# Neutrino Oscillations in Extended Anti-GUT Model[‡]

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**Abstract.** What we call the Anti-GUT model is extended a bit to include also right-handed neutrinos and thus make use of the see-saw mechanism for neutrino masses. This model consists in assigning gauge quantum numbers to the known Weyl fermions and the three see-saw right-handed neutrinos. Each family (generation) is given its own Standard Model gauge fields and a gauge field coupled to the $B-L$ quantum number for that family alone. Further we assign a rather limited number of Higgs fields, so as to break these gauge groups down to the Standard Model gauge group and to fit, w.r.t. order of magnitude, the spectra and mixing angles of the quarks and leptons. We find a rather good fit, which for neutrino oscillations favours the small mixing angle MSW solution, although the mixing angle predicted is closest to the upper side of the uncertainty range for the measured solar neutrino mixing angle.

An idea for making a "finetuning"-principle to "explain" the large ratios found empirically in physics, and answer such questions as "why is the weak scale low compared to the Planck scale?", is proposed. A further very speculative extension is supposed to "explain" why we have three families.

## I INTRODUCTION

Anti-GUT is the name which we have given to the model based on our favourite gauge group

$$AGUT = SMG \times SMG \times SMG \times U(1)_f. \tag{1}$$

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It began in the work of one of us (H.B.N.) with N. Brene, D. Bennett and I. Picek and others, but the inclusion of the $U(1)_f$ and the application to study the masses and mixing angles for quarks and leptons was together with C.D.F. and his students (G. Lowe, D. Smith, M. Gibson). Here the symbol $SMG$ stands for the Standard Model Group—and we may really think about it as a group [1] and not only a Lie algebra. It may then be assigned physical significance via the spectrum of representations it has (but one may just think of it as a Lie algebra, if one wants):

$$SMG = S(U(2) \times U(3)) = SU(2) \times SU(3) \times U(1). \tag{2}$$

In the present talk we shall actually present a slightly extended version of the "original" Anti-GUT model, which could thus be designated the extended Anti-GUT. This model may be described by saying that to each family we assign its own set of gauge fields, consisting of a set of Standard Model gauge fields plus a gauged $U(1)$ group coupling to the family in question through the $B-L$ quantum number (but only to that family, the remaining families being considered to have zero value for this $(B-L)$-charge and for Standard Model ones related to the considered family gauge bosons). Here $B-L$ stands for baryon number minus lepton number. That is to say the gauge group of the extended Anti-GUT—the model we shall consider in the present talk—is taken to be

$$(SMG \times U(1)_{B-L})^3 \approx SU(2)^3 \times SU(3)^3 \times U(1)^6. \tag{3}$$

Here it is meant that the subgroup $(SMG_i \times U(1)_{B-L,i}) = S(U(2) \times U(3)) \times U(1)$ $= SU(2) \times SU(3) \times U(1)^2$ only couples to the $i$th family and that, in the usual way, the quarks of family $i$ are triplets under the $SU(3)_i$ (but singlets under the other $SU(3)_j$'s, *i.e.* when $j \neq i$) and so on. The $(B-L)_i$ couples to the $B-L$ = baryon number $-$ lepton number for the $i$'th family.

This essentially describes the couplings in the model, but there is a little tricky point that was introduced, because it could be shown that otherwise we would have had a no-go theorem for fitting the quark spectra [2]: the right-handed up-type ("up-type"$= u$, $c$, and $t$) quarks of the second and third family are permuted. That is to say the right-handed components of what is experimentally seen as the third family particle—the top quark—are dominantly equal to the right-handed up-type components coupling to the second family gauge fields. Similarly the experimentally seen right-handed charm quark is identified with the formally right-handed top quark of our model, namely the components coupling to the third family gauge fields.

## II MOTIVATION FOR THE MODEL

But why should a model of the proposed type be expected to have a chance of being the right one?

Well, the group may be characterised by means of a few relatively simple requirements with some phenomenological support: It is the largest possible gauge group that transforms the known 45 Weyl fermions of the Standard Model plus three right-handed see-saw neutrinos into themselves, without unifying any irreducible representations under the Standard Model and with no anomaly troubles—neither gauge anomalies nor mixed anomalies. That is to say it may be characterised as the biggest group for which the gauge symmetry can be upheld without any Green-Schwarz anomaly cancellation, without unifying particles that are not already unified in the Standard Model and without transforming them into "fantasy particles" only existing in the model.

Looking for inspiration for what gauge group to believe in beyond the Standard Model, we are led to consider hints from the following remarkable features of the quark and lepton spectra:

- There are very big mass ratios from family to family and even to some extent within the families. This feature strongly suggests that different family particles should have different quantum numbers. How else should the quantum number differences between right- and left-handed components be so different that they can be used to mass protect some particles by much bigger factors than others?

- Pure $SU(5)$, or for that matter all the Grand Unified Theories (GUTs) extending $SU(5)$, predicts the GUT-scale mass relations:

$$\frac{m_\tau}{m_b} = \frac{m_\mu}{m_s} = \frac{m_e}{m_d} = 1 \tag{4}$$

  unless one helps it by introducing rather big representation Higgs fields, $\underline{45}$ say. However these relations, eq. (4), are not well fulfilled by the renormalisation group evolved experimental values. Only the third family mass ratio $m_\tau/m_b$ fits reasonably well with the simple version of $SU(5)$-GUT (or its SUSY extension). So we would be much better off w.r.t. fitting with experiment, if we could get the $SU(5)$ mass predictions only as predictions up to factors of order of unity, but not as exact relations. So we really do not want to unify the down-type quarks with the charged leptons, because the masses do not match except in the $\tau - b$ case.

We may classify gauge group proposals, restricted in each case to the factor group which acts on the 45 known Weyl quark and leptons (or 48 if you count the perhaps non-existing right-handed neutrinos), according to whether the group is small or big and according to whether it unifies or avoids unifying the various irreducible representations under the Standard Model (SM). Especially we can ask for the four groups obtained by requiring minimal or maximal degree of such group size and of the degree of unification. Under all circumstances we must of course, in order to have a consistent gauge field model, require that the anomalies cancel—in the present article we shall ignore the possibility of Green-Schwarz cancellation, with the excuse that we do not find any Green-Schwarz anomaly cancellation needed between the Standard Model gauge fields. Nature has even "carefully" chosen to make equally many quark and lepton families, thereby ensuring simple anomaly cancellation between the Weyl particle contributions without any Green-Schwarz mechanism.

The four corners of the set of gauge groups acting on the Standard Model Weyl fermions can be seen [3] to correspond to the following gauge groups:

- $\underline{\text{small, separating}}$ : $SMG$.
- $\underline{\text{small, unifying}}$ : $SU(5)$(= usual GUT).
- $\underline{\text{large, separating}}$ : $SMG^3 \times U(1)_f$ (= Anti-GUT)
- $\underline{\text{large, unifying}}$ : $SU(5)^3 \times U(1)_f$ (= Rajpoot Model with $U(1)_f$ [4]).

If we say that we want many approximately conserved gauge quantum numbers in order to produce many different big mass ratios, we are suggested to take "large" groups so as to have many possibilities for separating the particle masses order of magnitudewise. If we decide that really the mass predictions in simple GUT-$SU(5)$ are (except for the third family) not true—because we do not like the big $\underline{45}$ representation—then we favour a separating gauge group. In this way it is suggested that we should seek the right model in the $\underline{\text{large, separating}}$-corner; but that is just our Anti-GUT model!

Originally we started from ideas of what we called "confusion" [5,6]. This means that, if there were different gauge fields attached to isomorphic gauge groups, there is a mechanism (the "confusion") that could cause the gauge group with many identical cross product factors to break down to its diagonal subgroup. Thus only one group would survive from each class of isomorphic ones. The model used at that time was a chaotic lattice gauge theory and the breaking to the diagonal subgroup came about by speculating that, after some going around in the lattice, one would lose track of which group was which. In this way there should in practice only be one–the diagonal subgroup.

Such a model could even "explain" why the Standard Model group is essentially the cross product of the three lowest dimensional (simple) groups from which to build gauge groups, namely $U(1)$, $SU(2)$, and $SU(3)$. The explanation should be that on a very short scale level—Planck scale or fundamental scale say—there are many gauge groups of a lot of types, but the isomorphic ones get "confused" and we only find one representative of each class of isomorphic ones. Also there is a general expectation that, in order to survive at scales large compared to the fundamental scale, the gauge groups should have relatively small dimension with matter fields in small chiral representations.

We used this picture in connection with the assumption which we today would call the multiple point principle (MPP [7,8]). The MPP says that the coupling constants are—by some presumably yet to be explained effect (perhaps Baby-universe theory)—being finetuned by Nature to make a lot of phases meet each other just for those values of the couplings that are realized in Nature. This idea of a lot of phases meeting is analogous to the feature of a microcanonical ensemble that it will often obtain just the temperature of a phase transition. In English one has a special word for a mixture of ice and water, namely “slush”. So such mixtures must be quite common. When in equilibrium as a microcanonical ensemble, such slush has (under one atmosphere pressure) always the temperature that by definition is zero degrees Celsius. The microcanonical ensemble constitutes a model for how it can happen that this phase transition temperature $0^o$ becomes much more common than most other single temperatures; there is a delta function distribution of temperatures with the delta function peaks at the phase transitions. If there were a true analogy between the microcanonical ensemble making specific temperatures much more likely than others and a mechanism making coupling constants in Nature take those values that are just between the different phases of the theory, we would have a mechanism for our MPP.

We used a replacement for this slush analogous MPP-idea to postulate that, in Nature, the gauge couplings should take just those values that correspond to lattice artifact phase transitions—values that can be read out of lattice calculation literature. Combining this postulate (in an old version) with our favourite Anti-GUT gauge group we fitted the number of families—not yet known at that time—and tried to say that it was so accurately determined that we could almost see it was an integer, namely three. While one of us (H.B.N.) was giving talks about that, it was found at LEP that the neutrino decays of the $Z^0$-gauge boson showed that there were indeed only three families. So we had made a successful prediction!

## III ANTI-GRAND UNIFICATION THEORY AND ITS EXTENSION

In this section we review the Anti-GUT model and its extension describing neutrino masses and mixing angles.

The Anti-GUT model [9–14] has been put forward by ourselves and collaborators over many years, with several motivations. It is mainly justified by a very promising series of experimental agreements, obtained from fitting many of the SM parameters with rather few Anti-GUT parameters, even though most predictions are only made order of magnitudewise. The Anti-GUT model deserves its name in as far as its gauge group $SMG^3 \times U(1)_f$, which replaces, so to speak, the often-used GUT gauge groups such as $SU(5)$, $SO(10)$ *etc.*, can be specified by requiring that:

1. It should only contain transformations which change the known 45 (= 3 generations of 15 Weyl particles each) Weyl fermions—counted as left-handed say—into each other unitarily (*i.e.* it must be a subgroup of $U(45)$).

2. It should be anomaly-free even without using the Green-Schwarz [15] anomaly cancellation mechanism.

3. It should NOT unify the irreducible representations under the SM gauge group, called here $SMG = SU(3) \times SU(2) \times U(1)$.

4. It should be as big as possible under the foregoing assumptions.

In the present article we shall, however, allow for see-saw neutrinos—essentially right-handed neutrinos—whereby we want to extend the number of particles to be transformed under the group being specified to also include the right-handed neutrinos, even though they have not been directly "seen".

The extended group, which we shall use as the model gauge group replacing the unifying groups, can be specified by a similar set of assumptions to those used above by two of us [16] to specify the "old" Anti-GUT. We replace assumption 1 by a slightly modified assumption which only excludes unobserved fermions when they have nontrivial quantum numbers under the SM group, so that they are mass protected. The particles that are mass protected under the SM would namely be rather light and would likely have been seen. But see-saw neutrinos with zero SM quantum numbers could not be mass protected by the SM and could easily be so heavy as not to have been "seen".

