

CERN-TH/2000-379
FISIST/20-2000/CFIF
hep-ph/0012289

Split Fermions in Extra Dimensions and CP Violation

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Abstract

We discuss CP violation in the quark sector within a novel approach to the Yukawa puzzle proposed by Arkani-Hamed and Schmaltz, where Yukawa hierarchies result from localising the Standard Model quark field wave-functions, at different positions (in the extra dimensions) in a “fat-brane.” We show that at least two extra dimensions are necessary in order to obtain sufficient CP violation, while reproducing the correct quark mass spectrum and mixing angles.

arXiv:hep-ph/0012289v2 15 Jan 2001

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1 Introduction

Understanding the pattern of fermion masses and mixing angles is one of the fundamental, still unsolved, puzzles in Particle Physics. In the Standard Model (SM), Yukawa couplings are arbitrary free parameters and therefore, in order to shed any light into the fermion mass puzzle, physics beyond the SM is required.

The traditional approach to the fermion mass issue [1] consists of adding spontaneously broken flavour symmetries (gauged or global) to the SM. The symmetry and the pattern of symmetry breaking then leads to predictions for the fermion masses and mixing angles (or at least to correlations among them).

Recently, Arkani-Hamed and Schmaltz (AS) [2] suggested a novel approach to the flavour puzzle, in the framework of large, extra dimensions [3]. They suggest that we live in a “fat” four-dimensional subspace (fat-brane), which is infinite in the usual four space-time dimensions and possesses a finite volume in the extra, orthogonal dimensions. In this scenario, the Higgs boson and the gauge fields are free to propagate in the entire fat-brane, while fermions have higher dimensional wave-functions which are localised in specific points in the extra dimensions [4]. Therefore, the effective four-dimensional Yukawa coupling between two fermion species turns out to be (very efficiently) suppressed if the fermions are localised in different points, due to the small overlap of their respective wave-functions. Within this scenario, it is possible to start from a set higher-dimensional Yukawa couplings of order one, and obtain strong and specific fermion mass hierarchies simply by appropriately choosing the position of the fermionic fields. It has been shown that this unorthodox proposal not only is capable of reproducing the correct pattern of quark masses and mixing angles [5], but that it also has potential (and rather unique) experimental signatures [6].

In this paper, we address the question of CP violation in the AS scenario. We argue that, if the fermions are localised in different points in a one dimensional subspace (line), it is not possible to obtain sufficient CP violation in order to accommodate the current experimental data. We perform our analysis in the so-called nearest-neighbour-interaction (NNI) basis [7], and illustrate the source of difficulties which are encountered in the context of only one extra dimension.* Finally, we consider models with two extra dimensions and give an example where all quark masses and mixing angles are correctly reproduced, along with the required strength of CP violation.

This paper is organised as follows: in Sec. 2 we briefly review the AS scenario, paying special attention to the choice of weak basis and its interpretation. In Sec. 3 we show what is required in order to obtain the correct strength of CP violation, while reproducing the observed pattern of quark masses and mixing angles. In Sec. 4 we summarise our results and present our conclusions.

*Henceforth, we refer to the number of extra dimensions as the dimensionality of the space where fermions are localised. This does not necessarily agree with the total number of large, new dimensions, which is required to be larger than 1 if one is to properly address the hierarchy problem [3]. For example, our fat-brane may live in two compact extra dimensions, while the fermions are localised to one-dimensional “walls” within the brane.

