# Production of $3 \pi^{0}$ and $\eta 2 \pi^{0}$ from $\pi^{-} p$ Collision in GAMS* Experiment 

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#### Abstract

The data on the $\pi^{-} p \rightarrow M^{0} n$, with $M^{0}=\pi^{0} \pi^{0} \pi^{0}$ or $\pi^{0} \pi^{0} \eta$ obtained in the GAMS experiment may be useful to study the $\sigma(400 \sim 700)$ and $a_{1}^{\chi}(\sim 1000)$, which can be taken as chiral partners of $\pi$ and $\rho$, respectively. A preliminary analysis for $3 \pi^{0}$ invariant mass spectra gives a support for the assumed $a_{1}^{\chi}$ with a mass of $\sim 930 \mathrm{MeV}$ and $\Gamma \sim 170 \mathrm{MeV}$.


## §1. Introduction

GAMS (NA12) experiment was carried out in 1984 at CERN SPS for the charge exchange reaction

$$
\begin{equation*}
\pi^{-}(100 \mathrm{GeV} / \mathrm{c})+p\left(\text { Liquid } H_{2} \text { target }\right) \quad \rightarrow \quad \gamma^{\prime} s+n \tag{1}
\end{equation*}
$$

for different final states decaying to $\gamma$-rays. Many papers have been published on $4 \gamma^{\prime} s$, (2), (3) $:\left(\eta \pi^{0}\right)$ system, rare decay of $\eta, \rho(1405) \rightarrow \eta \pi^{0}, G(1590) \rightarrow \eta \eta, \eta \eta^{\prime}$. $5 \gamma^{\prime} s:\left(\omega \pi^{0}\right)$ system, $\rho(2150) \rightarrow \omega \pi^{0}$, etc.
$6 \gamma^{\prime} s(5),(6): \eta \pi^{0} \pi^{0}$ for searching exotic states
but not much on $6 \gamma^{\prime} s$ (especially $\pi^{0} \pi^{0} \pi^{0}$ ).
Recently, a new meson classification scheme incorporating an approximate chiral symmetry has been worked out by the two of the present authors and the collaborator (1).

This idea leads to an explanation of the $\sigma$ meson as a chiral partner of $\pi$, and predicts many new mesons. Especially, the following two states

$$
\begin{array}{ll}
\sigma(400 \sim 700), & I^{G}\left(J^{P C}\right)=0^{+}\left(0^{++}\right), \\
a_{1}^{\chi}(\sim 1000), & { }^{1} S_{0}, \text { chiral partner of } \pi(139), \\
I^{G}\left(J^{P C}\right)=1^{-}\left(1^{++}\right),
\end{array}{ }^{3} S_{1}, \text { chiral partner of } \rho(770),
$$

have attracted much interest.The reactions with the final $6 \gamma$ states of $\pi^{0} \pi^{0} \pi^{0}$ or $\eta \pi^{0} \pi^{0}$ may be useful to study these states. The reaction of Eq.(1) can be taken as a two step one (see Fig. 1) consisting of (i) $\pi^{-}$changing to $M^{0}$ by the exchange of a

[^0]particle $E^{-}$and (ii) decay of $M^{0}$ to $\pi^{0} \pi^{0} \pi^{0}$ or $\eta \pi^{0} \pi^{0}$. The exchange vertex should conserve G-parity, while the decay vertex may break it. Considering that only the process of $E$ with mass $\lesssim 1 \mathrm{GeV}$ is effective, there are two cases of $E^{-}\left(\rho^{-}, a_{0}^{-}\right)$. Then, as shown in Table I, $a_{1}^{\chi}(\sim 1000)$ should be allowed to decay to $\pi^{0} \pi^{0} \pi^{0}$ but not to $\eta \pi^{0} \pi^{0}$, in analogy to the decay of $a_{1}^{N}$ (normal $a_{1}$ ), which has the same quantum number as $a_{1}^{\chi} \cdot \sigma(400 \sim 700)$ decay to $2 \pi^{0} 8^{8}$ could be studied in both final states of $\pi^{0} \pi^{0} \pi^{0}$ and $\eta \pi^{0} \pi^{0}$. With such an expectation, we are digging up our old data for two final states of $\pi^{0} \pi^{0} \pi^{0}$ and $\eta \pi^{0} \pi^{0}$, since the data quality should be excellent.

