Space-time foam effects on particle interactions and the Greisen-Zatsepin-Kuzmin cutoff

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Modeling space-time foam using a noncritical Liouville-string model for the quantum fluctuations of *D*-branes with recoil, we discuss the issues of momentum and energy conservation in particle propagation and interactions. We argue that momentum should be conserved *exactly* during propagation and *on the average* during interactions, but that energy is conserved only *on the average* during propagation and *is in general not* conserved during particle interactions, because of changes in the background metric. We discuss the possible modification of the GZK cutoff on high-energy cosmic rays, in the light of this energy non-conservation as well as the possible modification of the usual relativistic momentum-energy relation.

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I. INTRODUCTION

Analysts of quantum gravity expect that the fabric of space-time should fluctuate over distance scales of the order of the Planck length l_P and over time scales of the order of the Planck time t_P [1]. The big issue is whether there might be any observable consequences of this ''space-time foam.'' Various ways of modeling certain aspects of quantum fluctuations in space-time have been proposed, including loop gravity [2], noncritical Liouville strings [3,4] and *D*-branes [5]. Within the latter framework, it has been suggested [4] that entropy might not be conserved at the microscopic level, during both particle propagation and interactions, and that energy might be conserved at best statistically. It was also on the basis of this calculational scheme that we proposed that the conventional special-relativistic relation between momentum and energy might be modified, reflecting a breakdown of Lorentz invariance and leading to an effective refractive index *in vacuo* [6,7]. Related effects on particle propagation, such as birefringence, have been suggested in the context of the loop approach to quantum gravity [8].

Subsequently and independently of these theoretical developments, it has been pointed out that a modification of the conventional special-relativistic relation between momentum and energy might dissolve the Greisen-Zatsepin-Kuzmin (GZK) cutoff [9] on high-energy cosmic rays [10,11]. Simply put, if $E \neq p$ for ultrarelativistic particles, the kinematical thresholds for reactions such as $N+\gamma \rightarrow \Delta \rightarrow N+\pi$ and $\gamma+\gamma \rightarrow e^- + e^+$ would be modified, altering the absorption lengths of ultrahigh-energy nucleons N and photons γ. The two lines of thought were brought together by Kifune [12], who pointed out that the modification of the energy-momentum relation proposed in the Liouville *D*-brane approach [4,6] might remove the GZK cutoff for ultrahigh-energy protons striking the cosmic microwave background radiation, and by Protheroe and Meyer [13], who made a similar observation for TeV photons striking the astrophysical infrared background.

Most previous works on this subject have not considered the possibility of energy and/or momentum non-conservation during particle *interactions*, such as $N+\gamma \rightarrow \Delta \rightarrow N+\pi$ or $\gamma+\gamma \rightarrow e^- + e^+$, which could also affect their kinematical thresholds, and hence the corresponding absorption lengths and GZK cutoffs. An exception is [14], where energy-momentum non-conservation was considered in the context of a generic space-time foam analysis, without a specific model of such effects. A detailed argument how such energy-momentum non-conservation effects could arise in specific circumstances, has not been presented anywhere in the literature, as far as we are aware. In this paper, we clarify the circumstances under which we would expect, on the basis of our Liouville-string *D*-brane model for space-time foam, whether and when the energy and/or momentum of observable particles should not be conserved, and by what amount. We extend our previous analyses to include particle interactions, and comment on the implications for analyses of the GZK cutoffs.

We first review our arguments that, during propagation, the momentum of an observable particle should be conserved *exactly* and its energy conserved *statistically*. On the other hand, we argue in this paper that, during interactions, the sum of the momenta of observable particles should only be conserved *statistically*, and their total energy *is not in general* conserved. The basic intuition behind the conservation of energy and momentum during particle propagation is that, although the energy of a particle modifies the background metric as ''recoil'' effect, and this metric modification in turn modifies the relativistic momentum-energy dispersion relation; this modification is space- and time-translation invariant. The particle feels an ''unusual'' non-flat metric, but this is a constant of motion. On the contrary, the ''recoil'' background metric changes during an interaction, leading in general to a violation of energy conservation. However, momentum remains conserved statistically, as a consequence of basic properties of correlation functions in Liouville string theory. The lack of energy conservation during particle interactions modifies the kinematical threshold for particle production quantitatively, but the qualitative effects on particle absorption lengths and the GZK cutoffs [12] still remain.

