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POLARIZATION INSTABILITY SHIFT IN BIREFRINGENT SINGLE MODE OPTICAL FIBERS

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Abstract

An intensity-dependent change in ellipticity of an input light beam leads to a characteristic shift in polarization instability. The presence of dichroism gives rise to self-induced ellipticity effect in the polarization state of an intense input light oriented along the fast axis of a birefringent single mode optical fiber and results in a higher critical value of input power at which the fiber effective beat length becomes infinite.

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1 INTRODUCTION

When two or more optical waves copropagate inside a birefringent single-mode fiber they can differ not only in their wavelengths but also in their states of polarization. The optical pulses can further couple with each other through fiber nonlinearity and the polarization of each field can change during propagation as a result of optically induced nonlinear birefringence. The coupling of two waves with the same frequency but different polarizations gives rise to a number of interesting nonlinear effects in optical fibers. One of such effects is polarization instability. This instability manifests as large changes in the output state of polarization when the input power or the polarization state is changed slightly. At a critical value of the input power, nonlinear birefringence can cancel intrinsic birefringence completely and the effective beat length becomes infinite. It has been discovered $[1-3]$ that induced nonlinear birefringence is responsible for polarization instability.

As early as 1964, it was observed that an intense elliptically polarized beam propagating within an isotropic medium induces a nonlinear birefringence effect known as ellipse rotation which is a continuous precession of the orientation angle of the polarization ellipse, while leaving its shape and handedness unchanged assuming no dichroism [4]. Since then, self-induced polarization effects have been studied extensively, particularly in the context of optical Kerr effect or nonlinear refractive index and remains a topic of much interest given its practical and important consequences for laser propagation characteristics as well as its use as the basis for a wide variety of applications [5]. These nonlinear polarization effects in a birefringent single-mode optical fiber have led to practical applications through intensity discriminators, fiber-optic logic gates and kerr shutters [6] . Also, a new class of devices, among which a linear coherent amplifier mixer and an optically activated polarization switch, have been envisaged due to the intensity-dependent changes in the light polarization as it evolves along a lossless birefringent single mode optical fiber [7]. The problem of the interaction between a nonlinear and a dc-induced birefringence has been analytically solved by Sala for an isotropic medium [8]. The form of the refractive index change induced by the elliptically polarized pump beam corresponds to elliptical birefringence with the important consequence that an arbitrarily polarized probe beam in addition to reorientation experiences a change in the shape and handedness of its polarization ellipse. Winful presented exact solutions for the intensity-dependent polarization state of a light wave in a birefringent optical fiber taking into account both the intrinsic linear polarization evolution and the nonlinear ellipse rotation and showed that self-induced polarization changes can occur with equal excitation of the fiber principal axes [9]. While these effects on optical properties and propagation characteristics in birefringent fibers have long been known and examined in details, the problem of the resulting effects due to an intensity-dependent change in the ellipticity of the polarization ellipse has received very little attention. In their work, the authors assumed that the susceptibility tensor is real and therefore neglected dichroism which is related to the imaginary part of the suscetibility tensor and responsible for the intensity-dependent change in ellipticity.

In this paper, we are interested in the influence on the polarization nonlinear evolution when there is a change in the ellipticity of the polarization ellipse after including the effects of both intrinsic linear birefringence and optically induced nonlinear birefringence.

2 SHIFT IN POLARIZATION INSTABILITY

The monochromatic electric fields propagating along a birefringent single-mode optical fiber can be expressed as a superposition of the two orthogonal polarizations of the fundamental mode:

$$\mathbf{E} = \sum_{j=x,y} A_j\left(z,t\right)\mathbf{E}_j\left(x,y\right)\exp\left[i\left(k_j z-\omega t\right)\right] \quad + \quad c.c. \tag{1}$$

where $A_j\left(z,t\right)$ are the field amplitudes along the principal axes of the fiber, $\mathbf{E}_j\left(x,y\right)$ are the transverse mode distributions, and ω, t, and k_j have the usual interpretation of respectively frequency, time, and propagation constant. The longitudinal coordinate z is chosen to coincide with the fiber axis of symmetry along which the light wave propagates.

The nonlinear wave equation which describes the transverse electric fields evolution along a birefringent optical fiber is expressed as

$$\frac{d^2E_i}{dz^2}+\left(\frac{\omega}{c}\right)^2\epsilon_{ij}E_j = -4\pi\left(\frac{\omega}{c}\right)^2P_i^{NL}\left(\omega,\mathbf{r}\right) \tag{2}$$

where $P_i^{NL}\left(\omega,\mathbf{r}\right)$ is the spectral amplitude of the nonlinear induced polarization and ϵ_{ij} is the linear dielectric tensor which is diagonal in the principal axis representation.

Since the anisotropy of birefringent fibers is relatively weak $\left(\delta n \simeq 10^{-4}\right)$, the nonlinear polarization will not differ much from that of an isotropic medium, at least for effects third order in the electric field [9]. Thus, we can write the nonlinear induced polarization as

$$P_i^{NL}\left(\omega,\mathbf{r}\right) = 3\boldsymbol{\chi}_{ijkl}^{(3)}\left(\omega;\omega,\omega,-\omega\right)E_j\left(\omega,\mathbf{r}\right)E_k\left(\omega,\mathbf{r}\right)E_l^*\left(\omega,\mathbf{r}\right) \tag{3}$$

where the indices run over (x,y).

