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# A Distributed Frequency RF Scheme for Capture of Muons

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## Introduction

Bob Palmer [1] has proposed a scheme for the capture of muons using only medium frequency RF. It comprises of a decay and drift area, where the correlation between momentum and time is formed. The beam is then bunched using RF (at a frequency near to 200 MHz) where each bunch is formed at a different average momentum. By beating two close frequencies the average momentum of each bunch can be corrected. The clear advantage of such a scheme is that it uses standard medium frequency RF with which the accelerator community has much experience and high gradients have been obtained. However, it will be shown that the number of muons is not (so far) interesting.

As a starting point we imagine taking a muon beam with a longitudinal momentum centred around $P_0$ with total limits of $\pm\Delta P$. If we wish to produce $n_b$ bunches at a frequency of $f_0$, then the required drift length is

$$L_{drift} = \frac{cn_b}{f_0}\left(\sqrt{\frac{m^2}{(P_0-\Delta P)^2}+1}-\sqrt{\frac{m^2}{(P_0+\Delta P)^2}+1}\right)^{-1}$$

where $m$ is the muon mass. While bunching the beam, we can consider that each bunch centre (at a fixed momentum) will shift in time relative to the adjacent bunch. As the period between bunches increases it results in change in frequency per length $L$ of

$$\frac{\Delta f}{L} = \frac{f_0^2}{c}\left(\sqrt{\frac{m^2}{P_0^2}+1}-\sqrt{\frac{m^2}{(P_0+\delta P)^2}+1}\right)$$

where $\delta P$ is the momentum difference between two bunches. This assumes the variation of momentum with time along the macro-bunch is linear.

As the frequency cannot be continuously changed, the phase slippage of a bunch can be estimated for a fixed frequency, and by keeping this to a small value the distance over which the frequency can remain fixed (followed by a discrete jump) can be estimated from

$$\frac{\phi}{L} = \frac{2\pi f\Delta t}{L} = \frac{2\pi f}{c}\left(\sqrt{\frac{m^2}{P_0^2}+1}-\sqrt{\frac{m^2}{(P_0+\delta P)^2}+1}\right)$$

The procedure for adiabatic bunching requires too long a distance for the capture of high energy muons, so bunching is achieved here with a fixed gradient RF. In this case the frequency of the synchrotron oscillation ($f_s$) is

$$\left(\frac{f_s}{f_0}\right)^2 = \frac{qE_0 T\lambda\sin(-\varphi_s)}{2\pi mc^2\gamma^3\beta}$$

where $q$ is the muon charge, $E_0$ the RF gradient, $T$ the transit time factor, $\lambda$ the wavelength, $\varphi_s$ the synchronous phase and $\beta$ and $\gamma$ the Lorentz factors. This will give rise to a non-negligible variation in the synchronous oscillation frequency for different energy muon bunches.

Table 1. Important parameters for the bunching and momentum correction scheme.

| | | |
|---|---|---|
| $f_0$ | 176 | MHz |
| $P_0$ | 200 | MeV/c |
| $\Delta$P | 100 | MeV/c |
| $n_{bunch}$ | 30 | |
| $L_{drift}$ | 129.7 | m |
| $\Delta f/L$ | 424.3 | kHz/m |
| $\phi/L$ | 1.7 | $^{O}$/(m.bunch) |

The ratio of the number of synchronous oscillations ($n_{oscil}$) of a bunch with momentum $P$, to the number performed by a bunch with $P$=200 MeV/c, in a fixed distance, is given in Figure 1. It is clearly seen that if the bunching is designed to provide a bunched beam for the 200 MeV/c bunch, the beam around 125 MeV/c will again be debunched. In this case the correction of the average momentum by a RF field will become impossible.

If the amount of rotation is restricted to allow a different in the relative rotation of only 20%, the limits of capture would be for bunches between 180<P<230 MeV/c.

One further solution to the problem is to adapt the gradient of the RF in order to keep $f_s$ constant with the muon bunch momentum.