II. PARTICLE PROPAGATION: MODIFIED DISPERSION RELATIONS AND ENERGY-MOMENTUM CONSERVATION

We review in this section our use of the Liouville-string D-brane model of [4] to discuss the propagation of a closed-string state in the background of a single D particle, which we interpret as a model of (matter) particle propagation through a (dilute) space-time foam. The recoil of the D particle during the scattering distorts the surrounding space-time, inducing a non-trivial off-diagonal term in the metric [4]:

$$G_{0i} = -u_i = -(p_{0,i} - p_i)/M_D, \quad i = 1, \ldots, \tag{1}$$

where u_i denotes the recoil velocity of the D particle, where $i=1,\ldots,d(=3)$ is a spatial index, $M_D \sim 10^{19}$ GeV is the quantum-gravity scale, and p_0 (p) denotes the spatial momentum of the string state before (after) the scattering, in the frame where the D particle is initially at rest.[1]

After the collision, the string propagates in a background space-time with metric (1). We consider first the case of massless particles. The on-shell relation for a massless point-like particle in the metric (1) reads: $p_\mu p_\nu g^{\mu\nu} = 0 \rightarrow -E^2 + p_i^2 + E p_i u_i = 0$, from which we derive

$$E = |p_i| \left(1 + \frac{1}{4}(p_i u_i)^2 \right)^{1/2} + \frac{1}{2} p_i u_i . \tag{2}$$

Using $u_i = (p_o - p_i)/M_D$, we then find that

$$E = |p_i| + \frac{1}{2M_D} p_i (p_{0,i} - p_i) + \mathcal{O}\left(\frac{1}{M_D^2} \right). \tag{3}$$

We consider an average $\langle\langle \ldots \rangle\rangle$ over *both*: (i) statistical effects due to the sum over a gas of D particles in the background, as is appropriate for a space-time foam picture, and (ii) quantum effects, which are treated by summation over higher world-sheet topologies. We assume a random-walk model, averaging over the angle between the incident and scattered particles. Randomness implies that $\langle\langle p_{0,i} p_i \rangle\rangle = 0$ and $\langle\langle u_i \rangle\rangle = 0$. Note that, although the metric perturbation $h_{\mu\nu}$ (1) vanishes on the average, its two-point correlation does not vanish:

$$\langle\langle h_{\mu\nu} \rangle\rangle = 0, \quad \langle\langle h_{\mu\nu} h_{\rho\sigma} \rangle\rangle \neq 0 \tag{4}$$

as recently found also in other stochastic gravity models [15].

Because of momentum conservation during the scattering of the closed string with the D particle, which is rigorously true within the world-sheet σ-model approach, we find that

$$\langle\langle p_0 - p \rangle\rangle_i = \langle\langle M_D u_i \rangle\rangle = 0 \tag{5}$$

i.e., spatial momentum is conserved on the average. It is easy to see that this holds for an arbitrary number of local interactions involving N matter particles:

$$\left\langle \left\langle \sum_{n=1}^{N} p_i \right\rangle \right\rangle = M_D \langle\langle u_i \rangle\rangle = 0, \tag{6}$$

which is an important property of our space-time foam model that we use in the following.

Let us now consider the variance

$$(\Delta p_i)^2 = \langle\langle p_i p_i \rangle\rangle - \langle\langle p_i \rangle\rangle^2, \tag{7}$$

where we recall that the summation in $\langle\langle \ldots \rangle\rangle$ includes both quantum and statistical effects. We return later to a discussion of its possible value in a specific stringy model.

Performing the average $\langle\langle \ldots \rangle\rangle$ of Eq. (3), we obtain [4]

$$\langle\langle E \rangle\rangle \equiv \overline{E} = \overline{p}\left(1 - \frac{1}{2M_D}\overline{p} + \cdots \right) \; : \; \overline{p} \equiv \langle\langle |p_i| \rangle\rangle. \tag{8}$$

To make it clear that the statistical averaging, which depends on the details of the foam, cannot be performed quantitatively in terms of any known D-brane mass scale M_D, we replace $1/2M_D$ by $\xi/2M_D$, where we expect $\xi = \mathcal{O}(1)$ for a dilute-gas foam model. Thus, the following modified dispersion relation characterizes our foamy model:

$$\overline{E} = \overline{p}\left(1 - \frac{\xi}{2M_D}\overline{p} + \cdots \right), \quad \xi > 0. \tag{9}$$

This corresponds to a non-trivial refractive index, which is subluminal, as expected from the Born-Infeld dynamics of the D-brane foam:

$$c(\overline{E}) = \frac{\partial \overline{E}}{\partial \overline{p}} = 1 - \frac{\xi}{M_D}\overline{p}. \tag{10}$$

We have discussed elsewhere [7,16] how such a phenomenon can be tested using gamma-ray bursters and other intense astrophysical probes [17], by looking for delays in the arrival times of photons with different energies.

One may study in a similar manner the modification of the dispersion relation for a particle of mass $m \neq 0$. The on-shell relation now reads: $p_\mu p_\nu g^{\mu\nu} = -m^2 \rightarrow -E^2 + p_i^2 + E p_i u_i$

[1]Throughout this work, we use units where the (low-energy) light velocity *in vacuo* is $c_0 = 1$.