2 The Fat-Brane Scenario

In this section, we briefly present the fat-brane paradigm, and discuss how it can be used to solve the fermion mass puzzle. It is instructive to start from the 4 + 1-dimensional action [2]* for two fermion fields:

$$S \supset \int dx^4 dy \bar{Q}[i\Gamma_M \partial^M + \Phi^Q(y)]Q + \bar{U}[i\Gamma_M \partial^M + \Phi^U(y)]U + \kappa H Q^c U, \quad (2.1)$$

where Q and U are the “(anti)quark” fields, H is the (higher-dimensional) Higgs field and κ the higher-dimensional Yukawa coupling. Γ_M are the 4 + 1-dimensional version of the Dirac matrices ($M = 0\dots 4$). $\Phi^{Q,U}(y)$ are potentials for the quark fields, and are such that the quarks are confined to specific “points” in the extra dimension. We refer to [2] for details. After expanding Q , U and H in its “normal modes” (properly normalised), the Yukawa part of the action for the zero-modes is

$$S_{\text{Yuk}} = \int dx^4 \kappa h(x) q(x) u(x) \int dy \phi_q(y) \phi_u(y), \quad (2.2)$$

where $\phi_{q,u}(y)$ are the fifth dimensional wave-functions for the Q and U , and $h(x)$, $q(x)$, $u(x)$ are the four-dimensional Higgs, q and u fields, respectively. It is assumed that the Higgs zero-mode is independent of y , the extra dimension. Assuming $\phi_{q,u}(y)$ to be Gaussians centred at l_q and l_u , respectively, with width $1/(\sqrt{2}\mu)$ [2],

$$\int dy \phi_q(y) \phi_u(y) = \frac{\sqrt{2}\mu}{\sqrt{\pi}} \int dy e^{-\mu^2(y-l_q)^2} e^{-\mu^2(y-l_u)^2} = e^{-\mu^2(l_q-l_u)^2/2}, \quad (2.3)$$

where the second equality is valid when the thickness of the brane (range of y) is significantly larger than $1/\mu$ and $l_{q,u}$. The effective four-dimensional Yukawa coupling is then of the form $\lambda = \kappa e^{-\mu^2(l_q-l_u)^2/2}$. The exponential factor obtained from Eq. (2.3) is of key importance, since it allows for (exponentially) suppressed Yukawa couplings even when the original higher-dimensional couplings are of order one, if the different quark fields are confined at different points in the extra dimension. Such mechanism is capable of not only generating very small Yukawa couplings, but may also be used to suppress dangerous higher-dimensional operators mediating proton decay, $K^0 \leftrightarrow \bar{K}^0$ mixing, etc [2], which normally plague theories with a small fundamental scale.

A pertinent question is whether there is any geometrical configuration of quark fields which fits all quark masses and mixing angles. This issue was addressed in [5], and the answer is positive. Therefore, without assuming any flavour symmetry (*i.e.* all κ of order one), it is possible to accommodate the observed pattern of fermion masses and mixing angles simply by appropriately placing each quark field in a different position.

We address the issue of the quark Yukawa matrix in more detail in order to discuss a few relevant points. First, we choose a basis where the weak charged current is diagonal, such that

$$\mathcal{L}_{\text{Yuk}} = \lambda_u^{ij} Q_i U_j H + \lambda_d^{ij} Q_i D_j H^*, \quad (2.4)$$

*See [8] for a more detailed treatment of the “fat-brane” action.

where Q_i are the quark doublets, and U_i, D_i the up-type and down-type antiquark singlets, respectively ($i, j = 1, 2, 3$ for the three families). According to Eq. (2.3), $\lambda_u^{ij} = \kappa^{ij} \exp(-\mu^2(l_{q_i} - l_{u_j})^2/2)$ (repeated indexes not summed over). The same holds for λ_d^{ij} with l_{u_j} replaced by l_{d_j} .

Therefore, as was mentioned previously, if all κ^{ij} are of order one, the whole texture of the Yukawa matrix is dictated by the relative distance of q 's and u 's, d 's. An important comment is that the exponential factors dramatically affect the otherwise arbitrary moduli of the higher-dimensional Yukawa couplings, but they do nothing to potentially large complex phases. Therefore, in general terms, each element of $\lambda_{(u,d)}^{ij}$ is accompanied by arbitrary, unsuppressed complex phases.

