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In Pursuit of New Physics with B_s Decays

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Abstract

The presence of a sizeable CP-violating phase in B_s–$\overline{B}_s$ mixing would be an unambiguous signal of physics beyond the Standard Model. We analyse various possibilities to detect such a new phase considering both tagged and untagged decays. The effects of a sizeable width difference $\Delta\Gamma$ between the B_s mass eigenstates, on which the untagged analyses rely, are included in all formulae. A novel method to find this phase from simple measurements of lifetimes and branching ratios in untagged decays is proposed. This method does not involve two-exponential fits, which require much larger statistics. For the tagged decays, an outstanding role is played by the observables of the time-dependent angular distribution of the $B_s \to J/\psi[\to l^+l^-]\,\phi[\to K^+K^-]$ decay products. We list the formulae needed for the angular analysis in the presence of both a new CP-violating phase and a sizeable $\Delta\Gamma$, and propose methods to remove a remaining discrete ambiguity in the new phase. This phase can therefore be determined in an unambiguous way.

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1 Introduction

The rich phenomenology of non-leptonic B decays offers various strategies to explore the phase structure of the Cabibbo–Kobayashi–Maskawa (CKM) matrix [1] and to search for manifestations of physics beyond the Standard Model [2]. Concerning the latter aspect, CP violation in B_s–$\overline{B}_s$ mixing is a prime candidate for the discovery of non-standard physics. In the first place the B_s–$\overline{B}_s$ mixing amplitude is a highly CKM-suppressed loop-induced fourth order weak interaction process and therefore very sensitive to new physics. Moreover in the Standard Model the mixing-induced CP asymmetries in the dominant B_s decay modes practically vanish, because they are governed by the tiny phase $\arg(-V_{tb}V_{ts}^*/(V_{cb}V_{cs}^*))$. It does not take much new physics to change this prediction: already a fourth fermion generation[1] can easily lead to a sizeable new CP-violating phase in B_s–$\overline{B}_s$ mixing [4]. It is further possible that there are new flavour-changing interactions which do not stem from the Higgs-Yukawa sector. The phases of these couplings are not related to the phases of the CKM elements and therefore induce extra CP violation. An example is provided by generic supersymmetric models in which new flavour-changing couplings come from off-diagonal elements of the squark mass matrix [5]. While such new contributions are likely to affect also B_d–$\overline{B}_d$ mixing, they appear in the B_d system as a correction to a non-zero Standard Model prediction for the mixing-induced CP asymmetry, which involves the poorly known phase $\beta = \arg(-V_{tb}V_{td}^*/(V_{cb}V_{cd}^*))$. To extract the new physics here additional information on the unitarity triangle must be used. In the B_s system, however, the new physics contribution is a correction to essentially zero [6].

Indeed, the discovery of new physics through a non-standard CP-violating phase in B_s–$\overline{B}_s$ mixing may be achievable before the LHCb/BTeV era, in Run-II of the Fermilab Tevatron.

B_s-meson decays into final CP eigenstates that are caused by $\bar{b} \to \bar{c}c\bar{s}$ quark-level transitions such as $B_s \to D_s^+ D_s^-$, $J/\psi\, \eta^{(\prime)}$ or $J/\psi\, \phi$, are especially interesting [7–9]. The η and η' mesons in $B_s \to J/\psi\, \eta^{(\prime)}$ can be detected through $\eta \to \gamma\gamma$ and $\eta' \to \rho^0\gamma$, $\pi^+\pi^-\eta$, or through $\eta \to \pi^+\pi^-\pi^0$ [10]. These modes require photon detection. In the case of $B_s \to J/\psi[\to l^+l^-]\, \phi[\to K^+K^-]$, which is particularly interesting for B-physics experiments at hadron machines because of its nice experimental signature, the final state is an admixture of different CP eigenstates. In order to disentangle them, an angular analysis has to be performed [11, 12]. Experimental attention is also devoted to three-body final states [13]. B_s-meson decays triggered by the quark decay $\bar{b} \to \bar{c}u\bar{d}$ can likewise access a CP-specific final state, e.g. via $B_s \to D^0_{\mathrm{CP}+}[\to K^+K^-]K_S$, with a likewise negligibly small CP-violating phase in the Standard Model. The key point here is that there are many different decay modes which all contain the same information on the pursued new CP-violating phase ϕ. Furthermore, additional information on ϕ can be gained from analyses that require no tagging. Untagged studies determine $|\cos\phi|$ and are superior to tagged analyses in terms of efficiency, acceptance and purity. However, they require a sizeable width difference $|\Delta\Gamma|$ between the B_s mass eigenstates. On the other hand, from tagged analyses (such as CP asymmetries) $\sin\phi$ can be extracted, if the rapid B_s–$\overline{B}_s$ oscillation can be resolved. Both avenues should be pursued and their results combined,

[1] This scenario is still possible, though somewhat disfavoured by electroweak precision data [3].

because they measure the same fundamental quantities.

If we denote the Standard Model and the new physics contributions to the B_s–$\overline{B}_s$ mixing amplitude with $S_{\rm SM}$ and $S_{\rm NP}$, respectively, then the measurement of the mass difference Δm in the B_s system determines $|S_{\rm SM}+S_{\rm NP}|$. The knowledge of both Δm and the B_s–$\overline{B}_s$ mixing phase ϕ then allows to solve for both the magnitude and phase of $S_{\rm NP}$. Information on ϕ is especially valuable, if $|S_{\rm SM}|$ and $|S_{\rm NP}|$ are comparable in size and Δm agrees within a factor of 2 or 3 with the Standard Model prediction.

The purpose of this paper is twofold: we first identify useful measurements and show how the information from different decay modes and different observables can be combined in pursuit of a statistically significant "smoking gun" of new physics. Second we show how the B_s–$\overline{B}_s$ mixing phase can be identified unambiguously, without discrete ambiguities. The outline is as follows: after setting up our notation in Section 2 we consider untagged B_s decays and discuss various methods to determine $|\cos\phi|$ in Section 3. Tagged B_s decays are discussed in Section 4, whereas Section 5 shows how to resolve the discrete ambiguity in ϕ. Finally, we conclude in Section 6.

2 Preliminaries

In this section we define the various quantities entering the time evolution of B_s mesons and their decay amplitudes. We closely follow the notation of the BABAR-Book [1]. Some of the discussed quantities depend on phase conventions and enter physical observables in phase-independent combinations [14]. Since this feature is well understood and extensively discussed in the standard review articles [1], we here fix some of these phases for convenience and only briefly touch this issue where necessary.

We choose the following convention for the CP transformation of meson states and quark currents:[2]

$$CP|\,B_s\,\rangle = -|\,\overline{B}_s\,\rangle, \qquad CP\,\overline{q}_L\gamma_\mu b_L\,(CP)^{-1} \;=\; -\overline{b}_L\gamma^\mu q_L. \tag{1}$$

Hence the CP eigenstates are

$$|\,B_s^{\rm even}\,\rangle = \frac{1}{\sqrt{2}}\Big(|\,B_s\,\rangle - |\,\overline{B}_s\,\rangle\Big), \qquad \text{and} \qquad |\,B_s^{\rm odd}\,\rangle \;=\; \frac{1}{\sqrt{2}}\Big(|\,B_s\,\rangle + |\,\overline{B}_s\,\rangle\Big). \tag{2}$$

The time evolution of the B_s–$\overline{B}_s$ system is governed by a Schrödinger equation:

$$i\,\frac{d}{dt}\begin{pmatrix} |\,B_s(t)\,\rangle \\ |\,\overline{B}_s(t)\,\rangle\end{pmatrix} = \left(M - i\,\frac{\Gamma}{2}\right)\begin{pmatrix} |\,B_s(t)\,\rangle \\ |\,\overline{B}_s(t)\,\rangle\end{pmatrix} \tag{3}$$

with the mass matrix $M = M^\dagger$ and the decay matrix $\Gamma = \Gamma^\dagger$. Here $|\,B_s(t)\,\rangle$ denotes the state of a meson produced as a B_s at time $t = 0$, with an analogous definition for $|\,\overline{B}_s(t)\,\rangle$. The off-diagonal elements $M_{12} = M_{21}^*$ and $\Gamma_{12} = \Gamma_{21}^*$ correspond to B_s–$\overline{B}_s$ mixing. In the Standard Model

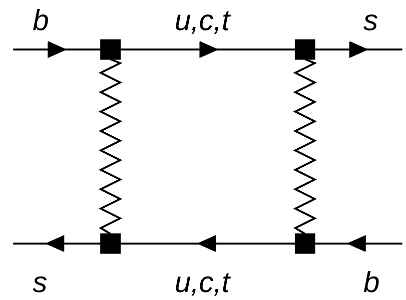

Figure 1: B_s–$\overline{B}_s$ mixing in the Standard Model.

the leading contributions to M_{12} and Γ_{12} stem from the box diagram in Fig. 1; Γ_{12} originates from the real final states into which both B_s and $\overline{B}_s$ can decay. It receives contributions from box diagrams with light u and c quarks. Since Γ_{12} is dominated by CKM-favoured tree-level decays, it is practically insensitive to new physics. On the other hand, M_{12} is almost completely induced by short-distance physics. Within the Standard Model the top quarks in Fig. 1 give the dominant contribution to B_s–$\overline{B}_s$ mixing. This contribution is suppressed by four powers of the weak coupling constant and two powers of $|V_{ts}| \simeq 0.04$. Hence new physics can easily compete with the Standard Model and possibly even dominate M_{12}. If the non-standard contributions to M_{12} are unrelated to the CKM mechanism of the three-generation Standard Model, they will affect the mixing phase

$$\phi_M = \arg M_{12}.$$

With our convention (1) the Standard Model prediction is $\phi_M = \arg(V_{tb}V_{ts}^*)^2$.

The mass eigenstates at time $t = 0$, $|\, B_L\,\rangle$ and $|\, B_H\,\rangle$, are linear combinations of $|\, B_s\,\rangle$ and $|\, \overline{B}_s\,\rangle$:

$$\text{lighter eigenstate:} \quad |\, B_L\,\rangle = p|\, B_s\,\rangle + q|\, \overline{B}_s\,\rangle$$

$$\text{heavier eigenstate:} \quad |\, B_H\,\rangle = p|\, B_s\,\rangle - q|\, \overline{B}_s\,\rangle, \qquad \text{with } |p|^2 + |q|^2 = 1. \tag{4}$$

We denote the masses and widths of the two eigenstates with $M_{L,H}$ and $\Gamma_{L,H}$ and define

$$\Gamma \;=\; \frac{1}{\tau_{B_s}} = \frac{\Gamma_H + \Gamma_L}{2}, \qquad \Delta m \;=\; M_H - M_L, \qquad \Delta\Gamma \;=\; \Gamma_L - \Gamma_H. \tag{5}$$

While $\Delta m > 0$ by definition, $\Delta\Gamma$ can have either sign. Our sign convention is such that $\Delta\Gamma > 0$ in the Standard Model. By examining the eigenvalue problem of $M - i\Gamma/2$ we find that the experimental information $\Delta m \gg \Gamma$ model-independently implies $|\Gamma_{12}| \ll |M_{12}|$. By expanding the eigenvalues and q/p in Γ_{12}/M_{12}, we find

$$\Delta m = 2|M_{12}|, \qquad \Delta\Gamma \;=\; 2\,|\Gamma_{12}|\cos\phi \qquad \text{and} \qquad \frac{q}{p} \;=\; -e^{-i\phi_M}\left[1 - \frac{a}{2}\right]. \tag{6}$$

[2]metric $g_{\mu\nu} = (1, -1, -1, -1)$

Here the phase ϕ is defined as

$$\frac{M_{12}}{\Gamma_{12}} = -\left|\frac{M_{12}}{\Gamma_{12}}\right| e^{i\phi}. \tag{7}$$

In (6) we have kept a correction in the small parameter

$$a = \left|\frac{\Gamma_{12}}{M_{12}}\right| \sin\phi, \tag{8}$$

but neglected all terms of order Γ_{12}^2/M_{12}^2 and do so throughout this paper. Since a can hardly exceed 0.01 we will likewise set it to zero in our studies of B_s decays into CP eigenstates and only briefly discuss a non-zero a in sect. 3.4.

