

Classical and Quantum Brane Cosmology¹

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Abstract. The first part of this lecture quickly touches upon some important but infrequently discussed issues in large extra dimension and warped extra dimension scenarios, with particular reference to effects in the early universe. The second part discusses a modification and extension of an earlier proposal by Brown and Teitelboim to relax the effective cosmological term by nucleation of fundamental membranes.

INTRODUCTION

The last two years have seen a ferment in our ideas concerning beyond-the-standard-model physics and cosmology, the essential new ingredient being the introduction of the brane concept. One realization was that in principle, the Standard Model (SM) could be localized on a brane, or set of branes, embedded in a higher-dimensional spacetime. Although such suggestions go back to the prescient work of Rubakov and Shaposhnikov [1, 2], the present activity was stimulated primarily by the papers of Arkani-Hamed *et al.* [3–5] (ADD), and Randall and Sundrum [6, 7] (RS). ADD showed that the fundamental gravitational scale (or if string theory is correct the string-scale) could be close to the TeV scale by invoking large (possibly sub-mm) new spatial dimensions transverse to the SM brane, implying a very rich new early universe and collider phenomenology. (In the context of string theory there were some noteworthy investigations of low-string-scale or semi-large, $R \sim (\text{TeV})^{-1}$, extra dimensions [8, 9], but without the important brane-localization of the SM degrees of freedom imagined by ADD.)

An important variant of the brane scenario involves so-called warped compactifications. In such compactifications the directions parallel and transverse to the branes do not factorize into a product space, leading to non-trivial dependence of the 4D graviton wavefunction upon the transverse coordinate(s), and possible localization of the bulk graviton. Warped (and even non-compact) “compactifications” were first considered in the 1980’s by a number of authors [2, 10–13], but it was only with the addition of the modern brane idea by RS that these speculations took off. The phenomenology of these warped models is in many ways quite different from those of ADD, having typically less striking signatures at colliders or in astrophysical processes, but concomitantly being less constrained. However, in the context of early universe cosmology there have been quite a large number of papers arguing that the early universe evolution of the RS models is dramatically different from that of standard FRW cosmology. Although, as I will explain, these statements are correct in a certain high-temperature, high-energy domain, they do not simply apply to sub-TeV early-universe evolution as many later authors have naively assumed. In this talk I will try to disentangle the situation, hopefully for the benefit of those freshly arriving (or just interested) in this active field.

In the second part of my talk I will discuss recent work on some of the quantum properties of branes and how these might impact upon cosmology. In this regard it is important to realize that almost all the brane-world activity (*e.g.*, the ADD and RS scenarios) take the branes to be *classical backgrounds* about which a small fluctuation analysis (particle emission for example) is undertaken. But as we learned for non-abelian gauge theories there are many significant features of a theory that are not captured by such a small-fluctuation treatment. In particular, I am thinking of instanton processes and false vacuum “bounce” decays, and monopole and other soliton-like solutions. Brane-world theories are richer in their dynamics than non-abelian gauge theories, and one should expect that similarly important phenomena await investigation and application to the physics of the early universe. The application I will describe involves false-vacuum decay enabled by branes [14]. As we will see there are properties of such brane-enabled processes that lead to an essential alteration of the usual field-theoretic vacuum-decay dynamics, and I will argue that this could have some bearing on the problem of the cosmological constant (and possibly lead to novel theories of inflation).

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SOME ASPECTS OF BRANE COSMOLOGY AND PHENOMENOLOGY

The two closely related scenarios of ADD and RS both imagine that the SM degrees of freedom are localized to a brane, or set of branes, embedded in an higher-dimensional spacetime. Here the branes are taken as a background configuration. If such a set-up is to be even a crude representation of our world, then it is vital that we consider a sufficiently rich structure with the potential to be at least a toy-model for the SM.

As an example of how this minimal requirement can impact one's view of the physics of the scenarios, consider the status of the "solution" of the hierarchy problem in the warped case. What is usually considered is a pure gravity set-up with localized brane 'sources' of tension, sometimes with an additional scalar field in the bulk to provide a stabilization mechanism if one is concerned with more than one brane. With the assumption of the bulk (AdS) cosmological constant and suitably fine-tuned brane tensions one certainly finds a metric of the form $d^2s = e^{-2\kappa|y|}\eta_{\mu\nu}dx^\mu dx^\nu + d^2y$, with an exponential warp factor. In the background of this warp factor it is quite correct to say that all mass scales associated to a theory located at some y_c in the 5th dimension are conformally rescaled by a factor $e^{-\kappa|y_c|}$. Does this constitute a solution to the hierarchy problem? The evidence is no for the following reason: One must consider the dynamics that localizes the (toy) SM degrees of freedom on the brane, and in all cases that have been studied this involves the introduction of further *bulk* degrees of freedom. For example, in string or supergravity investigations of branes in warped geometries there are always extra degrees of freedom (*e.g.*, the dilaton and other moduli) whose expectation values at the brane set the values of coupling constants on the brane. These additional degrees of freedom enter the bulk equations in a non-trivial way, and their effect to turn the exponential warp factor into a polynomial factor [15,16], thus eliminating the exponential hierarchy.

Following Herman Verlinde one argue for this behavior on general grounds by employing (a generalization of) the AdS/CFT correspondence. (See section 5 of Ref. [15]). Crudely speaking this correspondence maps the renormalization group properties of a brane-localized theory to the geometrical properties of the transverse bulk solution. The unperturbed AdS solution corresponds to an exact conformal field theory on the brane. But the SM degrees of freedom that we observe are certainly not conformal; moreover, the SM is unstable to the introduction of relevant operators (such as the Higgs mass) in the UV. In the geometrical bulk picture this instability maps to an instability of the exponential bulk solution to the couplings of the new bulk degrees of freedom. The new couplings must be fine-tuned to get a sufficiently large exponential warp factor, the degree of tuning being the same as that in the non-susy SM embedded in, say, a GUT theory. However, *one only sees this instability when the theory is sufficiently rich to have the potential of realizing the (toy, non-supersymmetric, non-conformal) SM on the brane.* Of course if one is happy to have supersymmetry at the TeV scale then no fine-tuning is needed. The moral of the story is that some important qualitative properties of brane-worlds are very dependent on how simplified a toy model one is working with.