$+m^2=0$, from which we infer, following an approach similar to the massless case in which we assume isotropic random foam,

$$\bar{E}=\bar{p}-\frac{\xi}{2M_D}\overline{p^2}+\left\langle\left\langle\frac{m^2}{2|p_i|}\right\rangle\right\rangle+\ldots,\quad \bar{p}=\langle\langle|p_i|\rangle\rangle \tag{11}$$

for highly-energetic massive particles. The quantity $\langle\langle m^2/|p_i|\rangle\rangle$ is not trivial to evaluate, because we have to invoke the precise meaning of a quantum-fluctuating momentum in our σ-model approach. We know that, in the case of a D particle, momentum can be represented as a coupling in the σ model, whose tree-level value represents the average momentum $\bar{p}$, and the summation over world-sheet topologies can be shown to correspond, in leading order, to Gaussian fluctuations δp_i. Working to leading order in small flucutations, we find $\langle\langle m^2/|p_i|\rangle\rangle\simeq m^2/\bar{p}$. A more detailed analysis is pending, but this assumption will prove sufficient for our qualitative description, so we therefore write the following modified dispersion relation:

$$\bar{E}=\bar{p}-\frac{\xi}{2M_D}\overline{p^2}+\frac{m^2}{2\bar{p}}+\cdots,\quad \bar{p}=\langle\langle|p_i|\rangle\rangle, \tag{12}$$

for a highly-energetic massive particle.

We now consider the issue of energy conservation during particle propagation through space-time foam. We consider a particle with incident (final) energy-momentum $(E_{1(2)},p_{1(2)})$ scattering off a D particle in an isotropic foamy model of the type introduced above. For simplicity, we concentrate on one-dimensional propagation: the extension to higher dimensions is straightforward. The quantity $\delta E=E_1-E_2$ measures energy non-conservation in a single scatering. Averaging over both quantum fluctuations and statistical foam effects, and using Eq. (9), one has

$$\begin{aligned}\langle\langle\delta E\rangle\rangle&=\bar{E}_1-\bar{E}_2=\bar{p}_1-\bar{p}_2\\&\quad-\frac{\xi}{2M_D}((\Delta p_1)^2-(\Delta p_2)^2+\bar{p}_1^2-\bar{p}_2^2)\\&=0,\end{aligned} \tag{13}$$

where we have taken into account Eq. (5) and we assume that the momentum variances of the incoming and outgoing particles are of the same order, as one would expect for energies $E<<M_D$.

Thus we arrive at the following conclusion:

Theorem 1. Momentum is conserved exactly and energy is conserved on the average in the propagation of observable matter particles in isotropic models of Liouville D-particle foam. However, as has been emphasized previously, this energy conservation is not absolute: it is a statement about expectation values, not an operator statement.

As also emphasized previously, this statistical energy conservation is a consequence of the renormalizability of the world-sheet σ-model theory. The statistical variance in the energy measured for a photon of given momentum may be related to quantum (loop) effects in the σ-model treatment of the D brane recoil [18]. We do not discuss this effect here in any detail, but note the conclusion, namely that we expect a variation ΔE in the measured energy of the generic form

$$\Delta E=g_s\frac{\zeta}{2M_D}\overline{p^2} \tag{14}$$

where we have indicated explicitly a factor of the string coupling g_s, and emphasize that the numerical coefficient ζ is unknown and distinct from the corresponding parameter ξ in the modified dispersion relation (12). In a weakly-coupled string model, the variation (14) might well be negligible, but it might be important in a strongly-coupled model. If it is important, it would have the effect of smearing out the effect on the GZK cutoff that we discuss later.

III. PARTICLE INTERACTIONS IN LIOUVILLE SPACE-TIME FOAM

We now extend the previous analysis to consider the decay of a highly-energetic particle with energy and momentum (E_1,p_1) into two other massless particles with energy-momentum vectors (E_2,p_2) and (E_3,p_3), in the context of our Liouville-string approach. Particle interactions are represented as correlators among vertex operators V_i in the world-sheet σ model, which represent the various particle excitations. It is known [19] that, in the context of noncritical Liouville strings [3], an N-point correlator of such vertex operators, $\mathcal{F}_N\equiv\langle V_{i_1}\ldots V_{i_N}\rangle$, where $\langle\ldots\rangle$ signifies a world-sheet expectation value in the standard Polyakov treatment, transforms under infinitesimal Weyl shifts of the world-sheet metric in the following way:

$$\delta_{\rm weyl}\mathcal{F}_N=\left[\delta_0+\mathcal{O}\left(\frac{s}{A}\right)\right]\mathcal{F}_N \tag{15}$$

where the standard part δ_0 of the variation involves a sum over the conformal dimensions h_i of the operators V_i that is independent on the world-sheet area A. The quantity s is the sum of the gravitational anomalous dimensions [3]:

$$s=-\sum_{i=1}^{N}\frac{\alpha_i}{\alpha}-\frac{Q}{\alpha},\quad \alpha_i=-\frac{Q}{2}+\frac{1}{2}\sqrt{Q^2+4(h_i-2)},$$

$$\alpha=-\frac{Q}{2}+\frac{1}{2}\sqrt{Q^2+8} \tag{16}$$

where Q is the central-charge deficit.