Another interesting point is that, since in the AS framework quark fields are localised in different places, families are distinguishable, at least in principle, even in the limit where all Yukawa couplings vanish. As a result, there is no freedom to rotate the fermion fields in family space (no $U(3)^5$ global symmetry). Ultimately a weak basis (WB) will be dictated by the localising mechanism for the fermion fields. However, one can still refer to different choices of WB which should be understood as corresponding to different assumptions about the underlying physics.

3 Realistic Quark Masses and CP Violation

Within the AS approach to the Yukawa puzzle the choice of WB plays a crucial rôle. It is worthwhile to comment on what WB choices are more appropriate for the fat-brane scenario.

Let us recall that in the standard approach to the fermion mass problem, flavour symmetries added to the SM lead to: i -zeros in the Yukawa matrices, resulting from terms forbidden by the flavour symmetry and/or ii -equality and/or relations between elements in the Yukawa matrices, which are connected by the flavour symmetry. In the AS approach, as discussed in the previous section, it is quite natural to obtain effective zeros in the Yukawa matrices, since they correspond to elements which connect fermions which are very “far” from one another. On the other hand, equalities or specific relations among elements of the Yukawa matrices are not “natural” in the AS scheme – they require fine-tuned choices for the positions of the fermion fields in the extra dimensions.

From the above considerations, one is led to conclude that, within the fat-brane framework, the most convenient WB are those which contain a large number of zeros, while bases where the fermion mass matrices are either symmetric or Hermitian are not adequate. Two especially interesting bases are the nearest-neighbour-interaction (NNI) basis,

$$M_d = \begin{pmatrix} 0 & a & 0 \\ a' & 0 & b \\ 0 & b' & c \end{pmatrix}; \quad M_u = \begin{pmatrix} 0 & d & 0 \\ d' & 0 & e \\ 0 & e' & f \end{pmatrix}, \quad (3.1)$$

and the triangular (T) basis,

$$M_d = \begin{pmatrix} g & k & l \\ 0 & m & n \\ 0 & 0 & p \end{pmatrix}; \quad M_u = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix}, \quad (3.2)$$

where M_d and M_u are the down-type and up-type mass-matrices (after electroweak symmetry breaking) with complex entries.

It should be emphasised that, in the SM, both the NNI [7] and T forms* correspond to a choice of WB, in the sense that starting from arbitrary mass matrices M_u and M_d one can always perform transformations of the type

$$M_u \rightarrow M'_u = W_L^\dagger M_u W_R^u, \quad (3.3)$$

$$M_d \rightarrow M'_d = W_L^\dagger M_d W_R^d, \quad (3.4)$$

where the W 's are unitary matrices, which transform M_u and M_d into the NNI or the T form while leaving the weak currents diagonal in flavour space (note that W_L is the same matrix in M_u and M_d).

3.1 One Extra Dimension

Recently, Mirabelli and Schmaltz [5] (MS) performed what they referred to as a “brute force” scan over the parameter space in the framework of one extra dimension, and found the following mass matrices [5]:

$$M_d = \begin{pmatrix} 0 & 16.974 & 0 \\ 14.510 & 0 & 123.42 \\ 0 & 1373.2 & 2370.2 \end{pmatrix} \text{ MeV}$$

$$M_u = \begin{pmatrix} 1.7630 & 0 & 0 \\ 0 & 576.06 & 2.7882 \times 10^{-3} \\ 5902.8 & 0 & 165900 \end{pmatrix} \text{ MeV} \simeq \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix}, \quad (3.5)$$

where the zeros correspond to elements strongly suppressed by exponential factors. These matrices lead to the correct values for quark masses and mixing angles. We now address the issue of CP violation, which was not discussed in [5]. In order to do so, we assume that all entries in M_u and M_d are complex, with arbitrary phases (as discussed in Sec. 2). From Eq. (3.5), one obtains, to a very good approximation,

$$H_u \equiv M_u M_u^\dagger \simeq \begin{pmatrix} m_u^2 & 0 & 0 \\ 0 & m_c^2 & 0 \\ 0 & 0 & m_t^2 \end{pmatrix},$$

$$H_d \equiv M_d M_d^\dagger = \begin{pmatrix} H_{11}^d & 0 & H_{13}^d \\ 0 & H_{22}^d & H_{23}^d \\ H_{13}^{d*} & H_{23}^{d*} & H_{33}^d \end{pmatrix}. \quad (3.6)$$

*The NNI basis should not be confused with the Fritzsch ansatz [9] which consists of taking the NNI form together with Hermiticity. NNI with this additional requirement does lead to physical predictions which have already been ruled out by experiment due to the observed large top quark mass.

