

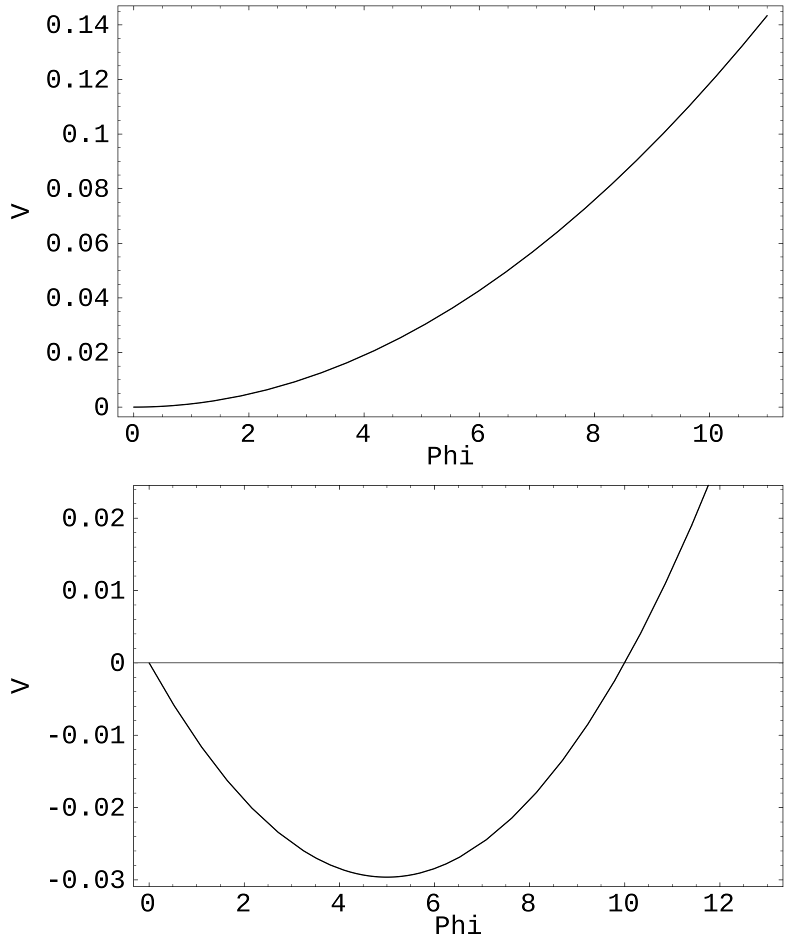

FIG. 4. Potential well for Φ generation, $A_{\perp 0} = 1.4, v_g = .9999$, a) $A_{0z} = 0$(above) b) $A_{0z} = .001$(below).

We see from Fig.4a that for $A_{0z} = 0, V(\Phi)$ is always positive. But for $A_{0z} \neq 0$ (Fig.4b), it is negative between $\Phi = 0$ and $\Phi = \Phi_{max}$, and then it becomes positive for higher values of Φ. It means, that in the absence of ambient potential, we have only the periodic solution but in the presence of the ambient potential we may have growing as well as localized periodic solutions for Φ. The solution is given by an elliptical integral in the form

$$\xi = \pm \int \frac{d\Phi}{\sqrt{-V(\Phi)}}, \tag{24}$$

where, $V(\Phi)$ is given by equation (23).

For $v_g \to 1, \quad \gamma_g \to \infty$, and thus $\frac{\gamma_{0\perp}^2}{\gamma_g^2} \to 0$. In that case, from the condition $V(\Phi) = 0$, we may find

$$\Phi_{max} \approx 2A_{0z} v_g \gamma_g^2. \tag{25}$$

Thus with an ambient potential A_{0z} in $e^{\pm}$ plasma, an electromagnetic radiation with the group velocity very close to the speed of light, may generate a huge electrostatic potential, which may accelerate secondary particles to high energies. With an intense ultra short laser radiation this phenomenon may be accomplished in the laboratory for the wake field acceleration of particles in plasmas with the species of similar masses.

Acknowledgments

This work was done within the framework of the Associateship Scheme of the Abdus Salam International Centre for Theoretical Physics, Trieste, Italy. Financial support from the Swedish International Development Cooperation Agency is acknowledged. The authors would also like to thank the Abdus Salam ICTP for hospitality where this work was done.

REFERENCES

[1] J. Arons and E. T. Scharlemann, ApJ, **231**, 854 (1979).

[2] P. A. Sturrock, ApJ, **164**, 179 (1971).

[3] M. A. Ruderman and P. G. Sutherland, ApJ, **196**, 51 (1975).

[4] R. G. Greaves, M. D. Tinkle and C. M. Surko, Phys. Plasmas, **1** 1439 (1994).

[5] C. M. Surko, M. Leventhal, and A. Passner, Phys. Rev. Lett. **62**, 901 (1989).

[6] R. E. Wagner, Q. Su, and R. Grobe, Phys. Rev. Lett., **84**, 3282 (2000).

[7] A. Sen and G. L. Johnston, Phys. Rev. Lett., **70**, 786 (1993).

[8] U. A. Mofiz, Phys. Rev. A **42**, 960 (1990); U. A. Mofiz and U. De Angelis, J. Plasma Phys. **33**, 107 (1985).

[9] P. Sprangle, E. Esarey, and A. Ting, Phys. Rev. A, **41**, 4463 (1990).