The model which we have in mind as the extended Anti-GUT model [17], that should inherit the successes of the "old" Anti-GUT model and in addition have see-saw neutrinos, is proposed to have the gauge group $SMG^3 \times U(1)_f \times U(1)_{\mathrm{B-L},1} \times U(1)_{\mathrm{B-L},23} \approx SU(3)^3 \times SU(2)^3 \times U(1)^6$. It is assumed to couple in the following way: The three SM groups $SMG = SU(3) \times SU(2) \times U(1)$ are supposed to be one for each family or generation. That is to say, for example, there is a first generation $SMG$ among the three; for which all the fermions in the second and third generations are in the trivial representations, and with zero charge, while the first generation particles couple to this first generation $SMG$ as if in the same representations (same charges too) as they are under the SM. For example, the left proto-electron and the proto-electron neutrino form a doublet under the $SU(2)_1$ belonging to the first generation (while they are in singlets w.r.t. the other two $SU(2)$'s) and have weak hypercharge w.r.t. the first generation $U(1)_1$, with a value $y_1/2 = -1/2$ analogous to the SM weak hypercharge being $y/2 = -1/2$ for left-handed leptons.

The $U(1)_f$-charge is assigned in a slightly complicated way which is, however, largely the only one allowed, modulo various permutations and rewritings, from the no-anomaly requirements. It is zero for all first-generation particles and for all particles usually called left-handed. The $U(1)_f$ charge value on a "right-handed" particle in the second generation is opposite to that on the corresponding one in the third generation. See Table 1 for the detailed assignment.

The two last $U(1)$-groups, $U(1)_{\mathrm{B-L},1}$ and $U(1)_{\mathrm{B-L},23}$, in our model have charge assignments corresponding to the quantum number $B-L$ (= baryon number minus lepton number), though in such a way that the charges of $U(1)_{\mathrm{B-L},1}$ are zero for the second and third generations. The $U(1)_{\mathrm{B-L},1}$ charges are only non-zero for the first generation, for which they then coincide with the baryon number minus the lepton number. Analogously the $U(1)_{\mathrm{B-L},23}$-charge assignments are zero on the first-generation quarks and leptons, while they coincide with the baryon number minus the lepton number for second and third generations. In the next section, we will discuss anomaly cancellation in the extended Anti-GUT model.

It is then further part of our model that this large gauge group is broken down spontaneously to the SM group, lying as the diagonal subgroup of the $SMG^3$ part of the group, by means of a series of Higgs fields. The quantum numbers of these fields have been selected mainly from the criterion of fitting the masses and mixing angles w.r.t. order of magnitude. The Abelian quantum numbers proposed for the "old" Anti-GUT Higgs fields were:

$$S: \quad (\frac{1}{6}, -\frac{1}{6}, 0, -1) \tag{5}$$

$$W: \quad (0, -\frac{1}{2}, \frac{1}{2}, -\frac{4}{3}) \tag{6}$$

$$T: \quad (0, -\frac{1}{6}, \frac{1}{6}, -\frac{2}{3}) \tag{7}$$

$$\xi: \quad (\frac{1}{6}, -\frac{1}{6}, 0, 0). \tag{8}$$

These four Higgs fields are supposed to have VEVs of the order of between a twentieth and unity compared to the fundamental scale supposed to be the Planck scale. In addition there was then the Higgs field under the Anti-GUT-group which should take the role of finally breaking the SM gauge group down to $U(3) = SU(3) \times U(1)_{em}$, *i.e.* play the role of the Weinberg-Salam Higgs field:

$$\phi_{WS}: \quad (0, \frac{2}{3}, -\frac{1}{6}, 1). \tag{9}$$

Here the quantum numbers were presented in the order of first giving the three different weak hypercharges corresponding to the three generations $y_i/2$ ($i = 1, 2, 3$), and then the $U(1)_f$-charge.

In reference [9] we fitted the parameters, being Higgs fields VEVs, to the masses and mixing angles for charged fermions and the values are as follows:

$$\langle S \rangle = 1 \ , \ \langle W \rangle = 0.179 \ , \ \langle \xi \rangle = 0.099 \ , \ \langle T \rangle = 0.071 \tag{10}$$

In the following we shall often abbreviate the expressions for these VEVs by deleting the $\langle \cdots \rangle$ around the Higgs fields, mostly with the understanding that $S$, $W$, ... then mean the VEV "measured in fundamental units".

In the Anti-GUT model, the old as well as the new, it is assumed that, at the fundamental (Planck) scale, particles exist with whatever quantum numbers are needed as propagators in the chain diagrams used to generate the fermion mass matrices [11], as discussed in section VIII. The fitted "suppression factors" are the VEVs in units of the "fundamental scale" particles.

It has to be checked that extending the group, to have the $U(1)_{\mathrm{B-L},1}$ and $U(1)_{\mathrm{B-L},23}$ factors, does not disturb the successful features of the model. This can be done by only giving the fields $\xi$ and $S$ non-zero charges under these "new" $U(1)$ groups, so as to get:

$$S: \quad (\frac{1}{6}, -\frac{1}{6}, 0, -1, -\frac{2}{3}, \frac{2}{3}) \tag{11}$$

$$\xi: \quad (\frac{1}{6}, -\frac{1}{6}, 0, 0, \frac{1}{3}, -\frac{1}{3}) \tag{12}$$

where the last two quantum numbers are the $U(1)_{\mathrm{B-L,1}}$ and $U(1)_{\mathrm{B-L,23}}$ charges respectively.

But now we also want to introduce two new Higgs fields $\phi_{B-L}$ and $\chi$ into the model: the first, $\phi_{B-L}$, is a Higgs field used to fit the new scale that comes in from neutrino oscillations giving the scale of the see-saw particle masses. When the left-right-transition (Dirac neutrino) mass matrix is of the same order as the usual charged fermion mass matrices, this scale is of the order $10^{12}$ GeV.

In our model we use the gauged $B-L$, in fact the total one because we break $U(1)_{B-L,1} \times U(1)_{B-L,23} \supseteq U(1)_{B-L,\mathrm{total}}$ at a much higher scale (near the Planck scale), to mass protect the right-handed neutrinos. These right-handed neutrinos are meant to function as see-saw particles, so they can be sufficiently light to give the "observed" left-handed neutrino masses by the see-saw mechanism. The breaking of the $U(1)_{B-L,\mathrm{total}}$, and thereby the setting of the see-saw scale, is then caused by our "new" Higgs field called $\phi_{B-L}$.

In order to get viable neutrino spectra we shall choose the quantum numbers of $\phi_{B-L}$ so that the effective $\overline{\nu_{\tau_R}}\, C\, \overline{\nu_{e_R}}^t + h.c.$ term gets a direct contribution and is thus not further suppressed. This is the way to avoid "factorised mass matrices"— *i.e.* matrices of the form

$$\begin{pmatrix} \phi_1^2 & \phi_1\phi_2 & \phi_1\phi_3 \\ \phi_1\phi_2 & \phi_2^2 & \phi_2\phi_3 \\ \phi_1\phi_3 & \phi_2\phi_3 & \phi_3^2 \end{pmatrix} \tag{13}$$

with different order unity factors, though, on different elements. Such factorised matrices are rather difficult to avoid otherwise. If we get such a "factorised matrix" and, as in our model, have mainly diagonal elements in the $\nu$-Dirac matrix, $M_\nu^D$, then we get the prediction that

$$\frac{\Delta m_\odot^2}{\Delta m_{\mathrm{atm}}^2} \approx (\sin\theta_{\mathrm{atm}})^4 \ , \tag{14}$$

which is not true experimentally. Therefore we choose $\phi_{B-L}$ to have the quantum numbers of $\overline{\nu_{\tau_R}}$ plus those of $\overline{\nu_{e_R}}$:

$$\begin{aligned} Q_{\phi_{B-L}} &= Q_{\bar\nu_{\tau_R}} + Q_{\bar\nu_{e_R}} \\ &= (0,0,0,0,1,0) + (0,0,0,1,0,1) \\ &= (0,0,0,1,1,1) \ . \end{aligned}$$

The other "new" Anti-GUT Higgs field we call $\chi$ and one of its roles is to help the $\langle\phi_{B-L}\rangle$ to give non-zero effective mass terms for the see-saw neutrinos, by providing a transition between $\nu_{\tau_R}$ and $\nu_{\mu_R}$. It also turns out to play a role in fitting the atmospheric mixing angle (to be of order unity). Its quantum numbers are therefore postulated to be the difference of those of these two see-saw particles

$$\begin{aligned} Q_\chi &= Q_{\nu_{\mu_R}} - Q_{\nu_{\tau_R}} \\ &= (0,0,0,1,0,-1) - (0,0,0,-1,0,-1) \\ &= (0,0,0,2,0,0) \ . \end{aligned}$$

# IV ANOMALY CANCELLATION

We introduce here an anomaly-free Abelian extension of the "old" AGUT, which we shall use below to obtain the neutrino mass spectrum and their mixing angles. The "new" Anti-GUT gauge group is

$$SMG^3 \times U(1)_f \times U(1)_{B-L,1} \times U(1)_{B-L,23} \tag{15}$$

and is broken, by a set of Higgs fields $S$, $W$, $T$, $\xi$, $\chi$ and $\phi_{B-L}$, down to the SM gauge groups. This diagonal $SMG$ group will finally be broken down by the field $\phi_{WS}$, playing the role of the Weinberg-Salam Higgs field, into $SU(3) \times U(1)_{em}$.

The requirement that all anomalies involving $U(1)_f$, $U(1)_{\mathrm{B-L},1}$ and $U(1)_{\mathrm{B-L},23}$ then vanish strongly constrains the possible fermion charges (we denote the $U(1)_f$ charges by $Q_f(t_R) \equiv t_R$ *etc.* and the $U(1)_{\mathrm{B-L}}$ charges by $Q_{\mathrm{B-L},1}(u_R) \equiv \bar{u}_R$, $Q_{\mathrm{B-L},23}(t_R) \equiv \tilde{t}_R$ *etc.* respectively). The anomaly cancellation conditions constrain the fermion $U(1)_f$ charges to satisfy the following equations:

$$\begin{aligned}
\mathrm{Tr}\ [\mathrm{SU_1(3)^2U(1)_f}] &= 2u_L - u_R - d_R = 0 \\
\mathrm{Tr}\ [\mathrm{SU_2(3)^2U(1)_f}] &= 2c_L - c_R - s_R = 0 \\
\mathrm{Tr}\ [\mathrm{SU_3(3)^2U(1)_f}] &= 2t_L - t_R - b_R = 0 \\
\mathrm{Tr}\ [\mathrm{SU_1(2)^2U(1)_f}] &= 3u_L + e_L = 0 \\
\mathrm{Tr}\ [\mathrm{SU_2(2)^2U(1)_f}] &= 3c_L + \mu_L = 0 \\
\mathrm{Tr}\ [\mathrm{SU_3(2)^2U(1)_f}] &= 3t_L + \tau_L = 0 \\
\mathrm{Tr}\ [\mathrm{U_1(1)^2U(1)_f}] &= u_L - 8u_R - 2d_R + 3e_L - 6e_R = 0 \\
\mathrm{Tr}\ [\mathrm{U_2(1)^2U(1)_f}] &= c_L - 8c_R - 2s_R + 3\mu_L - 6\mu_R = 0 \\
\mathrm{Tr}\ [\mathrm{U_3(1)^2U(1)_f}] &= t_L - 8t_R - 2b_R + 3\tau_L - 6\tau_R = 0 \\
\mathrm{Tr}\ [\mathrm{U_1(1)U(1)_f^2}] &= u_L^2 - 2u_R^2 + d_R^2 - e_L^2 + e_R^2 = 0 \\
\mathrm{Tr}\ [\mathrm{U_2(1)U(1)_f^2}] &= c_L^2 - 2c_R^2 + s_R^2 - \mu_L^2 + \mu_R^2 = 0 \\
\mathrm{Tr}\ [\mathrm{U_3(1)U(1)_f^2}] &= t_L^2 - 2t_R^2 + b_R^2 - \tau_L^2 + \tau_R^2 = 0 \\
\mathrm{Tr}\ [\mathrm{U(1)_f^3}] &= 6u_L^3 + 6c_L^3 + 6t_L^3 - 3u_R^3 - 3c_R^3 - 3t_R^3 - 3d_R^3 - 3s_R^3 \\
&\quad -3b_R^3 + 2e_L^3 + 2\mu_L^3 + 2\tau_L^3 - e_R^3 - \mu_R^3 - \tau_R^3 \\
&\quad -\nu_{e_R}^3 - \nu_{\mu_R}^3 - \nu_{\tau_R}^3 = 0
\end{aligned}$$

$$
\begin{aligned}
\text{Tr}\,[(\text{graviton})^2\text{U}(1)_\text{f}] &= 6u_L + 6c_L + 6t_L - 3u_R - 3c_R - 3t_R - 3d_R - 3s_R \\
&\quad -3b_R + 2e_L + 2\mu_L + 2\tau_L - e_R - \mu_R - \tau_R \\
&\quad -\nu_{e_R} - \nu_{\mu_R} - \nu_{\tau_R} = 0
\end{aligned}
$$