## §2. Experiment and Data reduction for $\mathbf{6} \gamma$ states

A $\pi^{-}$beam at $100 \mathrm{GeV} / \mathrm{c}\left(s^{1 / 2} \sim 14 \mathrm{GeV}\right)$ was injected to a liquid $H_{2}$ target ( 6 cm in diameter and 60 cm in length). The detector system is schematically sketched in Fig. 2. Positions and energies of all $\gamma$-rays emitted in the forward direction were measured with GAMS $4000^{2}$ ), an array of 4096 Pb -glass modules placed at 15 m from the target. Each lead glass module had a size of $3.8 \times 3.8 \times 45 \mathrm{~cm}^{3}$. Two modules on the beam line are missing to allow the non-interacting $\pi^{-}$beam pass


Fig. 1. Diagram of charge exchange reaction of Eq. (1) as two step processes.
Table I. List of allowed $E^{-}$and $M^{0}$ (see Fig. 1)

|  | $E^{-}$ | $M^{0}$ with $J^{P C}=0^{-+}$ | $1^{++}$ | $2^{-+}$ |
| :--- | :--- | :--- | :--- | :--- |
| $M^{0} \rightarrow \pi^{0} \pi^{0} \pi^{0}$ | $E^{-}=\rho^{-}(770), I^{G}=1^{+}$ |  | $a_{1}^{N}(1260)$ | $\pi_{2}(1670)$ |
| $\left(I^{G}=1^{-}\right)$ | G conserved |  | $a_{1}^{\chi}(\sim 1000)$ |  |
|  | $E^{-}=a_{0}^{-}(980), I^{G}=1^{-}$ | $\eta(549)$ |  |  |
|  | G broken | $\eta^{\prime}(958)$ |  |  |
| $M^{0} \rightarrow \pi^{0} \pi^{0} \eta$ | $E^{-}=a_{0}^{-}(980)$ | $\eta^{\prime}(958)$ | $f_{1}(1285)$ |  |
| $\left(I^{G}=0^{+}\right)$ | G conserved | $\eta(1295)$ | $f_{1}(1420)$ |  |
|  |  | $\eta(1440)$ |  |  |

through. The longitudinal vertex was determined within 4 cm in $\sigma$ by measuring the Cherenkov light of the incident $\pi^{-}$in the liquid $H_{2}$. The transverse vertex was obtained from the fit of $\pi^{0} \rightarrow 2 \gamma$ for all the existing $\pi^{0}$ 's. The excited baryons in the final states could be removed by detecting the deposited energy in the guard system (GS), which surrounded the target and consisted of plastic scintillators and Pb-glass. Data analysis was carried out according to the standard scheme (see Ref.6); Starting from raw data MT, DST and super-DST for physics analysis (called DSTA) were produced. Experimental and MC events were treated in parallel by using essentially the same analysis program. Fitting of events was carried out by minimizing

$$
\begin{align*}
& \chi^{2}=\chi_{0}^{2}-2 \lambda_{n}\left(M_{n}^{2}-m_{n}^{2}\right)-2 \sum \lambda_{i j}\left(M_{i j}^{2}-m_{i j}^{2}\right) \text { with } \\
& \chi_{0}^{2}=\sum_{I=1,6}\left\{\left[\left(x_{i}-\underline{x_{i}}\right) / \sigma\left(x_{i}\right)\right]^{2}+\left[\left(y_{i}-\underline{y_{i}}\right) / \sigma\left(y_{i}\right)\right]^{2}+\left[\left(E_{i}-\underline{E_{i}}\right) / \sigma\left(E_{i}\right)\right]^{2}\right\}, \tag{2}
\end{align*}
$$

where $x_{i}, y_{i}$ and $E_{i}$ are the coordinates and the energy, respectively, of the i-th photon measured with the GAMS detector, and $\underline{x_{i}}, \underline{y_{i}}$ and $\underline{E_{i}}$ are the corresponding

Table II. Statistics of the present data sample.

| (1) \# of Events in DST with $\geq 6 \gamma^{\prime}$ s | $1,088 \mathrm{~K}$ | events |
| :--- | ---: | :--- |
| (2) Vertex inside LH2 target | 664 K |  |
| (3) Reject $\gamma^{\prime}$ s close to central hole | 205 K |  |
| (4) $6 \gamma$ events | 117 K |  |
| (5) Chi-square fit to each final states (4C-fit) | 95 K events in total |  |
| $\pi^{0} \pi^{0} \pi^{0}$ | 40,540 events |  |
| $\pi^{0} \pi^{0} \eta$ | 27,800 |  |
| $\pi^{0} \eta \eta(10,060), \pi^{0} \eta \eta^{\prime}(6,200), \pi^{0} \pi^{0} \eta^{\prime}(4,520), \eta \eta \eta(2,120)$, |  |  |
| $\eta \eta \eta^{\prime}(1,940), \pi^{0} \eta^{\prime} \eta^{\prime}(1,360), \eta \eta^{\prime} \eta^{\prime}(580), \eta^{\prime} \eta^{\prime} \eta^{\prime}(50)$ |  |  |