For any arbitrary set of quark mass matrices, in the weak basis where H_u is diagonal, all $|H_d|_{ij}$ have physical meaning and can be expressed as a function of quark masses and mixing angles. In particular one has

$$H_{12}^d = m_d^2 V_{ud} V_{cd}^* + m_s^2 V_{us} V_{cs}^* + m_b^2 V_{ub} V_{cb}^*, \quad (3.7)$$

where V_{ij} are elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. Since in the MS ansatz H_{12}^d vanishes, from Eq. (3.7) together with unitarity of the CKM matrix, one obtains

$$\frac{m_s^2 - m_d^2}{m_b^2 - m_d^2} = \frac{|V_{ub}||V_{cb}|}{|V_{us}||V_{cs}|} \quad (3.8)$$

and

$$\arg(V_{ub}V_{cs}V_{us}^*V_{cb}^*) = \pi. \quad (3.9)$$

Eq. (3.8) corresponds to relation (7.12) of reference [5], which was derived in a slightly different way, while Eq. (3.9) implies that there is no CP violation in the solution found in [5] (namely, Eq. (3.5)). An alternative way of showing that CP is not violated is by evaluating the following WB invariant

$$\text{Tr}[H_u, H_d]^3 = 6i(m_u^2 - m_c^2)(m_t^2 - m_c^2)(m_t^2 - m_u^2)\text{Im}(H_{12}^d H_{23}^d H_{13}^{d*}) = 0, \quad (3.10)$$

where the first equality is obtained in the WB where H_u is diagonal while the second equality results from the fact that H_{12}^d vanishes in this ansatz. The vanishing of the above invariant is a necessary and sufficient condition [10] for CP -invariance in the SM. Note that M_d in Eq. (3.5) has the NNI form, and this is why H_{12}^d vanishes. This, in turn, implies that Eq. (3.8) has a much larger range of applicability than the ansatz presented in [5], being valid for any model where M_d has the NNI form while H_u is diagonal.

One may wonder whether our conclusion about the absence of CP violation in the MS ansatz is affected by the fact that M_u in Eq. (3.5) is not exactly diagonal. It can be readily seen that taking M_u diagonal is a very good approximation (as argued in [5]), so that when one considers the effect of the off-diagonal terms, the resulting strength of CP violation is always too small to account for the experimentally observed value of ϵ_K [11]. Indeed, if one allows for complex arbitrary phases in the entries of M_d , M_u in Eq. (3.5), one can readily derive an upper bound on the strength of CP violation, namely $J \equiv |\text{Im}(V_{ub}V_{cs}V_{us}^*V_{cb}^*)| \leq 5 \times 10^{-9}$. This is to be compared to $J \simeq 10^{-5}$, required by the experimental value of ϵ_K .

We now proceed with a systematic study of fermion mass matrices in the AS scheme. We shall work in the NNI basis defined at the beginning of this section by Eq. (3.1) and assume that the entries of M_u and M_d have arbitrary phases.

At this point one should mention that although the MS example of Eq. (3.5) is not explicitly written in the NNI basis, a simple WB transformation consisting of the permutation of the first two columns transforms their ansatz into the NNI form. Geometrically this transformation corresponds to the interchange of positions between U_1 and U_2 in the extra dimension. The relative positions of the quark wave-functions giving rise to the NNI basis in one dimension are depicted in Fig. 1, which is to be compared to Fig. 5 in [5].