The phase ϕ is physical and convention-independent; if $\phi = 0$, CP violation in mixing vanishes. In the Standard Model $\phi = \phi_M - \arg(-\Gamma_{12})$ is tiny, of order 1%. This is caused by two effects: first, Γ_{12} is dominated by the decay $b \to c\bar{c}s$ and $(V_{cb}V_{cs}^*)^2$ is close to the B_s–$\overline{B}_s$ mixing phase $\arg(V_{tb}V_{ts}^*)^2$. Second, the small correction to $\arg(-\Gamma_{12})$ involving $V_{ub}V_{us}^*$ is further suppressed by a factor of m_c^2/m_b^2. In the search for a sizeable new physics contribution to ϕ these doubly Cabibbo-suppressed terms proportional to $V_{ub}V_{us}^*$ can safely be neglected, as we do throughout this paper.

For a B_s decay into some final state f, we introduce the $|\Delta B| = 1$ matrix elements

$$A_f = \langle f|\, B_s\,\rangle \qquad \text{and} \qquad \overline{A}_f = \langle f|\, \overline{B}_s\,\rangle.$$

The key quantity for CP violation reads

$$\lambda_f = \frac{q}{p}\,\frac{\overline{A}_f}{A_f}. \tag{9}$$

The time evolution formulae and the expressions for the CP asymmetries in the forthcoming sections can be conveniently expressed in terms of

$$\mathcal{A}_{\rm CP}^{\rm dir} = \frac{1-|\lambda_f|^2}{1+|\lambda_f|^2}, \qquad \mathcal{A}_{\rm CP}^{\rm mix} = -\frac{2\,{\rm Im}\,\lambda_f}{1+|\lambda_f|^2} \qquad \text{and} \qquad \mathcal{A}_{\Delta\Gamma} = -\frac{2\,{\rm Re}\,\lambda_f}{1+|\lambda_f|^2}\,. \tag{10}$$

If f is a CP eigenstate, $CP|\,f\,\rangle = \pm|\,f\,\rangle$, then $\mathcal{A}_{\rm CP}^{\rm dir} \neq 0$ or $\mathcal{A}_{\rm CP}^{\rm mix} \neq 0$ signals CP violation: a non-vanishing $\mathcal{A}_{\rm CP}^{\rm dir}$ implies $|A_f| \neq |\overline{A}_f|$, meaning direct CP violation; $\mathcal{A}_{\rm CP}^{\rm mix}$ measures mixing-induced CP violation in the interference of $B_s \to f$ and $\overline{B}_s \to f$. The third quantity, $\mathcal{A}_{\Delta\Gamma}$, plays a role, if $\Delta\Gamma$ is sizeable. The three quantities obey the relation

$$\left|\mathcal{A}_{\rm CP}^{\rm dir}\right|^2 + \left|\mathcal{A}_{\rm CP}^{\rm mix}\right|^2 + |\mathcal{A}_{\Delta\Gamma}|^2 = 1.$$

The time-dependent decay rate $\Gamma(B_s(t) \to f)$ of an initially tagged B_s into some final state f is defined as

$$\Gamma(B_s(t) \to f) = \frac{1}{N_B}\,\frac{dN(B_s(t) \to f)}{dt}\,. \tag{11}$$

Quark decay	Hadronic decay	Remarks
$\overline{b} \to \overline{c}c\overline{s}$	$B_s \to \psi\phi$	
	$B_s \to \psi K^{(*)}\overline{K}^{(*)}$	
	$B_s \to \psi\phi\phi$	
	$B_s \to \psi\eta$	
	$B_s \to \psi\eta'$	
	$B_s \to \psi f_0$	CP-odd final state
	$B_s \to \chi_{c0}\phi$	CP-odd final state
	$B_s \to D_s^{(*)+}D_s^{(*)-}$	$D_s^+D_s^-$ is CP-even
	$B_s \to D^{(*)+}D^{(*)-}$ or $D^{(*)0}\overline{D}^{(*)0}$	non-spectator decays, $D\overline{D}$ is CP-even
$\overline{b} \to \overline{c}u\overline{d}$	$B_s \to K_S\overline{D}^{(*)0}$ [$\to \phi K_S$, $\rho^0 K_S$, $K\overline{K}$ or $\pi^+\pi^-$]	

Table 1: Some CKM-favoured B_s decay modes into CP-specific final states. Here, ψ represents J/ψ or $\psi(2S)$. Decays into two vector particles or into three-body final states with one or more vector particles require an angular analysis to separate the CP-even from the CP-odd component. The final states $D_s^{\pm}D_s^{*\mp}$ are dominantly CP-even [16] (see sect. 3).

Here $B_s(t)$ represents a meson at proper time t tagged as a B_s at $t = 0$; $dN(B_s(t) \to f)$ denotes the number of decays of $B_s(t)$ into the final state f occurring within the time interval $[t, t + dt]$; N_B is the total number of B_s's produced at time $t = 0$. An analogous definition holds for $\Gamma(\overline{B}_s(t) \to f)$. By solving the Schrödinger equation (3) using (6), we can find these decay rates [15]:

$$\Gamma(B_s(t) \to f) = \mathcal{N}_f \, |A_f|^2 \, \frac{1 + |\lambda_f|^2}{2} \, e^{-\Gamma t} \times \left[\cosh \frac{\Delta\Gamma \, t}{2} \, + \, \mathcal{A}_{\rm CP}^{\rm dir} \, \cos(\Delta m \, t) + \mathcal{A}_{\Delta\Gamma} \, \sinh \frac{\Delta\Gamma \, t}{2} + \mathcal{A}_{\rm CP}^{\rm mix} \, \sin(\Delta m \, t) \right], \tag{12}$$

$$\Gamma(\overline{B}_s(t) \to f) = \mathcal{N}_f \, |A_f|^2 \, \frac{1 + |\lambda_f|^2}{2} \, (1 + a) \, e^{-\Gamma t} \times \left[\cosh \frac{\Delta\Gamma \, t}{2} - \mathcal{A}_{\rm CP}^{\rm dir} \, \cos(\Delta m \, t) + \mathcal{A}_{\Delta\Gamma} \, \sinh \frac{\Delta\Gamma \, t}{2} - \mathcal{A}_{\rm CP}^{\rm mix} \, \sin(\Delta m \, t) \right]. \tag{13}$$

Here $\mathcal{N}_f$ is a time-independent normalization factor.

A promising testing ground for new physics contributions to ϕ_M are decays into CP eigenstates triggered by the quark decay $b \to c\overline{c}s$. Table 1 summarizes such CP-specific B_s decay modes. To estimate the size of the small Standard Model predictions consider first the decay amplitudes [17]:

$$A_f, \overline{A}_f \, \propto \, \left[1 + \left(\frac{\lambda^2}{1 - \lambda^2} \right) \, a_{\rm p} \, e^{i\theta} \, e^{\pm i\gamma} \right]. \tag{14}$$

Hence the weak phase factor $e^{i\gamma}$, which is associated with the quantity $a_{\rm p}e^{i\theta}$, is strongly Cabibbo-suppressed by two powers of the Wolfenstein parameter $\lambda \simeq |V_{us}| \simeq 0.22$ [18]. The "penguin parameter" $a_{\rm p}e^{i\theta}$ measures – sloppily speaking – the ratio of penguin- to tree-diagram-like topologies and is loop-suppressed. Since new-physics contributions to these decay amplitudes have to compete with a tree diagram, they are not expected to play a significant role. A detailed discussion for a left–right-symmetric model can be found in [9]. Since we are interested in large "smoking gun" new physics effects in B_s–$\overline{B}_s$ mixing, we account for the Standard Model contributions within the leading order of λ and set $|\overline{A}_f| = |A_f|$, neglecting direct CP violation. With the weak phase $\phi_{c\bar{c}s} = \arg(V_{cb}V_{cs}^*)$ one then finds

$$\frac{\overline{A}_f}{A_f} = -\eta_f e^{2i\phi_{c\bar{c}s}}. \tag{15}$$

Here η_f denotes the CP parity of f: $CP|\, f\,\rangle = \eta_f|\, f\,\rangle$. In Table 1 we also included decay modes driven by the quark level decay $b \to c\overline{u}d$. The weak phase of these modes involves the phases of the K and D decay amplitudes into CP eigenstates. The phases combine to $\arg(V_{cb}V_{ud}^*)+\arg(V_{ud}V_{us}^*)+\arg(V_{us}V_{cs}^*) = \arg(V_{cb}V_{cs}^*)$, i.e. the same result as for $b \to c\bar{c}s$. With (6) and (15) λ_f reads

$$\lambda_f = \frac{q}{p}\frac{\overline{A}_f}{A_f} = \eta_f\, e^{-i\phi}. \tag{16}$$

Here we have identified the phase $\arg(\eta_f\lambda_f) = \phi_M - 2\phi_{c\bar{c}s}$ with the phase ϕ defined in (7). This is possible, because $\arg(-\Gamma_{12}) = 2\phi_{c\bar{c}s}+O(\lambda^2)$ and we neglect the Cabibbo-suppressed contributions. The Standard Model contribution to $\phi = \phi_{\rm SM}+\phi_{\rm NP}$ equals $\phi_{\rm SM} = -2\eta\lambda^2$. Here η is the Wolfenstein parameter measuring the height of the unitarity triangle. Since our focus is a sizeable new physics contribution $\phi_{\rm NP}$, we can safely neglect $\phi_{\rm SM}$ and identify ϕ with $\phi_{\rm NP}$ in the following. That is, we neglect terms of order λ^2 and higher. Using (16) the quantities in (10) simplify to

$$\mathcal{A}_{\rm CP}^{\rm dir} = 0, \qquad \mathcal{A}_{\rm CP}^{\rm mix} = \eta_f \sin\phi \qquad \text{and} \qquad \mathcal{A}_{\Delta\Gamma} = -\eta_f\cos\phi. \tag{17}$$

The corrections to (17) from penguin effects can be found in [17]. We next specify to the PDG phase convention for the CKM matrix [19], in which $\arg(V_{cb}V_{cs}^*) = \mathcal{O}(\lambda^6)$. Then we can set $\phi_{c\bar{c}s}$ to zero and identify

$$\phi_M = \phi.$$

With this convention the mass eigenstates can be expressed as

$$\begin{aligned}
|\, B_L\,\rangle &= \frac{1+e^{i\phi}}{2}\,|\, B_s^{\rm even}\,\rangle - \frac{1-e^{i\phi}}{2}\,|\, B_s^{\rm odd}\,\rangle + \mathcal{O}(a), \\
|\, B_H\,\rangle &= -\frac{1-e^{i\phi}}{2}\,|\, B_s^{\rm even}\,\rangle + \frac{1+e^{i\phi}}{2}\,|\, B_s^{\rm odd}\,\rangle + \mathcal{O}(a)\,.
\end{aligned} \tag{18}$$

Whenever we use $B_s^{\rm even}$ and $B_s^{\rm odd}$ we implicitly refer to this phase convention. If formulae involving $B_s^{\rm even}$ and $B_s^{\rm odd}$ are used to constrain models with an extended quark sector, the phase convention used for the enlarged CKM matrix must likewise be chosen such that $\arg(V_{cb}V_{cs}^*) \simeq 0$.

3 Untagged Studies

3.1 Time Evolution

Whereas the width difference $\Delta\Gamma$ is negligibly small in the B_d system, it can be sizeable for B_s mesons. This has the consequence that the untagged B_s data sample bears information on CP violation [20]. Further the width difference itself is sensitive to the B_s–$\overline{B}_s$ mixing phase ϕ [21], as we can see from (6).