Similar conditions should be obeyed replacing $U(1)_f$ both by $U(1)_{B-L,1}$, and by $U(1)_{B-L,23}$, *i.e.* replacing the $t_R$, $b_R$, $\ldots$ by $\tilde{t}_R$, $\tilde{b}_R$, $\ldots$:

$$
\begin{aligned}
\text{Tr}\,[\text{SU}_1(3)^2\text{U}(1)_{\text{B-L},1}] &= 2\bar{u}_L - \bar{u}_R - \bar{d}_R = 0 \\
\text{Tr}\,[\text{SU}_2(3)^2\text{U}(1)_{\text{B-L},23}] &= 2\tilde{c}_L - \tilde{c}_R - \tilde{s}_R = 0 \\
\text{Tr}\,[\text{SU}_3(3)^2\text{U}(1)_{\text{B-L},23}] &= 2\tilde{t}_L - \tilde{t}_R - \tilde{b}_R = 0 \\
\text{Tr}\,[\text{SU}_1(2)^2\text{U}(1)_{\text{B-L},1}] &= 3\bar{u}_L + \bar{e}_L = 0 \\
\text{Tr}\,[\text{SU}_2(2)^2\text{U}(1)_{\text{B-L},23}] &= 3\tilde{c}_L + \tilde{\mu}_L = 0 \\
\text{Tr}\,[\text{SU}_3(2)^2\text{U}(1)_{\text{B-L},23}] &= 3\tilde{t}_L + \tilde{\tau}_L = 0
\end{aligned}
$$

But with several $U(1)$s, there will in addition be anomaly conditions for combinations between the different ones. Taking into account that $U(1)_{B-L,1}$ charges are zero for all second- and third-generation fermions, while $U(1)_{B-L,23}$ charges are zero for the first generation, the further conditions are:

$$
\begin{aligned}
\text{Tr}\,[\text{U}_1(1)^2\text{U}(1)_{\text{B-L},1}] &= \bar{u}_L - 8\bar{u}_R - 2\bar{d}_R + 3\bar{e}_L - 6\bar{e}_R = 0 \\
\text{Tr}\,[\text{U}_2(1)^2\text{U}(1)_{\text{B-L},23}] &= \tilde{c}_L - 8\tilde{c}_R - 2\tilde{s}_R + 3\tilde{\mu}_L - 6\tilde{\mu}_R = 0 \\
\text{Tr}\,[\text{U}_3(1)^2\text{U}(1)_{\text{B-L},23}] &= \tilde{t}_L - 8\tilde{t}_R - 2\tilde{b}_R + 3\tilde{\tau}_L - 6\tilde{\tau}_R = 0 \\
\text{Tr}\,[\text{U}_1(1)\text{U}(1)^2_{\text{B-L},1}] &= \bar{u}_L^2 - 2\bar{u}_R^2 + \bar{d}_R^2 - \bar{e}_L^2 + \bar{e}_R^2 = 0 \\
\text{Tr}\,[\text{U}_2(1)\text{U}(1)^2_{\text{B-L},23}] &= \tilde{c}_L^2 - 2\tilde{c}_R^2 + \tilde{s}_R^2 - \tilde{\mu}_L^2 + \tilde{\mu}_R^2 = 0 \\
\text{Tr}\,[\text{U}_3(1)\text{U}(1)^2_{\text{B-L},23}] &= \tilde{t}_L^2 - 2\tilde{t}_R^2 + \tilde{b}_R^2 - \tilde{\tau}_L^2 + \tilde{\tau}_R^2 = 0 \\
\text{Tr}\,[\text{U}(1)^3_{\text{B-L},1}] &= 6\bar{u}_L^3 - 3\bar{u}_R^3 - 3\bar{d}_R^3 + 2\bar{e}_L^3 - \bar{e}_R^3 - \bar{\nu}_{e_R}^3 = 0 \\
\text{Tr}\,[\text{U}(1)^3_{\text{B-L},23}] &= 6\tilde{c}_L^3 + 6\tilde{t}_L^3 - 3\tilde{c}_R^3 - 3\tilde{t}_R^3 - 3\tilde{s}_R^3 - 3\tilde{b}_R^3 \\
&\quad +2\tilde{\mu}_L^3 + 2\tilde{\tau}_L^3 - \tilde{\mu}_R^3 - \tilde{\tau}_R^3 - \tilde{\nu}_{\mu_R}^3 - \tilde{\nu}_{\tau_R}^3 = 0 \\
\text{Tr}\,[(\text{graviton})^2\text{U}(1)_{\text{B-L},1}] &= 6\bar{u}_L - 3\bar{u}_R - 3\bar{d}_R + 2\bar{e}_L - \bar{e}_R - \bar{\nu}_{e_R} = 0 \\
\text{Tr}\,[(\text{graviton})^2\text{U}(1)_{\text{B-L},23}] &= 6\tilde{c}_L + 6\tilde{t}_L - 3\tilde{c}_R - 3\tilde{t}_R - 3\tilde{s}_R - 3\tilde{b}_R \\
&\quad +2\tilde{\mu}_L + 2\tilde{\tau}_L - \tilde{\mu}_R - \tilde{\tau}_R - \tilde{\nu}_{\mu_R} - \tilde{\nu}_{\tau_R} = 0 \\
\text{Tr}\,[\text{U}(1)^2_\text{f}\text{U}(1)_{\text{B-L},1}] &= 6u_L^2\bar{u}_L - 3u_R^2\bar{u}_R - 3d_R^2\bar{d}_R + 2e_L^2\bar{e}_L - e_R^2\bar{e}_R - \nu_{e_R}^2\bar{\nu}_{e_R} = 0 \\
\text{Tr}\,[\text{U}(1)^2_\text{f}\text{U}(1)_{\text{B-L},23}] &= 6c_L^2\tilde{c}_L - 3c_R^2\tilde{c}_R - 3s_R^2\tilde{s}_R + 2\mu_L^2\tilde{\mu}_L - \mu_R^2\tilde{\mu}_R - \nu_{\mu_R}^2\tilde{\nu}_{\mu_R} \\
&\quad +6t_L^2\tilde{t}_L - 3t_R^2\tilde{t}_R - 3b_R^2\tilde{b}_R + 2\tau_L^2\tilde{\tau}_L - \tau_R^2\tilde{\tau}_R - \nu_{\tau_R}^2\tilde{\nu}_{\tau_R} = 0 \\
\text{Tr}\,[\text{U}(1)_\text{f}\text{U}(1)^2_{\text{B-L},1}] &= 6u_L\bar{u}_L^2 - 3u_R\bar{u}_R^2 - 3d_R\bar{d}_R^2 + 2e_L\bar{e}_L^2 - e_R\bar{e}_R^2 - \nu_{e_R}\bar{\nu}_{e_R}^2 = 0 \\
\text{Tr}\,[\text{U}(1)_\text{f}\text{U}(1)^2_{\text{B-L},23}] &= 6c_L\tilde{c}_L^2 - 3c_R\tilde{c}_R^2 - 3s_R\tilde{s}_R^2 + 2\mu_L\tilde{\mu}_L^2 - \mu_R\tilde{\mu}_R^2 - \nu_{\mu_R}\tilde{\nu}_{\mu_R}^2 \\
&\quad +6t_L\tilde{t}_L^2 - 3t_R\tilde{t}_R^2 - 3b_R\tilde{b}_R^2 + 2\tau_L\tilde{\tau}_L^2 - \tau_R\tilde{\tau}_R^2 - \nu_{\tau_R}\tilde{\nu}_{\tau_R}^2 = 0 \\
\text{Tr}\,[\text{U}(1)_1\text{U}(1)_\text{f}\text{U}(1)_{\text{B-L},1}] &= u_L\bar{u}_L - 2u_R\bar{u}_R + d_R\bar{d}_R - e_L\bar{e}_L + e_R\bar{e}_R = 0
\end{aligned}
$$

**TABLE 1.** All $U(1)$ quantum charges in the extended Anti-GUT model.

| | $SMG_1$ | $SMG_2$ | $SMG_3$ | $U(1)_f$ | $U_{B-L,1}$ | $U_{B-L,23}$ |
|---|---|---|---|---|---|---|
| $u_L, d_L$ | $\frac{1}{6}$ | 0 | 0 | 0 | $\frac{1}{3}$ | 0 |
| $u_R$ | $\frac{2}{3}$ | 0 | 0 | 0 | $\frac{1}{3}$ | 0 |
| $d_R$ | $-\frac{1}{3}$ | 0 | 0 | 0 | $\frac{1}{3}$ | 0 |
| $e_L, \nu_{e_L}$ | $-\frac{1}{2}$ | 0 | 0 | 0 | $-1$ | 0 |
| $e_R$ | $-1$ | 0 | 0 | 0 | $-1$ | 0 |
| $\nu_{e_R}$ | 0 | 0 | 0 | 0 | $-1$ | 0 |
| $c_L, s_L$ | 0 | $\frac{1}{6}$ | 0 | 0 | 0 | $\frac{1}{3}$ |
| $c_R$ | 0 | $\frac{2}{3}$ | 0 | 1 | 0 | $\frac{1}{3}$ |
| $s_R$ | 0 | $-\frac{1}{3}$ | 0 | $-1$ | 0 | $\frac{1}{3}$ |
| $\mu_L, \nu_{\mu_L}$ | 0 | $-\frac{1}{2}$ | 0 | 0 | 0 | $-1$ |
| $\mu_R$ | 0 | $-1$ | 0 | $-1$ | 0 | $-1$ |
| $\nu_{\mu_R}$ | 0 | 0 | 0 | 1 | 0 | $-1$ |
| $t_L, b_L$ | 0 | 0 | $\frac{1}{6}$ | 0 | 0 | $\frac{1}{3}$ |
| $t_R$ | 0 | 0 | $\frac{2}{3}$ | $-1$ | 0 | $\frac{1}{3}$ |
| $b_R$ | 0 | 0 | $-\frac{1}{3}$ | 1 | 0 | $\frac{1}{3}$ |
| $\tau_L, \nu_{\tau_L}$ | 0 | 0 | $-\frac{1}{2}$ | 0 | 0 | $-1$ |
| $\tau_R$ | 0 | 0 | $-1$ | 1 | 0 | $-1$ |
| $\nu_{\tau_R}$ | 0 | 0 | 0 | $-1$ | 0 | $-1$ |
| $\phi_{WS}$ | 0 | $\frac{2}{3}$ | $-\frac{1}{6}$ | 1 | 0 | 0 |
| $S$ | $\frac{1}{6}$ | $-\frac{1}{6}$ | 0 | $-1$ | $-\frac{2}{3}$ | $\frac{2}{3}$ |
| $W$ | 0 | $-\frac{1}{2}$ | $\frac{1}{2}$ | $-\frac{4}{3}$ | 0 | 0 |
| $\xi$ | $\frac{1}{6}$ | $-\frac{1}{6}$ | 0 | 0 | $\frac{1}{3}$ | $-\frac{1}{3}$ |
| $T$ | 0 | $-\frac{1}{6}$ | $\frac{1}{6}$ | $-\frac{2}{3}$ | 0 | 0 |
| $\chi$ | 0 | 0 | 0 | 2 | 0 | 0 |
| $\phi_{B-L}$ | 0 | 0 | 0 | 1 | 1 | 1 |

$$\mathrm{Tr}\,[\mathrm{U(1)_2U(1)_fU(1)_{B-L,23}}] = c_L\tilde{c}_L - 2c_R\tilde{c}_R + s_R\tilde{s}_R - \mu_L\tilde{\mu}_L + \mu_R\tilde{\mu}_R = 0$$
$$\mathrm{Tr}\,[\mathrm{U(1)_3U(1)_fU(1)_{B-L,23}}] = t_L\tilde{t}_L - 2t_R\tilde{t}_R + b_R\tilde{b}_R - \tau_L\tilde{\tau}_L + \tau_R\tilde{\tau}_R = 0$$