So forewarned, we now turn to cosmology, and consider first the scenario of ADD. This case is in part defined by the statement that the brane-parallel dimensions (for simplicity supposed to be just our usual 3+1 dimensions) and brane-transverse dimensions factorize into a product space.

$$ds^2 = g_{\mu\nu}^{(4)}(x)dx^\mu dx^\nu + R^2 g_{ij}^{(n)}(y)dy^i dy^j. \quad (1)$$

In particular the presence of the brane is assumed not to strongly perturb the structure of the extra dimensions. Usually one supposes, following ADD, that the extra dimensions are close to being flat, and of linear size R . As I will soon explain, this flatness is not a necessary assumption [17] (for example AdS spaces *are* allowed), and relaxing this assumption can, and does, have a strong effect on the phenomenology.

The new modes so introduced minimally include at low energy the Kaluza-Klein (KK) modes of the bulk graviton, while at high energies comparable to the (now much lower \gtrsim TeV) Planck scale whatever physics renders the theory consistent – string excitations of the brane and bulk states being the default assumption. The low energy modes are the ones responsible for the stringent astrophysical and cosmological constraints that these large extra dimension theories must satisfy, and of course therefore also have the potential to lead to interesting predictions. Before I go on it is worthwhile to discuss a much more general, and very useful description of the low-energy modes. Rather like condensed-matter physicists studying a material, let us consider a plot of the *density of states* $\rho(E)$ with respect to energy of our brane world scenario. In the simple case that the extra dimensions are an n -dimensional square torus,

$$\begin{aligned} \rho(E) &= 0 & E \leq 1/R \\ \rho(E) &= \frac{\text{Area}(S^n)}{2^n} R(ER)^{n-1} & E > 1/R. \end{aligned} \quad (2)$$

(This ignores the important question of zero modes to which I will return). The Gauss Law constraint, $M_{Pl}^2 = M_*^{(n+2)}R^n$, relating the traditional Planck scale M_{Pl} to the new fundamental scale, M_* , of gravity implies that there are of order $M_{Pl}^2/M_*^2 \sim 10^{32}$ modes in total, so an approximation by a continuum is good.

For the large extra dimension scenario we see that the density of KK states is a power-law function of the mass scale. The most important model-independent constraints arise from the production of light KK modes of the graviton, and so is controlled by the low-mass part of this spectrum. (More precisely for semi-thermal production from supernovae cores or at the big-bang nucleosynthesis epoch, various moments of this distribution convolved with a Boltzmann suppression factor.) Thus we should ask how much is the shape of the density of states fixed, and how much is model-dependent?

For geometrical compactifications, these KK modes are understood as the eigenmodes of the appropriate Laplace operator, Δ , on the internal space, and all the constraints depend on the form of the spectral density of this operator, which in turn depends completely on the topology and geometry of the internal space. For example there exist attractive alternate choices of the compactification implying significantly weaker constraints [17] (admitting in particular a standard 4d FRW evolution up to quite high temperatures, cf, the usual limit of a few MeV at best in the $n = 2$ flat case, and avoiding the supernovae bounds entirely). These alternate compactifications employ a topologically non-trivial internal space, specifically a n -dimensional compact hyperbolic manifold (CHM), with a large mass-gap to the first non-trivial KK mode. In other words the low part of the KK spectrum in Eq. (2), is *not* model independent. They are also as good, or as bad, as the warped models in explaining the large hierarchy between the traditional Planck and weak scales, since even though the volume of these manifolds is large their linear size L is only slightly larger than the new fundamental length scale ($L \sim 30M_*^{-1}$ for example), thus naively only requiring numbers of $\mathcal{O}(10)$ (but beware fine tuning in sufficiently rich models).

In fact there is no reason why the spectrum of “KK modes” should have a geometrical interpretation *at all*. (All that is necessary is that a few sum rules associated to the correct “running” of the gravitational interactions are satisfied. This is so that the gravitational interaction becomes of $\mathcal{O}(1)$ strength at the new fundamental scale of gravity M_* .) This is directly analogous to the asymmetric orbifold constructions in heterotic string theory which also did not have a geometrical interpretation of a compactification from 10 to 4 dimensions. Thus the proper, model-independent way of describing the physics of, and constraints on, the large extra dimension and warped dimension models is to introduce the density of states function $\rho(E)$ – the extra-dimensional analog of the parton distribution functions of the proton!

Finally, what about the possible zero modes I mentioned in passing? In discussing the physics of brane-world models one must be careful not to forget such modes since being massless they can of course have a dramatic effect on the long-wavelength, low energy behavior of the theory. A case in point is the radion mode of the ADD scenario (or the inter-brane separation mode in the original RS scenario). Before a stabilization mechanism for the size of the extra dimensions is introduced this mode is massless, and thus the low-energy theory is not pure gravity, but gravity plus a Brans-Dicke-like scalar. Such a theory can of course have very different evolution as compared to the usual Einstein-gravity FRW solution. Once a stabilization potential is introduced the radion gets a mass, m , and below this mass scale normal FRW evolution is guaranteed by the usual theorems, anomalous behavior only showing up at energies $E \gtrsim m$ where the radion can be excited. Again the moral of the story is that too simple a model can lead to misleading conclusions.

I now turn to a discussion of a dynamical process involving branes.