When B_s's and $\overline{B}_s$'s are produced in equal numbers, the untagged decay rate for the decay $B_s^{\mathrm{un}} \to f$ reads

$$\begin{aligned}\Gamma[f,t] &= \Gamma(B_s(t) \to f) + \Gamma(\overline{B}_s(t) \to f) \\ &= \mathcal{N}_f \left[e^{-\Gamma_L t} \left| \langle f |\, B_L \rangle \right|^2 + e^{-\Gamma_H t} \left| \langle f |\, B_H \rangle \right|^2 \right] + \mathcal{O}(a). \qquad (19) \\ &= \mathcal{N}_f \, |A_f|^2 \left[1 + |\lambda_f|^2 \right] \, e^{-\Gamma t} \left\{ \cosh \frac{\Delta\Gamma\, t}{2} + \, \sinh \frac{\Delta\Gamma\, t}{2} \, \mathcal{A}_{\Delta\Gamma} \right\} + \mathcal{O}(a). \qquad (20)\end{aligned}$$

Here the second expression is simply obtained by adding (12) and (13). In (19) the same result is expressed in terms of the mass eigenstates and nicely exhibits how the decay is governed by two exponentials. Using (11) we can relate the overall normalization to the branching ratio:

$$\begin{aligned}Br[f] &= \frac{1}{2} \int_0^\infty dt \; \Gamma[f,t] \qquad (21) \\ &= \frac{\mathcal{N}_f}{2} \, |A_f|^2 \, \left[1 + |\lambda_f|^2 \right] \frac{\Gamma + \mathcal{A}_{\Delta\Gamma} \, \Delta\Gamma/2}{\Gamma^2 - (\Delta\Gamma/2)^2} + \mathcal{O}(a). \qquad (22)\end{aligned}$$

Conforming with [19] we have normalized the event counting to $N_B + N_{\overline{B}} = 2N_B$, so that $Br[all] = 1$. Using (22) we rewrite (20) as

$$\Gamma[f,t] = 2\, Br[f] \; \frac{\Gamma^2 - (\Delta\Gamma/2)^2}{\Gamma + \mathcal{A}_{\Delta\Gamma} \, \Delta\Gamma/2} \; e^{-\Gamma t} \, \left[\cosh \frac{\Delta\Gamma\, t}{2} + \, \sinh \frac{\Delta\Gamma\, t}{2} \, \mathcal{A}_{\Delta\Gamma} \right] + \mathcal{O}(a). \qquad (23)$$

Now (23) is our master equation for the time evolution of the decay of an untagged B_s sample. If $\Gamma = 1/\tau_{B_s}$ is known, one could perform a two-parameter fit of the decay distribution to (23) and determine $\Delta\Gamma$ and $\mathcal{A}_{\Delta\Gamma}$. The latter determines ϕ through (17), if f is a CP eigenstate from a CKM-favoured decay. In practice, however, most data come from short times with $\Delta\Gamma\, t \ll 1$, and one is only sensitive to the product $\Delta\Gamma \cdot \mathcal{A}_{\Delta\Gamma}$:

$$\Gamma[f,t] = 2\, Br[f] \; \Gamma \, e^{-\Gamma t} \, \left[1 + \frac{\Delta\Gamma}{2} \, \mathcal{A}_{\Delta\Gamma} \, \left(t - \frac{1}{\Gamma} \right) \right] + \mathcal{O}\left((\Delta\Gamma\, t)^2 \right). \qquad (24)$$

We return to this point in sect. 3.3.

3.2 The Width Difference $\Delta\Gamma$ and Branching Ratios

The mass matrix M_{12} and the decay matrix Γ_{12} provide three rephasing invariant quantities: $|M_{12}|$, $|\Gamma_{12}|$ and the relative phase ϕ. In (6) we have related the two observables Δm and $\Delta\Gamma$ to $|M_{12}|$, $|\Gamma_{12}|$ and ϕ. Interestingly, it is possible to find a third observable, which determines $|\Gamma_{12}|$ and thus encodes additional information. We define

$$\Delta\Gamma_{\rm CP} \equiv 2|\Gamma_{12}| \;=\; 2 \sum_{f\in X_{c\overline{c}}} \left[\Gamma(B_s \to f_{\rm CP+}) \,-\, \Gamma(B_s \to f_{\rm CP-})\right]. \tag{25}$$

Here $X_{c\overline{c}}$ represents the final states containing a $(c,\overline{c})$ pair, which constitute the dominant contribution to $\Delta\Gamma_{\rm CP}$ stemming from the decay $b\to c\overline{c}s$. In (25) we have decomposed any final state f into its CP-even and CP-odd component, $|\,f\,\rangle = |\,f_{\rm CP+}\,\rangle + |\,f_{\rm CP-}\,\rangle$ and defined

$$\Gamma(B_s\to f_{\rm CP\pm}) = \mathcal{N}_f\,|\langle\, f_{\rm CP\pm}\,|B_s\rangle|^2 \;=\; \frac{|\langle\, f_{\rm CP\pm}\,|B_s\rangle|^2}{|\langle\, f\,|B_s\rangle|^2}\,\Gamma(B_s\to f).$$

$\mathcal{N}_f$ is the usual normalization factor originating from the phase-space integration. In order to prove the second equality in (25) we start from the definition of Γ_{12}:

$$\Gamma_{12} = \sum_f \mathcal{N}_f\,\langle\, B_s\,|f\rangle\langle\, f\,|\overline{B}_s\rangle \;=\; \frac{1}{2}\sum_f \mathcal{N}_f\left[\,\langle\, B_s\,|f\rangle\langle\, f\,|\overline{B}_s\rangle + \langle\, B_s\,|\overline{f}\rangle\langle\,\overline{f}\,|\overline{B}_s\rangle\,\right]. \tag{26}$$

In the second equation we have paired the final state $|\,f\,\rangle$ with its CP conjugate $|\,\overline{f}\,\rangle = -CP|\,f\,\rangle$. In the next step we trade f for $f_{\rm CP+}$ and $f_{\rm CP-}$ and use the CP transformation

$$\langle\, f_{\rm CP\pm}\,|\overline{B}_s\rangle = \mp\, e^{2i\phi_{c\overline{c}s}}\,\langle\, f_{\rm CP\pm}\,|B_s\rangle,$$

where $\phi_{c\overline{c}s} = \arg(V_{cb}V_{cs}^*)$ is the phase of the $b\to c\overline{c}s$ decay amplitude, which dominates Γ_{12}. Then (26) becomes

$$\begin{aligned} -\; e^{-2i\phi_{c\overline{c}s}}\,\Gamma_{12} &= \sum_{f\in X_{c\overline{c}}} \mathcal{N}_f\,\left[|\langle\, f_{\rm CP+}\,|B_s\rangle|^2 - |\langle\, f_{\rm CP-}\,|B_s\rangle|^2\right] \\ &= \sum_{f\in X_{c\overline{c}}} \left[\Gamma(B_s\to f_{\rm CP+}) \,-\, \Gamma(B_s\to f_{\rm CP-})\right]. \end{aligned} \tag{27}$$

Interference terms involving both $\langle\, f_{\rm CP+}\,|B_s\rangle$ and $\langle\, f_{\rm CP-}\,|B_s\rangle$ drop out when summing the two terms $\langle\, B_s\,|f\rangle\langle\, f\,|\overline{B}_s\rangle$ and $\langle\, B_s\,|\overline{f}\rangle\langle\,\overline{f}\,|\overline{B}_s\rangle$. In (27) both sides of the equation are rephasing-invariant. An explicit calculation of Γ_{12} reveals that the overall sign of the LHS of (27) is positive, which completes the proof of (25).

Loosely speaking, $\Delta\Gamma_{\rm CP}$ is measured by counting the CP-even and CP-odd double-charm final states in B_s decays. We specify this statement in the following and relate $\Delta\Gamma_{\rm CP}$ to measured observables in sect. 3.3.2. Our formulae become more transparent if we adopt the standard phase convention with $\arg(V_{cb}V_{cs}^*) \simeq 0$ and use the CP-eigenstates defined in (2). With

$|\,B_s\,\rangle = (|\,B_s^{\text{even}}\,\rangle + |\,B_s^{\text{odd}}\,\rangle)/\sqrt{2}$ one easily finds from (27):

$$\Delta\Gamma_{\text{CP}} = 2|\Gamma_{12}| \;=\; \Gamma\,(B_s^{\text{even}}) - \Gamma(B_s^{\text{odd}}). \tag{28}$$

Here the RHS refers to the total widths of the CP-even and CP-odd B_s eigenstates. We stress that the possibility to relate $|\Gamma_{12}|$ to a measurable quantity in (25) crucially depends on the fact that Γ_{12} is dominated by a single weak phase. For instance, the final state K^+K^- is triggered by $b \to u\overline{u}s$ and involves a weak phase different from $b \to c\overline{c}s$. Although K^+K^- is CP-even, the decay $B_s^{\text{odd}} \to K^+K^-$ is possible. An inclusion of such CKM-suppressed modes into (27) would add interference terms that spoil the relation to measured quantities. The omission of these contributions to Γ_{12} induces a theoretical uncertainty of order 5% on (28).

In the Standard Model the mass eigenstates in (18) coincide with the CP eigenstates (with $B_L = B_s^{\text{even}}$) and $\Delta\Gamma_{\text{SM}} = \Delta\Gamma_{\text{CP}}$. The effect of a non-zero B_s–$\overline{B}_s$ mixing phase ϕ reduces $\Delta\Gamma$:

$$\Delta\Gamma \;=\; \Delta\Gamma_{\text{CP}}\cos\phi, \tag{29}$$

while $\Delta\Gamma_{\text{CP}} \;=\; 2|\Gamma_{12}|$ is not sensitive to new physics. From the calculated Γ_{12} we can predict to which extent $\Gamma\,(B_s^{\text{even}})$ exceeds $\Gamma\left(B_s^{\text{odd}}\right)$ and this result does not change with the presence of a non-zero ϕ.

The theoretical prediction for $\Delta\Gamma_{\text{CP}}$ is known to next-to-leading order in both Λ_{QCD}/m_b [22] and the QCD coupling α_s [23]. It reads

$$\frac{\Delta\Gamma_{\text{CP}}}{\Gamma} = \left(\frac{f_{B_s}}{245\ \text{MeV}}\right)^2 \left[\,(0.234 \pm 0.035)\,B_S - 0.080 \pm 0.020\,\right]. \tag{30}$$

Here the coefficient of B_S has been updated to $m_b(m_b)+m_s(m_b) = 4.3$ GeV (in the $\overline{\text{MS}}$ scheme) and f_{B_s} is the B_s meson decay constant. Recently the KEK–Hiroshima group succeeded in calculating f_{B_s} in an unquenched lattice QCD calculation with two dynamical fermions [24]. The result is $f_{B_s} = (245 \pm 30)$ MeV. B_S parametrizes the relevant hadronic matrix element, with $B_S = 1$ in the vacuum saturation approximation. A recent quenched lattice calculation has yielded $B_S = 0.87 \pm 0.09$ [25] for the $\overline{\text{MS}}$ scheme. A similar result has been found in [26]. This analysis, however, calculates $\Delta\Gamma$ after normalizing (30) to the measured mass difference in the B_d–$\overline{B}_d$ system. This method involves $|V_{td}|$, which is obtained from a global CKM fit and thereby relies on the Standard Model. Since the target of our analysis is new physics, we cannot use the numerical prediction for $\Delta\Gamma$ of [26]. At present, studies of B_S are a new topic in lattice calculations and we can expect substantial improvements within the next few years. With these numbers one finds from (30):

$$\frac{\Delta\Gamma_{\text{CP}}}{\Gamma} = 0.12 \pm 0.06. \tag{31}$$

Here we have conservatively added the errors from the two lattice quantities linearly.

Since $\Delta\Gamma_{\text{CP}}$ is unaffected by new physics and $\Delta\Gamma_{\text{CP}} > 0$, several facts hold beyond the Standard Model: i) There are more CP-even than CP-odd final states in B_s decays. ii) The shorter-lived mass eigenstate is always the one with the larger CP-even component in (18). Its branching ratio into a CP-even final state $f_{\rm CP+}$ exceeds the branching ratio of the longer-lived mass eigenstate into $f_{\rm CP+}$, if the weak phase of the decay amplitude is close to $\arg V_{cb}V_{cs}^*$. For $\cos\phi > 0$ B_L has a shorter lifetime than B_H, while for $\cos\phi < 0$ the situation is the opposite [21]. iii) Measurements based on the comparison of *branching ratios* into CP-specific final states determine $\Delta\Gamma_{\rm CP}$ rather than $\Delta\Gamma$. Such an analysis has recently been performed by the ALEPH collaboration [27]. ALEPH has measured

$$2\,Br[D_s^{(*)+}D_s^{(*)-}] = 0.26^{+0.30}_{-0.15} \tag{32}$$

and related it to $\Delta\Gamma_{\rm CP}$. For this the following theoretical input has been used [16]:

i) In the heavy quark limit $m_c \to \infty$ and neglecting certain terms of order $1/N_c$ (where $N_c = 3$ is the number of colours) the decay $B_s^{\rm odd} \to D_s^{\pm}D_s^{*\mp}$ is forbidden. Hence in this limit the final state in $B_s^{\rm un} \to D_s^{\pm}D_s^{*\mp}$ is CP-even. Further in $B_s^{\rm un} \to D_s^{*+}D_s^{*-}$ the final state is in an S-wave.

ii) In the Shifman–Voloshin (SV) limit $m_c \to \infty$ with $m_b - 2m_c \to 0$ [28], $\Delta\Gamma_{\rm CP}$ is saturated by $\Gamma(B_s^{\rm un} \to D_s^{(*)+}D_s^{(*)-})$. With i) this implies that in the considered limit the width of $B_s^{\rm odd}$ vanishes. For $N_c \to \infty$ and in the SV limit, $2\Gamma(B_s^{\rm un} \to D_s^{(*)+}D_s^{(*)-})$ further equals the parton model result for $\Delta\Gamma_{\rm CP}$ (quark-hadron duality).