In standard critical string theory, the vertex operators corresponding to excitations are $\propto e^{ik_iX^i}$, where k_i is the momentum and the index i runs over both space and time. However, in the noncritical treatment [4] in which time is identified with the zero mode of the Liouville field ϕ, the index i is *only spatial*. The role of the energy is played in this approach by the gravitational anomalous dimension α_i, since the dressed excitation vertex V_i^L is[2]

$$V_i^L \sim e^{\alpha_i \phi} e^{i k_i X^i}. \tag{17}$$

It is then evident that the integration over the σ-model spatial coordinates in the world-sheet correlator $\mathcal{F}_N$, will always imply conservation of the *spatial momentum*.

There is a caveat in this argument, namely that the form (17) of the vertex operator describes an excitation on a flat space-time. This geometry is only an *average* effect in models of isotropic space-time foam (4). In general, quantum-gravity effects caused, e.g., by the recoil of a heavy D particle as we are considering here, distort the space-time. This has the inevitable result that the plane-wave expansion (17) breaks down. The corresponding conservation of momentum therefore is demoted to an average effect due to the statistical sum over space-time foam fluctuations.

Turning now to the energy, we observe from Eq. (17) that its conservation is not immediately apparent, because the (Liouville) energies are weighted by a real exponent in the exponential rather than a phase factor. However [4,19], there is an alternative way of imposing Liouville energy conservation in the mean, which follows from a careful treatment of Eq. (15), interpreting the (world-sheet) zero mode of the Liouville field, ϕ_0, as target time t. This zero mode appears inside the world-sheet covariant area A [19]: $\phi_0 \propto A^{1/2}$.

If $s \neq 0$, i.e., the Liouville energy is not conserved, one obtains a non-trivial time dependence on the associated correlator $\mathcal{F}_N$, which means that it can no longer be interpreted as a unitary S-matrix amplitude in a factorizable \$ matrix element [4,19]. On the other hand, the ordinary kinematics of particle interactions, that is commonly assumed in most approaches to the phenomenology of quantum gravity [10–13], holds only in models in which the noncriticality of the string does not imply a breakdown of \$-matrix factorization when the quantum-gravitational interactions are properly taken into account. This means that the complete matter + gravity system should behave like an ordinary quantum-mechanical system, as in conventional critical string theory.

This is not always feasible in practice, and in generic Liouville theory a precise mathematical description of quantum-gravity induced effects is still lacking. However, this is possible in simplified models of D-brane foam [4], where foamy effects are represented by looking at the back-reaction effects on the space-time geometry that arise from its distortion during the scattering of a propagating stringy matter mode on a heavy, non-relativistic D particle embedded in the space-time. In such a model, the recoil of the D-particle defect is described by a logarithmic conformal field theory on the world sheet. In this framework, the closed world sheet of the propagating stringy matter particle is torn apart by the presence of the defect during the scattering event [4], and the recoil of the defect is represented as a (non-conformal) logarithmic deformation of the world sheet [20]. Dressing with the Liouville field restores criticality, but also leads to a bulk gravitational field (1), which expresses the distortion of the space-time surrounding the scattered defect.

As a result of the recoil treatment, the effective stringy σ model becomes noncritical, since the corresponding deformations are relevant from a world-sheet renormalization group viewpoint [20]. This is the exclusive source of non-criticality, which implies that the induced central-charge deficit Q can be expressed in terms of the corresponding deformation couplings. The leading efffect comes from the deformation associated with the recoil velocity u_i (1), and is given by the kinetic energy of the recoiling D particle, to leading order in an approximation of small recoil velocity, as appropriate for non-relativistic heavy D particles. To develop this approach, we make the following formal steps.

(i) We first consider the gravitational anomalous dimension α_D of a recoiling D-particle excitation. In Liouville theory [3], there are two possibilities:

$$\alpha_D^{(\pm)} = -\frac{Q}{2} \pm \sqrt{\frac{Q^2}{4} + \frac{\epsilon^2}{2}} \tag{18}$$

where we took into account the fact [20] that the deformation of a recoiling D particle has an anomalous dimension $-\epsilon^2/2$ on a flat world-sheet, satisfying

$$\epsilon \sim [\ln(L/a)]^{-1/2} \to 0 \tag{19}$$

where $L(a)$ is the infrared (ultraviolet) world-sheet distance cutoff, and the relation (19) between ϵ and the cutoff is dictated by the closure of the logarithmic world-sheet algebra.

Then (ii) in standard Liouville theory, for excitations that appear as external legs in string amplitudes, it is only the $\alpha^{(+)}$ excitations that are kept. However, the D-particle recoil excitations, since they express virtual foam effects, can only appear as internal lines of string amplitudes, so the $\alpha_D^{(-)}$ solutions are in principle *allowed*. In fact, it is the $\alpha_D^{(-)}$ choice that should be selected in order to obtain statistical energy conservation, as we discuss next.

