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Transverse Space Charge Effect During Bunch Compression in PDAC1 and PDAC2 Compressor Ring

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1 2D Model of Bunch Compression

During bunch compression space charge forces and dispersion can become an important issue since they lead to an increase of the beam size. In the longitudinal plane the compression might be described analytically by using the envelope equation

$$z_m'' + k_{z0}^2 z_m - \frac{K_L}{z_m^2} - \frac{\epsilon_L^2}{z_m^3} = 0 \tag{1}$$

with $k_{z0}^2 = eZVh|\eta|/(2\,\pi\,R^2\,\gamma\,\beta^2\,A\,m\,c^2)$, $K_L = -3\,g\,N\,(Z^2/A)\,r_p\,\eta/(2\,\beta^2\,\gamma^3)$, g-factor $g = 0.5 + 2\,\ln(R_p/R_b)$, $\epsilon_L = |\eta| z_m \Delta p/p_0$, and $\eta = 1/\gamma_t^2 - 1/\gamma^2$. z_m is the half bunch length in the bunch frame.

Eq. 1 is based on the Neuffer[1] self consistent distribution and predicts according the initial parameters an extra coherent momentum spread [1]. However in a regime of weak longitudinal space charge, one can assume $K_L = 0$ and instead of solving Eq. 1 one can solve the single particle equation

$$z'' + k_{z0}^2 z = 0 \tag{2}$$

[1] Sometimes also called: "Hofmann-Pedersen" or "local elliptic" distribution

and find the envelope evolution during compression

$$z_m = z_i \sqrt{\cos^2(2\pi q_s n_t) + \chi^2 \sin^2(2\pi q_s n_t)} \tag{3}$$

with z_i initial envelope, and $\chi = z_f/z_i$ compression factor, q_s is the synchrotron tune, and n_t the number of turns.

As the simulation is 2D, one has to choose a relevant slice from the bunch. The most relevant section for the transverse space charge is the center of the bunch where the current is maximum. For this section the momentum spread can be computed by means of Eq. 3 and using the invariant $z_m \Delta p/p_0 = const.$ as

$$\frac{\Delta p}{p_0} = \left(\frac{\Delta p}{p_0}\right)_0 \frac{1}{\sqrt{\cos^2(2\pi q_s n_t) + \chi^2 \ \sin^2(2\pi q_s n_t)}} \tag{4}$$

where $(\Delta p/p_0)_0$ is the initial momentum spread. On the other hand, the peak current is given by $I_{peak} = 1.5eZN/\tau$ with N particles per bunch, and τ is the bunch length in seconds. Consequently we can write the evolution of the peak current as

$$I_{peak} = I_{peak,0} \frac{1}{\sqrt{\cos^2(2\pi q_s n_t) + \chi^2 \ \sin^2(2\pi q_s n_t)}} \tag{5}$$

The 2D modeling of the bunch compression is based on Eq. 4, and Eq. 5. In a computer code we consider a coasting beam which has the same characteristics of the central slice of the bunch: same momentum spread, same transverse distribution and and current I_{peak}. The coasting beam is tracked through the compressor ring and space charge is computed self consistently at each time step. In order to include the effect of the compression during the tracking, the off momentum and "charge state" of each macroparticle are changed according to Eq. 4, and Eq. 5, i.e.

$$q_m = \frac{q_{m0}}{\sqrt{\cos^2(2\pi q_s n_t) + \chi^2 \ \sin^2(2\pi q_s n_t)}} \tag{6}$$

$$\delta = \frac{\delta_0}{\sqrt{\cos^2(2\pi q_s n_t) + \chi^2 \ \sin^2(2\pi q_s n_t)}} \tag{7}$$

where q_m is the macroparticle charge state, and δ the macroparticle off momentum.

2 Simulation Results

Simulations of final bunch rotation for the two CERN proton driver scenarios PDAC1 [2] (10^{13} p/bunch, initial bunch length 60 ns, final bunch length 7.5 ns, initial momentum spread $7.5 \cdot 10^{-4}$, 2 GeV, 7.23 MHz) and PDAC2 [3,4] (1.1^{12} p/bunch, initial bunch length 13 ns, final bunch length 6 ns, initial momentum spread $7.5 \cdot 10^{-4}$, 2.2 GeV, 44 MHz) have been performed by using the MIMAC library developed at GSI. Space charge has been computed with a poisson solver for a square boundary box of sizes 0.2 m x 0.2 m. In each dimension 64 grid points are used to interpolate the electrostatic field. Space charge calculation and subsequent macroparticles tracking is updated each 1/200 of a period. The initial number of macroparticles is 10^5. Particles off a round beam pipe of radius 10 cm have been removed from the computation. The rotation has been performed until 180 degree to demonstrate the irreversible beam size growth.

In Fig. 1 are shown for the two scenarios, respectively: The horizontal beam envelope evolution, here computed as the farthest particle from the design orbit (two pictures at the top). The initial peak current is 40 A for PDAC1 and 20 A for PDAC2 scenario. For the PDAC1 scenario the compression factor is 8 and the peak current at 90 degree rotation is 320 A. Space charge weakens lattice focusing properties and consequently increases dispersion. The maximum growths of rms size is 44 mm which is much less than the maximum growth of the envelope. The envelope results for PDAC1 and PDAC2 has been proved not to sensitive to statistical fluctuations in the initial distribution. The difference between rms and envelope sizes is due to few macroparticles which during the compression are pushed out the beam core by nonlinear forces arised by the excitation of higher order eigenmodes [5]. A zoom around the 90 degree of rotation allows to identify the lattice locations of the peaks of the envelope oscillations (two central pictures). The two pictures at the bottom show for the two scenarios the vertical envelope evolution during compression.