Identifying $\Gamma(B_s^{\rm even} \to D_s^{(*)+}D_s^{(*)-}) \simeq \Delta\Gamma_{\rm CP}$ and $\Gamma(B_s^{\rm odd} \to D_s^{(*)+}D_s^{(*)-}) \simeq 0$ we find:

$$2\,Br[D_s^{(*)+}D_s^{(*)-}] \simeq \Delta\Gamma_{\rm CP}\left[\frac{1+\cos\phi}{2\,\Gamma_L} + \frac{1-\cos\phi}{2\,\Gamma_H}\right] = \frac{\Delta\Gamma_{\rm CP}}{\Gamma}\left[1+\mathcal{O}\left(\frac{\Delta\Gamma}{\Gamma}\right)\right]. \tag{33}$$

Thus the measurement in (32) is compatible with the theoretical prediction in (31). For $\phi = 0$, the expression used in Ref. [27], in which the Standard Model scenario has been considered, is recovered. The term in square brackets accounts for the fact that in general the CP-even eigenstate $|\,B_s^{\rm even}\,\rangle$ is a superposition of $|\,B_L\,\rangle$ and $|\,B_H\,\rangle$. It is straightforward to obtain (33): inserting (18) into (19) expresses $\Gamma[f,t]$ in terms of $\Gamma(B_s^{\rm even} \to f)$ and $\Gamma(B_s^{\rm odd} \to f)$. After integrating over time the coefficient of $\Gamma(B_s^{\rm even} \to f)$ is just the term in square brackets in (33).

When using (33) one should be aware that the corrections to the limits i) and ii) adopted in [16] can be numerically sizeable. For instance, in the SV limit there are no multibody final states like $D_s^{(*)}\overline{D}X_s$, which can modify (33). As serious would be the presence of a sizeable CP-odd component of the $D_s^{(*)+}D_s^{(*)-}$ final state, since it would be added with the wrong sign to $\Delta\Gamma_{\rm CP}$ in (33). A method to control the corrections to the SV limit experimentally is proposed in sect. 3.3.2. We further verify from (33) that the measurement of $Br[D_s^{(*)+}D_s^{(*)-}]$ determines $\Delta\Gamma_{\rm CP}$. Its sensitivity to the new physics phase ϕ is suppressed by another factor of $\Delta\Gamma/\Gamma$ and is irrelevant in view of the theoretical uncertainties.

3.3 Determination of $\Delta\Gamma$ and $|\cos\phi|$

There are two generic ways to obtain information on $\Delta\Gamma$:

i) The measurement of the B_s lifetime in two decay modes $B_s^{\rm un} \to f_1$ and $B_s^{\rm un} \to f_2$ with $\mathcal{A}_{\Delta\Gamma}(f_1) \neq \mathcal{A}_{\Delta\Gamma}(f_2)$.

ii) The fit of the decay distribution of $B_s^{\rm un} \to f$ to the two-exponential formula in (23).

As first observed in [21], the two methods are differently affected by a new physics phase $\phi \neq 0$. Thus by combining the results of methods i) and ii) one can determine ϕ. In this section we consider two classes of decays:

- flavour-specific decays, which are characterized by $\overline{A}_f = 0$ implying $\mathcal{A}_{\Delta\Gamma} = 0$. Examples are $B_s \to D_s^- \pi^+$ and $B_s \to X \ell^+ \nu_\ell$,
- the CP-specific decays of Table 1, with $\mathcal{A}_{\Delta\Gamma} = -\eta_f \cos\phi$.

In both cases the time evolution of the untagged sample in (23) is not sensitive to the sign of $\Delta\Gamma$ (or, equivalently, of $\cos\phi$). For the CP-specific decays of Table 1 this can be seen by noticing that

$$\mathcal{A}_{\Delta\Gamma} \sinh \frac{\Delta\Gamma\, t}{2} = -\,\eta_f \,|\cos\phi|\, \sinh \frac{|\Delta\Gamma|\, t}{2}.$$

Here we have used the fact that $\Delta\Gamma$ and $\cos\phi$ always have the same sign, because $\Delta\Gamma_{\rm CP} > 0$. Hence the untagged studies discussed here in sect. 3.3 can only determine $|\cos\phi|$ and therefore lead to a four-fold ambiguity in ϕ. The sign ambiguity in $\cos\phi$ reflects the fact that from the untagged time evolution in (23) one cannot distinguish, whether the heavier or the lighter eigenstate has the shorter lifetime (however, see sect. 5).

In order to experimentally establish a non-zero $\Delta\Gamma$ from the time evolution in (23) one needs sufficient statistics to resolve the deviation from a single-exponential decay law, see (24). As long as we are only sensitive to terms linear in $\Delta\Gamma\, t$ and $\Delta\Gamma/\Gamma$, we can only determine $\mathcal{A}_{\Delta\Gamma}\, \Delta\Gamma$ from (24). $\mathcal{A}_{\Delta\Gamma}\, \Delta\Gamma$ vanishes for flavour-specific decays and equals $-\eta_f \Delta\Gamma\, \cos\phi$ for CP-specific final states. Hence from the time evolution alone one can only determine $\Delta\Gamma\, \cos\phi$ in the first experimental stage. This determination is discussed in sect. 3.3.1. Once the statistical accuracy is high enough to resolve terms of order $(\Delta\Gamma)^2$, one can determine both $|\Delta\Gamma|$ and $|\cos\phi|$. Fortunately, the additional information from branching ratios can be used to find $|\Delta\Gamma|$ and $|\cos\phi|$ without resolving quadratic terms in $\Delta\Gamma$. The determination of $|\Delta\Gamma|$ and $|\cos\phi|$ is discussed in sect. 3.3.2.

3.3.1 Determination of Γ and $\Delta\Gamma \cos\phi$

Lifetimes are conventionally measured by fitting the decay distribution to a single exponential. Consider a decay which is governed by two exponentials,

$$\begin{aligned} \frac{\Gamma[f,t] + \Gamma[\overline{f},t]}{2} &= A\, e^{-\Gamma_L t} \,+\, B\, e^{-\Gamma_H t} \\ &= e^{-\Gamma t} \left[(A+B) \cosh \frac{\Delta\Gamma t}{2} + (B-A) \sinh \frac{\Delta\Gamma t}{2} \right], \end{aligned} \tag{34}$$

but fitted to a single exponential

$$F\,[f,t] = \Gamma_f \, e^{-\Gamma_f \, t}. \tag{35}$$

In (34) we have averaged over f and its CP-conjugate $\overline{f}$. Of course the coefficients depend on the final state: $A = A(f)$, $B = B(f)$. A maximum likelihood fit of (35) converges to [31]

$$\Gamma_f = \frac{A/\Gamma_L + B/\Gamma_H}{A/\Gamma_L^2 + B/\Gamma_H^2}. \tag{36}$$

We expand this to second order in $\Delta\Gamma$:

$$\Gamma_f = \Gamma + \frac{A-B}{A+B}\frac{\Delta\Gamma}{2} - \frac{2\,AB}{(A+B)^2}\frac{(\Delta\Gamma)^2}{\Gamma} + \mathcal{O}\left(\frac{(\Delta\Gamma)^3}{\Gamma^2}\right). \tag{37}$$

In flavour-specific decays we have $A = B$ (see (23)). We see from (37) that here a single-exponential fit determines Γ up to corrections of order $\Delta\Gamma^2/\Gamma^2$.

Alternatively, one can use further theoretical input and exploit that $\Gamma_{B_s}/\Gamma_{B_d} = 1 + \mathcal{O}(1\%)$ from heavy quark symmetry [22, 29, 30]. This relation can therefore be used to pinpoint Γ in terms of the well-measured B_d lifetime. New physics in the standard penguin coefficients of the effective $\Delta B = 1$ hamiltonian only mildly affects $\Gamma_{B_s}/\Gamma_{B_d}$ [30]. The full impact of new physics on $\Gamma_{B_s}/\Gamma_{B_d}$, however, has not been studied yet.

With (23) and (34) we can read off A and B for the CP-specific decays of Table 1 and find $A(f_{\rm CP+})/B(f_{\rm CP+}) = (1+\cos\phi)/(1-\cos\phi)$ and $A(f_{\rm CP-})/B(f_{\rm CP-}) = (1-\cos\phi)/(1+\cos\phi)$ for CP-even and CP-odd final states, respectively. Our key quantity for the discussion of CP-specific decays $B_s^{\rm un} \to f_{\rm CP}$ is

$$\Delta\Gamma'_{\rm CP} \equiv -\eta_f \mathcal{A}_{\Delta\Gamma}\,\Delta\Gamma \;=\; \Delta\Gamma\,\cos\phi \;=\; \Delta\Gamma_{\rm CP}\,\cos^2\phi. \tag{38}$$

With this definition (37) reads for the decay rate $\Gamma_{{\rm CP},\eta_{\rm f}}$ measured in $B_s^{\rm un} \to f_{\rm CP}$:

$$\Gamma_{{\rm CP},\eta_{\rm f}} = \Gamma + \eta_f\,\frac{\Delta\Gamma'_{\rm CP}}{2} - \sin^2\phi\,\frac{(\Delta\Gamma)^2}{2\Gamma} + \mathcal{O}\left(\frac{(\Delta\Gamma)^3}{\Gamma^2}\right).$$

That is, to first order in $\Delta\Gamma$, comparing the $B_s^{\rm un}$ lifetimes measured in a flavour-specific and a CP-specific final state determines $\Delta\Gamma'_{\rm CP}$. Our result agrees with the one in [21], which has found (38) by expanding the time evolution in (34) and (35) for small $\Delta\Gamma\,t$. Including terms of order $(\Delta\Gamma)^2$, lifetime measurements in a flavour-specific decay $B_s^{\rm un} \to f_{\rm fs}$ determine [31]:

$$\Gamma_{\rm fs} = \Gamma - \frac{(\Delta\Gamma)^2}{2\Gamma} + \mathcal{O}\left(\frac{(\Delta\Gamma)^3}{\Gamma^2}\right).$$

This implies $\Gamma_{\rm fs} < \Gamma$. Despite the heavy quark symmetry prediction $\Gamma_{B_s}/\Gamma_{B_d} \simeq 1$, a large $\Delta\Gamma$ leads to an excess of the B_s lifetime measured in $B_s^{\rm un} \to f_{\rm fs}$ over the B_d lifetime [31]. From (37) one finds

$$\Gamma_{{\rm CP},\eta_{\rm f}} - \Gamma_{\rm fs} = \frac{\Delta\Gamma'_{\rm CP}}{2}\left(\eta_f + \frac{\Delta\Gamma'_{\rm CP}}{\Gamma}\right) + \mathcal{O}\left(\frac{(\Delta\Gamma)^3}{\Gamma^2}\right). \tag{39}$$

Hence for a CP-even (CP-odd) final state the quadratic corrections enlarge (diminish) the difference between the two measured widths. A measurement of $\Delta\Gamma'_{\rm CP}$ at Run-II of the Tevatron seems to be feasible. The lifetime measurement in the decay mode $B_s^{\rm un} \to J/\psi\phi$ has been studied in simulations [32, 33]. This decay mode requires an angular analysis to separate the CP-odd (P-wave) from the CP-even (S-wave and D-wave) components. The angular analysis is discussed in sect. 4.2. With 2 fb^{-1} integrated luminosity CDF expects 4000 reconstructed $B_s^{\rm un} \to J/\psi[\to \mu\overline{\mu}]\phi$ events and a measurement of $\Delta\Gamma'_{\rm CP}/\Gamma$ with an absolute error of 0.052. This simulation assumes that $\Gamma - (\Delta\Gamma)^2/(2\Gamma)$ (see (37)) will be measured from flavour-specific decays with an accuracy of 1% [33] and uses the input $\Delta\Gamma'_{\rm CP}/\Gamma = 0.15$. When combining this with other modes in Table 1 and taking into account that an integrated luminosity of 10–20 fb^{-1} is within reach of an extended (up to 2006) Run-II, the study of $\Delta\Gamma'_{\rm CP}$ at CDF looks very promising. The LHC experiments ATLAS, CMS and LHCb expect to measure $\Delta\Gamma'_{\rm CP}/\Gamma$ with absolute errors between 0.012 and 0.018 for $\Delta\Gamma'_{\rm CP}/\Gamma = 0.15$ [34]. An upper bound on $\Delta\Gamma'_{\rm CP}$ would be especially interesting. If the lattice calculations entering (31) mature and the theoretical uncertainty decreases, an upper bound on $|\Delta\Gamma'_{\rm CP}|$ may show that $\phi \neq 0, \pi$ through

$$\frac{\Delta\Gamma'_{\rm CP}}{\Delta\Gamma_{\rm CP}} = \cos^2\phi. \tag{40}$$

Note that conversely the experimental establishment of a non-zero $\Delta\Gamma'_{\rm CP}$ immediately helps to constrain models of new physics, because it excludes values of ϕ around $\pi/2$. This feature even holds true, if there is no theoretical progress in (31).

