

# Example Beam Dynamic Designs for a Neutrino Capture and Phase Rotation Line using 50 m, 100m and 200m Long Induction Linacs

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## Layout

In this note the results of some simulations are reported for a capture and phase rotation using only solenoids and Induction Linac elements.

FNAL have previously proposed a similar scheme and have calculated for the pion/muon beam derived from a higher energy proton beam at 16 or 24 GeV [1]. The simulations are based on the following layout (see Figure 1).

1. Pions ( $\pi^+$ ) are collected from the target using a tapered solenoid (max  $B=20$  T) as favoured by the Muon Collaboration. The pions are generated by 2 GeV (kinetic) protons incident on the target with a very short bunch length. The production of pions and the first 3.3 m of tapered solenoid was modelled with GFLUKA [2]. The target consists of 2.6 cm of Hg, which is only 20% of the interaction length. A full target would be of the order of 30 cm in length.
2. The solenoid is continued at 1.4 T with a diameter of 60 cm to allow decay from pions to muons and the correlation of energy with time to develop. No RF is included in this region. This lack of a first phase rotation means there is no enhancement of the polarisation. The drift/decay solenoid is 200 m in length.
3. An induction linac is applied for the correction of the momentum spread to an average value of 200 MeV/c. The number of muons collected by restricting the Linac length to 50, 100 and 200 m will be given. A  $\Delta P/P=20\%$  limit is applied for the muons at the output of this region to give an well defined momentum spread at the input of the following buncher. The beam is focused in the induction linac by a continuous solenoid at 1.4 T. The maximum radius is continued through the whole bunch rotation scheme at 30 cm.

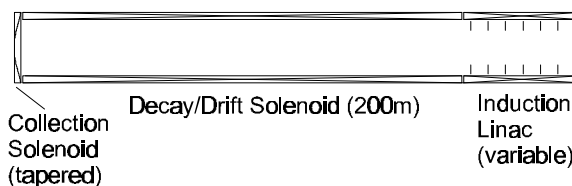
## Induction Linac Implementation

Correction of the energy spread by the induction linac is split into three domains. At the entrance of each domain the average momentum of the muons as a function of time is calculated from the surviving muons. From this data the energy required to correct the muons to

200 MeV/c is found. This correcting field is then applied over the over the number of meters required to correct half the momentum spread at the maximum gradient of  $\pm 2$  MV/m. The process is then repeated. In the final domain the momentum is fully corrected.

The simulations have been performed using ICOOL 2.02. The induction linac model has been modified to allow the introduction of a polynomial to represent  $E_z(t)$  up to the eighth order, although only the sixth order has been used here. In addition the linac has a “switch on” time, before which no electric field is present. The electric field is limited only to the longitudinal component and does not vary with the radius. A limit of  $\pm 2$  MV/m for the accelerating gradient is imposed on the induction linac.

The transverse emittance is only limited by the acceptance of the 1.4 T solenoid with an aperture of 60 cm. At the output the number of muons inside an emittance of 9 and 14 mrad (rms-normalised) will be estimated.



**Figure 1. Scheme of the collection and phase rotation studied herein.**

## Results

The pion production from the target, as modelled with GFLUKA, is favoured towards  $\pi^+$ . From  $10^6$  protons incident on the target, 6984  $\pi^+$  are produced and only 3611  $\pi^-$ , inside longitudinal moment cuts of  $50 < P_z < 1000$  MeV/c. For the simulations presented here 5000 pions were used at the input.

After the 200 m decay and drift solenoid 2825 muons are left to enter the Induction Linac, the majority of the particles are lost in the first few meters of the solenoid due to impact on the wall.

All three linac lengths are sufficient to correct the low energy muons. The only limit to the transmission was in the number of high momentum muons that could be captured.

The main parameters of the muon beam at the exit of the induction linacs are presented in Table 1. The output parameters apply to muons within a momentum cut at the output of  $180 < P_z < 220$  MeV/c.

The number of muons per proton per GeV (scaled for the longer target) should be compared with a requirement of 0.00875 given in the “Draft Parameters” report for the PJK scheme [3] at the end of the phase rotation section.

The results clearly demonstrate that increasing the linac length over 50 m does not greatly enhance the muon yield.

The Momentum-Time phase spaces at the output of the 100 m induction linac, before and after momentum cuts are shown in Figure 2. A profile of the muon pulse is given in Figure 3 for the same 100 m long linac.

The number of muons into an acceptance of 9 and 14 mm.rad (interesting figures for the following cooling channel) have been estimated as ( $N_{\mu/p}$  per GeV scaled linearly for target length) 0.0049 and 0.0095 respectively. Hence the larger emittance option still has sufficient particles to be interesting.

All the ICOOL input files are available from:

<http://nicewww.cern.ch/~scrivens/nufactrs/nufactrs.html>

**Table 1. Beam parameters for two different induction linac lengths.**

Length of Linac	50 m	100 m	200 m
$N\pi^+$ input	5000	5000	5000
$N\mu^+$ output	1574	1610	1678
$N_{\mu/p}$	0.00220	0.00225	0.00234
$N_{\mu/p}$ per GeV	0.00110	0.00112	0.00117
$N_{\mu/p}$ per GeV\$	0.0127	0.0130	0.0135
ICOOL $\epsilon$ (mm norm)	24	24	24
LBL $\epsilon$ (mm norm)	18	18	18
$\Delta P/P$ rms	3.1%	3.1%	3.4%
$\Delta P/P$ total	20%	20%	20%
Muon length (tot $\mu$ s)	0.58	0.47	0.52
Muon length (95% $\mu$ s)*	0.33	0.34	0.40

\$ Linearly scaled by the ratio 2.6cm/30cm to estimate the effect of increasing the target length.