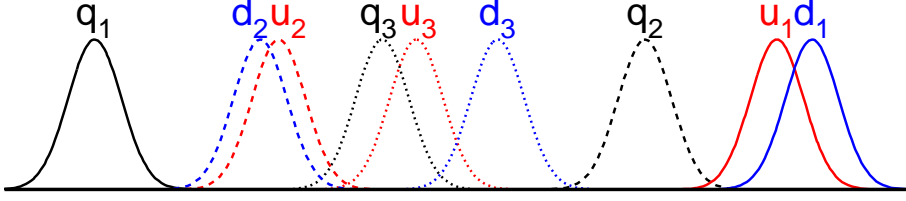


Figure 1: Quark wave-functions in the one extra dimension solution presented by MS, rotated to the NNI-form.

Since, in the SM, there are no right-handed currents, the mass spectrum and the CKM-matrix only depend on $H_u \equiv M_u M_u^\dagger$ and $H_d \equiv M_d M_d^\dagger$, which have the following form in the NNI basis:

$$H_{(u,d)} = \begin{pmatrix} H_{11}^{(u,d)} & 0 & H_{13}^{(u,d)} \\ 0 & H_{22}^{(u,d)} & H_{23}^{(u,d)} \\ H_{13}^{(u,d)*} & H_{23}^{(u,d)*} & H_{33}^{(u,d)} \end{pmatrix}. \quad (3.11)$$

By making a transformation of the type

$$H_u \rightarrow K^\dagger H_u K; \quad H_d \rightarrow K^\dagger H_d K, \quad (3.12)$$

where K is a diagonal unitary matrix, it is possible to eliminate all complex phases from H_u , while the off diagonal elements of H_d still have arbitrary phases. In this case, both matrices are diagonalized in the following way,

$$O_u^\dagger H_u O_u = \text{diag}(m_u^2, m_c^2, m_t^2), \quad (3.13)$$

$$O_d^\dagger K'^\dagger H_d K' O_d = \text{diag}(m_d^2, m_s^2, m_b^2), \quad (3.14)$$

where the O 's are orthogonal matrices and K' is a unitary diagonal matrix whose rôle is to eliminate the phases in H_d . Without loss of generality, one may choose $K' = \text{diag}(1, e^{i\phi}, e^{i\sigma})$.

The CKM matrix is, therefore, given by

$$V_{CKM} = O_u^\dagger K' O_d. \quad (3.15)$$

The relevant elements for our discussion can be explicitly written as

$$V_{us} = O_{11}^u O_{12}^d + O_{21}^u O_{22}^d e^{i\phi} + O_{31}^u O_{32}^d e^{i\sigma}, \quad (3.16)$$

$$V_{ub} = O_{11}^u O_{13}^d + O_{21}^u O_{23}^d e^{i\phi} + O_{31}^u O_{33}^d e^{i\sigma}, \quad (3.17)$$

$$V_{cb} = O_{12}^u O_{13}^d + O_{22}^u O_{23}^d e^{i\phi} + O_{32}^u O_{33}^d e^{i\sigma}, \quad (3.18)$$

$$V_{cs} = O_{12}^u O_{12}^d + O_{22}^u O_{22}^d e^{i\phi} + O_{32}^u O_{32}^d e^{i\sigma}, \quad (3.19)$$

$$V_{td} = O_{13}^u O_{11}^d + O_{23}^u O_{21}^d e^{i\phi} + O_{33}^u O_{31}^d e^{i\sigma}. \quad (3.20)$$

In order to check whether it is possible to obtain, with only one extra dimension, sufficient CP violation, we have done a series of guided trials using Eqs. (3.16)-(3.20) and taking into account the observed quark mass spectrum. Soon it became apparent that this it is not possible with only one extra dimension. This can be understood by the following argument. Since we are working in the NNI basis (where H_{12}^d vanishes), the only way to obtain sufficient CP violation is by having, in the same basis, H_u significantly deviating from the diagonal. This in turn requires more proximity between the U_i 's and Q_j 's. Once the U_i 's get close enough to the Q_j 's there is no room to place the D_k 's along the same line (since the Q_j 's are now closer to each other) while at the same time obtaining the correct masses and mixing. Indeed with only one extra dimension, we have not found any solution significantly different from the one proposed by MS, which would correctly reproduce the quark masses and mixing angles. This confirms the uniqueness of the MS solution (modulo WB transformations on right handed quark fields), in the case of only one extra dimension.