BRANE NUCLEATION AND THE COSMOLOGICAL TERM

An interesting approach to the problem of the cosmological term is the proposal that it is relaxed by jumps (saltation) associated with exotic dynamics. There is an important conceptual advantage to having the relaxation connected to some non-continuous dynamics that responds only to a source taking the form of an effective cosmological term. For if the dynamics responds to several influences, it is difficult to see how a particularly simple value for one partial determinant of its behavior can become overwhelmingly preferred. Note that this logic also applies, in connection with *continuous* relaxation of scalar fields, to self-interactions, including kinetic terms [18].

Such saltation of the cosmological term has been previously considered in the context of stepwise false vacuum decay [19,20] in a quasi-periodic staircase potential by Abbott [21], and by Brown and Teitelboim (BT) through nucleation of fundamental membrane degrees of freedom [22,23]. This second formulation is close in spirit to the one discussed in [14] and which I review here.

Basic Mechanism of Saltation

The essential ingredients of the BT model are a 4-form gauge field strength $F_4 = dA_3$, and a fundamental membrane degree of freedom (a 2-brane) whose world-volume couples to A_3 . Such a field strength F_4 has no local dynamics in 4-dimensional spacetime, but its expectation value contributes to the effective cosmological term. The presence of

an expectation value for F_4 induces the nucleation of the 2-brane to which it couples, in a manner analogous to the Schwinger mechanism for electric field decay through nucleation of e^+e^- pairs. When a membrane is nucleated, say as a spherical shell, the effective value of the cosmological term on the inside differs from the previous value on the outside, by an amount proportional to the coupling constant of the membrane. If a membrane of the correct sign is nucleated, the contribution of the 4-form to the effective value of the cosmological term will be reduced. Thus if the steps between adjacent false vacua are sufficiently small, and if there is a reason why vacuum decay stops or slows down dramatically as $\Lambda_{\text{eff}} \rightarrow 0$, this mechanism can in principle relax a large microscopic cosmological term (arising from all sources other than the membrane- F field dynamics) to a value within observational bounds. One possible virtue of saltatory relaxation is that, in contrast to mechanisms involving symmetries or continuous relaxation, a small non-zero value automatically emerges as a consequence of the non-vanishing jump size.

Specifically, BT considered gravity in 4 spacetime dimensions with a 2-brane coupled to a 3-form gauge potential A_3 . The Euclidean space action is

$$S_E = \tau_2 \int_{\xi} \sqrt{\det g_{ab}} + \rho_2 \int_{\xi} A_3 - \frac{1}{2} \int d^4x \sqrt{G} (F_4^2 + M^2(-R + 2\lambda)) + \text{surface terms}, \quad (3)$$

where the ξ^a parameterize the membrane world-volume, and $g_{ab} = \partial_a X^\alpha \partial_b X^\beta G_{\alpha\beta}$ is the induced world-volume metric. The surface terms ensure that the action has well-defined functional derivatives with respect to the metric and gauge field. A most important point is that in four dimensions the 4-form field strength contains no independent propagating degrees of freedom, its value, up to a constant, being fully determined by the background of sources charged with respect to A_3 . The parameters entering this action (with 4-d mass dimension Δ) are: 1) the 2-brane tension $\tau_2, \Delta = 3$; 2) the 2-brane charge density, $\rho_2, \Delta = 2$; 3) the Planck mass, $M = (8\pi G)^{-1/2}, \Delta = 1$; and, 4) the bare cosmological constant, λ , with $\Delta = 2$.

The instanton solution is a membrane that divides space into two regions, an outside O and an inside I . In each region, the field strength is a constant $\langle F_4^{O,I} \rangle = c_{O,I} \epsilon^{(4)} / \sqrt{G}$ and the field strengths are matched across the membrane boundary via $c_I = c_O - \rho_2$, so that the effective cosmological terms are

$$\Lambda_{O,I} = \lambda + \frac{1}{2M^2} c_{O,I}^2, \quad (4)$$

where the field strength contribution follows from Einstein's equations. From Eq. (4), it is clear that if the bare cosmological term is to be canceled, it must be negative, and therefore $\lambda < 0$ is assumed.

In this BT realization of saltation the tunneling probability is $P \sim e^{-B}$, where the bounce action for this false vacuum decay is

$$B = \begin{cases} \infty, & \text{if } \Lambda_O, \alpha_O < 0 \\ 12\pi^2 M^2 \left[\frac{1}{\Lambda_O} (1 - b\alpha_O) - \frac{1}{\Lambda_I} (1 - b\alpha_I) \right], & \text{otherwise.} \end{cases} \quad (5)$$

Here the bubble radius, defined so that the area of the bubble slice when continued back to Minkowski signature is $4\pi b^2$, is $b = (\Lambda_{O,I}/3 + \alpha_{O,I}^2)^{-1/2}$ with $\alpha_{O,I} = [c_O - (1/2 \pm 3x^2/4)\rho_2]/3xM$ and $x = \tau_2/\rho_2 M$. (Often Planck units, $M \equiv 1$, will be used.) The associated change in the effective cosmological constant across the membrane is given by

$$\Lambda_I = \Lambda_O + \frac{1}{2M^2} (\rho_2^2 - 2\rho_2 c_O). \quad (6)$$

As ρ_2 will necessarily be small, while c_O is large so as to (in the end) cancel the bare cosmological term, the second term inside the parentheses of Eq. (6) dominates.

Note that for $\Lambda_O < 0$, transitions can take place only when $\alpha_O > 0$. It is not hard to show that this constraint, along with the reality condition on b , implies that further lowering of the effective cosmological term will not occur beyond the first anti-de Sitter step if the tension is sufficiently large. This remarkable result is closely related to the Coleman-de Luccia [20] gravitational suppression of false vacuum decay. BT hypothesize that our Universe is at the endpoint of such an evolution.