From these equations we can get the following solutions:

$$(u_L, u_R, d_R, e_L, e_R, \nu_{e_R}) = (0, 0, 0, 0, 0, 0)$$
$$(c_L, c_R, s_R, \mu_L, \mu_R, \nu_{\mu_R}) = (0, 1, -1, 0, -1, 1)$$
$$(t_L, t_R, b_R, \tau_L, \tau_R, \nu_{\tau_R}) = (0, -1, 1, 0, 1, -1)$$
$$(\bar{u}_L, \bar{u}_R, \bar{d}_L, \bar{d}_R, \bar{e}_L, \bar{e}_R, \bar{\nu}_{e_L}, \bar{\nu}_{e_R}) = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, -1, -1, -1, -1)$$
$$(\tilde{c}_L, \tilde{c}_R, \tilde{s}_L, \tilde{s}_R, \tilde{b}_L, \tilde{b}_R, \tilde{t}_L, \tilde{t}_R) = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3})$$
$$(\tilde{\mu}_L, \tilde{\mu}_R, \tilde{\tau}_L, \tilde{\tau}_R, \tilde{\nu}_{\mu_L}, \tilde{\nu}_{\mu_R}, \tilde{\nu}_{\tau_L}, \tilde{\nu}_{\tau_R}) = (-1, -1, -1, -1, -1, -1, -1, -1)$$

We summarise the Abelian gauge quantum numbers of our model for fermions and scalars in Table 1. However, the following three points should be kept in mind; then the information in Table 1 and these three points describe our whole model:

1. We have only presented here the six $U(1)$-charges in our model. The non-Abelian quantum charge numbers are to be derived from the following rule: find in the table $y_i/2$ ($i = 1, 2, 3$ is the generation number), then find that Weyl particle in the SM for which the SM weak hypercharge divided by two is $y/2 = y_i/2$ and use its $SU(2)$ and $SU(3)$ representation for the particle considered in the table. But now use it for $SU(2)_i$ and $SU(3)_i$.

2. Remember we imagine that at the "fundamental" scale (presumed to be $\simeq$ the Planck scale) we have essentially all particles that can be imagined with couplings of order unity. But we do not want to be specific about these very heavy particles, in order not to decrease the likelihood of our model being right. We are only specific about the particles in Table 1 and the gauge fields.

3. The 39 gauge bosons correspond to the group (equation (15)) and are not included in Table 1.

# V IMPROVED CHARGE FORMULATION

The system of charges just presented may seem a little complicated and arbitrary. However the fermion charge combinations are so restricted, by the anomaly conditions and the connection to the Standard Model, that they can essentially only be permuted in various ways. We can though transform these charges into some linear combinations that come to look nicer and easier to remember; but the physical content of the theory is of course the same in the reformulated version.

Indeed it turns out that the $U(1)_f$-charge, $Q_f$, contains the information which corresponds to letting even the second and third families have their separate $(B-L)$-charges. We can define generally the second and third family $(B-L)$-charges:

$$(B-L)_2 = \frac{1}{2}(B-L)_{23} + \frac{y_2}{2} - \frac{y_3}{2} - \frac{Q_f}{2} \tag{16}$$

and

$$(B-L)_3 = \frac{1}{2}(B-L)_{23} + \frac{y_3}{2} - \frac{y_2}{2} + \frac{Q_f}{2} \tag{17}$$

for the Weyl particles (the fermions in the Standard Model with the charges in our scheme). The $(B-L)_i$ charges then take on their well-known values as given in Table 2. So they should be no problem to remember and one can easily reconstruct the $U(1)_f$-charges, from the above two formulae (16,17), in case one should want them.

One can also formally use the same formulae for the quantum numbers of the Higgs fields which we have proposed and obtain their values in the new notation, as given in Table 2.

**TABLE 2.** All $U(1)$ quantum charges in the re-extended Anti-GUT model.

| | $SMG_1$ | $SMG_2$ | $SMG_3$ | $U_{B-L,1}$ | $U_{B-L,2}$ | $U_{B-L,3}$ |
|---|---|---|---|---|---|---|
| $u_L, d_L$ | $\frac{1}{6}$ | 0 | 0 | $\frac{1}{3}$ | 0 | 0 |
| $u_R$ | $\frac{2}{3}$ | 0 | 0 | $\frac{1}{3}$ | 0 | 0 |
| $d_R$ | $-\frac{1}{3}$ | 0 | 0 | $\frac{1}{3}$ | 0 | 0 |
| $e_L, \nu_{e_L}$ | $-\frac{1}{2}$ | 0 | 0 | $-1$ | 0 | 0 |
| $e_R$ | $-1$ | 0 | 0 | $-1$ | 0 | 0 |
| $\nu_{e_R}$ | 0 | 0 | 0 | $-1$ | 0 | 0 |
| $c_L, s_L$ | 0 | $\frac{1}{6}$ | 0 | 0 | $\frac{1}{3}$ | 0 |
| $c_R$ | 0 | $\frac{2}{3}$ | 0 | 0 | $\frac{1}{3}$ | 0 |
| $s_R$ | 0 | $-\frac{1}{3}$ | 0 | 0 | $\frac{1}{3}$ | 0 |
| $\mu_L, \nu_{\mu_L}$ | 0 | $-\frac{1}{2}$ | 0 | 0 | $-1$ | 0 |
| $\mu_R$ | 0 | $-1$ | 0 | 0 | $-1$ | 0 |
| $\nu_{\mu_R}$ | 0 | 0 | 0 | 0 | $-1$ | 0 |
| $t_L, b_L$ | 0 | 0 | $\frac{1}{6}$ | 0 | 0 | $\frac{1}{3}$ |
| $t_R$ | 0 | 0 | $\frac{2}{3}$ | 0 | 0 | $\frac{1}{3}$ |
| $b_R$ | 0 | 0 | $-\frac{1}{3}$ | 0 | 0 | $\frac{1}{3}$ |
| $\tau_L, \nu_{\tau_L}$ | 0 | 0 | $-\frac{1}{2}$ | 0 | 0 | $-1$ |
| $\tau_R$ | 0 | 0 | $-1$ | 0 | 0 | $-1$ |
| $\nu_{\tau_R}$ | 0 | 0 | 0 | 0 | 0 | $-1$ |
| $\phi_{WS}$ | 0 | $\frac{2}{3}$ | $-\frac{1}{6}$ | 0 | $\frac{1}{3}$ | $-\frac{1}{3}$ |
| $S$ | $\frac{1}{6}$ | $-\frac{1}{6}$ | 0 | $-\frac{2}{3}$ | $\frac{2}{3}$ | 0 |
| $W$ | 0 | $-\frac{1}{2}$ | $\frac{1}{2}$ | 0 | $-\frac{1}{3}$ | $\frac{1}{3}$ |
| $\xi$ | $\frac{1}{6}$ | $-\frac{1}{6}$ | 0 | $\frac{1}{3}$ | $-\frac{1}{3}$ | 0 |
| $T$ | 0 | $-\frac{1}{6}$ | $\frac{1}{6}$ | 0 | 0 | 0 |
| $\chi$ | 0 | 0 | 0 | 0 | $-1$ | 1 |
| $\phi_{B-L}$ | 0 | 0 | 0 | 1 | 0 | 1 |

A technical detail that can be useful in checking our tables and the mass matrices is the regularity

$$(B-L)_i = -\frac{y_i}{2} \pmod{\frac{1}{2}} \quad \text{for} \quad \text{i}= 1,2,3. \tag{18}$$

which follows from the facts that the quarks have $(B-L)_i = 1/3$ for their family $i$ say (and zero for the other family $(B-L)_j$ 's) and that their family weak hypercharge satisfies

$$\frac{y_i}{2} = -\frac{\text{“triality”}}{3} \pmod{\frac{1}{2}} \tag{19}$$

As one can see from Table 2, most of the Higgs fields which we have proposed also have quantum number assignments obeying the rules of eq. (18), but the two Higgs fields $W$ and $T$ do <u>not</u> obey the rules. In fact they do obey the rule for the first family *i.e.* for $i=1$, but have deviations of just opposite sign for the second and third family—so that the total $B-L$ and the total weak hypercharge $\frac{y}{2}$ obey the rule (18) even for the $W$ and $T$ fields. In fact $W$ has $(B-L)_2 + y_2/2 = -1/3 + (-1/2) = 1/6\,(\mathrm{mod}\,1/2)$ and $(B-L)_3 + y_3/2 = \frac{1}{3} + 1/2 = 1/3\,(\mathrm{mod}\,1/2)$, while $T$ has $(B-L)_2 + y_2/2 = 0 + (-1/6) = 1/3(\mathrm{mod}\,1/2)$ and $(B-L)_3 + y_3/2 = 0 + 1/6 = 1/6(\mathrm{mod}\,1/2)$. It then follows with the above assignment of charges that, in order to make transitions between left- and right-handed quarks or leptons, the fields $W$ and $T$ can only occur in such combinations that the quantities $(B-L)_i + y_i/2$ add up to zero modulo 1/2. That is to say combinations like $T^3$, $T^2W^\dagger$, $T(W^\dagger)^2$, $(W^\dagger)^3$, $T^\dagger W^\dagger$, ... are allowed as well as their conjugates, but *e.g.* just $T$ or just $W$ or $W^\dagger$ would not be allowed to occur as mass matrix elements.

## VI MASS MATRICES WITHIN THE ANTI-GUT MODEL

In the "old" Anti-GUT model we have only the usual SM fermions at low energies, but in our "new" version we assume that there exist very heavy right-handed neutrinos, all of them having already decayed and not being observable in our world. They function as see-saw particles and thus give rise to an effective Majorana-type mass matrix for the left-handed neutrinos. These three "right-handed"neutrinos get masses from the VEV of $\phi_{B-L}$ ($10^{12}$ GeV), $\xi$ and also $\chi$ Higgs fields (the latter two having a VEV of order 1/10 in Planck units).

The effective mass matrix elements, left-left, for the left-handed neutrinos—the ones we "see" experimentally—then come about using the $\nu_R$ see-saw propagator surrounded by left-right transition neutrino mass matrices. The latter are rather analogous to the charged lepton and quark mass matrices which are proportional to the VEV of the Weinberg-Salam Higgs field, being components of $\phi_{WS}$ (with VEV $\sim 173$ GeV) in our model.

The Higgs field $\phi_{\mathrm{B-L}}$ breaks the total $B-L$ quantum number, as well as the first and third family $(B-L)_i$ quantum numbers. Thus the effective Majorana mass terms are added into the Lagrange density using the Higgs field $\phi_{B-L}$. The part of the effective Lagrangian we have to consider is:

$$\begin{aligned}-\mathcal{L}_{\mathrm{lepton-mass}} &= \bar{\nu}_L\, M_\nu^D\, \nu_R + \frac{1}{2}(\overline{\nu_L})^c\, M_L\, \nu_L + \frac{1}{2}(\overline{\nu_R})^c\, M_R\, \nu_R + h.c.\\ &= \frac{1}{2}(\overline{n_L})^c\, M\, n_L + h.c. \qquad (20)\end{aligned}$$

where

$$n_L \equiv \left( \begin{array}{c} \nu_L \\ (\nu_L)^c \end{array} \right) \;,\quad M \equiv \left( \begin{array}{cc} M_L & M_\nu^D \\ M_\nu^D & M_R \end{array} \right) \;; \qquad (21)$$

$M_\nu^D$ is the standard $SU(2)\times U(1)$ breaking Dirac mass term, and $M_L$ and $M_R$ are the isosinglet Majorana mass terms for left-handed and right-handed neutrinos, respectively.

Supposing that the left-handed Majorana mass $M_L$ terms are comparatively negligible, because of SM gauge symmetry protection, a naturally small effective Majorana mass for the light neutrinos (predominantly $\nu_L$) can be generated by mixing with the heavy states (predominantly $\nu_R$) of mass $M_{\nu_R}$. With no left-left term, $M_L = 0$, the light eigenvalues of the matrix $M$ are

$$M_{\text{eff}} \approx M_\nu^D \, M_R^{-1} \, (M_\nu^D)^T \ . \tag{22}$$

This result is the well-known see-saw mechanism [18]: the light neutrino masses are quadratic in the Dirac masses and inversely proportional to the large $\nu_R$ Majorana masses. Notice that if some $\nu_R$ are massless or light they would not be integrated away but simply added to the light neutrinos.

