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**CONDUCTANCE FLUCTUATIONS AND DISTRIBUTION
AT METAL-INSULATOR TRANSITION INDUCED
BY ELECTRIC FIELD IN DISORDERED CHAIN**

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Abstract

A simple Kronig-Penney model for 1D mesoscopic systems with δ peak potentials is used to study numerically the influence of a constant electric field on the conductance fluctuations and distribution at the transition. We found that the conductance probability distribution has a system-size independent form with large fluctuations in good agreement with the previous works in 2D and 3D systems.

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1 Introduction

During the last two decades several works have been devoted to understand the transport properties in mesoscopic systems [1-8]. Experiments performed in such systems showed that the conductance g is not a self-averaged quantity [3] and then fluctuates as the function of the Fermi energy, chemical potential and the sample size. In the metallic regime experimental [2-4] and theoretical [5-8] studies indicated that the conductance fluctuations are of order of e^2/h and are universal (i.e. independent of the system size or the amount of disorder).

In the localized (insulating) regime, the conductance exhibits strong fluctuations which tend to diverge for large system sizes [9,10]. At critical point of the metal-insulator transition (MIT), large fluctuations were predicted analytically for $d = 2 + \varepsilon$ ($\varepsilon \ll 1$) [11,12] but not detected numerically by Markos and Kramer in 2D and 3D systems [13,14]. They found that the large conductance fluctuations are not the general feature of the metal-insulator transition. It was also found that the variance of $\log(g)$ is of order of its mean value and the standard deviation of g is also $\sim \langle g \rangle$ [15]

Since the conductance does not obey to the central limit theorem [1] not only its mean value but its whole probability distribution has to be studied. Numerical results in 2D and 3D disordered systems showed that the conductance is gaussian distributed in the metallic regime [16] while for strongly localized systems (insulating regime) a log-normal distribution was found [16]. The correct form of the probability distribution at the transition is not well known. In such regime, it was proved that the conductance distribution is independent of the microscopic details of the model (determined by the distribution of the disorder), of the system size and of the position of the critical point which separates the metallic and the localized regime in the space of external parameters (energy, disorder). This universality of the conductance distribution was studied and confirmed for 2D and 3D models [13-14]. The system-size invariance of $P(g)$ at the critical points of the MIT was confirmed for 3D and 4D orthogonal systems [17, 18]. It was also found that that the critical distribution depends on the dimension [15], the symmetry of the system [15,17] and on the boundary conditions [19].

The electric field was shown to delocalize the electronic states in 1D disordered systems (where all the states are localized) [20, 21]. In a previous work, we have shown that the Anderson transition may occur for strong fields [22].

In this work, we use the Kronig-Penney model to test the size-independent probability distribution of the conductance at the transition in disordered mesoscopic chain under the influence of an electric field.

2 Model description

We consider a Kronig-Penney model applied to a 1D system of equally spaced potential $V(x)$ with random strengths under a constant electric field F . The corresponding Schrodinger

equation can be read:

$$\left\{ -\frac{d^2}{dx^2} + \sum_n \beta_n \delta(x - n) - eFx \right\} \Psi(x) = E\Psi(x) \quad (1)$$

Here $\Psi(x)$ is the single particle wavefunction at x , β_n the potential strength of the n -th site and E the single particle energy in units of $\hbar^2/2m$ with m the electronic effective mass. The electronic charge e and the lattice parameter a are taken here for simplicity to be unity. The chain length is identical to the number of scatterers ($L = N$). The two ends of the system are assumed to be connected ohmically to ideal leads (where the electron moves freely) and maintained at a constant potential difference $V = FL$. The potential strength β_n is uniformly distributed between $-W/2$ and $W/2$ (W being the degree of disorder).

The exact solution of the equation (1) is Airy function-like. In order to reduce the computational time consuming we use the so-called ladder approximation which is valid only for weak fields [20, 21]. For strong fields, we use the multistep function approximation [23] which is very accurate and use plane waves instead of the Airy functions. The second order differential equation (1) can be mapped by means of the Poincaré map representation in the ladder approximation [20]

$$\Psi_{n+1} = \left[\cos(k_{n+1}) + \frac{k_n \sin(k_{n+1})}{k_{n+1} \sin(k_n)} \cos(k_n) + \beta_n \frac{\sin(k_{n+1})}{k_{n+1}} \right] \Psi_n - \frac{k_n \sin(k_{n+1})}{k_{n+1} \sin(k_n)} \Psi_{n-1} \quad (2)$$

where Ψ_n is the value of the wavefunction at site n and $k_n = \sqrt{E + Fn}$ is the electron wave number at the site n . The solution of equation (2) is carried out iteratively by taking the two initial wave functions at sites 1 and 2 : $\Psi_1 = \exp(-ik)$ and $\Psi_2 = \exp(-2ik)$. We consider here an electron having a wave number k incident at site $N + 3$ from the right (by taking the chain length $L = N$, i.e. $N + 1$ scatterers). The transmission coefficient (T) reads

$$T = \frac{k_0}{k_L} \frac{|1 - \exp(-2ik_L)|^2}{|\Psi_{N+2} - \Psi_{N+3} \exp(-ik_L)|^2} \quad (3)$$

where $k_0 = \sqrt{E}$ and $k_L = \sqrt{E + FN}$.