The described method to obtain $\Delta\Gamma'_{\rm CP}$ can also be used, if the sample contains a known ratio of CP-even and CP-odd components. This situation occurs e.g. in decays to $J/\psi\phi$, if no angular analysis is performed or in final states, which are neither flavour-specific nor CP eigenstates. We discuss this case below in sect. 3.3.2 with $B_s^{\rm un} \to D_s^{\pm} D_s^{(*)\mp}$. A measurement of the B_s lifetime in $B_s^{\rm un} \to J/\psi\phi$ has been performed in [35], but the error is still too large to gain information on $\Delta\Gamma'_{\rm CP}$. Note that the comparison of the lifetimes measured in CP-even and CP-odd final states determines $\Delta\Gamma'_{\rm CP}$ up to corrections of order $(\Delta\Gamma/\Gamma)^3$.

3.3.2 Determination of $|\Delta\Gamma|$ and $|\cos\phi|$

The theoretical uncertainty in (31) dilutes the extraction of $|\cos\phi|$ from a measurement of $\Delta\Gamma'_{\rm CP}$ alone. One can bypass the theory prediction in (31) altogether by measuring both $\Delta\Gamma'_{\rm CP}$ and $|\Delta\Gamma|$ and determine $|\cos\phi|$ through

$$\frac{\Delta\Gamma'_{\rm CP}}{|\Delta\Gamma|} = |\cos\phi|. \tag{41}$$

To obtain additional information on $\Delta\Gamma$ and ϕ from the time evolution in (23) requires more statistics: the coefficient of t in (24), $\Delta\Gamma\,\mathcal{A}_{\Delta\Gamma}/2$, vanishes in flavour-specific decays and is equal to $-\eta_f \Delta\Gamma'_{\rm CP}/2$ in the CP-specific decays of Table 1. Therefore the data sample must be large enough to be sensitive to the terms of order $(\Delta\Gamma\, t)^2$ in order to get new information on $\Delta\Gamma$ and ϕ. We now list three methods to determine $|\Delta\Gamma|$ and $|\cos\phi|$ separately. The theoretical uncertainty decreases and the required experimental statistics increases from method 1 to method 3. Hence as the collected data sample grows, one can work off our list downwards. The first method exploits information from branching ratios and needs no information from the quadratic $(\Delta\Gamma\, t)^2$ terms.

Method 1: We assume that $\Delta\Gamma'_{\rm CP}$ has been measured as described in sect. 3.3.1. The method presented now is a measurement of $\Delta\Gamma_{\rm CP}$ using the information from branching ratios. With (40) one can then find $|\cos\phi|$ and subsequently $|\Delta\Gamma|$ from (41). In the SV limit the branching ratio $Br[D_s^{(*)+}D_s^{(*)-}]$ equals $\Delta\Gamma_{\rm CP}/(2\Gamma)$ up to corrections of order $\Delta\Gamma/\Gamma$, as discussed in sect. 3.2 [16]. Corrections to the SV limit, however, can be sizeable. Yet we stress that one can control the corrections to this limit experimentally, successively arriving at a result which does not rely on the validity of the SV limit. For this it is of prime importance to determine the CP-odd component of the final states $D_s^{\pm}D_s^{*\mp}$ and $D_s^{*+}D_s^{*-}$. We now explain how the CP-odd and CP-even component of any decay $B_s^{\rm un} \to f$ corresponding to the quark level transition $b \to c\overline{c}s$ can be obtained. This simply requires a fit of the time evolution of the decay to a single exponential, as in (35). Define the contributions of the CP-odd and CP-even eigenstate to $B_s \to f$:

$$\Gamma(B_s^{\rm odd} \to f) \;\equiv\; \mathcal{N}_f\, |\langle f |\, B_s^{\rm odd} \rangle|^2, \qquad\qquad \Gamma(B_s^{\rm even} \to f) \;\equiv\; \mathcal{N}_f\, |\langle f |\, B_s^{\rm even} \rangle|^2. \tag{42}$$

It is useful to define the CP-odd fraction x_f by

$$\frac{\Gamma(B_s^{\rm odd} \to f)}{\Gamma(B_s^{\rm even} \to f)} = \frac{\left|\langle f |\, B_s^{\rm odd} \rangle\right|^2}{|\langle f |\, B_s^{\rm even} \rangle|^2} \;=\; \frac{\left|\langle \overline{f} |\, B_s^{\rm odd} \rangle\right|^2}{\left|\langle \overline{f} |\, B_s^{\rm even} \rangle\right|^2} \;=\; \frac{x_f}{1-x_f}. \tag{43}$$

The time evolution $(\Gamma[f,t]+\Gamma[\overline{f},t])/2$ of the CP-averaged untagged decay $B_s^{\rm un} \to f, \overline{f}$ is governed by a two-exponential formula:

$$\frac{\Gamma[f,t]+\Gamma[\overline{f},t]}{2} = A(f)\, e^{-\Gamma_L t} + B(f)\, e^{-\Gamma_H t}. \tag{44}$$

With (18) and (19) one finds

$$\begin{aligned} A(f) &= \frac{\mathcal{N}_f}{2}\, |\langle f |\, B_L \rangle|^2 + \frac{\mathcal{N}_f}{2}\, |\langle \overline{f} |\, B_L \rangle|^2 \\ &= \frac{1+\cos\phi}{2}\, \Gamma(B_s^{\rm even} \to f) \,+\, \frac{1-\cos\phi}{2}\, \Gamma(B_s^{\rm odd} \to f) \end{aligned}$$

$$B(f) = \frac{\mathcal{N}_f}{2}\,|\langle f|\,B_H\,\rangle|^2 \;+\; \frac{\mathcal{N}_f}{2}\,|\langle \overline{f}|\,B_H\,\rangle|^2$$
$$= \frac{1-\cos\phi}{2}\,\Gamma(B_s^{\text{even}} \to f) \;+\; \frac{1+\cos\phi}{2}\,\Gamma(B_s^{\text{odd}} \to f). \tag{45}$$

With (43) we arrive at

$$\frac{A(f)}{B(f)} = \frac{(1+\cos\phi)\Gamma(B_s^{\text{even}} \to f) + (1-\cos\phi)\Gamma(B_s^{\text{odd}} \to f)}{(1-\cos\phi)\Gamma(B_s^{\text{even}} \to f) + (1+\cos\phi)\Gamma(B_s^{\text{odd}} \to f)} \;=\; \frac{1+(1-2x_f)\cos\phi}{1-(1-2x_f)\cos\phi}. \tag{46}$$

In (45) and (46) it is crucial that we average the decay rates for $B_s^{\text{un}} \to f$ and the CP-conjugate process $B_s^{\text{un}} \to \overline{f}$. This eliminates the interference term $\langle\, B_s^{\text{odd}}\,|f\rangle\langle\, f\,|B_s^{\text{even}}\rangle$, so that $A(f)/B(f)$ only depends on x_f. The single exponential fit with (35) determines Γ_f. Equations (37) and (46) combine to give

$$2\,(\Gamma_f - \Gamma) = (1-2x_f)\,\Delta\Gamma\,\cos\phi \;=\; (1-2x_f)\,\Delta\Gamma_{\text{CP}}\,\cos^2\phi \;=\; (1-2x_f)\,\Delta\Gamma'_{\text{CP}} \tag{47}$$

up to corrections of order $(\Delta\Gamma)^2/\Gamma$. In order to determine x_f from (47) we need $\Delta\Gamma'_{\text{CP}}$ from the lifetime measurement in a CP-specific final state like $D_s^+ D_s^-$ or from the angular separation of the CP components in $B_s^{\text{un}} \to \psi\phi$. The corrections of order $(\Delta\Gamma)^2/\Gamma$ to (47) can be read off from (37) with (46) as well. Expressing the result in terms of Γ_f and the rate Γ_{fs} measured in flavour-specific decays, we find

$$1-2x_f \,=\, 2\,\frac{\Gamma_f - \Gamma_{\text{fs}}}{\Delta\Gamma'_{\text{CP}}}\,\left[1 \,-\, 2\,\frac{\Gamma_f - \Gamma_{\text{fs}}}{\Gamma}\right] + \mathcal{O}\left(\frac{(\Delta\Gamma)^2}{\Gamma^2}\right). \tag{48}$$

In order to solve for $\Gamma(B_s^{\text{even}} \to f)$ and $\Gamma(B_s^{\text{odd}} \to f)$ we also need the branching ratio $Br[f]+Br[\overline{f}]$. Recalling (22) one finds from (44) and (45):

$$Br[f] + Br[\overline{f}] = \quad \Gamma(B_s^{\text{even}} \to f)\left[\frac{1+\cos\phi}{2\Gamma_L} + \frac{1-\cos\phi}{2\Gamma_H}\right]$$
$$+\,\Gamma(B_s^{\text{odd}} \to f)\left[\frac{1-\cos\phi}{2\Gamma_L} + \frac{1+\cos\phi}{2\Gamma_H}\right]. \tag{49}$$

By combining (43) and (49) we can solve for the two CP components:

$$\Gamma(B_s^{\text{even}} \to f) = \left[\Gamma^2 - (\Delta\Gamma/2)^2\right]\,\left(Br[f] + Br[\overline{f}]\right)\,\frac{1-x_f}{2\Gamma \,-\, \Gamma_f}$$
$$= (1-x_f)\,\left(Br[f] + Br[\overline{f}]\right)\,\Gamma + \,\mathcal{O}\,(\Delta\Gamma)$$
$$\Gamma(B_s^{\text{odd}} \to f) = \left[\Gamma^2 - (\Delta\Gamma/2)^2\right]\,\left(Br[f] + Br[\overline{f}]\right)\,\frac{x_f}{2\Gamma \,-\, \Gamma_f}$$
$$= x_f\,\left(Br[f] + Br[\overline{f}]\right)\,\Gamma \,+\, \mathcal{O}\,(\Delta\Gamma)\,.$$

From (28) we now find the desired quantity by summing over all final states f:

$$\Delta\Gamma_{\rm CP} \;=\; \Gamma\left(B_s^{\rm even}\right) - \Gamma\left(B_s^{\rm odd}\right) = 2\left[\Gamma^2 - (\Delta\Gamma/2)^2\right] \sum_{f\in X_{c\bar{c}}} Br[f]\,\frac{1-2\,x_f}{2\Gamma\,-\,\Gamma_f} \tag{50}$$

$$= 2\,\Gamma \sum_{f\in X_{c\bar{c}}} Br[f]\,(1-2\,x_f)\,\left[1\,+\,\mathcal{O}\left(\frac{\Delta\Gamma}{\Gamma}\right)\right]. \tag{51}$$

It is easy to find $\Delta\Gamma_{\rm CP}$: first determine $1-2x_f$ from (48) for each studied decay mode, then insert the result into (50). The small quadratic term $(\Delta\Gamma/2)^2 = \Delta\Gamma_{\rm CP}\Delta\Gamma_{\rm CP}'/4$ is negligible. This procedure can be performed for $Br[D_s^{\pm}D_s^{*\mp}]$ and $Br[D_s^{*+}D_s^{*-}]$ to determine the corrections to the SV limit. In principle the CP-odd P-wave component of $Br[D_s^{*+}D_s^{*-}]$ (which vanishes in the SV limit) could also be obtained by an angular analysis, but this is difficult in first-generation experiments at hadron colliders, because the photon from $D_s^* \to D_s\gamma$ cannot be detected. We emphasize that it is not necessary to separate the $D_s^{(*)+}D_s^{(*)-}$ final states; our method can also be applied to the semi-inclusive $D_s^{(*)\pm}D_s^{(*)\mp}$ sample, using $\Delta\Gamma_{\rm CP}'$ obtained from an angular separation of the CP components in $B_s^{\rm un} \to \psi\phi$. Further one can successively include those double-charm final states which vanish in the SV limit into (50). If we were able to reconstruct all $b\to c\bar{c}s$ final states, we could determine $\Delta\Gamma_{\rm CP}$ without invoking the SV limit. In practice a portion of these final states will be missed, but the induced error can be estimated from the corrections to the SV limit in the measured decay modes. By comparing $\Delta\Gamma_{\rm CP}$ and $\Delta\Gamma_{\rm CP}'$ one finds $|\cos\phi|$ from (40). The irreducible theoretical error of method 1 stems from the omission of CKM-suppressed decays and is of order $2|V_{ub}V_{us}/(V_{cb}V_{cs})| \sim 5\%$.