(iii) To this end, we consider the sum of the anomalous dimensions of N matter particles plus a recoiling D particle excitation, with gravitational dimension $\alpha_D^{(-)}$, which contributes to the central charge deficit $Q^2 = C - C^*$, where C^* represents the critical-string central charge, e.g., $C^* = 25$ in bosonic strings. The deficit C is calculated at the end of the scattering with the D particle, which, in the terminology of [20], occurs at a (large) time $t \sim 1/\epsilon^2$, consistent with Eq. (19) and the identification of the Liouville mode with the target time [4]. The deficit C is given [21] by the standard Zamolodchikov C theorem [22] applied to the open sector of the σ model:[3] $C \simeq C^* + \int_{t_0}^{t} \beta^i \mathcal{G}_{ij} \beta^j$, where the indices i,j

[2]We recall that Liouville dressing is necessary in order to restore world-sheet conformal invariance (criticality) [3].

[3]We recall that the Liouville dressing is applied to the closed sector, by rewriting the boundary recoil deformation as a total derivative on the bulk world sheet. In order to compute the central charge deficit Q, which is an entity that exists prior to Liouville dressing, to leading order one has to take into account only the boundary conformal field theory of the recoil deformations, and apply Zamolodchikov's C-theorem there.

$=C,D$ run over the pair of recoil operators V_i describing the collective coordinates (position and momenta) of the recoiling D particle [20], both with anomalous dimensions $-\epsilon^2/2$ in flat world sheets. The function $\mathcal{G}_{ij}\sim\langle V_i V_j\rangle$ is the Zamolodchikov metric in theory space of these operators [22], while the world-sheet renormalization-group β functions are written as: $\beta^i=-(\epsilon^2/2)g^i+\hat{\beta}^i$, and $\mathcal{G}_{ij}\sim(1/\epsilon^2)\mathcal{G}^{(1)}_{ij}+\cdots$, as in standard theory [4], with $\mathcal{G}^{(1)}_{ij}$ the residue of the simple poles in ϵ^2. In the problem at hand, one has $\hat{\beta}^C=u^i$, $\hat{\beta}^D=0$, $\mathcal{G}^{(1)}_{CC}\propto-\epsilon^2$ and $\mathcal{G}^{(1)}_{CD}\sim 1$. Hence, to leading order in $\epsilon^2\to 0$, the central charge deficit C is then given by

$$Q^2=Q_0^2+\frac{1}{\epsilon^2}(u^i)^2, \quad |u_i|\ll 1 \tag{20}$$

where Q_0^2 is a combination of constants which identifies the ‘‘critical’’ theory, defined as the one with $u_i\to 0$, i.e., no recoil. In the $u\to 0$ limit, the central charge of the σ model may be identified with that of a four-dimensional string theory, and the deficit Q is then just a numerical constant, corresponding to a non-trivial dilaton background that varies linearly with the Liouville zero mode-time. Such theories with linear dilatons have been considered in [23], where it was argued that they provide examples of expanding universes in string theory. However, simple linear dilaton models with flat metric space-times, such as the ones derived in the limit $u\to 0$ above, actually describe a non-expanding string universe if the lengths are measured by string rods [24]. In this sense, the critical four-dimensional model obtained in the no-recoil limit $u\to 0$ describes an ‘‘equilibrium’’ situation.

Finally, (iv) the constant Q_0^2 on the right-hand side (RHS) of Eq. (20) will be *fixed* by the requirement of *energy conservation* in the complete matter + recoiling D-brane system, as we discuss below.

IV. ENERGY NON-CONSERVATION DURING INTERACTIONS

We now examine the question whether energy is conserved during interactions, in the D-particle foam picture outlined above. Consider $s=0$ in Eq. (16), where in s one now includes the D-particle anomalous dimension $\alpha_D^{(-)}$, with Q given by Eq. (20). According to the previous discussion, this guarantees unitarity of the complete matter + recoiling D-particle system. Expanding the appropriate denominators up to leading order in $(u^i)^2/\epsilon^2$, one obtains

$$\sum_{i=1}^{N} E_i\simeq\frac{1}{2}M_D u^2 \tag{21}$$

where $i=1,\dots,N$ runs over all the particles in the interaction, e.g., $N=3$ in the $1\to 2$-body decay process considered here, and the E_i denote the corresponding energies, which are identified with the gravitational anomalous dimensions α_i, once one identifies time with the Liouville zero mode ϕ_0 [4].

The constants in Q_0^2 in the expression (20) for Q are fixed so as to provide the correct normalization for the kinetic energy of the recoiling D particle of the RHS of Eq. (21). Such constants are attributed to properties of the equilibrium background space-time, in the limit $u^i\to 0$, and, as such, they are to be associated with the specification of the rest energy of the background D-brane foam. Equation (21), then, expresses the net change in energy induced by the recoil kinetic energy of the D brane affected by the highly-energetic particles. We shall concentrate on this quantity from now on, as a measure of the induced energy non-conservation in the observable particle subsystem.