The dimensionless 4-probe conductance ($g = \frac{G}{e^2/h}$) can be obtained from the transmission coefficient T via the Landauer formula for 1D systems [24]:

$$g = \frac{2T}{1 - T} \quad (4)$$

where the factor two arises from the two possible states of the electron spin.

3 Results and discussion

In this section we discuss the numerical results of the conductance fluctuations and distribution at the transition regime. In a previous work [22], we have shown that the real metal-insulator transition does not occur at $E \approx V$ as believed from the results of Mato and Caro

[25] but this regime can be obtained for strong fields. To obtain the probability distribution of the conductance, we construct a statistical ensemble of 10^4 samples which differ only in the realization of the disorder. The electric field has been shown to delocalize the electronic states in 1D disordered systems where the wave function becomes power-law decaying [20, 21], while for sufficiently large field strengths, the eigenstates become extended [22]. In Figure 1, we show the variance of $\text{Ln}(g)$ as a function of the applied electric field (for large field strengths $F > 1$). The conductance fluctuations has two pronounced peaks for $F = 1.465$ and $F = 3$. These large fluctuations are a sign of localized states even for large field strengths. We expect that the metal-insulator transition may occur at these regions. To check the nature of the regime, we show in Figure 2 the probability distribution of $\text{Ln}(g)$ (Fig.2a) and g (Fig.2b) for different sizes of the system ($L = 500, 700, 800$ and 900) and for $F = 3$. The conductance distribution is neither normal nor lognormal. The L-independence of the conductance distribution observed for 2D and 3D systems [13-15, 18] is confirmed here. This behaviour is not observed in the metallic and insulating regime (see Fig.3) where the conductance distribution depends on the size of the system. The long tail of the distribution in Fig.1 is representative for large fluctuations in good agreement with the results of Shapiro [11] and Shapiro and Cohen [12] for such regime. The distribution of g in Fig.2b has a similar shape as found recently at the crossover region between the metallic and the insulating regime in disordered quasi one-dimensional wire [26]. For an electric field strength corresponding to the first peak in Fig.1, the conductance distribution has a size dependent form (not shown). We can conclude that the MIT regime occur at $F = 3$ for $E = 0.4$ and $W = 0.25$. We can also observe that this regime is characterized by very large conductances $\langle g \rangle \simeq 58$ (Fig.4 and Fig.2) This was not observed in the metallic regime where the mean conductance is small $\langle g \rangle \simeq 0.4$ [22]. We presented in Table I mean value of $\log g$ and its variance. We note clearly that $\text{Var}(\text{Ln}(g))$ is not of order of $\text{Ln}(g)$ which is typical for the localized states as found for 3D and 4D systems [15]. This shows that in our system the extended states dominates due to electric field.

We can also observe that $P(g)$ has a hole at small g in agreement with the analytical result in the ε expansion [11, 12] and with the numerical results for 2D [27] and 3D systems [18, 28].

4 Conclusion

We have used the Kronig-Penney model in a simple 1D disordered system in the presence of an electric field to examine the conductance fluctuations and the size-independence of its distribution at the metal-insulator transition. The results are in good agreement with the previous works in 2D and 3D systems for other models [13-14] for the metal-insulator transition. It is important to study the universality of the conductance distribution at the transition in a system of finite width potentials where the conductance fluctuations are less important in comparison to the present model [22] and to find the critical points of the transition in the (energy-disorder-

electric field) space and for different kinds of disorder. These problems will be the subject of a forthcoming paper.

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Table I Mean conductance $\langle Ln(g) \rangle$ and its variance for different system size L .

L	$\langle Ln(g) \rangle$	$Var(Ln(g))$
500	3.9557	0.20548
600	3.9314	0.20474
700	3.9596	0.21149
800	3.9632	0.21717

Figure Captions

Fig.1 Variance of $\text{Ln}(g)$ versus applied electric field for $L = 500$, $E = 0.4$ and $W = 0.25$.

Fig.2a Probability distribution of $\text{Ln}(g)$ for $F = 3$, $E = 0.4$ and $W = 0.25$ and different system sizes ($L = 500, 700, 800$ and 900) compared with a gaussian with same mean and variance (dashed curve).

Fig2b Distribution of g for the same parameters as in Fig1a.

Fig.3 Distribution of $-\text{Ln}(g)$ in the insulating regime for $F = 0$, $E = 5$ and $W = 2$ and different system sizes ($L = 500, 700$ and 800) from left to right compared with a gaussian with same mean and variance (dashed curves).

Fig.4 Mean conductance $\langle \text{Ln}(g) \rangle$ versus applied electric field for the same parameters as in Fig.1.