Problems with Abbott-Brown-Teitelboim

The ABT mechanism is an intriguingly different approach to the problem of the cosmological term, with a number of striking features. However there are of course a number of problems to be overcome. In rough order of increasing seriousness they are:

- **Ad-hoc degrees of freedom.** Dynamical entities are postulated *ad hoc*, with no richer context to connect them to other principles and problems of physics.
- **Small parameters.** The value of the cosmological term at present is bounded by $\Lambda_{\text{obs}} M^2 \lesssim (2 \times 10^{-3} \text{ eV})^4 \sim 10^{-120} M^4$. For the observed value to be natural in the framework of brane nucleation, the spacing between allowed values of the effective cosmological term, near the observed value, cannot be much larger. This translates into the condition

$$\rho_2 \lesssim \frac{\Lambda_{\text{obs}}}{|\lambda|^{1/2}}. \quad (7)$$

This is an extremely stringent condition on the microphysics, even for plausible $|\lambda| \ll 1$, since the observed cosmological constant is so small. For example, even if the bare cosmological constant is generated only at the TeV scale through low-energy supersymmetry breaking, so $|\lambda| \sim 10^{-60}$, one requires $\rho_2 \lesssim 10^{-90}$.

Note, however, that if the rest of the ABT mechanism could be made to work then, even without explaining this very small number, one has still made considerable progress. The reason for this is that one would have exchanged a technically unnatural fine-tuning of Λ , for a tiny, but technically natural, coupling ρ_2 .

- **Stopping the saltation.** The cosmological term must not only relax to within its observational bounds, it must also stop evolving once it reaches this interval. For de Sitter space, additional bubble nucleations are always possible. But in the case of AdS, due to the Coleman-deLuccia stopping, a sufficient condition to ensure absolute vacuum stability is $\tau_2^2 > 4\rho_2 c_0/3$ or, in terms of the tension to charge density ratio,

$$x = \frac{\tau_2}{M \rho_2} \gtrsim \sqrt{\frac{|\lambda|}{\Lambda_{\text{obs}}}}. \quad (8)$$

Unfortunately, even for the smallest plausible $|\lambda|$, a large hierarchy between tension and charge density is required if one is to employ such stopping.

- **Penultimate step and an empty universe.** However more problematic still (and from a phenomenological viewpoint the most serious defect of the mechanism), is that, as BT showed, upon combining the stability and step-size conditions, the time required to reach the endpoint is extraordinarily large, so large that even the glacially slow inflation that occurs in the penultimate vacuum would leave the universe entirely devoid of matter and energy.

There are a number of possible responses to these problems. The path advocated in Ref. [14] is to consider modifications of the ABT proposal motivated by the richer dynamics of the branes of string theory. Recent developments in string theory have emphasized that a wide variety of extended objects (which naturally couple to higher-form gauge fields of the type needed by the BT mechanism) play a fundamental role. Moreover, such a string theory embedding introduces an essentially new feature, which may allow a solution of the penultimate step problem of the ABT mechanism. The new feature is that there can exist exponentially large density of states factors associated with semi-classical brane processes involving coincident branes. This is because such coincident branes support low-energy internal degrees of freedom. These degrees of freedom are not directly visible in a naive semi-classical membrane calculation (analogous to the microstates that are deduced to exist for black-holes from the Bekenstein-Hawking entropy). The dynamics of relaxation is greatly altered by such large density of states factors.

In addition since the theory is naturally formulated in higher dimensions, the properties of these membranes as seen in four dimensions are determined in terms of the fundamental couplings together with properties of the extra-dimensional compactification. Large degeneracy factors and small tension, which give the most rapid relaxation, appear to arise for complex, near singular configurations. Just these configurations are favored for dynamical reasons. Unfortunately as I will discuss, the simple cases studied so far do not seem to allow an accommodation of the small coupling necessary for the small step size – at least not in a theory that contains the SM in a straightforward way.

Finally, regardless of whether membrane nucleation is the solution, or perhaps an ingredient, in solving the riddle of the cosmological constant, nucleation processes involving extended objects generically occur in string theory, and studying these processes could lead to insight into the question of vacuum selection.

(MODIFIED) SALTATION MOTIVATED BY STRING THEORY

I shall start with a lightening-quick review of some relevant features of string/M theory [24]. In its long wavelength approximation, M theory supports BPS M2-branes and M5-branes. The M2-branes couple electrically to the 3-index

gauge potential A_3 of 11-dimensional supergravity. Defining a dual gauge potential A_6 via $F = dA_3 = *F_7 = *dA_6$, one finds that the M5-branes couple magnetically to A_3 , or directly to A_6 . The simplest way to arrive at a 4-dimensional world is via compactification on a 7-dimensional internal space \mathcal{M}_7 . (There are also other ways of arriving at a 4-dimensional world involving F-theory or warped solutions that do not fit into the above framework. These may be useful in addressing the step-size problem.)

Two-branes can be obtained from either the fundamental M2-branes, or by wrapping M5-branes on a 3-cycle a_3 of the internal space \mathcal{M}_7 . Rather than discuss the properties of such branes in the general framework of M theory, let's make contact with a possibly more familiar framework by descending to string theory.

If \mathcal{M}_7 has the form $S^1 \times K$ then for a small S^1 , M theory reduces to type IIA string theory, and the M-branes lead to a spectrum of Dirichlet (D) and Neveu-Schwarz (NS) branes, and gauge fields that couple to them. Specifically, focus upon the Ramond-Ramond (RR) field strengths F_2 and F_4 , and in addition their 8-form and 6-form magnetic duals. These $(p+2)$ -form RR field strengths couple electrically to dynamical p -branes.