Taking into account spatial momentum conservation in the presence of the recoil D-particle deformation, which is rigorous in the context of the σ-model logarithmic conformal field theory treatment of D-brane recoil [21], as discussed earlier, we may write

$$u=\sum_{i=1}^{N}\frac{p_i}{M_D} \tag{22}$$

from which we may write Eq. (21) in the form

$$\sum_{i=1}^{N} E_i=\frac{1}{2}\sum_{i=1}^{N}\frac{p_i^2}{M_D}+\sum_{i<j}\frac{p_i p_j}{M_D}. \tag{23}$$

We now consider an average $\langle\langle\ldots\rangle\rangle$ over quantum (higher world-sheet genus) and statistical foam effects. We concentrate first on the three-particle interaction in the presence of a recoiling D particle, making the isotropy assumption (4). We then have from Eq. (23)

$$\delta\overline{E}_D\equiv\overline{E}_1-\overline{E}_2-\overline{E}_3=\frac{M_D}{2}\langle\langle u^2\rangle\rangle. \tag{24}$$

Using spatial momentum conservation (6), we then find

$$\delta\overline{E}_D=\frac{\xi_I}{M_D}(\overline{p_2^2}+\overline{p_3^2}+\overline{p_2}\,\overline{p_3})+\frac{\xi_I}{M_D}\langle\langle p_2p_3\rangle\rangle-\frac{\xi_I}{M_D}\langle\langle p_1p_3\rangle\rangle-\frac{\xi_I}{M_D}\langle\langle p_1p_2\rangle\rangle+\frac{\xi_I}{M_D}\sum_{i=1}^{3}(\Delta p_i)^2 \tag{25}$$

where ξ_I is a new numerical parameter ($\xi_I\neq\xi$ in general) that characterizes the effects of the statistical sum in $\langle\langle\ldots\rangle\rangle$.[4]

To leading order in the inverse string or D-brane scale M_D, the matrix element for any interaction agrees with that obtained in the framework of special relativistic quantum field theory without gravitation, e.g., QED in the example of the $\gamma+\gamma\to e^++e^-$ process considered here. On the other

[4]In the above, all the E_i are positive, which explains the sign conventions.

hand, the back-reaction effects of the recoiling D particle on the space-time are expected to modify the kinematics of this field-theoretic result. We examine their effects by factorizing multi-particle processes as products of three-point vertices, and treating the latter in a generic way without specifying the nature of the particles involved. We take into account spatial momentum conservation, but allow isotropic fluctuations, as implied in our models of foam.

In the limit when D-particle recoil effects are ignored, one may replace in Eq. (25) products of the form $\langle\langle p_i p_j\rangle\rangle$ by the appropriate average (mean-field) momenta $\overline{p_i}\,\overline{p_j}$, up to quantum uncertainty effects of order $\chi_{ij}=\langle\langle p_i p_j\rangle\rangle-\overline{p_i}\,\overline{p_j}$:

$$\langle\langle p_i p_j\rangle\rangle=\overline{p_i}\,\overline{p_j}+\chi_{ij}, \quad \chi_{ij}=\langle\langle p_i p_j\rangle\rangle-(\overline{p_i}\,\overline{p_j}). \tag{26}$$

Then, using momentum conservation for the mean momenta: $\overline{p_1}=\overline{p_2}+\overline{p_3}$, it is straightforward to see that the RHS of (25) consists only of the uncertainty variances χ_{ij}, thereby implying the known flat space-time field-theory result of energy conservation during decay.

On the other hand, incorporating the recoil of the background D particle introduces a distortion of space-time during the decay, changing the situation. In calculating such corrections in the presence of a random distribution of recoiling D particles, we assume that the statistical average over the D-brane foam generates a random distribution of angles between the momentum of a particle produced during the decay, in our case particle 2, and the momenta of each of the external particles (1,3). This random distribution is compensated by the random distribution of angles of the recoiling D particles, and hence spatial momentum is conserved on the average. Such processes provide minute fluctuating *corrections* to the mean-field relativistic field theory result on flat space-times.

In this case, the RHS of the pertinent three-particle interaction formula (25) becomes

$$\delta\overline{E}_D=\frac{\xi_I}{M_D}\overline{p_2^2}+\frac{\xi_I}{M_D}\sum_{i=1}^{3}(\Delta p_i)^2+\frac{\xi_I}{M_D}\chi_{13} \tag{27}$$

where $\overline{p_2}$ is a typical momentum of a light particle that may be exchanged in a four-body interaction, $\chi_{13}\equiv\langle\langle p_1 p_3\rangle\rangle-\overline{p_1}\,\overline{p_3}$, and again we took into account spatial momentum conservation (6). For all practical purposes, the contributions from the terms depending on variances Δp_i and χ_{23} may be assumed negligible compared to p_2^2/M_D. In any case, they can only make *additive* contributions to the RHS of Eq. (27), and hence can never cancel its first term.

Thus we arrive at the following conclusion:

Theorem 2: Observable momentum is conserved statistically and energy is violated during particle interactions, due to recoil effects in D-particle space-time foam. The amount of energy violation is, in order of magnitude and up to quantum uncertainties, equal to the typical kinetic energy of the recoiling heavy (non-relativistic) D particle, with energy being conserved in the complete system, where the back-reaction on the space-time geometry of the recoiling D particle is taken into account.