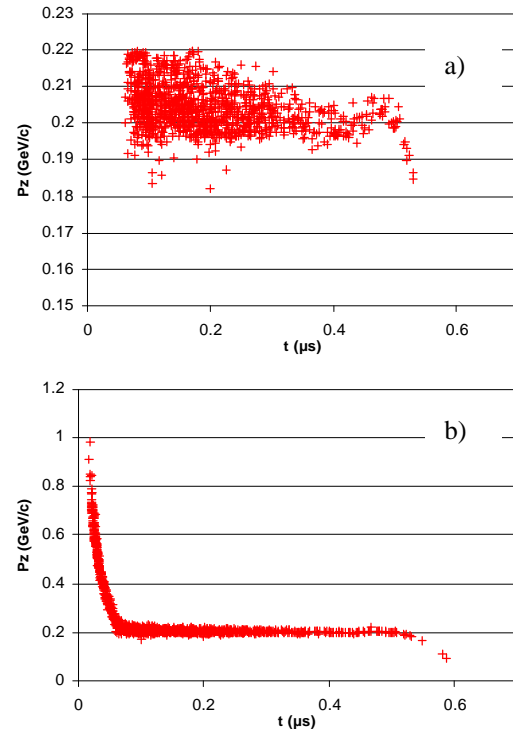
\* Pulse length containing 95% of muons.

## Comments on an Induction Linac Phase Rotation

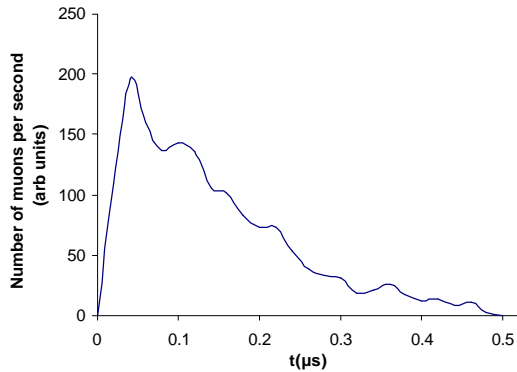
1. The scheme eliminates the possibility of using multiple bunches from a synchrotron or accumulator ring, mainly as the bunches would overlap. If the dynamics are designed to keep the bunches separated, pulsing the induction linac in burst mode at the frequency of the bunches from the proposed accumulator ring (3.6 MHz) would be very difficult, but perhaps not impossible. The

induction linac probably limited to the kHz rep-rate regime.

2. The power consumption of the muon-cooling RF will probably not allow a rep-rate into the kHz regime to be used. The condensing of the number of bunches in the accumulator to a single bunch regime could be achieved with different path length delivery lines to the target area, allowing all the proton bunches to strike the target simultaneously.
3. In this note no comment is offered on the feasibility of building and operating such a long induction linac, especially at such high repetition rates. It is only noted that reliability of the entire structure is aided by the fact that most of the elements would be identical in design and that the failure of a few of the elements would not lead to a large muon beam loss.



**Figure 2. Longitudinal phase space at the output of a 100 m long induction linac. a) With and b) without momentum cuts.**



**Figure 3. Particle density of muons as a function of time at the output of the 100 m induction linac, after momentum cuts.**

## Further Studies

Clearly there is still much simulation work to perform and some of the following optimisation should be addressed if a scheme similar like this is to be seriously considered for a neutrino factory.

- Calculate linac structures shorter than 50 m.
- Include a first phase rotation RF (with moderate fields) to enhance the number of muons captured.
- Add a mini-cooling section to allow the initial drift/decay solenoid to be shortened and decrease the muon transverse emittance.

In addition the following approximations in the ICOOL model should be addressed to have more accurate beam properties at the output of the linac.

- Reduce the initial drift/decay solenoid length, which is rather long in this case. In particular the distances should be reduced to assess the number of muons captured inside a time window consistent with the accumulator scheme (i.e. 300 ns between bunch centres).
- Include a more realistic magnetic field description with closely separated solenoids in the decay/drift region and a solenoid design compatible with the internal workings of an induction linac [1].
- Include more realistic field description of the induction linac cells, including transverse electric fields.
- Include pion beam parameters using a realistic length of target and initial proton beam parameters (in particular the bunch length).
- Calculate the beam dynamics with space-charge.

## Conclusion

With 4 MW on target at 2 GeV the resulting muon per proton number of 0.019 corresponds to  $2.4 \times 10^{21}$   $\mu$ /year in a  $10^7$ s year in an emittance of 14 mm.rad, at the output of the phase rotation, with a RMS momentum spread less than 4%. The bunch length would be  $\sim 0.3$   $\mu$ s.

## References

1. V. Balbekov and N Holtkamp, Phase Rotation of Muons using an Induction Linac, Muon Collider, Note 59 and V. Balbekov, Optimisation of Phase Rotation using an Induction Linac, Muon Collider Note 63. See <http://www-mucool.fnal.gov/notes/notes.html>
2. Pion Production modeled by N. Vassilopoulos using GFLUKA.
3. B. Palmer, Neutrino Factory Draft Parameters, Muon Collider Note 46. See <http://www-mucool.fnal.gov/notes/notes.html>