After compactification to $(3+1)$ dimensions on, say, a Calabi-Yau manifold K , the RR fields F_n can acquire $(3+1)$ -Lorentz invariant expectation values $\langle F_2 \rangle = v_2^i \omega_i^{(2)}$ and $\langle F_4 \rangle = v_4^i \omega_i^{(4)} + v_4 \varepsilon^{(4)}$, where $\omega_i^{(2,4)}$ are the harmonic 2- and 4-forms on K , and $\varepsilon^{(4)}$ is the spacetime volume element. Denote the expectation values collectively by v_a .

Generically, when $v_a \neq 0$, supersymmetry is broken and a scalar potential is generated that depends on the Calabi-Yau moduli T_i and the expectation values v_a . The critical points of this scalar potential are of two general types: either the Calabi-Yau runs away to infinite volume, or the theory is driven to conifold-like points where homology cycles of K degenerate (classically approach zero volume). Moreover, since the configurations with $v_a \neq 0$ are not iso-potential, there are dynamical processes whereby the values of the v_a 's change. The v_a 'discharge' by the nucleation and expansion of charged membranes, the D-branes of string theory. What do such false-vacuum decay processes involving D-branes look like?

Density of states for coincident branes

As mentioned above, the most important new feature introduced by the string theory branes is the existence of density of state factors. Calculations [28] performed in the context of checking duality between type I and heterotic $SO(32)$ string theory demonstrate that D-branes do make contributions that can be interpreted semi-classically as incorporating degeneracy factors reflecting the non-Abelian structure of coincident D-branes. Another aspect of this is that many coincident branes with large total charge can be described in appropriate limits as 'black' objects, similar to black holes, with event horizons, and with associated Bekenstein-Hawking (BH) entropy [29].

As a warm-up consider first the case of k coincident D3-branes. Such a configuration possesses a $U(k)$ super Yang-Mills (SYM) gauge theory on its world-volume. In the limit where the interactions with the bulk string theory are weak, and where the temperature (or excitation energy) of the SYM is small, one can compute the entropy of this system. When the effective SYM gauge coupling $g_{\text{eff}}^2 \simeq k g_{\text{YM}}^2$ is small, the entropy of the gas of massless gauge bosons and their superpartners at temperature T is simply

$$S_3 = \frac{2\pi^2}{3} k^2 V T^3, \quad (9)$$

where V is the spatial volume of the 3-branes. What happens when the effective coupling is large? In this case one can use the type IIB supergravity solution describing the k coincident 3-branes. These classical solutions with the asymptotic geometry and quantum numbers appropriate for k coincident 3-branes contain a non-extremality parameter upon which their masses and horizon areas depend. If one associates the area of the horizon with BH entropy, one can derive a temperature by taking an appropriate derivative. By this procedure, the supergravity picture yields the strong coupling form of the entropy. In this case, the entropy agrees with the preceding weak coupling formula up to a numerical prefactor $3/4$ (which is then a prediction for the strong coupling behavior of the theory).

For the more interesting case of k coincident D2-branes the UV theory (in the decoupling limit) is again a weakly-coupled $U(k)$ SYM theory, so the UV entropy again scales as k^2 . However one requires the IR entropy since, as I will soon argue, the physically motivated temperatures are the ambient de Sitter temperatures which are small (vanishingly small as $\Lambda_{\mathcal{O}} \rightarrow \Lambda_{\text{obs}}$). In the IR the SYM theory on the $(2+1)$ -dimensional world-volume becomes strongly coupled, so one must switch over to the supergravity description. As shown in Ref. [30], the theory flows in the far IR to that of the M2 brane with BH entropy inferred from the horizon area given by [29]

$$S_2 \simeq k^{3/2} A T^2, \quad (10)$$

with A the 2-brane area. This strongly suggests that such strongly-coupled brane configurations support $\mathcal{O}(k^{3/2})$ light degrees of freedom, though the physical nature of these degrees of freedom remains somewhat mysterious.

Given this entropy the probability for a semi-classical process involving k coincident D2-branes is multiplied by a density of states factor of the form $N_k \sim \exp(k^{3/2}AT^2)$ in the IR limit $T \rightarrow 0$. A more exact treatment requires, among other things, an additional analysis of the way in which the exponent scales as we approach the IR, as the physically motivated temperature is small but non-zero.

In any case, if this reasoning is accepted, the probability for a semi-classical process involving k coincident 2-branes should be multiplied by a density of states factor of the form

$$N_k \sim \exp(k^\beta AT^2), \quad (11)$$

for an appropriate temperature T . The exponent β likely lies between $\beta = 2$ and $\beta = 3/2$. (Although not utilized here, there might also be the possibility of k^3 scaling in the M5-brane limit.) A larger β exponent implies more complete saltation, so $\beta = 3/2$ is the more ‘conservative’ choice.

Temperature ambiguity and a black hole analogy

The only temperatures intrinsic to our scenario are the de Sitter temperatures $T_{O,I} = H_{O,I}/2\pi = (\Lambda_{O,I}/3)^{1/2}/2\pi$. Ambient ordinary matter might be at a much higher temperature, but the branes are in very poor thermal contact with it. If the initial cosmological term is much larger than the change brought about by the k -bounce, $\Lambda_O \gg k\rho_2 c_O$, then the de Sitter temperatures before and after nucleation are almost identical, $T_O \simeq T_I$, and we may use either one in calculating the density of states factor.

On the other hand, in the case that a given transition produces large changes in the effective cosmological constant, an ambiguity arises. One possibility is that the temperature scale for tunneling from a highly curved (high temperature) de Sitter space to a less curved de Sitter space (or even to a flat or anti-de Sitter space) is substantially set by the high temperature. In this case one would take $T \sim T_O$ in the density of states factor of Eq. (11).