The evolution equations describing the state of the light polarization can thus be expressed in compact form as [10, 11]

$$\frac{d\mathbf{S}}{dz} = \left[\Omega_L\left(z\right)+\Omega_{NL}\left(z,\mathbf{S}\right)\right]\times\mathbf{S} \tag{4}$$

Here, $\mathbf{S} = (S_1, S_2, S_3)$ is the Stokes vector which is associated with the polarization state of the fields and Ω with projections Ω_1, Ω_2, and Ω_3 is a vector in Stokes space related to the material characteristics and depend on the stokes parameters if the medium is nonlinear. In one of the standard conventions [12], the Stokes parameters are defined as

$$S_i = A_j^*\left(\sigma_i\right)_{jk}A_k \qquad\qquad i = 1,2,3 \tag{5}$$

where σ_i are the Pauli spin matrices.

If we assume small anisotropy along the direction of propagation and that the cubic nonlinearity is isotropic, the nonlinear evolution equations will take the following form

$$\frac{dS_1}{dz} = -\left(\frac{12\pi\omega}{nc}\boldsymbol{\chi}^{(3)}_{xxyy}\right) S_2 S_3 \tag{6}$$

$$\frac{dS_2}{dz} = \frac{\omega}{2nc}\left(\epsilon_{xx} - \epsilon_{zz} + 24\pi\boldsymbol{\chi}^{(3)}_{xxyy} S_1\right) S_3 \tag{7}$$

$$\frac{dS_3}{dz} = -\left\{\frac{\omega}{2nc}\left(\epsilon_{xx} - \epsilon_{zz}\right)\right\} S_2 \tag{8}$$

The exact analytical solutions to this system of equations have been obtained in terms of elliptic functions [8]:

$$S_3 = \frac{2\mathrm{pk}f}{q}\mathbf{cn}\left(R_0 f z \ + \mathcal{C} \ ; \mathrm{k}\right) \tag{9}$$

$$S_2 = \frac{2\mathrm{pk}f^2}{q}\left[\mathbf{sn}\left(R_0 f z \ + \mathcal{C} \ ; \mathrm{k}\right)\right]\mathbf{dn}\left(R_0 f z \ + \mathcal{C} \ ; \mathrm{k}\right) \tag{10}$$

and

$$S_1 = \frac{f^2}{q}\left\{1 - 2m\left[\mathbf{sn}^2\left(R_0 f z \ + \mathcal{C} \ ; \mathrm{k}\right)\right]\right\} - 1 \tag{11}$$

where **cn**, **sn**, and **dn** are Jacobian elliptic functions and $R_0 = \omega\left(\epsilon_{xx} - \epsilon_{zz}\right)/2nc$ with $f = \left[\left(1 + qS_{10}\right)^2 - q^2 S_{20}^2\right]^{\frac{1}{4}}$. Also, $p = sgn\left(S_{30}\right)$ and $q = 24\pi\boldsymbol{\chi}^{(3)}_{xxyy}/\left(\epsilon_{xx} - \epsilon_{zz}\right)$. The Jacobian modulus k is given by

$$\mathrm{k} = \left[\frac{1}{2} + \frac{1}{4f^2}\left(q^2 - 1 - f^4\right)\right]^{\frac{1}{2}} \tag{12}$$

and

$$-\mathrm{Re}\left[K\left(m\right)\right] \leq \mathcal{C} \leq +\mathrm{Re}\left[K\left(m\right)\right] \tag{13}$$

with $K\left(m\right)$ denoting the Jacobian quarter-period. Here, the Jacobian parameter $m = \mathrm{k}^2$.

Clearly, the output polariztion state of the optical beam is dependent on the Jacobian modulus k which in turn depends on the input polarization of the light as well as the nonlinear susceptibility tensor and the anisotropic tensor. Since the period of the elliptic function determines the effective beat length of the fiber [2], the beat length is dependent on input intensity and input polarization. The variation of the effective beat length as a function of input power for beams polarized along the fast and slow axes of the fiber has been obtained [2]. We obtain in Fig.[1], as was observed in [2] that, when the input beam is polarized along the fast axis of the fiber, the effective beat length becomes infinite at a critical input power because of complete cancellation between the intrinsic and nonlinear birefringences. However, we further observe as also shown in Fig.[1] that for a different initial ellipticity, a characteristic shift to higher critical input power occurs even though the orientation and handedness remain unchanged. This shift in the polarization instability results from an intensity-dependent change in the ellipticity of the polarization ellipse.

3 DISCUSSION AND CONCLUSIONS

We have reported a characteristic shift in instability of the polarization state of an intense light beam oriented along the fast axis of a birefringent single mode optical fiber. This is due mainly to an intensity-dependent change in ellipticity resulting from the presence of dichroism in the fiber. As the intensity of the input beam is increased, the size of the polarization ellipse also increases thereby altering the shape of the ellipse and hence the ellipticity. Thus, we note that an intensity-dependent change in the shape of the polarization ellipse will give rise to a different critical value of the input power at which the effective beat length becomes infinite. Infact, the input power at which the instability shift occurs can double that when the ellipticity is intensity independent. In addition, the critical input power at which polarization instability shift is observed, the induced birefringence can be twice the existing intrinsic birefringence. An optical fiber is dichroic and thus the nonlinear susceptibility tensor $\boldsymbol{\chi}^{(3)}_{ijkl}$ is a complex quantity and not purely real. Therefore, the polarization ellipse self-rotation effect of an incident elliptically polarized light will be accompanied by an intensity-dependent change in ellipticity. The origin of the shift in polarization instability is an intensity dependent ellipticity. We conclude that, when an incident beam undergoes an intensity-dependent change in ellipticity, a shift in polarization instability will occur and yield a different critical input power value at which fiber effective beat length becomes infinite.

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Figure 1: Effective beat length as a function of input power for intense beam polarized along the fast axis of a birefringent optical fiber. The solid curve represents variation when the ellipticity does not depend on input intensity and the dotted curve represents when there is an intensity-dependent change in ellipticity of input polarization.