Neutrino Factory Note 13

February 11, 2000

Geometry of the muon storage ring

André Verdier

Keywords:

Summary

Possible geometries of a muon storage ring have been determined from the requirement of the position of the detectors. If the detector distances are specified and not their azimuth, there remains one free parameter to determine the geometry. It can be used for instance to optimise the slope of the machine plane. On the CERN site the maximum height of the storage ring is limited by the thickness of the molasse to less than 300m. Then the maximum length of the straight section pointing to the far detector is about 800m. For a ring of triangular shape with two equal sides, this length represents about 36% of the ring circumference using 4T dipoles in the arcs. The overall maximum tunnel slope is 33°.

1 Introduction

The experiment working group requested recently that one large straight section of the muon storage ring point to a detector 3000km apart (the far detector) and another one points possibly to a detector 1000km apart (the near detector). The optional case with a detector at 9000km has not been considered as the slope of the large straight section pointing to this detector is already 45° and the maximum slope of the tunnel is even larger as seen below.

For a construction on the CERN site, the total height of the machine must be less than 300m so that the tunnel stays inside the molasse layer.

These conditions leave little freedom to determine the geometry of the machine.

In what follows, the procedure to design this geometry is first explained. Then some tables of possible parameters are given. A tentative cost is evaluated based on that of existing machines.

2 Procedure to determine the geometry of the muon storage ring

We follow merely the machine ring starting from the straight section pointing to the far detector, which is the main request.

Using a spherical model for the earth, two parameters determine how to start from a given point from the surface of the sphere to reach the surface again at a given distance via a straight line : the azimuth of the plane containing the line and the angle the line makes with the local tangent plane. We take arbitrarily the plane of the line pointing to the far detector as the reference vertical plane {Oy,Oz} of the local coordinate system used in the MAD program [1]. In our case the beam pointing to the far detector goes downwards and makes an angle ϕ (this is the name of the variable used in the SURVEY command in MAD) with the axis Oz.

At the end of this straight section, we enter a first arc. The angle the machine plane makes with the plane {Oy,Oz} is determined by the parameter ψ of the SURVEY command. If ψ is zero, the machine plane is perpendicular to the plane {Oy,Oz}. For a given ψ in a relevant range, there exist a value of the bending angle of the arc such that the exit trajectory points to the near detector.

If we assume that the third straight section is equal to the second one, the entire ring is determined.

ψ/rad.	max. slope	slope of SS3	SS1/$\mathcal{C}$	SS2/$\mathcal{C}$	$\mathcal{C}$/m	alt. dif./m
0.235466	19.07°	18.31°	0.297	0.297	2690.89	281.59
0.3	21.80°	21.80°	0.336	0.270	2378.01	273.43
0.4	26.44°	25.37°	0.369	0.248	2168.29	272.05
0.535	33.3°	33.1°	0.390	0.234	2051.52	277.04

Table 1: Tunnel slope and lengths ratio for some geometries with two equal straight section. SS1 is the straight section pointing to the far detector its length is 800m and its slope is 13.6°. SS2 points to the near detector, its slope is 4.50°. SS3 is the last straight section with the same length as SS2. The first line of the table refers to a 3-fold symmetric ring (SS1=SS2). For a race-track machine with the same arc and straight section of 800m, the ratio SS/$\mathcal{C}$ is 0.423, $\mathcal{C}$ being the ring length. The computations were done for slopes smaller than 33.3°.

3 Results

The length of the straight section pointing to the far detector should be about 800m to keep the ring inside the molasse layer. A filling factor of 0.6 has been assumed for the arcs. This means that the lengths of the dipoles are given by $\rho/\theta/0.6$, where ρ is the bending radius (e.g. 27.778m for particles at 50GeV and a field of 6T) and θ the bending angle. Each arc is described by a sequence of ten sector bends. The length of the dipoles in the first arc is varied to obtain the required slope at its end. The third arc is identical to the first one. The bending angle of the second arc has the value required to close the machine.

3.1 6T dipoles in the arcs

For a high field of 6T in the arc dipoles, the slopes and lengths of the storage rings are given in table (1). The maximum slope assumed is that of standard stairs, i.e. 33.3°. Larger slopes can be considered if it does not hamper the installation. For the 33.3° maximum slope, the ratio of the length of the large straight section to the circumference is already 39%, i.e. well above the specifications of the experiments working group which was 25%. It has not been attempted to adjust the altitude difference in the ring to 300m exactly. This altitude difference is obtained for a length of SS1 of 880m. This length represents 0.395 of the circumference. Thus for an increase of the length of the straight section of 10%, the increase of the ratio is not that large, which indicates that the straight section is probably a little too long. Actually the determination of this length should be included in a cost optimisation process involving the whole factory system, which is well outside the scope of this note.

For a triangle shape with three equal straight sections (first line in table (1)), the length ratio is 29.7%, i.e. about what is requested by the experiments working group.

3.2 Scaling the dipole field

If the bending radius is changed, nothing else needs changing to obtain a closed machine. The slopes are the sames as in table (1). The lengths ratio have been computed for the last line of table (1). They are given in table (2). We see that the efficiency of the large straight section SS1 of length 800m does not change very much, e.g. from 39% for 6T to 36% for 4T with the restriction that the machine height reaches 302m, i.e. the limit associated with the molasse layer, for the case of 4T dipoles.

Dip. field/T	SS1/$\mathcal{C}$	SS2/$\mathcal{C}$	$\mathcal{C}$/m	alt. dif./m
6	0.390	0.234	2051.52	277.04
5	0.379	0.227	2109.69	287.20
4	0.364	0.218	2196.96	302.44

Table 2: Lengths ratio for the case with the maximum slope of 33.3°. The lengths of the straight sections are the sames as in the last line of table (1).

4 Cost estimate

The cost is spread over three items : tunnel, arc, straight sections.

For a tunnel similar to that of LEP, the price is 19MSF/km including the extra costs associated with this sort of civil engineering.

An upper limit of the cost of the straight sections is that of the LEP machine, i.e. 14MSF/km.

The cost of the arc should be similar to that of RHIC (well below that of LHC which is about 100MSF/km), i.e. 20MSF/km.

For a machine with 4T dipoles in the arcs, the total cost is about 77MSF. This cost is much smaller than that of the recirculator. This means that there is little optimisation to do with it. In particular the arc dipoles do not have to be designed with particularly high fields, in opposite with the preliminary expectation of the working group.

5 Conclusion

It appears that the request of sending neutrino beams to detectors 3000km and 1000km from the source, together with the constraints associated with the CERN site and acceptable tunnel slopes leave little freedom for the geometry of the muon storage ring.

With a 4T field in the arc dipoles and a maximum tunnel slope of 33° and a length of the long section of 800m, the efficiency of this section, pointing to the far detector, is about 36% and that of the straight section pointing to the less far detector 21%. The circumference length is 2.2km. The price of this machine is around 80MSF, without taking into account the possible recuperation of the LHC cryoplant.

References

[1] H. Grote and F.C. Iselin, The MAD program (Methodical Accelerator Design) version 8.16, User's reference manual, CERN/SL/90-13(AP), (rev. 4) (March 27, 1995).