Method 1 is experimentally simple: at the first stage (relying on the SV limit) it amounts to counting the $B_s^{\rm un}$ decays into $D_s^{(*)+}D_s^{(*)-}$. A first simulation indicates that CDF will be able to separate the B_s decay modes into $D_s^+D_s^-$, $D_s^{*\pm}D_s^{\mp}$ and $D_s^{*+}D_s^{*-}$ [36]. The corrections to the SV limit are obtained by one-parameter fits to the time evolution of the collected double-charm data samples. This sample may include final states from decay modes which vanish in the SV limit, such as multiparticle final states. No sensitivity to $(\Delta\Gamma\,t)^2$ is needed. A further advantage is that $\Delta\Gamma_{\rm CP}$ is not diminished by the presence of new physics.

Method 2: In the Standard Model the decay into a CP eigenstate $f_{\rm CP}$ is governed by a single exponential. If a second exponential is found in the time evolution of a CKM-favoured decay $B_s^{\rm un} \to f_{\rm CP}$, this will be clear evidence of new physics [20]. To this end we must resolve the time evolution in (23) up to order $(\Delta\Gamma\,t)^2$. At first glance this seems to require a three-parameter fit to the data, because $\Gamma[f,t]$ in (23) depends on Γ, $\Delta\Gamma$ and (through $\mathcal{A}_{\Delta\Gamma}$, see (17)) on ϕ. It is possible, however, to choose these parameters in such a way that one of them enters $\Gamma\left[f_{\rm CP},t\right]$ at order $(\Delta\Gamma)^3$, with negligible impact. The fit parameters are Γ' and Y. They are chosen such that

$$\Gamma[f_{\rm CP+},t] = 2\,Br[f_{\rm CP+}]\;\Gamma' e^{-\Gamma' t}\,\left[1 + Y\,\Gamma'\,t\,\left(-1+\frac{\Gamma' t}{2}\right) + \mathcal{O}\left((\Delta\Gamma)^3\right)\right]. \tag{52}$$

Here we have considered a CP-even final state, for which a lot more data are expected than for CP-odd states. With (52) we have generalized the lifetime fit method described in sect. 3.3.1 to the order $(\Delta\Gamma\, t)^2$. A non-zero Y signals the presence of new physics. The fitted rate Γ' and Y are related to Γ, $\Delta\Gamma$ and ϕ by

$$Y \;=\; \frac{(\Delta\Gamma)^2}{4\Gamma'^2}\sin^2\phi, \qquad\qquad \Gamma' = \Gamma(1-Y) + \frac{\cos\phi}{2}\Delta\Gamma. \tag{53}$$

Note that for $|\cos\phi| = 1$ the rate Γ' equals the rate of the shorter-lived mass eigenstate and the expansion in (52) becomes the exact single-exponential formula. After determining Γ' and Y we can solve (53) for Γ, $\Delta\Gamma$ and ϕ. To this end we need the width Γ_{fs} measured in flavour-specific decays. We find

$$|\Delta\Gamma| \;=\; 2\sqrt{(\Gamma'-\Gamma_{\text{fs}})^2+\Gamma_{\text{fs}}^2 Y}\left[1+\mathcal{O}\left(\frac{\Delta\Gamma}{\Gamma}\right)\right], \qquad \Gamma \;=\; \Gamma_{\text{fs}} + \frac{(\Delta\Gamma)^2}{2\Gamma} + \mathcal{O}\left(\left(\frac{\Delta\Gamma}{\Gamma}\right)^3\right)$$

$$\Delta\Gamma'_{\text{CP}} \;=\; 2\left[\Gamma' - \Gamma\,(1-Y)\right]\left[1+\mathcal{O}\left(\left(\frac{\Delta\Gamma}{\Gamma}\right)^2\right)\right], \qquad |\sin\phi| \;=\; \frac{2\Gamma\sqrt{Y}}{|\Delta\Gamma|}\left[1+\mathcal{O}\left(\frac{\Delta\Gamma}{\Gamma}\right)\right]. \tag{54}$$

The quantity $\Delta\Gamma'_{\text{CP}}$, which we could already determine from single-exponential fits, is now found beyond the leading order in $\Delta\Gamma/\Gamma$. By contrast, $\Delta\Gamma$ and $|\sin\phi|$ in (54) are only determined to the first non-vanishing order in $\Delta\Gamma/\Gamma$.

In conclusion method 2 involves a two-parameter fit and needs sensitivity to the quadratic term in the time evolution. The presence of new physics can be invoked from $Y \neq 0$ and does not require to combine lifetime measurements in different decay modes.

Method 3: Originally the following method has been proposed to determine $|\Delta\Gamma|$ [20, 21]: The time evolution of a B_s^{un} decay into a flavour-specific final state is fitted to two exponentials. This amounts to resolving the deviation of $\cosh(\Delta\Gamma\, t/2)$ from 1 in (23) in a two-parameter fit for Γ and $|\Delta\Gamma|$. If one adopts the same parametrization as in (52), Γ' and Y are obtained from (53) by replacing ϕ with $\pi/2$. The best suited flavour-specific decay modes at hadron colliders are $B_s^{\text{un}} \to D_s^{(*)\pm}\pi^{\mp}$, $B_s^{\text{un}} \to D_s^{(*)\pm}\pi^{\mp}\pi^{+}\pi^{-}$ and $B_s^{\text{un}} \to D_s^{(*)\pm}X\ell^{\mp}\nu$. Depending on the event rate in these modes, method 3 could be superior to method 2 in terms of statistics. On the other hand, to find the "smoking gun" of new physics, the $|\Delta\Gamma|$ obtained must be compared to $\Delta\Gamma'_{\text{CP}}$ from CP-specific decays to prove $|\cos\phi| \neq 1$ through (41). Since the two measurements are differently affected by systematic errors, this can be a difficult task. First upper bounds on $|\Delta\Gamma|$ using method 3 have been obtained in [37].

The L3 collaboration has determined an upper bound $|\Delta\Gamma|/\Gamma \leq 0.67$ by fitting the time evolution of fully inclusive decays to two exponentials [38]. This method is quadratic in $\Delta\Gamma$ as well. The corresponding formula for the time evolution can be simply obtained from (34) with $A = \Gamma_L$ and $B = \Gamma_H$.

3.4 CP Violation in Mixing and Untagged Oscillations

In the preceding sections we have set the small parameter a in (8) to zero. CP violation in mixing vanishes in this limit. The corresponding "wrong-sign" CP asymmetry is measured in flavour-specific decays and equals

$$a_{\rm fs} = \frac{\Gamma(\overline{B}_s(t) \to f) - \Gamma(B_s(t) \to \overline{f})}{\Gamma(\overline{B}_s(t) \to f) + \Gamma(B_s(t) \to \overline{f})} \;=\; a \qquad \text{for} \quad \overline{A}_f = 0 \quad \text{and} \quad |A_f| = |\overline{A}_{\overline{f}}|. \tag{55}$$

A special case of $a_{\rm fs}$ is the semileptonic asymmetry, where $f = X\ell^+\nu$. A determination of a gives additional information on the three physical quantities $|M_{12}|$, $|\Gamma_{12}|$ and ϕ characterizing B_s–$\overline{B}_s$ mixing. Measuring Δm, $\Delta\Gamma_{\rm CP}$, $\Delta\Gamma'_{\rm CP}$ and a overconstrains these quantities.

The "right-sign" asymmetry vanishes:

$$\Gamma(B_s(t) \to f) - \Gamma(\overline{B}_s(t) \to \overline{f}) \;=\; 0 \qquad \text{for} \quad \overline{A}_f = 0 \quad \text{and} \quad |A_f| = |\overline{A}_{\overline{f}}|. \tag{56}$$

This implies that one can measure $a_{\rm fs}$ from *untagged* decays. This observation was already made in [39]. It is easily verified from the sum of (12) and (13) that to order a the time evolution of untagged decays exhibits oscillations governed by Δm. Since a is small, one must be concerned to which accuracy $|A_f| = |\overline{A}_{\overline{f}}|$ holds in flavour-specific decays in the presence of new physics. For example in left–right-symmetric extensions of the Standard Model, small CP-violating corrections to the decay amplitude could eventually spoil this relation at the few per mille level. Further, a small production asymmetry $\epsilon = N_{\overline{B}}/N_B - 1$ also leads to oscillations in the untagged sample. To first order in the small parameters a, ϵ and $|A_f|/|\overline{A}_{\overline{f}}| - 1$ one finds

$$\begin{aligned} a_{\rm fs}^{unt} &= \frac{\Gamma[f,t] - \Gamma[\overline{f},t]}{\Gamma[f,t] + \Gamma[\overline{f},t]} \\ &= \frac{|A_f|^2 - |\overline{A}_{\overline{f}}|^2}{|A_f|^2 + |\overline{A}_{\overline{f}}|^2} + \frac{a}{2} - \frac{a+\epsilon}{2}\,\frac{\cos(\Delta m\, t)}{\cosh(\Delta\Gamma t/2)} \qquad \text{for} \quad \overline{A}_f = 0 \quad \text{and} \quad |A_f| \approx |\overline{A}_{\overline{f}}|. \end{aligned} \tag{57}$$

For $|A_f| = |\overline{A}_{\overline{f}}|$ and $\epsilon = 0$ one recovers the formula derived in [39]. Note that the production asymmetry between B_s and $\overline{B}_s$ cannot completely fake the effect of a non-zero a in (57): while both $a \neq 0$ and $\epsilon \neq 0$ lead to oscillations, the offset from the constant term indicates new CP-violating physics either in B_s–$\overline{B}_s$ mixing (through $a \neq 0$) or in the studied decay amplitude (through $|A_f| \neq |\overline{A}_{\overline{f}}|$). The latter effect, which is theoretically much less likely, can be tested in $B^\pm$ decays and can therefore be disentangled from $a \neq 0$.

The ratio $\Delta\Gamma_{\rm CP}/\Gamma \leq 0.22$ from (31) and the current experimental limit $\Delta m \geq 14.9\,{\rm ps}^{-1}$ [40] imply that $|a| \leq 0.01$. CDF expects sufficiently many reconstructed $B_s^{\rm un} \to D_s^{(*)\pm}\pi^\mp$ and $B_s^{\rm un} \to D_s^{(*)\pm}\pi^\mp\pi^+\pi^-$ events at Run-II after collecting 2 fb^{-1} of integrated luminosity to achieve a statistical error at the few permille level. From (8) and (6) we can relate a to $|\Delta\Gamma|$, Δm and ϕ:

$$a = \frac{|\Delta\Gamma|}{\Delta m}\,\frac{\sin\phi}{|\cos\phi|}.$$

Note, however, that the measurement of the sign of a determines the sign of $\sin\phi$. This reduces the four-fold ambiguity in ϕ from the measurement of $|\cos\phi|$ to a two-fold one. It is interesting that, at order a, without tagging one can in principle gain information which otherwise requires tagged studies. Of course $\sin\phi$ can be measured more directly from tagged decays, as discussed in the forthcoming section 4.