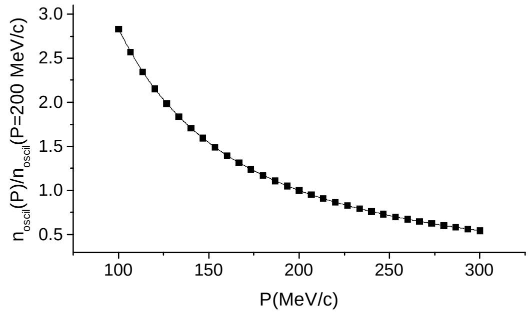


Figure 1. Number of synchronous rotations of a muon bunch as a function of momentum, relative to the number of rotations achieved by a bunch with a momentum of 200 MeV/c.

After the beam has been bunched, the average momentum can be corrected using two closely spaced frequencies, i.e.

$$E = E_0 \cos(\overline{\omega} t)\cos(\Delta\omega t)$$

where the two RF cavities oscillate at $\varpi+\Delta\omega$ and $\varpi-\Delta\omega$ with equal fields. A plot of the phase correcting RF wave is shown in Figure 2. Using the linear part of the beat frequency such that the bunches only traverse ±pi/4, the frequency difference of the cavities is fixed as

$$\Delta f = \frac{f_0}{4n_b}$$

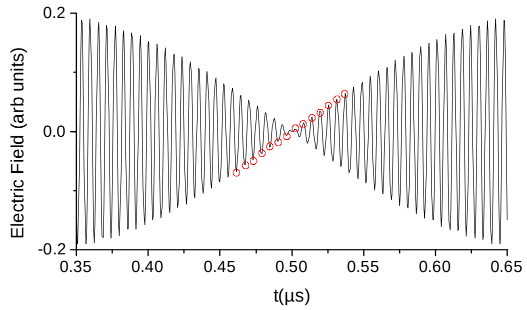


Figure 2. Scheme of the beat-wave momentum correction. Bunches are indicated as circles.

## Simulations for 200±100 MeV/c capture

For the first case a scheme for the collection of muons with a wide momentum spread was tested. Within this spread a sufficient number of muons are available to be compared with the induction linac scenarios. Simulations were performed with ICOOL 2.03 and the input pion distribution was the same as used for calculations of the induction linac scheme presented in [2].

After the target and a drift of 5.7 m in a collection solenoid scheme, 8m of RF cavities at 5 MV/m and 60 MHz are applied to enhance the number of muons in a momentum window from 100 – 300 MeV/c.

Then the drift is continued to 130 m. The time-momentum correlation is shown in Figure 6.

The bunching then takes place at a gradient of 5 MV/m over a distance of 6 m. The transverse focusing is provided by a 1.42 T longitudinal field. After a further drift of 3 m the time-momentum phase space shown in Figure 4 is found. In this case the under and over-rotation of certain bunches is clear.

The fact that all bunches are not rotated to a comparable degree makes the further makes the momentum correction of the individual bunches impossible. As an illustration a beat-wave is applied for 1 m at 100 MV/m as an illustration that the bunches cannot be easily corrected. The output after this stage is shown in Figure 5.

## Simulations for 200±25 MeV/c capture

In order to demonstrate that such a scheme can work (hypothetically), it has been calculated for a reduced input momentum spread. In this case the rotation velocity of the bunches does not vary by more than 20% when compared to the average bunch momentum. This is at the expense of a greatly reduced number of muons that can possibly be captured. The total scheme is shown in Figure 3.

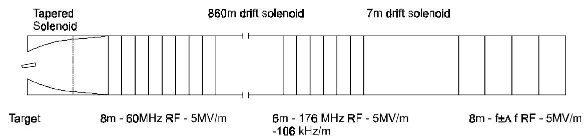


Figure 3. Scheme of the distributed RF capture system for a reduced muon momentum range. For final bunch momentum correction region, f=174.6219 MHz – 0.106 MHz/m and Δf=1.4543 MHz. The region to the left of the dashed line was modelled with FLUKA.

In order to enhance the number of muons, an 8m series of 60 MHz RF cavities providing 5 MV/m gradient is installed, 6m downstream of the target. To build the momentum-time correlation and provide 30 bunches, a total of 860 m of drift is required. At this point a cut is performed and only muons in the momentum range $180<P_z<230$ MeV/c are accepted.

The beam is then bunched over 6m of distributed frequency RF and an additional 7m of drift. At the output of this region, the moment-time distribution is shown in Figure 4.