We have already given the quantum charges of the Higgs fields, $S$, $W$, $T$, $\xi$, $\phi_{WS}$, $\phi_{B-L}$ and $\chi$ in Table 1. With this quantum number choice of Higgs fields the mass matrices are given for the uct-quarks:

$$M_U \simeq \frac{\langle\phi_{\text{WS}}\rangle}{\sqrt{2}} \begin{pmatrix} S^\dagger W^\dagger T^2 (\xi^\dagger)^2 & W^\dagger T^2 \xi & (W^\dagger)^2 T \xi \\ S^\dagger W^\dagger T^2 (\xi^\dagger)^3 & W^\dagger T^2 & (W^\dagger)^2 T \\ S^\dagger (\xi^\dagger)^3 & 1 & W^\dagger T^\dagger \end{pmatrix} \tag{23}$$

the dsb-quarks:

$$M_D \simeq \frac{\langle\phi_{\text{WS}}\rangle}{\sqrt{2}} \begin{pmatrix} SW (T^\dagger)^2 \xi^2 & W (T^\dagger)^2 \xi & T^3 \xi \\ SW (T^\dagger)^2 \xi & W (T^\dagger)^2 & T^3 \\ SW^2 (T^\dagger)^4 \xi & W^2 (T^\dagger)^4 & WT \end{pmatrix} \tag{24}$$

the charged leptons:

$$M_E \simeq \frac{\langle\phi_{\text{WS}}\rangle}{\sqrt{2}} \begin{pmatrix} SW (T^\dagger)^2 \xi^2 & W (T^\dagger)^2 (\xi^\dagger)^3 & W T^4 (\xi^\dagger)^3 \chi \\ SW (T^\dagger)^2 \xi^5 & W (T^\dagger)^2 & W T^4 \chi \\ S (W^\dagger)^2 T^4 \xi^5 & (W^\dagger)^2 T^4 & WT \end{pmatrix} \tag{25}$$

the Dirac neutrinos:

$$M_\nu^D \simeq \frac{\langle\phi_{\text{WS}}\rangle}{\sqrt{2}} \begin{pmatrix} S^\dagger W^\dagger T^2 (\xi^\dagger)^2 & W^\dagger T^2 (\xi^\dagger)^3 & (W^\dagger) T^2 (\xi^\dagger)^3 \chi \\ S^\dagger W^\dagger T^2 \xi & W^\dagger T^2 & (W^\dagger) T^2 \chi \\ S^\dagger W^\dagger T^\dagger \xi \chi^\dagger & W^\dagger T^\dagger \chi^\dagger & W^\dagger T^\dagger \end{pmatrix} \tag{26}$$

and the Majorana neutrinos:

$$M_R \simeq \langle\phi_{\text{B-L}}\rangle \begin{pmatrix} S^\dagger \chi^\dagger \xi & \chi^\dagger & 1 \\ \chi^\dagger & S \chi^\dagger \xi^\dagger & S \xi^\dagger \\ 1 & S \xi^\dagger & S \chi \xi^\dagger \end{pmatrix} \tag{27}$$

Note that the random complex order of unity and factorial factors, which are supposed to multiply all the mass matrix elements, are not represented here. We will discuss these factors in section VIII.

**TABLE 3.** Typical fit, $\alpha = -1$, $\beta = 1$, $\gamma = 1$, $\delta = 1$.

| | Fitted | "Experiment" |
|---|---|---|
| $m_u$ | 3.1 MeV | 4 MeV |
| $m_d$ | 6.6 MeV | 9 MeV |
| $m_e$ | 0.76 MeV | 0.5 MeV |
| $m_c$ | 1.29 GeV | 1.4 GeV |
| $m_s$ | 390 MeV | 200 MeV |
| $m_\mu$ | 85 MeV | 105 MeV |
| $M_t$ | 179 GeV | 180 GeV |
| $m_b$ | 7.8 GeV | 6.3 GeV |
| $m_\tau$ | 1.29 GeV | 1.78 GeV |
| $V_{us}$ | 0.21 | 0.22 |
| $V_{cb}$ | 0.023 | 0.041 |
| $V_{ub}$ | 0.0050 | 0.0035 |
| $J_{CP}$ | $1.04 \cdot 10^{-5}$ | $(2\ -\ 3.5) \cdot 10^{-5}$ |
| "$\chi^2$" | 1.46 | |

# VII CHARGED FERMION SPECTRUM

The mass matrices for the quarks, $M_U$ and $M_D$, happen not to have been changed at all by the introduction of the "new" Higgs fields $\chi$ (and $\phi_{B-L}$, but that has so little VEV compared to the Planck scale that it could never compete). Even in the charged lepton mass matrix the appearance of $\chi$ only occurs on off-diagonal matrix elements. These elements are already small and remain so small as to have no significance for the charged lepton mass predictions, as long as $\chi$ is of the order $\langle\chi\rangle \approx 0.07$ as we need for fitting $\theta_{\rm atm}$.

Therefore all the fits of the "old" Anti-GUT model are valid and we can still use the parameter values obtained by these earlier fits to $S$, $W$, $T$, $\xi$, presented above in equation (10). The result of a more recent fit [19] to the charged fermion spectrum, without imposing the constraint $\langle S\rangle = 1$, is shown in Table 3.

The quark and charged lepton fit of Table 3 is for the Higgs charge assignments chosen according to a set of discrete parameters

$$\alpha = 1, \quad \beta = 1, \quad \gamma = -1, \quad \delta = 1. \tag{28}$$

However, since the effect of this choice only comes in via a correction which we call the "factorial correction" [19] and only makes changes of order unity in principle, these discrete parameters should not be counted as variable parameters in the fit—in fact we only allowed them to have the values -1, 0, 1. The Higgs VEV parameters for this choice (28), with the inclusion of the "factorial correction", are

$$\langle W\rangle = 0.0894, \quad \langle T\rangle = 0.0525, \quad \langle S\rangle = 0.756, \quad \langle\xi\rangle = 0.0247. \tag{29}$$

It is seen that it is a rather good fit to the data, from the point of view that it is only expected to work up to order of unity factors. The worst case is the strange quark mass, since the expression for the Jarlskog-triangle-area, which is a measure of CP-violation $J_{CP}$, contains so many factors that we expect the uncertainty in our prediction for it to be relatively large. The trouble with the strange quark is due to the fact that the model has an order of magnitude family degeneracy built into the diagonal elements of the mass matrices. Thus, apart from the charm and the top quarks whose masses are dominated by off-diagonal mass matrix elements, the charged fermion masses are predicted to have an order of magnitude family degeneracy. So we cannot avoid having *e.g.* the $SU(5)$ relations (4) order-of-magnitudewise. Still having eq. (4) only as an order of magnitude relation is much better than getting it as an exact prediction!

In fact we can even predict [19] the accuracy with which we expect our predictions to work—namely that the uncertainty in the logarithm shall be equal to that for the distribution of the logarithm of a Gaussian distribution. Our fits actually do rather a bit too well, and even agree with the prediction concerning the skewness of the distribution: that the worst deviation should be that some mass(es) turn out to be too <u>small</u> experimentally compared to the prediction—the deviation of the strange quark mass fits in this way.

# VIII CALCULATION OF $M_{\mathrm{EFF}}$

In this section we calculate the effective neutrino mass matrix for left-handed components. Since, strictly speaking, our model only predicts orders of magnitude, a crude calculation is in principle justified. This calculation is presented in the first subsection, and then in the next subsection we make "statistical calculations" with random order-one factors and "factorial factors".

## A Crude calculation

From equation (27) we see, to the first approximation, that there are one massless and two degenerate right-handed neutrinos coming from the VEV of the $B-L$ breaking Higgs field, $\langle\phi_{\mathrm{B-L}}\rangle$. The mass splitting between the two almost degenerate see-saw neutrinos is $M_{31}\langle S\rangle\langle\chi\rangle\langle\xi\rangle$, where $M_{31}\approx\langle\phi_{\mathrm{B-L}}\rangle$ is the approximately common mass of the two heaviest see-saw neutrinos. The third lightest see-saw neutrino is dominantly "proto second generation" and has the mass $\langle\phi_{\mathrm{B-L}}\rangle\langle\chi\rangle\langle\xi\rangle$.

For the left-handed neutrinos, to the first approximation, we get the effective mass matrix as follows:

$$M_{\mathrm{eff}}\approx\frac{W^2T^2\langle\phi_{WS}\rangle^2}{2\langle\phi_{\mathrm{B-L}}\rangle}\begin{pmatrix}\frac{T^2\xi^5}{\chi} & \frac{T^2\xi^2}{\chi} & T\xi^2\\ \frac{T^2\xi^2}{\chi} & \frac{T}{\xi} & \frac{T}{\xi}\\ T\xi^2 & \frac{T}{\xi} & \frac{\chi}{\xi}\end{pmatrix}\ ,\tag{30}$$

But we have to emphasise here that this approximation is *good enough to calculate only the heaviest left-handed neutrino.* This is because all the mass matrix elements, to this approximation, come from the propagator contribution of the lightest see-saw particle, so that they really form a degenerate matrix of rank one. Using this contribution only would lead to two left-handed massless neutrinos and one massive left-handed neutrino. But we can still obtain the mixing angles $\theta_{13}$ and $\theta_{23}$ and the heaviest mass from $M_{\text{eff}}$:

$$\theta_{13} = \theta_{e,\text{heavy}} \approx \frac{T}{\chi}\xi^3 \tag{31}$$

$$\theta_{23} = \theta_{\mu,\text{heavy}} \approx \begin{cases} \frac{T}{\chi} & \text{when } \chi \gtrsim T \\ 1 & \text{when } \chi \lesssim T \end{cases} \tag{32}$$

$$M_{\nu_L \text{heavy}} \approx \begin{cases} \frac{W^2T^2\langle\phi_{WS}\rangle^2}{2\langle\phi_{\text{B-L}}\rangle}\frac{\chi}{\xi} & \text{when } \chi \gtrsim T \\ \frac{W^2T^2\langle\phi_{WS}\rangle^2}{2\langle\phi_{\text{B-L}}\rangle}\frac{T}{\xi} & \text{when } \chi \lesssim T \end{cases} \tag{33}$$

From these equations we can restrict the region of $\chi$ by comparing with Super-Kamiokande experimental data; $\chi$ must be almost of the same order as $T$. Thus we know the mixing angle $\theta_{13}$ of the first and third generations must be of the order of $\xi^3$.

However, to get the much lower neutrino masses we cannot use the contribution from the lightest see-saw propagator, but we have to use the propagator terms from the two approximately equally heavy see-saw particles. This contribution to the propagator matrix is

$$M_R^{-1}\Big|_{\substack{heavy \\ see-saws}} \approx \frac{1}{\langle\phi_{\text{B-L}}\rangle}\begin{pmatrix} \chi\xi & \xi & 1 \\ \xi & \chi\xi & \chi \\ 1 & \chi & \chi\xi \end{pmatrix} \tag{34}$$

where the $\chi\xi/\langle\phi_{B-L}\rangle$ comes from the mass difference of the almost degenerate see-saw particles.