However, when the change in the nominal de Sitter temperature is comparable to the temperature itself, the thermal description of the tunneling process is internally inconsistent. A similar situation has been encountered before, in black hole physics [31]. The problem arises in its most acute form for near-extremal holes, as the temperature approaches zero. If one uses the temperature of the initial hole, one finds a significant probability for radiating a quantum that will take the hole past extremality to a naked singularity. A more refined analysis [32,33] shows that radiation is *not* thermal with regard to the initial temperature, and in particular that radiation beyond extremality is forbidden.

If an analogy is made between maximally homogeneous cosmologies and black holes based on their temperatures, then de Sitter spaces correspond to ordinary holes, flat space corresponds to an extremal hole, and anti-de Sitter spaces to naked singularities. This analogy suggests, in view of the previous paragraph, that we should not consider finite temperature branes that mediate transitions from de Sitter to anti-de Sitter spaces. As a very crude working hypothesis, which interpolates smoothly to this suggestion, one can employ the geometric mean of the de Sitter temperatures $T \sim \sqrt{T_O T_I}$, instead of $T \sim T_O$.

The dynamics of both these possibilities is explored below.

TWO SALTATION SCENARIOS

Let us now gather the pieces and attempt to envisage if they may be assembled into a complete scenario. The cosmological constant evolves from some initial value through multi-bounce transitions. The probability for such transitions is $P \sim e^D e^{-B}$, with the bounce action B is given by Eq. (5), with $\rho_2 \rightarrow k\rho_2$ and $\tau_2 \rightarrow k\tau_2$, where k is the bounce number. The density of states prefactor is specified by $D = k^\beta AT^2$, where $A = 4\pi b^2$ is the 2-brane area, and T is the temperature. For concreteness, the low ‘conservative’ value of $\beta = 3/2$ is assumed, but simple extensions of the following analysis holds more generally, for example, for $\beta = 2$ or larger.

Since the temperature T is not under good theoretical control it is useful to explore both of the broad alternatives mentioned previously.

Single-step relaxation

First consider the possibility that the temperature is given by the scale of the initial (outside) de Sitter temperature, so $T \sim T_O \sim \sqrt{\Lambda_O}$. One begins with some bare cosmological constant $\lambda < 0$. Assume that the initial field strength gives a similar contribution to the effective cosmological constant, so $\Lambda_O, c_O^2 \sim |\lambda|$. Also assume that some mechanism

provides a very small charge density $\rho_2 \sim \Lambda_{\text{obs}}/\sqrt{|\lambda|}$, consistent with the naturality condition discussed earlier, and that for simplicity $x \simeq 1$.

With such initial conditions and brane properties, the maximal bounce action is $B \sim 1/|\lambda|$, while the degeneracy factor may be as large as $D \sim \lambda^2/\Lambda_{\text{obs}}^2$. Recall that $\Lambda_{\text{obs}} \sim 10^{-120}$, while λ is plausibly in the range of 1 to 10^{-60} . Thus, the degeneracy enhancement overpowers the bounce action suppression, and tunneling proceeds rapidly.

It is not difficult to show that D is maximized for $k\rho_2 \sim c_O$, *i.e.*, for field strength step sizes of the right order to neutralize the field strength contribution to the effective cosmological constant. Indeed the most probable tunneling events nucleate bubbles of deep anti-de Sitter space. Such events produce small, short-lived interior universes, so the meaning of ‘probable’ in this context must be carefully qualified. Among universes that live a long time and even remotely resemble ours, the exponentially most favored are those closest to having zero effective cosmological term.

This scenario invokes a form of the anthropic principle. It is a uniquely weak one, however, in the following sense. Anthropic bounds on the cosmological term are highly asymmetrical [18]. For positive cosmological terms, the formation of sufficiently large gravitational condensations requires cosmological terms below ~ 100 in units of ρ_c , the critical density. For negative cosmological term, the lifetime of the universe requires cosmological terms roughly above -1 . Thus if the spacing between allowed near-zero saltatory values of the cosmological term in units of ρ_c is, say, 3 and allows the values $\dots, -2, 1, 4, \dots$, then among values that can be experienced by sentient observers, 1 is by far the most likely.

Now suppose such a transition to nearly flat space has occurred. As discussed before, absolute stability is possible only for anti-de Sitter spaces, and only for extremely large x . However, absolute stability is not required on empirical grounds. One need only require that the effective value of the cosmological term is at present stable on cosmological time scales. For a starting effective cosmological constant of $\Lambda_O \sim \Lambda_{\text{obs}}$, the degeneracy factor is highly suppressed by small T , and the bounce action is dominant. The stability of the vacuum is then determined solely by B .

The bounce action can become very large even for almost flat de Sitter spaces. The least suppressed transition (and so most dangerous from the point of view of vacuum instability) is that mediated by $k = 1$. For small ρ_2 and small Λ_O , the $k = 1$ bounce action is

$$B \approx \frac{27\pi^2 x^4}{2} \left[\frac{\rho_2}{c_O^3} - \frac{\rho_2^2}{c_O^2 \Lambda_O} \right], \quad \rho_2, \sqrt{\Lambda_O} \ll c_O. \quad (12)$$

Neglecting numerical factors, this gives $B \simeq x^4 \Lambda_{\text{obs}}/\lambda^2$. If $|\lambda| \sim 1$, then the action is very small, and there is no effective stability. On the other hand, if supersymmetry is broken at a low scale, then we expect $|\lambda| \ll 1$. When does $B \gtrsim 1$? This translates into

$$|\lambda_{\text{halting}}| M^2 \lesssim x^2 M^2 \sqrt{\Lambda_{\text{obs}}} \simeq x^2 (2 \times 10^{-3} \text{ eV})^2 (2.4 \times 10^{18} \text{ GeV})^2 \simeq x^2 (2 \text{ TeV})^4, \quad (13)$$

In a more careful analysis, one may require $B \gg 1$ for stability. However, the required supersymmetry breaking scale will not differ significantly from the above estimates, as B goes as the inverse 8th power of the energy scale appearing in $|\lambda|M^2$. In any case, given the present experimental lower bounds on the supersymmetry breaking scale, this suggests that the stability of the vacuum in this scenario requires low scale supersymmetry breaking, and relates the cosmological constant, Planck, and weak scales according to $M_{\text{weak}}^2 \sim (10^{-3} \text{ eV})(M_{\text{Planck}})$ in accord with observation.