The beat-wave RF is then applied to correct the each bunch average momentum to 200 MeV/c. This RF is applied over 8 m and the frequency must also be distributed in frequency. The momentum-time diagram of the bunches is shown in Figure 5, where all muons outside a ±45° limit with respect to the bunch centre have been removed.

Based on the production and capture file sol_fl99_2gev.dat, the number of muons per proton per GeV is

$N_{\mu/p}$ per GeV
$= (1635/5000)*(1/10)*(6984/10^6)*(30/2.6)/2$
$= 1.32 \times 10^{-3}$

or in terms of muon per pion

$N_{\mu/\pi} = 0.0327$

The value for number of muons per proton GeV (0.0013) is about one third of the number suggested for the muon decay ring, in the PJK scenario.

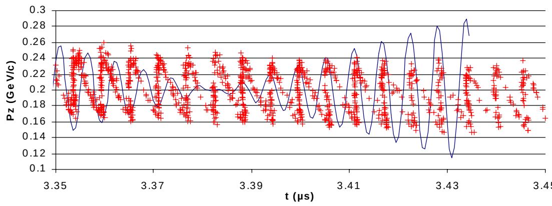


Figure 4. Momentum-time plot of bunches before entering momentum correction region. Beat-wave RF is shown super-imposed.

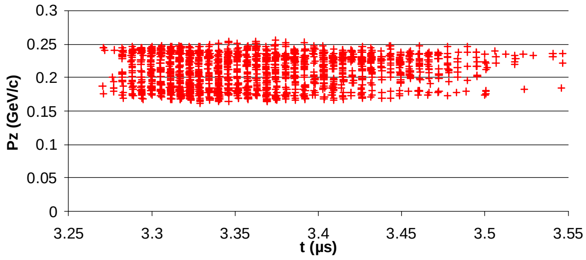


Figure 5. Output bunched beam, capturing from $180<P_z<230$ MeV/c.

## Conclusions

It has been demonstrated that a muon beam with a momentum range from 180 to 230 MeV/c can be captured (from a beam dynamics point of view) by applying a distributed RF scheme where the frequency is adjusted every metre.

Extending this scheme to capture a momentum spread from 100 to 300 MeV/c proves to be very difficult as the difference in synchrotron rotation means that the bunches are not all rotated to the same degree at the point of the momentum correction. One possible solution is to reduce the cavity gradient to keep the rotation speed constant. However, with the scheme already requiring 14 difference frequencies, the prospect of simultaneously changing the gradient would make the scheme very complicated. However, the simulated scheme above provides less than 1/3 of the requirement of a neutrino factory and while it remains an interesting option, it does not seem worthwhile to continue this line as of present.

## References

[1] B. Palmer, transparency Workshop on Exotic m± Cooling Methods, p229, UCLA, Feb 22-24, 1999.

[2] K. Hanke and R. Scrivens, Simulations of an Induction Linac with Realistic Field Configuration, CERN NF Note 17.

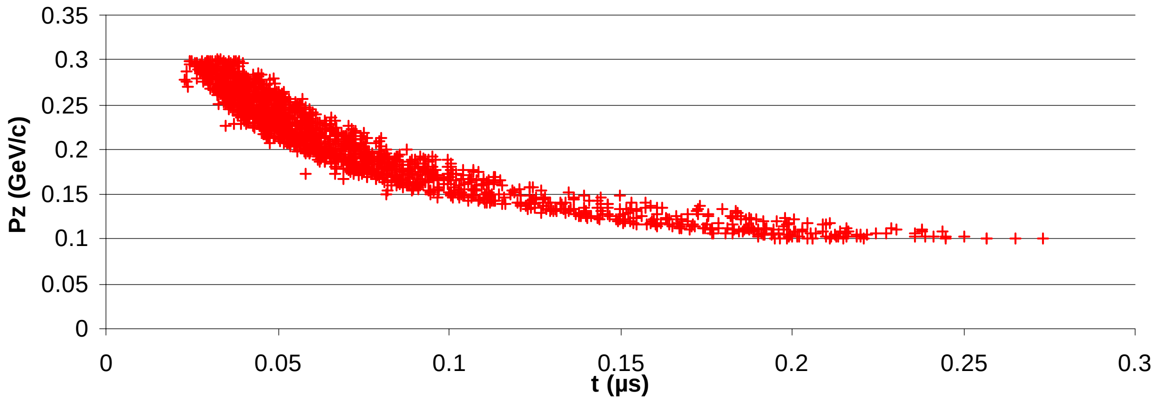


Figure 6. Time-momentum phase space of muons after an initial capture RF and the 130 m drift/decay solenoid