3.2 Two Extra Dimensions

In the search for a solution with two extra dimensions, we shall continue to work in the NNI basis and look for configurations of the quark fields where the CKM matrix arising from Eq. (3.15), although dominated by O_d , receives a significant contribution from O_u . As we have previously argued, for NNI quark mass matrices, the existence of a non-negligible contribution from O_u is essential in order to be able to generate sufficient CP violation. It is clear from Eqs. (3.13),(3.14) that the orthogonal matrices O_u , O_d are determined by the relative positions of the quark fields in the extra dimension space, while the phases ϕ and σ are free parameters. Both the modulus and the argument of V_{us} crucially depend on the phase ϕ , while σ does not play much of a rôle in the determination of V_{us} , due to the smallness of the factor $O_{31}^u O_{32}^d$ (as will be seen later). On the other hand, for the elements V_{ub} and V_{cb} , both ϕ and σ potentially play an important rôle.

We have found an interesting set of locations for the quark fields which leads to the right spectrum of quark masses and pattern of mixing angles, while allowing for the right strength of CP violation. The locations of the quark fields in the two extra dimensions are depicted in Fig. 2. Explicitly,

$$q_i = \frac{1}{\mu} \begin{pmatrix} 5.941; 0 \\ -4.008; 0 \\ 0; 0 \end{pmatrix}, u_i = \frac{1}{\mu} \begin{pmatrix} -8.347; 0 \\ 1.815; 0 \\ -0.941; 0 \end{pmatrix}, d_i = \frac{1}{\mu} \begin{pmatrix} -8.421; 0 \\ 2.219; 2.332 \\ -1.253; 2.767 \end{pmatrix}, \quad (3.21)$$

which lead to the following masses matrices, assuming $\kappa^{ij}v = 1.5m_t$, for all i and j (see [5] for details regarding this choice):

$$M_d = \begin{pmatrix} 0 & 16.112 & 0 \\ 14.690 & 0 & 121.77 \\ 0 & 1400 & 2467.8 \end{pmatrix} \text{ MeV},$$

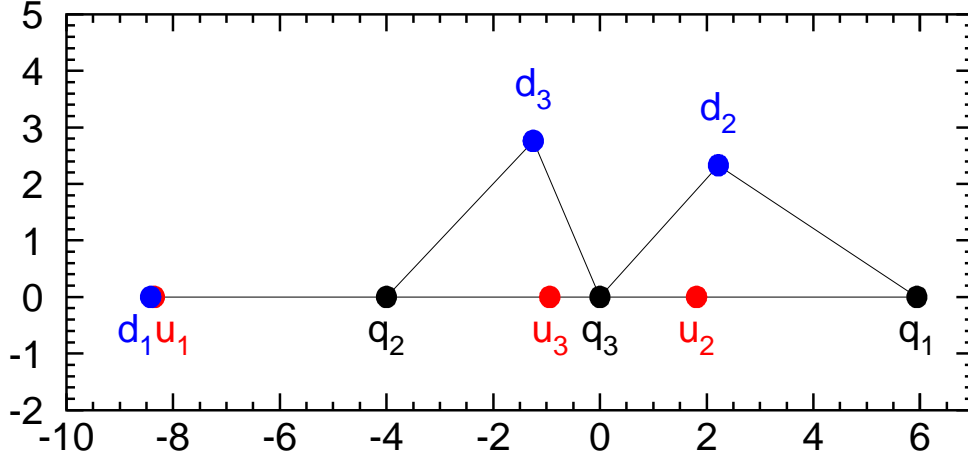


Figure 2: Locations of the quark wave-functions corresponding to Eq. (3.21). Distances are measured in units of μ^{-1} (see text). The lines indicate distances which are dictated by the nonzero entries of M_d , M_u given in Eq. (3.22). Note that once the u_i 's and q_i 's are placed on the same straight line, d_2 and d_3 are forced into the second dimension.