4 Tagged Decays

4.1 The CP-Violating Observables of $B_s \to D_s^+ D_s^-$ and $J/\psi\,\eta^{(\prime)}$

For a B_s decay into a CP eigenstate f the B_s–$\overline{B_s}$ oscillations lead to the following time-dependent CP asymmetry:

$$a_{\rm CP}(t) \equiv \frac{\Gamma(\overline{B_s}(t)\to f)-\Gamma(B_s(t)\to f)}{\Gamma(B_s(t)\to f)+\Gamma(\overline{B_s}(t)\to f)} \;=\; -\,\frac{\mathcal{A}_{\rm CP}^{\rm dir}\cos(\Delta m\,t)+\mathcal{A}_{\rm CP}^{\rm mix}\sin(\Delta m\,t)}{\cosh\left(\Delta\Gamma\,t/2\right)+\mathcal{A}_{\Delta\Gamma}\sinh\left(\Delta\Gamma\,t/2\right)}. \quad (58)$$

Here the mass and width difference Δm and $\Delta\Gamma$ can be found in (5) and $\mathcal{A}_{\rm CP}^{\rm dir}$, $\mathcal{A}_{\rm CP}^{\rm mix}$ and $\mathcal{A}_{\Delta\Gamma}$ have been defined in (10). We have set the small parameter a in (8) to zero and will continue to do so. The final states $B_s\to D_s^+D_s^-,\ \psi\,\eta^{(\prime)}, \psi f_0$ or $\chi_{c0}\phi$ in Table 1 are CP eigenstates. Their CP eigenvalue η_f reads $\eta_{D_s^+D_s^-}=\eta_{\psi\eta'}=\eta_{\psi\eta}=+1$ and $\eta_{\psi f_0}=\eta_{\chi_{c0}\phi}=-1$. With (17) we then find from (58):

$$a_{\rm CP}(t) = -\,\frac{\eta_f\sin\phi\,\sin(\Delta m\,t)}{\cosh\left(\Delta\Gamma\,t/2\right)-\eta_f|\cos\phi|\sinh\left(|\Delta\Gamma|\,t/2\right)}. \quad (59)$$

Since $\Delta\Gamma$ and $\cos\phi$ have the same sign (see (29)) we could replace these quantities by their absolute values in the denominator of (59). This displays that the ambiguity in the sign of $\cos\phi$ cannot be removed by measuring $a_{\rm CP}$. Its measurement determines $\sin\phi$ and leaves us with a two-fold ambiguity in ϕ. Then we still do not know whether the heavier or lighter mass eigenstate is shorter-lived. The resolution of this ambiguity will be discussed in Section 5.

4.2 The CP-violating Observables of $B_s \to J/\psi\,\phi$ and $D_s^{*+}D_s^{*-}$

The situation in the decay $B_s\to J/\psi\,\phi$, which is very promising for B-physics experiments at hadron machines because of its nice experimental signature, is a bit more involved than in the case of the pseudoscalar–pseudoscalar modes $B_s\to D_s^+D_s^-$ and $J/\psi\,\eta^{(\prime)}$, since the final state is an admixture of different CP eigenstates. In order to disentangle them, we have to make use of the angular distribution of the decay products of the decay chain $B_s\to J/\psi[\to l^+l^-]\,\phi[\to K^+K^-]$, which can be found in [11, 12]. In that paper, also appropriate weighting functions are given to extract the observables of the angular distribution in an efficient way from the experimental data. For an initially, i.e. at time $t=0$, present B_s-meson, the time-dependent angular distribution can be written generically as

$$f(\Theta, \Phi, \Psi; t) = \sum_k \mathcal{O}^{(k)}(t)\, g^{(k)}(\Theta, \Phi, \Psi), \tag{60}$$

where we have denoted the angles describing the kinematics of the decay products of $J/\psi \to l^+l^-$ and $\phi \to K^+K^-$ by Θ, Φ and Ψ. The observables $\mathcal{O}^{(k)}(t)$ describing the time evolution of the angular distribution (60) can be expressed in terms of real or imaginary parts of certain bilinear combinations of decay amplitudes. In the case of decays into two vector mesons, such as $B_s \to J/\psi\,\phi$, it is convenient to introduce linear polarization amplitudes $A_0(t)$, $A_\parallel(t)$ and $A_\perp(t)$ [41]. Whereas $A_\perp(t)$ describes a CP-odd final-state configuration, both $A_0(t)$ and $A_\parallel(t)$ correspond to CP-even final-state configurations. The observables $\mathcal{O}^{(k)}(t)$ of the corresponding angular distribution are given by

$$|A_f(t)|^2 \quad \text{with} \quad f \in \{0, \parallel, \perp\}, \tag{61}$$

as well as by the interference terms

$$\mathrm{Re}\,\{A_0^*(t)A_\parallel(t)\} \quad \text{and} \quad \mathrm{Im}\,\{A_f^*(t)A_\perp(t)\} \quad \text{with} \quad f \in \{0, \parallel\}. \tag{62}$$

For our consideration, the time evolution of these observables plays a crucial role. In the case of the observables (61), which correspond to "ordinary" decay rates, we obtain

$$|A_0(t)|^2 = |A_0(0)|^2 e^{-\Gamma t}\left[\cosh\frac{\Delta\Gamma\, t}{2} - |\cos\phi|\sinh\frac{|\Delta\Gamma|\, t}{2} + \sin\phi\,\sin(\Delta m\, t)\right] \tag{63}$$

$$|A_\parallel(t)|^2 = |A_\parallel(0)|^2 e^{-\Gamma t}\left[\cosh\frac{\Delta\Gamma\, t}{2} - |\cos\phi|\sinh\frac{|\Delta\Gamma|\, t}{2} + \sin\phi\,\sin(\Delta m\, t)\right] \tag{64}$$

$$|A_\perp(t)|^2 = |A_\perp(0)|^2 e^{-\Gamma t}\left[\cosh\frac{\Delta\Gamma\, t}{2} + |\cos\phi|\sinh\frac{|\Delta\Gamma|\, t}{2} - \sin\phi\,\sin(\Delta m\, t)\right], \tag{65}$$

whereas we have in the case of the interference terms (62):

$$\begin{aligned}\mathrm{Re}\,\{A_0^*(t)A_\parallel(t)\} &= |A_0(0)|\,|A_\parallel(0)|\,\cos(\delta_2-\delta_1)\,e^{-\Gamma t}\\ &\times\left[\cosh\frac{\Delta\Gamma\, t}{2} - |\cos\phi|\sinh\frac{|\Delta\Gamma|\, t}{2} + \sin\phi\,\sin(\Delta m\, t)\right]\end{aligned} \tag{66}$$

$$\begin{aligned}\mathrm{Im}\,\{A_\parallel^*(t)A_\perp(t)\} &= |A_\parallel(0)|\,|A_\perp(0)|\,e^{-\Gamma t}\\ &\times\left[\sin\delta_1\,\cos(\Delta m\, t) - \cos\delta_1\,\cos\phi\,\sin(\Delta m\, t) - \cos\delta_1\,\sin\phi\,\sinh\frac{\Delta\Gamma\, t}{2}\right]\end{aligned} \tag{67}$$

$$\begin{aligned}\mathrm{Im}\,\{A_0^*(t)A_\perp(t)\} &= |A_0(0)|\,|A_\perp(0)|\,e^{-\Gamma t}\\ &\times\left[\sin\delta_2\,\cos(\Delta m\, t) - \cos\delta_2\,\cos\phi\,\sin(\Delta m\, t) - \cos\delta_2\,\sin\phi\,\sinh\frac{\Delta\Gamma\, t}{2}\right]\end{aligned} \tag{68}$$

In (66)–(68), δ_1 and δ_2 denote CP-conserving strong phases, which are defined as follows [11, 12]:

$$\delta_1 \equiv \arg\Big\{A_\parallel(0)^* A_\perp(0)\Big\}, \quad \delta_2 \equiv \arg\Big\{A_0(0)^* A_\perp(0)\Big\}. \tag{69}$$

The time evolutions (63)–(68) generalize those given in [11, 12] to the case of a sizeable B_s–$\overline{B}_s$ mixing phase ϕ to cover the pursued case of new physics. A further generalization taking into account also the small penguin contributions can be found in [42]. It should be emphasized that new physics manifests itself *only* in the observables $\mathcal{O}^{(k)}(t)$, while the $g^{(k)}(\Theta,\Phi,\Psi)$'s are not affected.

We may use the same angles Θ, Φ and Ψ to describe the kinematics of the decay products of the CP-conjugate transition $\overline{B_s} \to J/\psi\,\phi$. Consequently, we have

$$\overline{f}(\Theta,\Phi,\Psi;t) = \sum_k \overline{\mathcal{O}}^{(k)}(t)\, g^{(k)}(\Theta,\Phi,\Psi). \tag{70}$$

Within this formalism, CP transformations relating $B_s \to [J/\psi\,\phi]_f$ to $\overline{B_s} \to [J/\psi\,\phi]_f$ $(f \in \{0, \|, \perp\})$ are taken into account in the expressions for the $\mathcal{O}^{(k)}(t)$ and $\overline{\mathcal{O}}^{(k)}(t)$, and do not affect the form of the $g^{(k)}(\Theta,\Phi,\Psi)$. Therefore the same functions $g^{(k)}(\Theta,\Phi,\Psi)$ are present in (60) and (70) (see also [43, 44]). The CP-conjugate observables $\overline{\mathcal{O}}^{(k)}(t)$ take the following form:

$$|\overline{A}_0(t)|^2 = |A_0(0)|^2 e^{-\Gamma t}\left[\cosh\frac{\Delta\Gamma\, t}{2} - |\cos\phi|\sinh\frac{|\Delta\Gamma|\, t}{2} - \sin\phi\,\sin(\Delta m\, t)\right] \tag{71}$$

$$|\overline{A}_\parallel(t)|^2 = |A_\parallel(0)|^2 e^{-\Gamma t}\left[\cosh\frac{\Delta\Gamma\, t}{2} - |\cos\phi|\sinh\frac{|\Delta\Gamma|\, t}{2} - \sin\phi\,\sin(\Delta m\, t)\right] \tag{72}$$

$$|\overline{A}_\perp(t)|^2 = |A_\perp(0)|^2 e^{-\Gamma t}\left[\cosh\frac{\Delta\Gamma\, t}{2} + |\cos\phi|\sinh\frac{|\Delta\Gamma|\, t}{2} + \sin\phi\,\sin(\Delta m\, t)\right] \tag{73}$$

$$\begin{aligned}\mathrm{Re}\,\{\overline{A}_0^*(t)\overline{A}_\parallel(t)\} &= |A_0(0)|\,|A_\parallel(0)|\,\cos(\delta_2-\delta_1)\,e^{-\Gamma t}\\ &\quad\times\left[\cosh\frac{\Delta\Gamma\, t}{2} - |\cos\phi|\sinh\frac{|\Delta\Gamma|\, t}{2} - \sin\phi\,\sin(\Delta m\, t)\right]\end{aligned} \tag{74}$$

$$\begin{aligned}\mathrm{Im}\,\{\overline{A}_\parallel^*(t)\overline{A}_\perp(t)\} &= |A_\parallel(0)|\,|A_\perp(0)|\,e^{-\Gamma t}\\ &\quad\times\left[-\sin\delta_1\,\cos(\Delta m\, t) + \cos\delta_1\,\cos\phi\,\sin(\Delta m\, t) - \cos\delta_1\,\sin\phi\,\sinh\frac{\Delta\Gamma\, t}{2}\right]\end{aligned} \tag{75}$$

$$\begin{aligned}\mathrm{Im}\,\{\overline{A}_0^*(t)\overline{A}_\perp(t)\} &= |A_0(0)|\,|A_\perp(0)|\,e^{-\Gamma t}\\ &\quad\times\left[-\sin\delta_2\,\cos(\Delta m\, t) + \cos\delta_2\,\cos\phi\,\sin(\Delta m\, t) - \cos\delta_2\,\sin\phi\,\sinh\frac{\Delta\Gamma\, t}{2}\right]\end{aligned} \tag{76}$$

Note that one can determine $\sin\delta_{1,2}$, $\cos(\delta_1-\delta_2)$, $\sin\phi$, $\cos\delta_i\cos\phi$, Δm and $|\Delta\Gamma|$ from (63)-(76). Using $\cos(\delta_2-\delta_1)=\cos\delta_1\cos\delta_2+\sin\delta_1\sin\delta_2$ in (66) and (74) one realizes that these equations are invariant, if the signs of $\cos\phi$, $\Delta\Gamma$, and $\cos\delta_{1,2}$ are flipped simultaneously. Hence an overall two-fold sign ambiguity persists and the sign of $\cos\phi$ remains undetermined.

The time evolution of the full three-angle distribution of the products of the decay chain $B_s\to J/\psi[\to\ l^+l^-]\,\phi[\to\ K^+K^-]$ provides many interesting CP-violating observables [12, 42]. The expressions for three-angle angular distributions can be obtained by inserting (63-76) into Eqs. (64) and (70) of [12].