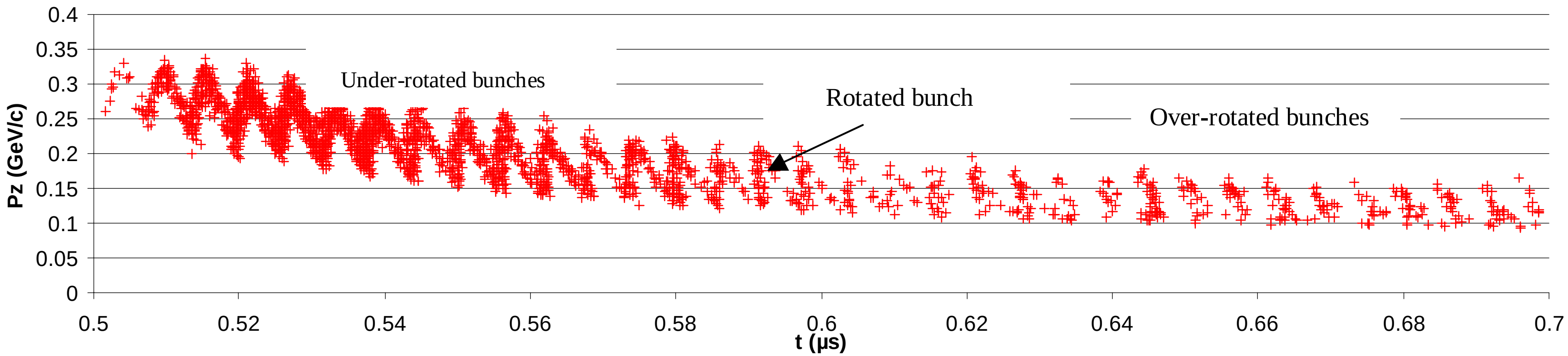


Figure 7. Time-momentum phase space of muons after bunching RF.

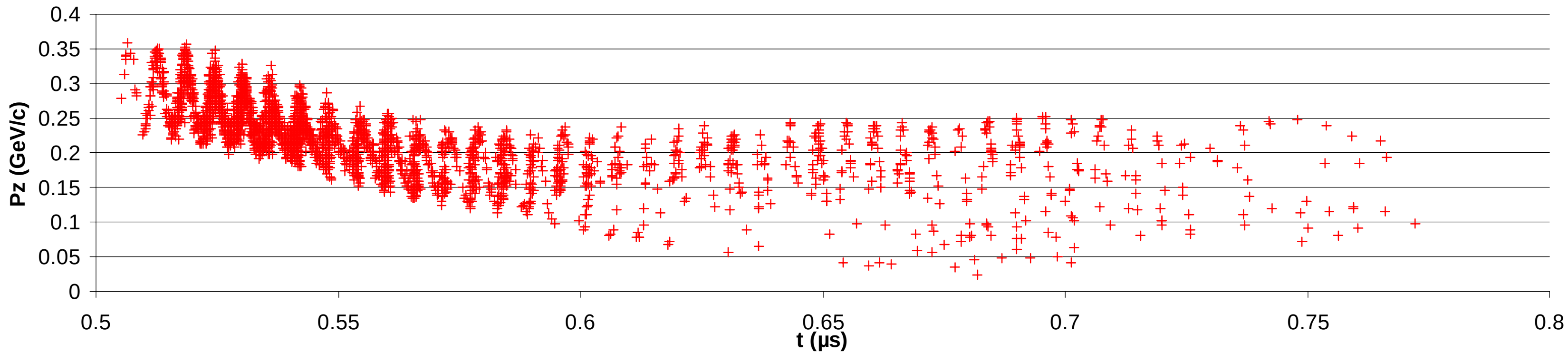


Figure 8. Time-momentum phase space of muons after attempting a momentum correction with a beat-wave.