V. IMPLICATIONS FOR THE GZK CUTOFF

As a non-trivial example that illustrates the physical interest of the above analysis, consider now the process $\gamma_H\gamma_L\rightarrow e^+e^-$, where γ_H is a highly energetic photon, and γ_L an infrared background photon, with energy $\omega\sim 0.025$ eV [13]. We recall that, in our framework, spatial momentum is conserved on the average, but the energy-momentum dispersion relations are modified (9). Also, as discussed in the previous section, energy is violated in 3- and 4-particle decays and interactions (27). We may, for our purposes, factorize the photon-photon scattering four-point amplitude into a product of two 3-body interactions mediated by the exchange of an off-shell electron with momentum p_2, as in standard QED. The relevant energy-momentum (non-)conservation equations are[5]

$$E_1+\omega=E_2+E_3+\delta E_D,$$
$$p_1-\omega=p_2+p_3. \tag{28}$$

Inserting the modified energy-momentum dispersion relations for both massless and massive particles in the first of the above equations, and using the second, one obtains, in the case of highly-energetic particles:

$$2\omega+\frac{m_1^2}{2p_1}-\frac{m_2^2}{2p_2}-\frac{m_3^2}{2p_3} =\delta E_D^{(4)}+\frac{\xi}{M_D}(p_2p_3+(p_2+p_3)\omega+\omega^2). \tag{29}$$

For the purposes of the threshold calculation, we assume that $\omega\ll p_1\simeq E_{th}=2p_2=2p_3$, $m_1\simeq 0$, $m_2=m_3=m_e$. Then, from Eq. (29) we obtain

$$2\omega E_{th}-2m_e^2=E_{th}\left(\delta E_D^{(4)}+\frac{\xi}{M_D}\left(\frac{1}{2}E_{th}+\omega\right)^2\right) \tag{30}$$

where the ω-dependent terms on the RHS of Eq. (30) are negligible for our purposes.

The amount of energy violation $\delta E_D^{(4)}$ in a four-particle interaction such as $\gamma+\gamma\rightarrow e^++e^-$ or $N+\gamma\rightarrow\Delta\rightarrow N+\pi$ is given by the sum of the corresponding violations in each of the two three-body interactions (27), assuming factorization of the relevant amplitudes, via virtual $e^{\pm}$ or Δ exchange, respectively. To leading order, and ignoring the stochastic quantum uncertainties in the propagating particle energies, excitation of a massive D brane essentially determines $\delta E^{(4)}$:

$$\delta E^{(4)}\simeq\frac{\xi_I}{2M_D}E_{th}^2+\cdots \tag{31}$$

where ξ_I parametrizes energy loss during interactions, and is *a priori* distinct from the propagation parameter ξ.

[5]From now on, for simplicity we omit overlines in the notation, since all the quantities below denote such averages.

The fact that $\delta E^{(4)} \geqslant 0$, i.e., energy is *lost* during particle decays in the presence of the *D*-particle foam, follows from the sign of the flat-world-sheet anomalous dimension $-\epsilon^2/2$ of the recoil deformation appearing inside the square root in the expression (18) for α_D. In our formalism, this sign is *positive*, because the corresponding operator is relevant in a world-sheet renormalization-group sense [20]. Should this operator have been *irrelevant* instead, its anomalous dimension would have appeared with the opposite sign inside Eq. (18), and one would have encountered a (rather unphysical) situation in which there would be energy *gain* in particle decays in the presence of *D*-particle foam. In such a case, $\delta E^{(4)} < 0$, and using the estimate (31), which would still be valid up to a change in sign, one might in principle have a complete *cancellation* on the RHS of the threshold equation (30), leaving intact the conventional Lorentz-invariant GZK cutoff. However, our argument for the sign of ξ_I excludes this possibility.

In the high-energy limit $E_{th} >> \omega$, (30) becomes a cubic equation:

$$E_{th} - \frac{m_e^2}{\omega} \simeq \frac{(\xi_I + \xi/2)E_{th}^3}{4M_D\omega} + \cdots \qquad (32)$$

where the $\cdots$ denote higher-order and quantum-uncertainty effects. Equation (32) always has a real solution that determines the kinematic threshold. For example, for the values $\omega \simeq 0.025$ eV, $m_e \sim 0.5$ MeV and $M_D \sim 10^{19}$ GeV, two of the three real solutions of the cubic equation for E_{th} are positive. The physical solution for $\xi/2 = \xi_I = 1$ is $E_{th,2}/M_D \simeq 2.2 \times 10^{-15}$, which implies a non-trivial modification of the conventional GZK cutoff, namely an increase to ~ 20 TeV, which is consistent with the observation of energetic photons from Mk421.

The fact that there is energy loss in particle interactions, as a result of the non-trivial recoil of excitations in the string/*D*-particle foam, distinguishes our approach from models [10–13] where energy is assumed to be conserved, as well as momentum. Our approach raises the GZK cutoff, but it is lower than that predicted in models with $\xi_I = 0$. Setting $\delta E_D^{(4)} = 0$ in Eq. (30), the resulting cubic equation for the threshold, analogous to Eq. (32), would imply $E_{th}/M_D \simeq 3.14 \times 10^{-15}$ if $\xi/2 = 1$.