Surrounding this propagator contribution with the "Dirac $\nu$"-mass matrix we get

$$\begin{aligned} M_{\text{eff}}\Big|_{\substack{heavy \\ see-saws}} &\approx M_\nu^D\, M_R^{-1}\Big|_{\substack{heavy \\ see-saws}} (M_\nu^D)^T \\ &\approx \frac{W^2T^2\langle\phi_{WS}\rangle^2}{2\langle\phi_{\text{B-L}}\rangle}\begin{pmatrix} T^2\xi^6 & T\xi^3 & T\xi^2 \\ T\xi^3 & T^2\chi\xi & T\chi \\ T\xi^2 & T\chi & \chi\xi \end{pmatrix} \quad . \end{aligned} \tag{35}$$

It is from this contribution that the two lightest left-handed neutrino masses and their mixing angle, $\theta_{12}$, are obtained:

$$M_{\nu_L \text{medium}} \approx \frac{W^2T^2\chi\xi\langle\phi_{WS}\rangle^2}{2\langle\phi_{\text{B-L}}\rangle} \tag{36}$$

$$\theta_{12} = \theta_{e,\text{medium}} \approx \begin{cases} \frac{T}{\chi}\xi & \text{when } \chi \gtrsim T \\ \xi & \text{when } \chi \lesssim T \ . \end{cases} \tag{37}$$

Note that the lightest mass particle is dominantly the $\nu_{e_L}$ neutrino and its small mixing is mainly with the medium mass neutrino ($\theta_{e,\text{medium}}/\theta_{e,\text{heavy}} \approx \xi^{-2} \gg 1$). So we should approximately identify the solar oscillation mixing angle with the mixing to the medium heavy neutrino:

$$\theta_{\odot} \simeq \theta_{e,\text{medium}} \approx \xi \tag{38}$$

and the solar mass squared difference as:

$$\Delta m^2_{\odot} \approx M^2_{\nu_L \text{medium}} \approx \frac{W^4 T^4 \chi^2 \xi^2 \left\langle \phi_{WS} \right\rangle^4}{4 \left\langle \phi_{\text{B-L}} \right\rangle^2} \ . \tag{39}$$

The atmospheric mixing angle goes between the heaviest and the medium mass neutrino:

$$\begin{aligned} \Delta m^2_{\text{atm}} &\approx M^2_{\nu_L \text{heavy}} - M^2_{\nu_L \text{medium}} \\ &\approx \frac{W^4 T^4 \chi^2 \left\langle \phi_{WS} \right\rangle^4}{4 \left\langle \phi_{\text{B-L}} \right\rangle^2 \xi^2} \end{aligned} \tag{40}$$

From equations (39) and (40), we find that the ratio of solar and atmospheric neutrino mass squared differences must be of the order of $\xi^4$, say about $10^{-4}$.

## B Statistical calculation using random order unity factors

In this subsection we will discuss the numerical calculation. The elements of the mass matrices are determined, up to factors of order one, to be a product of several Higgs VEVs measured in units of the fundamental scale $M_{\text{Planck}}$—the Planck scale.

We imagine that the mass matrix elements, *e.g.* for the right-handed neutrino masses or for the mass matrix $M^D_\nu$, are given by chain diagrams [11] consisting of a backbone of fermion propagators for fermions with fundamental masses, with side ribs (branches) symbolising a Yukawa coupling to one of the Higgs field VEVs.

We know neither the Yukawa couplings nor the precise masses of the fundamental mass fermions. However it is a basic assumption of the naturalness of our model that these couplings are of order unity and that, also, the fundamental masses deviate from the Planck mass by factors of order unity. In the numerical evaluation of the consequences of the model, we explicitly take into account these uncertain factors of order unity by providing each matrix element with an explicit random number $\lambda_{ij}$—with a distribution so that its average $\langle \log \lambda_{ij} \rangle \approx 0$ and its spread is a factor of two.

Then the calculation is performed with these numbers time after time, with different random number $\lambda_{ij}$-values, and the results averaged in logarithms. A crude realisation of the distribution of these $\lambda_{ij}$ could be a flat distribution between $-2$ and $+2$, then provided also with a random phase (with flat distribution).

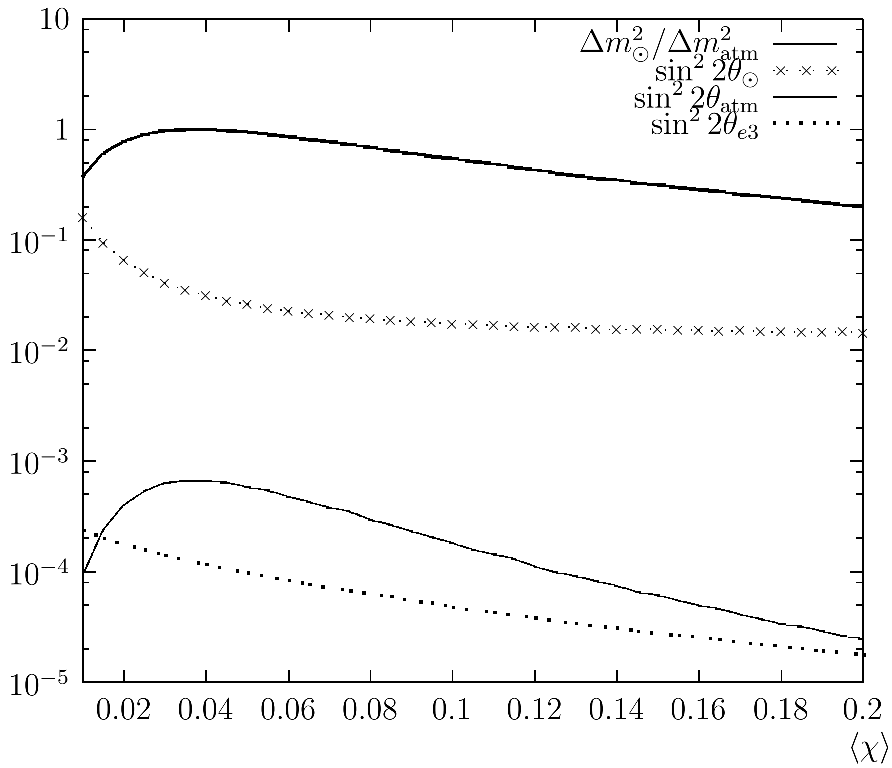


**FIGURE 1.** The numerical results for the ratio of the solar neutrino mass squared difference to that for the atmospheric neutrino mass squared difference. Also shown are the squared sine of the double of the solar neutrino mixing angle, the atmospheric neutrino mixing angle and the mixing angle $\theta_{e3}$.

Another "detail" is the use of a factor $\sqrt{\#\text{diagrams}}$ multiplying the matrix elements, to take into account that, due to the possibility of permuting the Higgs field attachments in the chain-diagram, the number of different diagrams is roughly proportional to the number of such permutations #diagrams. This is the correction introduced and studied for the charged fermion mass matrices by D. Smith and two of us [19]. In the philosophy of each diagram coming with a random order unity factor, the sum of #diagrams different diagrams gets of the order $\sqrt{\#\text{diagrams}}$ bigger than a single diagram of that sort*. It turns out that these factors are especially important for some elements involving the electron-neutrino in the matrix $M_\nu^D$, which are suppressed by several factors, as then many permutations can be

*) We have counted these permutations ignoring the field $S$. If we allowed both $S$ and $S^\dagger$ in the same diagram, the $\sqrt{\#\text{diagrams}}$ factor could give arbitrarily large numbers if $< S >= 1$. Actually, in the fits with these factorial factors, even the $S$ and $S^\dagger$ are somewhat suppressed and so the wild blow-up does not occur in practice. Also we may argue that sub-chains, with equally many $S$ and $S^\dagger$ insertions, can be put in all graphs and their effect is just to renormalize the parameters of the model.

made.

Yet another detail is that the symmetric mass matrices—occurring for the Majorana neutrinos—give rise to the same off-diagonal term twice in the right-handed neutrino matrix in the effective Lagrangian. So we must multiply off-diagonal elements with a factor 1/2. But in the $M_\nu^D$-matrix columns and rows are related to completely different Weyl fields and, of course, a similar factor 1/2 should not be introduced.

Concerning the $\sqrt{\#\text{diagrams}}$ factor for the diagonal mass matrix terms in the symmetric matrices, *e.g.* $M_R$, we shall remember that (contrary to what we shall do in non-symmetric matrices such as $M_\nu^D$ and the charged lepton ones) we must count diagrams with the Higgs fields attachment assigned in opposite order as only one diagram. The backbone in the diagram has no orientation and we shall count diagrams obtained from each other by inverting the sequence of the attached Higgs fields as only ONE diagram. Thus the diagonal elements will tend to have only half as many diagrams.

We give in Figure 1 results, obtained as the average of $50,000$ random combinations of order unity factors, as a function of the small VEV $\langle\chi\rangle$ of the new Higgs field $\chi$. In order to get an atmospheric mixing angle of the order of unity, the range for $\langle\chi\rangle$ around the "old" Anti-GUT VEV $\langle T\rangle \approx 0.07$ is suggested; so only this range is presented. Finally we present here the orders of magnitude of the right-handed neutrino masses and the mixing angle $\theta_{e3}$ (see Figure 1):

$$M_{R_{\nu_1}} \approx 10^{11}\ \ \text{GeV}\ \ , \tag{41}$$

$$M_{R_{\nu_2}} \approx 10^{13}\ \ \text{GeV}\ \ , \tag{42}$$

$$M_{R_{\nu_3}} \approx 10^{13}\ \ \text{GeV}\ \ , \tag{43}$$

$$\sin^2 2\theta_{e3} \approx 10^{-4}\ \ . \tag{44}$$

## IX THE PROBLEM OF SCALES

When we, as here, just postulate the see-saw scale to have the value needed by introducing an appropriate Higgs field, $\phi_{B-L}$ in our model, and fit its vacuum expectation value, here to $10^{13}$ GeV, we have just another scale problem.

This new scale problem is of an exactly analogous character to that of the famous Hierarchy Problem of why the weak energy scale is so low compared to, say, the Planck scale (or the unified scale in GUT theories). So, one could say, we have got one more hierarchy-problem! It has been popular to identify the see-saw scale with the GUT-scale but, to do so, one has to make use of the big mass ratios occurring between the right-handed neutrinos. When it IS fitted, it is always the heaviest of the see-saw neutrinos that gets a mass at the GUT-scale. This heaviest right-handed neutrino is that one which does not have any phenomenological consequences for the (left-handed) neutrino oscillations, and thus is very model dependent. To stretch the scale of the see-saw neutrinos from the needed scale, around the masses presented here of $10^{11}$ to $10^{13}$ GeV, up to the usual GUT-scale at $10^{15}$ to $10^{17}$ GeV is possible but not strongly suggested. But, phenomenologically, it is not possible to push the see-saw scale up to the Planck scale. Thus we must introduce at least one new scale, in addition to the Planck scale, the weak scale and the strong scale. This may be very important to think about in connection with solving the hierarchy problem, because we now have to look for a solution that can provide yet another scale—the see-saw scale or, if we drop the see-saw model altogether, the neutrino oscillation scale directly.

Preferably we should have a mechanism that could put these different scales at exponential values relative to, say, the Planck scale. This would be much in the same way mathematically as the strong interaction scale is understood to be exponential due to the renormalization group running of $\alpha_S$–the QCD-gauge coupling (squared and divided by $4\pi$).

We are indeed working on such a project (L.V. Laperashvili [20] is also included in this project), using the multiple point principle (MPP) introduced in section II. Once we have such a principle that fixes the couplings and masses in a certain way, we are not really solving the problem of avoiding finetuning, but rather we have postulated a finetuner that finetunes the parameters for us, namely the postulated MPP-mechanism. That should make it much easier to "solve" the scale problem or rather problems—now we include the neutrino oscillations—since what we need is "only" that, when the couplings and masses are being adjusted to make several degenerate minima as the MPP postulates, then as a consequence of that requirement they tend to also organize hierarchies of scales.

The argument on which we work goes roughly like this:

The MPP is taken to postulate that the effective potential $V_{eff}(\phi_{WS})$ shall have (at least) two minima as a function of $|\phi_{WS}|^2$, with the same value of the effective potential, for non-zero values of this Standard Model Higgs field $\phi_{WS}$. This is an interpretation of the somewhat vague statement that there shall be "many" minima with the same energy density. This obviously implies that the second derivative of this effective potential with respect to $|\phi_{WS}|^2$ has two places on the positive $|\phi_{WS}|^2$ axis where it is positive—namely the minima. It then follows that there is one place in between where the second derivative is negative—namely at the maximum that must occur between the two minima, for topological reasons so to speak. For our argument here the degeneracy of the minima does not seem particularly relevant—it is just that the MPP is a statement about degenerate minima.