Multi-step relaxation

Motivated by the black hole analogy, consider now an effective temperature that is the geometric mean of the initial and final de Sitter temperatures. In this case, tunneling to (and through) anti-de Sitter space is forbidden by fiat. However, the requirements of rapid tunneling to the observed cosmological constant and its stability are non-trivial constraints, and we now investigate their implications.

As in the previous scenario, we consider initial conditions $c_O^2, \Lambda_O \sim |\lambda|$. Now, however, the density of states factor D is typically maximized for Λ_I within an order of magnitude of Λ_O . To see this, a very rough estimate may be obtained by neglecting the bubble radius dependence on k and approximating $\Lambda_O - \Lambda_I = (2\rho_2 c_O - \rho_2^2)/2 \sim \rho_2 c_O$. We then have $D \propto \sqrt{\Lambda_O \Lambda_I} (\Lambda_O - \Lambda_I)^{3/2}$, which is maximized for $\Lambda_I = \Lambda_O/4$.

For $\Lambda_I \sim \Lambda_O$,

$$D_{\text{max}} \sim \frac{\Lambda_O^{5/2}}{|\lambda|^{7/4} \rho_2^{3/2}}. \quad (14)$$

It is not hard to verify that this degeneracy factor dominates the bounce suppression when $\Lambda_O \sim |\lambda|$. Thus, initially the effective cosmological constant tunnels rapidly as in the previous scenario, but in contrast to the previous case, the cosmological constant relaxes through several steps, with values roughly following a geometric series.

The effective cosmological constant will relax as described until $\Lambda_O \ll c_O^2$, when Eq. (12) holds. At this point, the condition that tunneling continue is the requirement $D_{\max} \gtrsim B$, or, since $B \sim \Lambda_O/\lambda^2$,

$$\Lambda_O^{3/2} \gtrsim |\lambda|^{-1/4} \rho_2^{3/2}. \quad (15)$$

For vanishing ρ_2 , tunneling may continue to arbitrarily small Λ_O . However, if we require stability from $B \gtrsim 1$, we find, from Eq. (12),

$$\rho_2 \gtrsim c_O^3 \sim |\lambda|^{3/2}, \quad (16)$$

so ρ_2 cannot be arbitrarily small. Combining Eqs. (15) and (16), we find that tunneling stops when

$$\Lambda_O \gtrsim |\lambda|^{4/3}. \quad (17)$$

Thus, even for $|\lambda| \sim 10^{-60}$, although the effective cosmological constant is reduced by a factor of 10^{20} , one membrane cannot suppress it to the observed value.

In general, however, it is important to note that several different 2-branes with various fundamental charge densities may be expected to arise from different degenerating cycles. Suppose that another brane begins nucleating as the first membrane reaches its endpoint. The initial conditions for this new membrane are identical to those for the first brane, except that now the role of the initial bare cosmological constant is played by $\Lambda_O \sim |\lambda|^{4/3}$. For appropriate charge densities, n branes may reduce the cosmological constant to $|\lambda|^{(4/3)^n}$. For $|\lambda| \sim 10^{-60}$, three branes are sufficient to reduce the cosmological constant to its observed value.

So far we have considered only the ‘conservative’ $\beta = 3/2$ case. For larger values of β more complete relaxations of the cosmological term are possible. For general β , a single membrane may relax the cosmological constant to $\Lambda_O \gtrsim |\lambda|^{2-\beta^{-1}}$. Thus, even for the $\beta = 2$ case, only two stages are required. Note also that in these multi-brane scenarios, in principle quite complex dynamics can arise, with periods of slow relaxation interspersed with more rapid changes.

Membrane tension and charge density

So far we have taken the membrane tension, and especially charge density, ρ_2 , as freely adjustable (and very small!) parameters. What is the situation for these quantities in string theory? In particular is there a mechanism that allows, or even prefers, such small values?

In (9+1)-dimensions, the tension and charge density of type II Dp-branes is $T_p = \rho_p = \frac{2\pi}{\ell_s^{p+1}}$, where the string length ℓ_s is defined in terms of the fundamental string tension through $T_{F1} = 1/(2\pi\alpha') = 2\pi/\ell_s^2$. However we wish to know the situation in 4-dimensions. There are two possibilities for how an effective 2-brane can arise in 4-dimensions.

First consider the case of a D2-brane in 10 dimensions descends directly to a 2-brane in 4 dimensions. The 10-dimensional supergravity action is compactified on a Calabi-Yau manifold with string-frame volume V_6 . The physical effective tensions and charge densities are then determined in 4-dimensional Einstein frame, with conventional normalizations of all kinetic terms. For the 2-brane case of interest a short calculation leads to the 4-dimensional effective charge density

$$\rho_2|_{4D, \text{eff}} = \frac{2\sqrt{2}\pi g_s^2}{M\ell_s^3}. \quad (18)$$

where $g_s = e^{(\phi)}$ is the string coupling. The tension is $\tau_2 = M\rho_2/\sqrt{2}$.