fitting parameters. $\sigma\left(x_{i}\right)$ and $\sigma\left(y_{i}\right)$ were taken to be $1 \mathrm{~cm} /\left(E_{i} \text { in } G e V\right)^{1 / 2}$. $\lambda_{n}$ and $\lambda_{i j}$ 's are also fitting parameters. $M_{n}\left(M_{i j}\right)$ is the neutron ( $\pi^{0}$ or $\eta$ or $\eta^{\prime}$ meson) mass calculated from the fitted variables and $m_{n}\left(m_{i j}\right)$ is the corresponding constants given by PDG. The degree of freedom, four, is equal to the number of the mass constraints. We required for the accepted events (i) $\chi^{2} \leq 12$ and (ii) longitudinal vertex $=-25 \sim+25 \mathrm{~cm}$ from the target center. The statistics of the accepted events 6) is given in Table II. If we require the total energy deposit (GSSUM) in GS $\leq 150$ MeV (noise being less than 50 MeV in each module) for neutron as recoil baryon, the statistics is reduced by half. This cut is switched off if we allow contamination of excited nucleons.

## §3. Preliminary result on $a_{1}^{\chi}$ and $\sigma$ for the $3 \pi^{0}$ final state

The $3 \pi^{0}$ mass spectrum (Fig.3a) has a broad $\pi_{2}(1670)$ and a sharp $\eta(548)$ peaks. A small but clear $\mathrm{X}(2050)$ peak (bump) has not yet been studied deeply. Another peak is seen at around 950 MeV , which cannot be explained by the sharp $\eta^{\prime}(958)$.

The fitting of the $3 \pi^{0}$ mass spectrum after acceptance correction was carried out in the variant mass and width (VMW) method ${ }^{(1)}$. The amplitude $M$ is given by

$$
\begin{equation*}
M(s)=\sum_{j} r_{j} \exp \left(i \theta_{j}\right) \Delta_{j} \quad \text { with } \quad \Delta_{j}=m_{j} \Gamma_{j} /\left(s-m_{j}^{2}+i m_{j} \Gamma_{j}\right) \tag{3}
\end{equation*}
$$

where $r_{j}$ and $\theta_{j}$ are the amplitude and phase of the $j$-th resonance, respectively. We took the three mesons, $M^{0}=a_{1}^{\chi}, a_{1}^{N}$ and $\pi_{2}(1670)$, into the amplitude which lead to the $3 \pi^{0}$ states as: (i) $a_{1}^{\chi} \rightarrow \sigma \pi^{0}$, (ii) $a_{1}^{N} \rightarrow \sigma \pi^{0}, f_{0}(980) \pi^{0}$, and (iii) $\pi_{2} \rightarrow$ $\sigma \pi^{0}, f_{0}(980) \pi^{0}, f_{0}^{*}(1270,1285) \pi^{0}$, with $\sigma, f_{0}, f_{0}^{*}$ all decaying to $2 \pi^{0}$. $\eta^{\prime}(958)$ was not included because of the absence of sharp peak in the spectrum and the too small BR to $3 \pi^{0}(0.15 \%)$. Dividing the $-t=0 \sim 0.7$ region into seven at a step of 0.1 , the mass and width of $\pi_{2}$ in each $(-t)$ region were first determined from the fit of $\pi_{2}$ peak in $1.5 \sim 1.8 G e V$. Fixing the $\pi_{2}$ parameters, we then fitted all the seven $(-t)$ regions using the common parameters (mass and width) for $a_{1}^{N}$ and $a_{1}^{\chi}$. As seen in Fig. 4, the fit with $a_{1}^{N}, a_{1}^{\chi}$ and $\pi_{2}$ gives much better $\chi_{2}^{2} / \mathrm{DF}(456 / 248=1.84)$ than that with only $a_{1}^{N}$ and $\pi_{2}(852 / 264=3.23)$. The fit gives ${ }^{* *}$ a mass of $931 \pm 3 \mathrm{MeV}$ and $\Gamma$

[^1]of $166 \pm 6 \mathrm{MeV}$ for $a_{1}^{\chi}$, while a mass of 1100 MeV and $\Gamma$ of 592 MeV for $a_{1}^{N}$. The evidence of $\sigma$ was searched for in the $2 \pi^{0}$ invariant mass spectra for different $3 \pi^{0}$ mass regions. The $2 \pi^{0}$ mass spectrum has a large $f_{2}(1270)$ peak, a small $K_{s}(498)$ peak which may come from $\pi^{-} p \rightarrow \Lambda K \pi^{0}$, etc., and the threshold structure due to the phase space of $\eta \rightarrow 3 \pi^{0}$ (see Figs.3b and 3c). The significance of $\sigma$ is not very clear and is still under study.