$$M_u = \begin{pmatrix} 0 & 50.0 & 0 \\ 20.3 & 0 & 2258 \\ 0 & 48000 & 160000 \end{pmatrix} \text{ MeV}, \quad (3.22)$$

where the zeros correspond to strongly suppressed matrix elements. From Eq. (3.22) one readily obtains O_d and O_u . The allowed range of values for ϕ is essentially dictated by the experimental value of $|V_{us}|$,

$$|V_{us}| = 0.219 \text{ to } 0.226. \quad (3.23)$$

Indeed from Eq. (3.22) it follows that

$$\begin{aligned} O_{11}^u &= 0.9973; & O_{21}^u &= 0.0735; & O_{31}^u &= -0.0010; \\ O_{12}^d &= 0.2157; & O_{22}^d &= -0.9758; & O_{32}^d &= 0.0358. \end{aligned} \quad (3.24)$$

From Eqs. (3.16), (3.23), and (3.24), it follows that ϕ is constrained to be in the range

$$83.6^\circ \leq \phi \leq 89.4^\circ, \quad (3.25)$$

while σ does not play much of a rôle in the determination of $|V_{us}|$, since $|O_{31}^u O_{32}^d| \sim 10^{-5}$. This situation is depicted in Fig. 3. We choose ϕ in the first quadrant in order to obtain the appropriate sign for $\text{Im}(V_{ub}V_{cs}V_{us}^*V_{cb}^*)$.

It is remarkable that in order to obtain the correct value of $|V_{us}|$ one is led to a large value of ϕ which in turn is crucial to have a sufficient amount of CP violation.

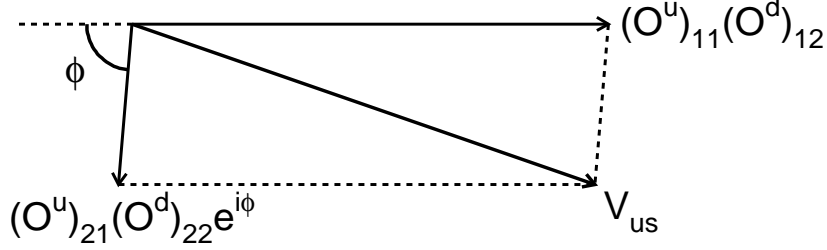


Figure 3: V_{us} in the complex plane (see Eq. (3.16) and text for details).

For $\phi = 85^\circ$, $\sigma = 0^\circ$, we have obtained

$$|V_{CKM}| = \begin{pmatrix} 0.9753 & 0.2208 & 0.0034 \\ 0.2205 & 0.9746 & 0.0384 \\ 0.0108 & 0.0370 & 0.9993 \end{pmatrix} \quad (3.26)$$

The strength of CP violation can be readily evaluated:

$$J \equiv |\text{Im}(V_{ub}V_{cs}V_{us}^*V_{cb}^*)| \simeq 2.2 \times 10^{-5}. \quad (3.27)$$

The values of the quark masses implied by Eq. (3.22) are

$$\begin{aligned} m_u &= 1.5 \text{ MeV}, m_d = 3.2 \text{ MeV}, m_s = 63.3 \text{ MeV}, \\ m_c &= 651 \text{ MeV}, m_b = 2839 \text{ MeV}, m_t = 167059 \text{ MeV}, \end{aligned} \quad (3.28)$$

corresponding to the quark masses evaluated at the common scale m_t . These are in agreement with the experimental values of the quark masses (computed in the $\overline{\text{MS}}$ renormalization scheme, with up, down, and strange quark masses evaluated at a scale of 2 GeV and the others evaluated at a scale equal to their $\overline{\text{MS}}$ mass), as compiled by the Particle Data Group [11]:

$$\begin{aligned} m_u &= 1.5 \text{ to } 5 \text{ MeV}, m_d = 3 \text{ to } 9 \text{ MeV}, m_s = 75 \text{ to } 170 \text{ MeV}, \\ m_c &= 1150 \text{ to } 1350 \text{ MeV}, m_b = 4000 \text{ to } 4400 \text{ MeV}, \\ m_t &= 166000 \pm 5000 \text{ MeV}, \end{aligned} \quad (3.29)$$

when one takes the effect of renormalization group running into account. In order to do this we use scaling factors η_i such that $\eta_i \equiv m_i(m_i)/m_i(m_t)$ for $i = c, b, t$ and $\eta_i \equiv m_i(2 \text{ GeV})/m_i(m_t)$ for $i = u, d, s$. These have been computed to three loops in QCD and one loop in QED [12], and are given by

$$\eta_u = 1.84, \eta_d = 1.84, \eta_s = 1.84, \eta_c = 2.17, \eta_b = 1.55, \eta_t = 1.00. \quad (3.30)$$

One comment which should be made at this point is that the fact that H_u in the example discussed is not diagonal leads to a correction of order 10% in the value of m_s implied by Eq. (3.8).

Finally, we would like to comment on the possibility of having sufficient CP violation with the triangular basis. As far as obtaining the correct mass spectrum and mixing angles, it was already pointed out [5] that the T basis requires at least two extra dimensions. Although we have not studied this case in detail, we do not anticipate, a priori, any difficulty in obtaining the right amount of CP violation as well.

4 Summary and Conclusions

A novel approach to the fermion mass puzzle was recently proposed in the context of large extra dimensions by Arkani-Hamed and Schmaltz (AS) [2] and further analysed by Mirabelli and Schmaltz [5]. In this approach, the fermion mass hierarchy and mixing pattern are a consequence of the fact that different fermionic fields are localised in slightly different points in the higher dimensional space. Although at this stage there is no fundamental understanding of why various fermions would be localised at different positions, the proposed paradigm provides a new and drastically different approach to the Yukawa puzzle.

In this paper we addressed the issue of CP violation in the AS framework. First, we considered a specific example proposed by Mirabelli and Schmaltz (MS) which leads to the correct quark mass spectrum and mixing angles, in the framework of one extra dimension. We showed that in the MS example, even if one allows for complex mass matrices with arbitrary phases, one can never generate sufficient CP violation through the Kobayashi-Maskawa mechanism. We performed our analysis in the nearest-neighbour-interaction (NNI) weak basis (WB), which is well suited for understanding fermion masses in the AS scheme. In fact, we showed that although the MS quark mass matrices are not written in the NNI basis, they can be transformed into that form by a simple WB transformation interchanging two columns of the up quark mass matrix. We also showed that a specific relation between quark masses and mixing angles (see Eq. (3.8)) obtained by MS, can be derived in a more general framework which goes beyond the specific MS ansatz. A systematic search was conducted with one extra dimension and quark mass matrices written in the NNI weak basis and it was shown that again it is not possible to generate sufficient CP violation. We then considered the case of two extra dimensions and constructed an example where the location of the fermion fields leads to the correct mass spectrum and mixing angles, while allowing for the generation of sufficient CP violation to account for the experimental value of ϵ_K . In the example considered, one is led to a striking connection between the value of $|V_{us}|$ and the strength of CP violation.

Finally, we would like to comment on lepton masses and mixing. Although the fat-brane approach can also provide an understanding of the hierarchy of lepton masses, the observed large mixing in the neutrino sector, together with the fact that

neutrinos can also be Majorana particles represents a further challenge [13] to the AS approach.

Acknowledgements

We thank Nima Arkani-Hamed and Martin Schmaltz for comments and words of encouragement. GCB and MNR thank the CERN Theory Division for hospitality.

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