The situation is considerably simplified in the case of the one-angle distribution, which takes the following form [11, 12]:

$$\frac{d\Gamma(t)}{d\cos\Theta}\propto\left(|A_0(t)|^2+|A_\parallel(t)|^2\right)\frac{3}{8}\left(1+\cos^2\Theta\right)+|A_\perp(t)|^2\,\frac{3}{4}\sin^2\Theta\,. \tag{77}$$

Here Θ describes the angle between the decay direction of the l^+ and the z axis in the J/ψ rest frame; the z axis is perpendicular to the decay plane of $\phi\to K^+K^-$. With the help of this one-angle distribution, the observables $|A_0(t)|^2+|A_\parallel(t)|^2$ and $|A_\perp(t)|^2$, as well as their CP conjugates, can be determined. They provide the following CP asymmetries:

$$\frac{\left[|\overline{A}_0(t)|^2+|\overline{A}_\parallel(t)|^2\right]-\left[|A_0(t)|^2+|A_\parallel(t)|^2\right]}{\left[|\overline{A}_0(t)|^2+|\overline{A}_\parallel(t)|^2\right]+\left[|A_0(t)|^2+|A_\parallel(t)|^2\right]}\;=\;\frac{-\sin\phi\,\sin(\Delta m\,t)}{\cosh(\Delta\Gamma\,t/2)-|\cos\phi|\sinh(|\Delta\Gamma|\,t/2)} \tag{78}$$

$$\frac{|\overline{A}_\perp(t)|^2-|A_\perp(t)|^2}{|\overline{A}_\perp(t)|^2+|A_\perp(t)|^2}\;=\;\frac{\sin\phi\,\sin(\Delta m\,t)}{\cosh(\Delta\Gamma\,t/2)+|\cos\phi|\sinh(|\Delta\Gamma|\,t/2)}. \tag{79}$$

In contrast to these CP-violating observables, untagged data samples are sufficient to determine the following quantities:

$$\begin{aligned}&\left[|A_0(t)|^2+|A_\parallel(t)|^2\right]+\left[|\overline{A}_0(t)|^2+|\overline{A}_\parallel(t)|^2\right]\\&\quad=\;2\,\left[|A_0(0)|^2+|A_\parallel(0)|^2\right]\,e^{-\Gamma t}\,\left[\cosh\frac{\Delta\Gamma\,t}{2}-|\cos\phi|\sinh\frac{|\Delta\Gamma|\,t}{2}\right]\end{aligned} \tag{80}$$

$$|A_\perp(t)|^2+|\overline{A}_\perp(t)|^2\;=\;2\,|A_\perp(0)|^2\,e^{-\Gamma t}\,\left[\cosh\frac{\Delta\Gamma\,t}{2}+|\cos\phi|\sinh\frac{|\Delta\Gamma|\,t}{2}\right]. \tag{81}$$

Since ϕ is tiny in the Standard Model, a striking signal of new-physics contributions to B_s–$\overline{B_s}$ mixing would be provided by a sizeable $\sin\phi$ either from a fit of the tagged observables (63) – (68), (71) – (76), or from the CP-violating asymmetries in (59), (78) and (79), or if the untagged observables (80) and (81) should depend on *two* exponentials. Note that in (80) the coefficient of $\sinh(|\Delta\Gamma|\,t/2)$ is always negative. Phrased differently, the coefficient of the exponential $\exp(-(\Gamma+|\Delta\Gamma|/2)t)$ with the larger rate is always larger than the coefficient of $\exp(-(\Gamma - |\Delta\Gamma|/2)t)$. In (81) the situation is reversed. This feature can be used as an experimental consistency check, once $\Delta\Gamma \neq 0$ is established.

Let us finally note that the formalism developed in this subsection applies also to the mode $B_s \to D_s^{*+} D_s^{*-}$, where the subsequent decay of the $D_s^{*\pm}$-mesons is predominantly electromagnetic, i.e. $D_s^{*\pm} \to D_s^{\pm}\gamma$. The corresponding angular distribution can be found in [11, 12]. The analysis of this decay requires the capability to detect photons and appears to be considerably more challenging than that of $B_s \to J/\psi\,\phi$, which is one of the “gold-plated” channels for B-physics experiments at hadron machines. Higher D_s resonances exhibiting all-charged final states, for instance $D_{s1}(2536)^+ \to D^{*+}[\to D\pi^+]\,K$, may be more promising in this respect [44]. If photon detection is not possible, one can still distinguish $D_s^{*\pm}$'s from $D_s^{\pm}$'s through the energy smearing associated with the escaped photon [36]. Then one can use the lifetime method introduced in sect. 3.3.2 to find the CP-odd fraction x ($\propto |A_\perp(0)|^2$) and the CP-even fraction $1-x$ ($\propto |A_0(0)|^2 + |A_\parallel(0)|^2$) of the $D_s^{*+}D_s^{*-}$ data sample through (47). If $x \neq 1/2$ there are still non-vanishing CP asymmetries, although they are diluted by $1-2x$. The corresponding formula for the CP asymmetry of this weighted average of CP-even and CP-odd final states can readily be obtained from (63)–(65) and (71)–(73):

$$\frac{\Gamma(\overline{B}_s(t) \to D_s^{*+}D_s^{*-}) - \Gamma(B_s(t) \to D_s^{*+}D_s^{*-})}{\Gamma(\overline{B}_s(t) \to D_s^{*+}D_s^{*-}) + \Gamma(B_s(t) \to D_s^{*+}D_s^{*-})} = \frac{-(1-2x)\,\sin\phi\,\sin(\Delta m\,t)}{\cosh(\Delta\Gamma\,t/2)\,-\,(1-2x)\,|\cos\phi|\sinh(|\Delta\Gamma|\,t/2)}. \tag{82}$$

The same procedure can be done with the $D_s^{\pm}D_s^{*\mp}$ data sample or any other of the decay modes in Table 1.

A complete angular analysis for the three-body decays in Table 1 is more involved than the analysis for $B_s \to \psi\phi$. For example in $B_s \to \psi K_S K_S$, the K_S pair does not necessarily come from a vector resonance and could be in an S- or D-wave or even have a larger angular momentum. In such cases one might restrict oneself to a one-angle transversity analysis of [45] or even satisfy oneself with the diluted asymmetries in (82).

5 The Unambiguous Determination of ϕ

While $\sin\phi$ can be measured by conventional methods, this section shows that even $sign(\cos\phi)$ can be determined. That determination is important for various reasons. It is not only necessary for a complete extraction of magnitude and phase of the new physics contributions to B_s–$\overline{B}_s$ mixing, ϕ must also be known to extract the CKM angle γ from $B_s \to D_s^{\pm}K^{\mp}$. Even if $\sin\phi$ is found to be consistent with zero, the determination of $sign(\cos\phi)$ is necessary to distinguish the Standard Model prediction $\cos\phi \simeq 1$ from $\cos\phi \simeq -1$. In the advent of new physics, $sign(\cos\phi)$ completes our knowledge about ϕ. There are several methods to extract $\cos\phi$.

Method 1: The previous section revealed that angular correlation studies of $B_s \to \psi\phi$ determine

$$\cos\delta_i \cos\phi. \tag{83}$$

Once $sign(\cos\delta_i)$ is known, $sign(\cos\phi)$ follows immediately. The former can be deduced from theory, once first-principle calculations of δ_i have progressed sufficiently [46]. Alternatively, one can infer $sign(\cos\delta_i)$ from their SU(3) counterparts occurring in $B_d \to \psi K^*[\to \pi^0 K_S], \psi\rho^0, \psi\omega$ decays [denoted by $sign(\cos\widehat{\delta}_i)$], as follows:

The angular correlations of those B_d modes are sensitive to [12, 45]

$$\cos\widehat{\delta}_i \cos 2\widetilde{\beta}.$$

By applying the SU(3) relation

$$sign(\cos\delta_i) = sign(\cos\widehat{\delta}_i),$$

the relative sign between $\cos 2\widetilde{\beta}$ and $\cos\phi$ can be determined, but not yet the absolute sign of $\cos\phi$. That absolute sign can be determined, since there are methods which extract the B_d–$\overline{B}_d$ mixing phase $2\widetilde{\beta}$ unambiguously, even in the presence of new physics [47–51]. In the absence of new physics, $\widetilde{\beta}$ equals the angle β of the CKM unitarity triangle. In Ref. [52], basically the same approach was used to determine the sign of $\cos 2\widetilde{\beta}$. However, in that paper it was assumed that ϕ is negligibly small, as in the Standard Model. On the other hand, in method 1 we assume that $2\widetilde{\beta}$ is known unambiguously, allowing the determination of $\cos\phi$. Using a theoretical input [46] to determine $sign(\cos\delta_i)$ as noted above, the angular distribution of the $B_d \to J/\psi(\to l^+l^-)K^{*0}(\to \pi^0 K_S)$ decay products considered in Ref. [52] also allows an unambiguous determination of $2\widetilde{\beta}$ in the presence of $\phi \neq 0$.

Method 2: Consider certain three- (or n-) body modes f that can be fed from both a B_s and a $\overline{B}_s$, and where the $\overset{(-)}{B}_s$ -decay amplitude is a sum over a non-resonant contribution and several contributions via quasi two-body modes. The strong phase variation can be modelled by Breit-Wigners and is known, so that $\cos\phi$ can be extracted. Such a method was suggested in determining $\cos 2\alpha$ and $\cos 2\widetilde{\beta}$ in B_d decays [51].

An additional method can be found elsewhere [53].

6 Conclusions

In this paper we have addressed the experimental signatures of a non-vanishing CP-violating phase ϕ in the B_s–$\overline{B}_s$ mixing amplitude. Since ϕ is negligibly small in the Standard Model, but sizeable in many of its extensions, it provides an excellent ground for the search of new physics. We have discussed the determination of ϕ from both untagged and tagged decays in CP-specific B_s decay modes triggered by the dominant quark level decays $\overline{b} \to \overline{c}c\overline{s}$ and $\overline{b} \to \overline{c}u\overline{d}$. From lifetime measurements in these modes one can find the product of $\cos\phi$ and the width difference $\Delta\Gamma$ in the B_s system. The previously proposed methods to separately determine $|\Delta\Gamma|$ and $|\cos\phi|$ from untagged decay modes require two-exponential fits to the time evolution of either flavour-specific or CP-specific decay modes. In both cases terms of order $(\Delta\Gamma)^2$ must be experimentally resolved, which requires a substantially higher statistics than needed to measure $\Delta\Gamma\cos\phi$. We have proposed a new method to measure $|\Delta\Gamma|$ and $|\cos\phi|$, which only requires lifetime fits to the collected data samples with double-charm final states. This method does not require sensitivity to $\mathcal{O}((\Delta\Gamma)^2)$ terms. It is based on the observation that the measurement of $\Delta\Gamma$ from branching ratios discussed in [16] and performed in [27] is almost unaffected by new physics. These branching ratios and $\Delta\Gamma\cos\phi$ obtained from the lifetime fits allow one to solve for $|\Delta\Gamma|$ and $|\cos\phi|$. In this context we have stressed that the lifetime measurements also allow one to determine the size of the CP-even and CP-odd components of $D_s^{*+}D_s^{*-}$ and $D_s^{\pm}D_s^{*\mp}$ final states. This is relevant for experiments which cannot detect photons well enough and therefore cannot separate these components with angular analyses. We have further mentioned that a non-zero phase ϕ leads to tiny $\Delta m\, t$ oscillations in untagged data samples. This implies that in principle the measurement of CP violation in mixing from flavour-specific decays does not require tagging.

For the tagged analyses we have generalized the formulae for the CP asymmetries to the case of a non-zero ϕ. Here we have discussed in detail the expressions needed for the angular analysis in $B_s \to \psi\phi$ decays or other final states composed of two vector particles. Finally we have shown how the discrete ambiguities in ϕ encountered with the measurements of $|\cos\phi|$ and $\sin\phi$ can be resolved and ϕ can be determined unambiguously. This is important, even if $\sin\phi$ is found to be consistent with zero, because it distinguishes the Standard Model case $\phi \simeq 0$ from the case $\phi \simeq \pi$. If there are new particles which couple to quarks with the same CKM elements as W bosons, there can be new contributions to the B_s–$\overline{B}_s$ mixing amplitude with larger magnitude, but opposite sign than the Standard Model box diagram. In this case one encounters $\phi \simeq \pi$. This situation can occur in multi-Higgs doublet models and in supersymmetric models with flavour universality. From a measurement of Δm alone the contributions from the Standard Model and from new physics to the B_s–$\overline{B}_s$ mixing amplitude cannot be separated. The new contribution can only be determined by combining the measurements of Δm and ϕ. Consider, for example, that Δm is measured in agreement with the Standard Model prediction: the new physics contribution to B_s–$\overline{B}_s$ mixing then varies between 0 and twice the Standard Model prediction, if ϕ is varied between 0 and $\pm\pi$.

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