In the case of ultrahigh-energy cosmic rays, it was thought that the GZK cutoff [9] should result from the scattering of ultrahigh-energy protons off the cosmic microwave background radiation to produce a Δ resonance. There is no observational evidence for such a cutoff: the present ultrahigh-energy cosmic-ray data extend up to energies $\sim 3 \times 10^{20}$ eV, beyond which the current experiments have insufficient statistics to see the rapidly-falling flux, whether or not there is a cutoff. We find a displaced GZK cutoff above 3×10^{20} eV if $\xi_I + \xi/2$ exceeds about $10^{-16} - 10^{-17}$, which is much smaller than that required to move the GZK cutoff for γ rays above 20 TeV. Hence, if the γ-ray data are to be explained by a violation of Lorentz invariance along the lines discussed here, we would also expect the GZK cutoff for protons to be displaced, unless there is some (unnatural?) cancellation in the sum $\xi_I + \xi/2$ for proton-γ interactions. However, the linearized leading-order approximation for the quantum-*D*-brane recoil effects, in which we only keep terms linear in E/M_D (where E a typical low-energy scale), fails at energies of order 10^{19} eV. In the ultra-high-energy cosmic-ray case one needs a resummation of higher-order corrections before arriving at a definite conclusion on the effects of quantum foam on the GZK proton cutoff, which is not feasible within our perturbative logarithmic conformal field theory approach to Liouville string. However, it is our belief that the qualitative effect of an increased cutoff will remain. It will be interesting to see whether future experiments with sensitivities to even higher-energy cosmic rays will continue to see a smoothly-falling spectrum with no cutoff, or whether a cutoff will appear somewhere above 3×10^{20} eV.

VI. DISCUSSION

In the equations (28) above, we have ignored quantum uncertainties in the energies and momenta, assuming them to be small compared to the mean-field effects of *D*-brane recoil. This is a feature of the specific *D*-brane recoil model we have examined here [4]. In more generic approaches to quantum-gravitational foam, however, such an assumption may not be valid, and quantum uncertainties associated in the energies and momenta may be comparable to the terms leading to modifications of the GZK cutoff discussed above. Even so, we would not expect such stochastic uncertainties to dominate over the modification of the usual dispersion relation.

However, such a possibility has recently been examined in [25], where energy and momentum uncertainties of the following generic form were assumed:

$$\Delta p_i \sim \Delta E \sim \left(\frac{E}{M_P}\right)^{\alpha} E \quad \text{and} \quad \Delta p_i - \Delta E = \varepsilon \frac{E^{1+\alpha}}{2M_D^{\alpha}}. \qquad (33)$$

In the approach of [25], the parameters α and ε are phenomenological, and should be bounded or determined by analyzing high-energy cosmic-ray data. This case (for $\alpha = 1$) may be recovered in our analysis above by setting $\xi \to 0$ in the appropriate expressions (28), (30), etc., and keeping only the terms associated with the quantum uncertainties, assuming the energy-momentum dependent form (33) above.

One way to distinguish the approach of [25] from ours, in which modified dispersion relations occur and probably dominate, is to look for observable effects in the arrival times of energetic photons from distant astrophysical sources such as GRBs [6,7]. In our approach, in which stochastic fluctuations are not expected to dominate, one would expect searches for different arrival times for photons in *different* energy ranges to be fruitful, for example pulses emitted by GRBs in different energy channels [7,16]. On the other hand, in approaches to quantum-gravitational foam like that of [25], in which quantum uncertainties in energies and momenta are dominant, one should observe energy-dependent spreads in the arrival times of photons *of the same energy*, which could be probed by comparing the widths of pulses emitted by GRBs in different energy channels [18,16].

A final remark is that our string approach to quantum space-time foam is compatible with the general principles of relativity, despite the appearance of a fundamental (Planckian) mass scale M_D. In string theory, this mass scale is an observer-independent quantity *by construction*. Our modifications of Lorentz symmetry, as expressed in the modified dispersion relations discussed above, pertain to a specific background of string theory, so such Lorentz violations should be considered as *spontaneous*. In the Liouville approach to time [4], although the time coordinate is special, in the sense of being associated with a renormalization-group scale on the world sheet of the string, and thus irreversible [22], there is no contradiction with general coordinate invariance, given that Liouville strings do embody that principle [4] in their σ-model framework. The irreversibility of the Liouville time is no stranger than time irreversibility in cosmological models, which are compatible with general coordinate invariance, being specific solutions of the gravitational field equations.

In this respect, we question the need for extra axioms [26] in order to formulate theories with an observer-independent minimal length. In our stringy approach, where the induced violations of Lorentz symmetry are due to the (dynamical) choice of background, there is a frame-dependence of the energy scale associated with modifications of the light velocity, but this is a reflection of the spontaneous violation of Lorentz invariance, and the underlying (Planckian) mass scale is universal. The ensuing predictions are compatible with the general principles of relativity.

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