Then we go through, loop-order for loop-order, to see if it is possible, under reasonable assumptions, that the second derivative $d^2V_{eff}(|\phi_{WS}|^2)/d(|\phi_{WS}|^2)^2$ could have the alternating sign behaviour, first positive then negative and then positive again, required for the presence of two minima. *In 0-loop approximation*:

$$\frac{d^2V_{eff}(|\phi_{WS}|^2)}{d(|\phi_{WS}|^2)^2} = \frac{\lambda}{2} = constant \tag{45}$$

*i.e.* it cannot have the required sign shifts except if equal to zero. *In 1-loop approx-*

*imation*:

$$\frac{d^2V_{eff}(|\phi_{WS}|^2)}{d(|\phi_{WS}|^2)^2} \approx \frac{\lambda_{run}(|\phi_{WS}|^2)}{2} + constant \tag{46}$$

In the approximation that we look for $|\phi_{WS}|^2 \gg \mu^2$ (the Higgs mass squared parameter), the running $\lambda$ is proportional to a single $\log|\phi_{WS}|^2$ with a coefficient that gets positive terms from the bosons in the field theory and negative terms from the fermions. Ignoring strange solutions with both minima very close to each other, we have a single monotonous logarithm that cannot switch back and forth in sign. So the only solution is, like in the 0-loop case, to put the coefficient zero and kill to zero the whole 1-loop contribution to the second derivative. *In 2-loop approximation*: It is possible to achieve an effective potential second derivative $d^2V_{eff}(|\phi_{WS}|^2)/d(|\phi_{WS}|^2)$ which can switch sign as needed to have two minima.

In fact we have found [21] that it is completely possible to have a scenario with coupling constants and masses leading to two minima. It is even completely within the experimentally allowed range of parameters to have the second minimum 1) at the Planck scale, and 2) degenerate as the MPP requires. We obtain this particular scenario when the top mass is 173 GeV and the Higgs mass is 135 GeV, namely the smallest Higgs mass allowed if the pure Standard Model is valid up to the Planck scale. Since the indirect estimates of the Higgs mass favour a mass lower than the experimental limit, they really suggest it should be as low as possible and thus just our prediction of 135 GeV, provided no new physics comes in below the Planck scale allowing a lower Higgs mass.

In fact it is a bit surprising that our calculation, even with a rather crude value for the Planck scale only assuming its order of magnitude for the second minimum, yields a top-mass of 173 GeV with an accuracy similar to the experimental accuracy of about $\pm 6$ GeV.

So it looks that with two loops we can easily get the two minima required, but with lower loop accuracy it is impossible. However, the various couplings are experimentally so small that the two-loop terms contribution is only a correction and not usually qualitatively important. If now it is needed that the two-loop-terms must be important, in order to have the required two minima (using MPP), then they must be boosted to importance by sufficiently big logarithms: In a leading log expansion, the two loop terms can become comparable or beat out 0-loop and 1-loop approximation terms if the logs are, so to speak, as big as the couplings are weak. But the log here must be, for instance, the logarithm of the ratio of the two minima we required. If that gets as big as the couplings are weak, it means that, say, the lowest minimum field value must go down relative to the other one by the <u>exponential</u> of the inverse couplings.

This would be very similar to what is achieved by means of the renormalisation group for the strong scale, but should now numerically also work for the lower minimum that determines the weak scale. That is potentially a solution of the scale problem!

There is though a little "technical" problem with our "solution" of the scale problem: At the second minimum it turns out, say in our scenario with the Higgs mass being 135 GeV, that the running self-coupling $\lambda_{run}$ for the Higgs field runs very very small. If it is allowed to have so small a value, the argument of crude order of magnitude character given above does not quite work. Then the logical argumentation for the need of the big ratio of scales fails, even when the requirement (MPP) of two degenerate vacua is postulated. Interestingly enough there is a better hope of coming through successfully with several hierarchy problems at once. So the see-saw scale problem could turn out being helpful in bringing our idea for solving the scale problem to work.

## X LOOKING FOR TOP-QUARK ANALOGUES

If we accepted the idea that we wanted the postulate of several minima—or even degenerate minima—to explain large scale ratios, then we need to have the zero- and one-loop approximation to cancel approximately [22], so that the two-loop terms can come to be qualitatively significant. Remember it was only the two-loop terms that could produce the required minima. But the one loop contribution was a logarithm multiplied by positive terms from the bosons and negative ones from the fermion loops. This is the reason why we need a strongly coupled top-quark, *i.e.* with a Yukawa coupling of order unity.

With this very speculative requirement, that each time we have a Higgs field with a small vacuum expectation value it is due to our postulated MPP, we can then use it as a guide in seeking the structure of the model that realizes our assumptions. As already mentioned the top quark does the job of cancelling the one-loop correction so as to allow two minima perfectly, as far as we yet know.

Luckily enough there is, in our above sketched model for the mass of the see-saw neutrinos coming from the field $\phi_{B-L}$, just a combination of right-handed neutrinos that can "play the role of the top-quark in the Standard Model", because it couples with of order unity Yukawa coupling. In fact the Higgs field $\phi_{B-L}$ (which has to have a very small VEV compared to the Planck scale, in fact of order $10^{13}$ GeV) couples in our model to the transition between the third and the first family right-handed neutrinos. Really it can convert a right-handed electron neutrino to a CP-antiparticle of the right-handed $\tau$-neutrino. This coupling is therefore unsuppressed. That is quite non-trivial in our model, since there can be so many suppressions of effective couplings due to some charges needing to be broken to generate the vertex. But nevertheless we have a top-quark-role-player, needed for the working of our two minima machinery, for the $\phi_{B-L}$ field.

Now we might say: but even the Higgs fields which in our model give small hierarchies—the fields T, W, $\xi$, $\chi$ having only VEV's of the order of 1/10 in Planck units—strictly speaking also represent hierarchy-like problems. Why are their expectation values so small compared to the fundamental (presumably Planck) scale. Even a small number of the order of 1/10 needs an explanation. So they should also have their "top-quark-role-players". In other words they should have unsuppressed couplings to some fermion, which is mass protected but gets a mass from just the Higgs field in question. Then that fermion could help cancel the one-loop contribution, so as to make the two-loop one become important.

Such a rule of requiring a "top-quark-role-player", for each Higgs field with very small expectation value relative to the presumed fundamental scale, will require the existence of some chiral fermions that then can get mass from the Higgs field in question. But chiral fermion fields, in turn, will give rise to anomalies that must be cancelled. So we obtain a series of consistency requirements that can be useful in building a model from the bottom up.

We start the construction by seeking Higgs fields to explain the quark-lepton mass and mixing angle suppressions. Then, in turn, we look for "top-quark-role-players" for these Higgs fields and, next, for further Weyl-fermions to cancel the anomalies coming from the "top-quark-role-players". The idea then is that the many consistency requirements shall be used to find a scheme that works self-consistently. Then all the small-VEV-Higgs-fields will have their degenerate minima and fermions, which they give mass to and which play the top quark role. Furthermore the gauge and mixed anomalies of these fermions will cancel.

One might feel that the story of the "top-quark-role-players" has a little too many speculative assumptions to be trustable. It could then be that there is a slightly different reason for the same sort of fermion fields to exist—a reason that could actually be a reformulation of the same model—each Higgs boson could be imagined and required to be a bound state of a pair of chiral fermions. That would lead to about the same requirements as the "top-quark-player" rule: For each low mass (or low VEV) Higgs/scalar field, there must be some constituents able to form it w.r.t. quantum numbers. They should typically be fermions and need to be mass protected to the scale of the mass of the boson to be constructed. The coupling has to be strong for the constituents to the bound state.

The requirements for a boson being a bound state of a couple of chiral fermions become very similar indeed to the requirements that the boson have them as its "top-quark-role players".

## XI AN EXTRA FAMILY OF WEYL FERMIONS

But what fermions could play the "top-quark-role" for the scalar boson fields $W$, $T$, $\xi$ , $\chi$? It would have to be fermions that actually obtain a mass from the fields for which they do the job of the top quark. But that means they end up having masses of the order of the Higgs field scale in question (the VEV). So we need, in our model, Weyl particles combining themselves to each other, so as to obtain masses at the scale of our Higgs fields $W$, $T$, $\xi$ ,... which is about 1/10th of the Planck scale. The mass protection for these Weyl particles must be due to some of the gauge quantum numbers in our model. However they should not be mass protected by the Standard Model quantum numbers, because then they would have much much lower masses and perhaps should have been seen. They must not even have the total $B-L$ as a mass protecting quantum number, because then they would get masses of the see-saw scale and not just a factor of 10 under the Planck scale, as we need them. They can though have these quantum numbers in a vectorial way, *i.e.* there could be equally many right and left Weyl particles with a given combination of the Standard Model quantum numbers and the total $B-L$.

For instance we could look at the Higgs field $\chi$, which was introduced for the sake of fitting the neutrino oscillations. Since it gives rise to a suppression of the order of a factor 10 to 15, it must have mass and VEV at about a factor 10 to 15 under the Planck scale (taken as the fundamental one). Its quantum numbers in our model are quite simple: It has no weak hypercharges and then, according to our rule for the non-abelian quantum numbers, also no non-abelian couplings. It only has the family-$(B-L)$-quantum numbers: $(B-L)_2 = -1$ and $(B-L)_3 = 1$, the rest being zero.

In order for a pair of Weyl fermions to couple to the $\chi$ field in an unsuppressed way, it is needed that their quantum numbers add up to those of the $\chi$ field modulo certain quantum number combinations. These quantum number combinations correspond to the ones sitting on the Higgs fields, like the $S$ field in our model, having expectation values of the order of the fundamental scale. Otherwise there would be a need for some Higgs field VEV to provide the lacking quantum number/charge in the coupling and it would be suppressed. In that case the field would couple too weakly and the pair of Weyl fermions could not play its top-quark-role.

If we seek to keep the quantum numbers from being too large, the simplest would be to have one fermion with all quantum numbers zero except for say $(B-L)_2 = -1$, and another fermion with all zero except for $(B-L)_3 = 1$. But we can also have that these two Weyl fermions in addition carry other quantum numbers, but such that one of these two particles carry just the opposite further charges to those of the other one.

In fact we are forced to assume that the two particles that should play the top-quark-role for $\chi$ have other charges. Otherwise they would be mass protected by the total $(B-L)$-charge and get masses at the see-saw scale, rather than the required one-tenth of the Planck scale. Indeed they must each have a $(B-L)$-charge of yet another family so as to get the total $(B-L)$ be zero. The highly suggested quantum numbers for these Weyl particles, counted as left-handed, are thus

$$(0,0,0;-1,0,1) \text{ and } (0,0,0;1,-1,0). \tag{47}$$

Now is it possible to build such a pair of Weyl particles into a scheme with mass protected particles cancelling the anomalies and not carrying the charges that would bring them to the low mass level, where they would be directly or indirectly seen? That is to say, we should not mass protect the particles in this scheme with Standard Model charges. This is most easily done by letting them all have the total weak hypercharge $y/2 = y_1/2 + y_2/2 + y_3/2 = 0$. If we also want them not to show up at the see-saw scale, we should not let them be mass protected by the total

**TABLE 4.** Quantum numbers of added set of 15 Weyl particles.

| $y_1/2$ | $y_1/2$ | $y_3/2$ | $(B-L)_1$ | $(B-L)_2$ | $(B-L)_3$ | |
|---|---|---|---|---|---|---|
| 0 | 1 | -1 | 0 | 0 | 0 | **(-1;0)** |
| -1 | 0 | 1 | 0 | 0 | 0 | **(-2;0)** |
| 1 | -1 | 0 | 0 | 0 | 0 | **(-3;0)** |
| 0 | 0 | 0 | 0 | 1 | -1 | **(0;-1)** |
| 0 | 1 | -1 | 0 | 1 | -1 | **(-1;-1)** |
| -1 | 0 | 1 | 0 | -1 | 1 | **(-2;1)** |
| 1 | -1 | 0 | 0 | -1 | 1 | **(-3;1)** |
| 0 | 0 | 0 | -1 | 0 | 1 | **(0;-2)** |
| 0 | -1 | 1 | -1 | 0 | 1 | **(1;-2)** |
| 1 | 0 | -1 | 1 | 0 | -1 | **(2;2)** |
| -1 | 1 | 0 | 1 | 0 | -1 | **(3;2)** |
| 0 | 0 | 0 | 1 | -1 | 0 | **(0;-3)** |
| 0 | -1 | 1 | 1 | -1 | 0 | **(1;-3)** |
| 1 | 0 | -1 | -1 | 1 | 0 | **(2;3)** |
| -1 | 1 | 0 | -1 | 1 | 0 | **(3;3)** |

$(B-L) = (B-L)_1 + (B-L)_2 + (B-L)_3$. Again the most easy way of ensuring this would be to let them not have the total $B-L$ at all, *i.e.* to have $B-L=0$ for them.