Clearly, we can only obtain a sufficiently small charge density by taking extreme values for ℓ_s and/or g_s . For example, with the canonical choice of string scale $\ell_s \simeq (10^{17} \text{ GeV})^{-1}$, we find that a charge density of $\rho_2 \lesssim 10^{-90}$ requires $g_s \lesssim 10^{-44}$. Alternatively we could take ℓ_s to be a larger length scale. These non-canonical cases include the ‘large extra dimension’ scenario with $g_s \sim 1$ and some number of sub-millimeter dimensions. However, given the success of quantum field theory at colliders such as LEP and the Tevatron, $\ell_s \lesssim (\text{TeV})^{-1}$ at the very best. Although an improvement, this still requires a tiny string coupling $g_s \lesssim 10^{-23}$ to generate a sufficiently small ρ_2 . It is difficult to understand how to accommodate the Standard Model in such an extreme corner of string theory moduli space.

In principle a more promising alternative employs branes wrapped on homology cycles of the compactification, and whose volume approaches zero classically, as for conifolds [25]. Indeed wrapping on such degenerating cycles leads to nearly tensionless branes, but unfortunately small charge densities are not achieved.

Specifically, if a Dp -brane of tension T_p wraps a k -cycle a_k of the compactification manifold, where $k \leq p$ and the volume of a_k is $\text{Vol}(a_k)$, then the result in the effective 4-dimensional theory is a $(p-k)$ -brane of tension $\tau_{(p-k)} \sim T_p \cdot \text{Vol}(a_k)$. If $\text{Vol}(a_k)$ approaches zero, *i.e.*, a_k is a degenerating cycle, a nearly tensionless object exists in the 4-dimensional theory. This is consistent with the higher-dimensional quantization rules for brane properties. Note, however, that as we wish to keep the Planck scale M fixed, these statements do not apply to the case where a $(p-2)$ -cycle of a factorizable compactification, such as a T^{p-2} in $K \simeq T^6$ or $K \simeq K3 \times T^2$, degenerates. Rather vanishing tension (with respect to M) applies to a non-trivial CY 3-fold with degenerating cycles. However a detailed calculation shows that in this non-factorizable case the relation between the tension and charge density picks up a dependence on the size of the cycle, the end result being that the charge density only logarithmically depends on the volume of the vanishing cycle (at least for all cases studied). Thus only very slightly smaller charge densities than in the direct descent case are possible.

Nevertheless, the phenomenon of classically vanishing tensions arising from the wrapping of branes on degenerating cycles is intriguing. One aspect that is worth noting is that the classical phenomenon of true degeneration and corresponding tensionless, or massless, states is typically not realized in the full quantum theory. Instead the effective (3+1)-dimensional 2-brane will have a dynamically generated non-perturbative tension, which may be exponentially small. It is known that in some cases a tension is generated from the dynamics of the would-be massless particle states arising from a D2-brane wrapping the 2-cycle. These particle states realize a non-Abelian gauge theory, presumably in a sector hidden with respect to the Standard Model, whose low-energy non-perturbative dynamics can break supersymmetry. (This sector is conceptually distinct from hidden sectors postulated to provide supersymmetry breaking for the supersymmetric Standard Model.) This leads to a potential for the volume modulus of the cycle, which stabilizes it at a scale $\Lambda \sim \exp(-8\pi^2/b_1 g_{YM}^2) M$ [26,27]. Once the cycle is stabilized at this small scale, membranes wrapping this cycle have tension that are also proportional to Λ and thus can be very small, even for $g_{YM} \sim 1$.

Thus, in summary, given our present understanding we just have to accept the exceedingly small charge density ρ_2 as an input parameter of a phenomenological model that awaits to be explained.

Summary and comments

Any viable scenario for the solution of the cosmological constant problem based on a relaxation principle must satisfy two basic constraints that are in tension with one another: the cosmological constant must relax sufficiently quickly from high scales, but must be stable on cosmological time scales at its present value. The mechanism of Abbott and Brown and Teitelboim unfortunately fails this test. However, as I have outlined, the enhancement of multi-step jumps due to large density of states factors, which typically leads to large tunneling probabilities, provides a possible solution to this dilemma.

I discussed two representative scenarios differing in the treatment of the effective temperature entering the density of states factor. In the simplest scenario, with $T \sim T_O$, the exponentially most probable transition, excluding extremely short-lived universes, is to universes that are most nearly flat. By requiring that this new vacuum be sufficiently stable, a non-trivial constraint for a mechanism of this kind was derived. This constraint provides an intriguing relationship, plausibly though not necessarily satisfied in Nature, between the supersymmetry breaking scale and the geometric mean of the present-day effective cosmological constant and Planck scales.

Alternatively, an analogy with black holes suggests a richer dynamics, in which flat space plays a distinguished role and tunneling to anti-de Sitter space is forbidden. In contrast to the previous scenario, the cosmological constant relaxes through a several jumps, roughly following a geometric series. The constraint of stability limits the range over which the cosmological constant may be relaxed by any given membrane. However, two or more types of branes with radically different scales may relax the cosmological constant to within observational bounds, and appeal to the anthropic principle may be avoided.

However, the saltation theory outlined above is clearly seriously incomplete. Not only are there many questions of principle to be addressed, such as the stopping mechanism, the correct form of the density of state enhancement, and the resolution of the temperature ambiguity, and the origin of the extremely small membrane coupling ρ_2 , but there are also phenomenologically important extensions that have not yet been explored. For example we have not attempted to incorporate saltation into a realistic FRW cosmological model including matter. Thus, in particular, we have not addressed the dynamics of relaxation following a phase transition, with its attendant potential problems (and opportunities for inflation). It is also of course important to explore the variety of observational and experimental consequences of the possible existence of light membrane degrees of freedom.

It is interesting to note that most model building in string/M theory has been based, implicitly or explicitly, on the paradigm of minimizing a potential, while the saltatory mechanism suggests a different principle, based on the dynamics of relaxation of the cosmological term. Moreover such a principle might prefer complex, near-singular configurations, of the type most favored by string model-builders. Finally it is intriguing that the density of states and effective temperature considerations suggest possible deep connections with the theory of black holes.

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