In the 44 MHz scenario PDAC2 the lower number of particle per bunch reduce sensibly the effect of space charge during the compression. The maximum horizontal envelope for a compression performed in 6 turns reaches 45 mm while on the vertical plane $y-$envelope remains practically unchanged.

3 Conclusion

The new PDAC2 compressor requires sensibly less horizontal aperture. Halo formation control requires to understand which nonlinear space charge resonances are excited during bunch compression. Further studies are needed.

4 References

1 G. Franchetti, I. Hofmann, and G. Rumolo, Phys. Rev. ST Accel. Beams **3**, 084201 (2000).

2 http://ppi.home.cern.ch/ppi/PDAC/PDACparam.html.

3 http://ppi.home.cern.ch/ppi/PDAC/PDAC2_1.html.

4 B. Autin et al. Design of a 2.2 GeV Accumulator and Compressor for a Neutrino Factory, Proc. of the 7th European Particle Accelerator Conference, 26-30 June 2000, Vienna, Austria

5 J.A. Holmes, V.V. Danilov, J.D. Galambos, D. Jeon, and D.K. Olsen, Phys. Rev. ST Accel. Beams **2**, 114202 (1999).

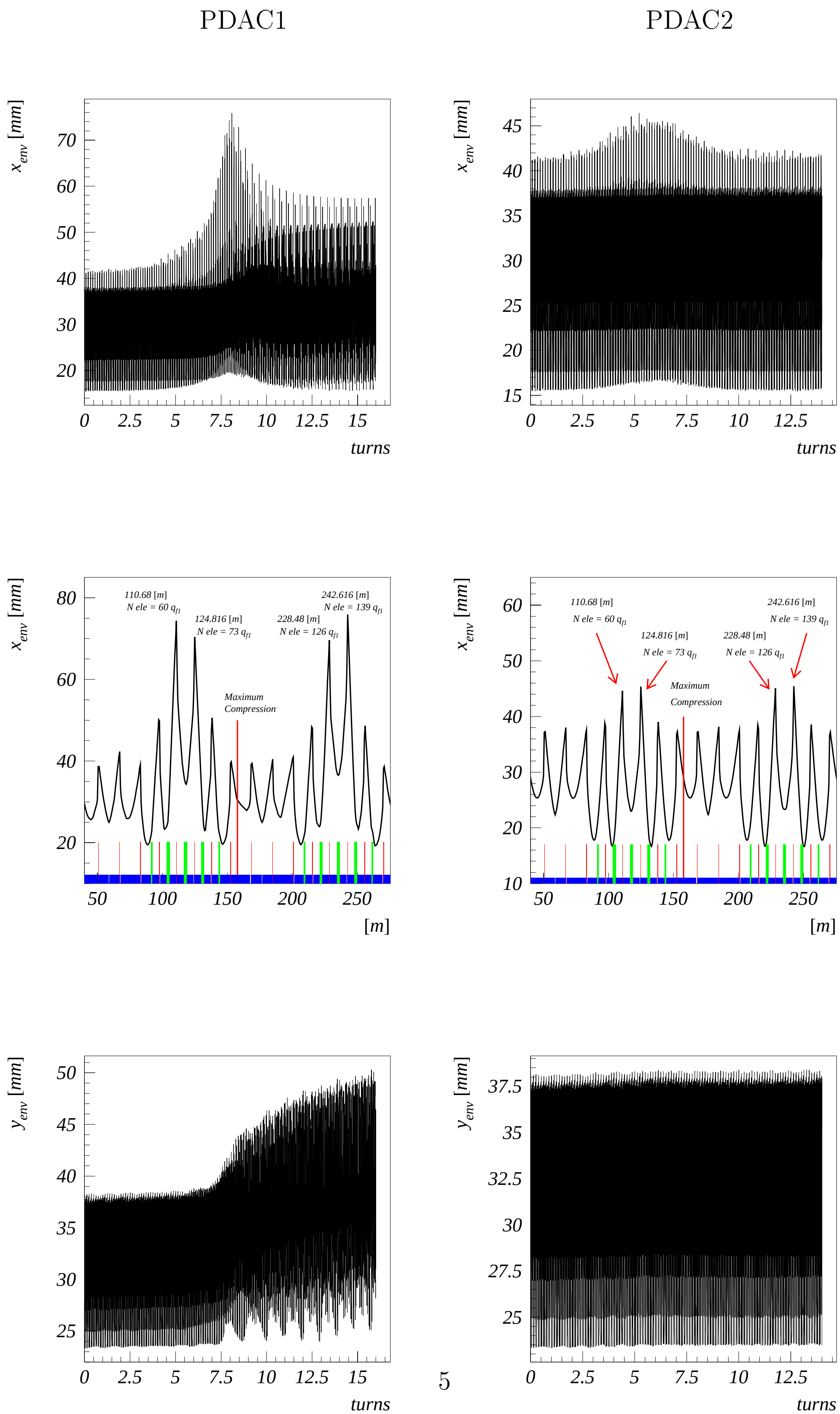

Figure 1: PDAC1 and PDAC2 tracking results. Top row: Horizontal envelope (defined by the outermost particles) tracking trough complete lattice. Middle row: zoom on region of maximum compression. Peaks occur in QF1 quadrupoles. Bottom row: vertical envelope tracking.