## §4. $\pi^{0} \pi^{0} \eta$ final state

Search for $\sigma$ in the $\pi^{0} \pi^{0}$ sub-system is underway. We will mention general features seen in this channel (see ${ }^{\sqrt{~}}$ ) for details); 3-clear peaks are seen on a broad peak in the $\pi^{0} \pi^{0} \eta$ mass spectrum; $\eta^{\prime}(958), f_{1}(1285)($ more dominant than $\eta(1295)$ since the moment $T_{L M}$ with $L \geq 2$ is large $) \rightarrow a_{0}(980) \pi^{0}$, and $\mathrm{X}(1440)\left(\eta(1440)\right.$ or $\left.f_{1}(1420)\right) \rightarrow$ $a_{0}(980) \pi^{0}$. The production cross section ratio after acceptance correction is

$$
\begin{aligned}
& \sigma\left(\pi^{-} p \rightarrow \eta^{\prime}(958) n\right) \mathbf{B}\left(\eta^{\prime} \rightarrow \pi^{0} \pi^{0} \eta\right) / \sigma\left(\pi^{-} p \rightarrow f_{1}(1285) n\right) \mathbf{B}\left(f_{1} \rightarrow \pi^{0} \pi^{0} \eta\right) \\
& / \sigma\left(\pi^{-} p \rightarrow X(1440) n\right) \mathbf{B}\left(X \rightarrow \pi^{0} \pi^{0} \eta\right)=100 /(29.7 \pm 1.7) /(13.5 \pm 1.4)
\end{aligned}
$$

The broad peak around 1600 MeV should contain significant contributions of meson peaks, since it cannot be explained by the acceptance-corrected phase space. The $\pi^{0} \pi^{0}$ invariant mass spectrum consists of (i) a dominant peak of $f_{2}(1270)$, (ii) a small $K_{s}(498)$ peak, (iii) a broad peak around $700 \mathrm{MeV} / \mathrm{c}^{2}$, which may contain $\sigma$ meson, and (iv) a threshold structure due to the decay of $\eta^{\prime}(958)$. The $\pi^{0} \eta$ invariant mass spectrum contains two dominant peaks of $a_{0}(980)$ and $a_{2}(1320)$.

## §5. Summary

(1) We are preparing $\pi^{-} p \rightarrow \pi^{0} \pi^{0} \pi^{0} n, \pi^{0} \pi^{0} \eta n$ data taken in the GAMS experiment in order to serve them for the study of

$$
\begin{aligned}
& \sigma(400 \sim 700), I^{G}\left(J^{P C}\right)=0^{+}\left(0^{++}\right),{ }^{1} S_{0} \text {, chiral partner of } \pi(139), \\
& a_{1}^{\chi}(\sim 1000), I^{G}\left(J^{P C}\right)=1^{-}\left(1^{++}\right),{ }^{3} S_{1} \text {, chiral partner of } \rho(770),
\end{aligned}
$$

(2) A preliminary analysis of $3 \pi^{0}$ invariant mass spectra gives a support for the $a_{1}^{\chi}$ with a mass of $\sim 930 \mathrm{MeV}$ and $\Gamma \sim 170 \mathrm{MeV}$.
(3) Analysis of $2 \pi^{0}$ invariant mass spectra in $\pi^{-} p \rightarrow \pi^{0} \pi^{0} \pi^{0} n$ is more complicated since the background increases due to 3 different combinations of $\pi^{0} \pi^{0}$. Preliminary analysis is not inconsistent with the existence of $\sigma(400 \sim 700)$.
(4) The $\pi^{0} \pi^{0} \eta$ channel may be convenient in searching for $\sigma \rightarrow 2 \pi^{0}$, since the ( $\pi^{0} \pi^{0}$ ) combination is unique.

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Fig. 2. A schematic sketch of the GAMS detector.


Fig. 3. (a) $3 \pi^{0}$ and (b) $2 \pi^{0}$ invariant mass spectra in the $\pi^{-} p \rightarrow \pi^{0} \pi^{0} \pi^{0} n$ reaction for $-t \geq$ $0.06 \mathrm{GeV}^{2}$. (c) A plot of $\pi^{0} \pi^{0} \pi^{0}$ in the $\left(2 \pi^{0}, 3 \pi^{0}\right)$ invariant mass plane for -t $\leq 0-1 \mathrm{GeV}^{2}$. All figures are for $\chi^{2} \leq 12$ and $\operatorname{GSSUM} \leq 150 \mathrm{MeV}$.


Fig. 4. Comparison between the fits of $3 \pi^{0}$ invariant mass spectra after acceptance correction with $a_{1}$ and $\pi_{2}$ (on the left), and with $a_{1}, a_{1}^{\chi}$ and $\pi_{2}$ (on the right).


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[^1]:    * In Fig. 4 only the four (out of seven) figures are shown.
    ** We also made a preliminary analysis including $\eta^{\prime}(958)$ and found that its contribution to the $a_{1}^{\chi}$ peak is less than $10 \%$ and the values of mass and width are changed scarcely.