It is in fact possible to find such a set of Weyl-particles, cancelling anomalies and containing the two particles suggested. This relatively elegant scheme of particles contains 15 Weyl particles—just the same number as the particles in a Standard Model family without the right-handed neutrino. The quantum numbers of this set of 15 Weyl particles are listed in Table 4.

It can easily be checked that the here proposed particles have anomalies that cancel among themselves. The last column in the table contains a shorthand notation for the quantum numbers, in the way that the first signed number from 0 to 3 enumerates some simple combinations of family weak hypercharges, so that:

$$\mathbf{0} = (0,0,0) \tag{48}$$
$$\mathbf{1} = (0,-1,1) \tag{49}$$
$$-\mathbf{1} = (0,1,-1) \tag{50}$$
$$\mathbf{2} = (1,0-1) \tag{51}$$
$$-\mathbf{2} = (-1,0,1) \tag{52}$$
$$\mathbf{3} = (-1,1,0) \tag{53}$$
$$-\mathbf{3} = (1,-1,0) \tag{54}$$

The same shorthand notation is used for the ordered sets of the three family $(B-L)_i$'s. So that *e.g.* $(-\mathbf{2};\mathbf{1})$ means $(y_1/2, y_2/2, y_3/2; (B-L)_1, (B-L)_2, (B-L)_3)$ $= (-1,0,1;0,-1,1)$.

This shorthand notation can be used to recognise a certain amount of regularity in the found scheme and can be used to quickly see the cancellation of many of the

**TABLE 5.** The fifteen Weyl particle quantum numbers as an array.

| | | | |
|---|---|---|---|
| - | $(-\mathbf{1};\mathbf{0})$ | $(-\mathbf{2};\mathbf{0})$ | $(-\mathbf{3};\mathbf{0})$ |
| $(\mathbf{0};-\mathbf{1})$ | $(-\mathbf{1};-\mathbf{1})$ | $(-\mathbf{2};+\mathbf{1})$ | $(-\mathbf{3};+\mathbf{1})$ |
| $(\mathbf{0};-\mathbf{2})$ | $(+\mathbf{1};-\mathbf{2})$ | $(+\mathbf{2};+\mathbf{2})$ | $(+\mathbf{3};+\mathbf{2})$ |
| $(\mathbf{0};-\mathbf{3})$ | $(+\mathbf{1};-\mathbf{3})$ | $(+\mathbf{2};+\mathbf{3})$ | $(+\mathbf{3};+\mathbf{3})$ |

anomalies. You should bear in mind that we have so many anomaly constraints that it is a bit of an art to find any chiral scheme, with anomaly cancellations for all the many choices of three charges with potential triangle anomalies. In fact the scheme is written - moderately - nicely in a 4 by 4 array, as given in Table 5.

The regularity in this Table is already a little bit complicated: There are fifteen Weyl particle quantum numbers listed—one for each of the sixteen combinations, except for the totally sterile combination $(\mathbf{0},\mathbf{0})$ with no charges at all. These are obtained by combining one of the following four combinations of a number for the hypercharges and a sign related to the $(B-L)_i$'s:

$$[\mathbf{0};-],\quad [\mathbf{1};-],\quad [\mathbf{2};+],\quad [\mathbf{3};+] \tag{55}$$

with one from the following series of a sign related to the family weak hypercharges and a number for the family $(B-L)_i$'s:

$$[-;\mathbf{0}],\quad [-;\mathbf{1}],\quad [+;\mathbf{2}],\quad [+;\mathbf{3}] \tag{56}$$

The idea here is that, for example, when we combine $[\mathbf{2};+]$ from the first series with $[-;\mathbf{1}]$ from the second series, we construct the shorthand symbol $(-\mathbf{2};+\mathbf{1})$. This then, in turn, is translated into the combination of charges supposed to be sitting on that left-handed Weyl particle with quantum numbers $(-\mathbf{2};+\mathbf{1}) = (y_1/2, y_2/2, y_3/2; (B-L)_1,\ (B-L)_2, (B-L)_3) = (-1,0,1;0,-1,1)$.

If we now, for example, will check that there are no anomalies corresponding to a triangle diagram, with the three external gauge boson couplings being the ones coupling to the weak hypercharges for family 1 and family 2 and the $(B-L)_2$, it means that we want to check the no anomaly condition

$$\sum_{\text{the 15 Weyl particles}} \frac{y_1}{2}\cdot\frac{y_2}{2}\cdot(B-L)_2 \tag{57}$$

$$= \sum_{\{(-\mathbf{3};+\mathbf{1}),(+\mathbf{3};+\mathbf{3})\}} \frac{y_1}{2}\cdot\frac{y_2}{2}\cdot(B-L)_2 \tag{58}$$

$$= -1 \sum_{\{+\mathbf{1},+\mathbf{3}\}} (B-L)_2 = 0 \tag{59}$$

It is, namely, easily seen that we can only get contributions to the product $\frac{y_1}{2}\frac{y_2}{2}$ from the weak hypercharge shorthand symbols that are neither having the numbers $\pm\mathbf{1}$ nor $\pm\mathbf{2}$. Further to have a $(B-L)_2$ non-zero factor, we need to avoid the

$(B-L)$-related shorthand symbol being $\pm\mathbf{2}$. In our formulation this cancellation takes place because the sign of $y_1 y_2/4$ is the same for all the $\pm\mathbf{3} = \pm(-1,1,0)$ weak hypercharge combinations, which are those to which the $y_1 y_2/4$ can give a contribution.

In the fifteen Weyl-particle system just presented, one also finds candidates for the "top-quark-role-player" for a field with the quantum numbers of the field combination $\xi S^\dagger$, which has the simple quantum numbers $(0,0,0;1,-1,0)$.

In our fit the Higgs field $S$ has a vacuum expectation value of order unity. So, in first approximation, there would be essentially no phenomenological consequences if we replaced the model by a related one, in which the field $\xi$ had got the quantum numbers of this combination $\xi S^\dagger$. Then, when there is use for a Higgs field with the $\xi$ quantum numbers, one would make use of an extra $S$-field. Since that does not cost any extra suppression, that would make no great difference and it could be an equally good model. But it could have the advantage that we could now find, among the particles which we are tempted to postulate to exist, some "top-quark-role-players" for the slightly modified $\xi$-field.

But now we must admit that further manipulation of the model into a related one seems to be needed, if there should be any chance of getting the suggested requirements of "top-quark-role-players", no anomaly fermions and all that to work. However, that some rudiments of the requirements come about, by means of almost phenomenologically invented fields, is a good sign. It suggests that further investigation in that direction could yield a model that hangs together and satisfies the requirements.

Let us also remark that this system of fifteen Weyl particles has some similarity with a usual family of Weyl particles in the Standard Model. At least it has the similarity that it consists of just 15 particles, just the number in a usual family. In this way there would be some sense in talking about such a set of particles as a "crossing fourth family". However it is, of course, really quite misleading, in as far as this "crossing family" couples to completely different gauge fields compared to what a proper fourth family should do.

# XII ON WHY THREE FAMILIES

It should be noted that the above proposed, and very speculatively phenomenologically called for, system of 15 particles, with masses of the order of a factor ten or so under the Planck mass scale, has some connection to the question of why we have just three families:

Indeed the construction of a system with just the number of particles corresponding to a single family, in the way we did it above, is specific to the case of just three families! We might think of the abelian gauge groups, that were used in the construction of the suggested system of 15 particles, as analogous to the Cartan algebra of the Standard Model group. If we do that, we see that it is natural that we made use of just four linearly independent abelian charges. Well, at first it looks like we used 6 charge-species, namely 3 weak hypercharges and 3 family $(B-L)_i$'s. However, we had to avoid the risk that this system of 15 particles gets mass protection by the Standard Model charges or the total $(B-L)$-charge. So we had to let the charge assignments for these 15 particles obey the rules that the Standard Model quantum numbers, as well as the total $(B-L)$-charge, be zero. That imposes two constraints—of zero $y/2$ and $(B-L)$—upon the charges and there are thus only $6-2=4$ independent charges.

Now note that, in order that the above construction of the "crossing fourth family" should work, we got the condition that the number of families multiplied by two—one for the family weak hypercharge $y_i/2$ and one for the $(B-L)_i$—shall be just two more than the rank of the Standard Model group. This constraint can only be satisfied for the case of just 3 Standard Model families!

Once we take it as a good idea that there be such a "crossing fourth family", we may again argue for the three families by a slightly different method: In the following section we shall shortly put forward an idea of thinking of some especially "nice" linear combinations of the Cartan algebra generators—the "Han-Nambu-like charges". We thereby get an argument for the number of Weyl-particles in a Standard Model family being just a power of two minus unity—or simply a power of two if we include a right-handed neutrino.

Once we observe that there are just $2^4-1=15$ particles in a family, we may ask ourselves whether, by analogy, it would not be expected that also the number of families and/or the total number of Weyl particles under the Planck scale should be of the form $2^n-1$. In this way we are led to propose a picture of the Weyl fermion charge assignments for all the particles that are mass protected by some charge in the system:

In addition to the three known families there is a bunch of Weyl-particles, with the scheme of charge assignments given in Tables 4 and 5 above, which are equal in number to those of a Standard Model family without its right-handed neutrino, namely 15. Together this makes $3(15+1)+15 = 63 = 2^6-1$ particles. In other words: The three known families, each associated to one speculated see-saw neutrino ( = right-handed neutrino) make up $3(15+1) = 48$ Weyl particles. Adding to them the fantasy extra family – that is not at all a real Standard Model family – suggested by the "top-quark-role-player" arguments consisting of 15 particles, we reach a total of $48+15 = 63$. This total of 63 particles can be considered remarkable by being a power of 2 minus one, just in analogy to the number of Weyl-particles in a Standard Model family.

But why should these $2^n-1$ systems of particles be so important and likely to occur in nature? We have – of course – no really convincing argument for the moment, but we present below a weak motivation in what is really an anomaly counting modulo 2.

# XIII HAN-NAMBU-LIKE CHARGES

In this section we should like to propose an idea for how one might make more specific the statement that the representations of the Standard Model gauge group realized in nature are *very small representations*.

If you look at the non-abelian representations in the Standard Model you find, except for the gauge fields themselves, *nothing but the trivial representation and the lowest dimensional representations after the trivial one*. This can be said to mean that the representations chosen by Nature are really remarkably *small*, as far as the non-abelian groups $SU(2)$ and $SU(3)$ are concerned!

Can we say the same thing in some sense when we think of abelian group(s)? For abelian groups you cannot so easily use the dimension of the representation, since abelian groups always have one-dimensional representations and thus dimension makes no distinction; instead the natural suggestion would be to use the ratio of the charges of the representation realized compared to the quantum of charge—the Millikan-like charge value of which all charges must be an integer multiple. One might now think of looking at the unique abelian *invariant* subgroup in the Standard Model gauge group, the weak hypercharge; but taken literally that idea does *not* work well, if you hope to show that the Standard Model charges are remarkably small. The problem is that the quantum for half the weak hypercharge $y/2$ is $1/6$, while the right-handed charged leptons have the value $y/2 = -1$. So it is 6 times as large, but 6 is not the smallest integer after zero!

However there may be no good reason for looking only at the invariant abelian subgroups. So we suggest taking into account also the non-invariant abelian subgroups. But there are a lot of abelian subgroups of the Standard Model group; really the Standard Model group has infinitely many abelian subgroups of course. Think for example of two non-invariant abelian subgroups of the Standard Model group, such as the one generated by the electric charge and the one generated by the Gell-Mann $\lambda_8$ or say $\sqrt{3}\lambda_8$. The electric charge in the Standard Model really has a quantum equal to $1/3$ in units of the Millikan quantum, because the quarks have non-integer charges, while $\sqrt{3}\lambda_8 = \mathrm{diag}(1, 1, -2)$ has a quantum of charge equal to 1. If we, however, now make a linear combination with complicated rational coefficients, we easily get a combined charge or generator in the Standard Model Lie algebra which can have a very small quantum of charge. So it is easy, by forming linear combinations of Lie algebra generators (essentially the same as charges), to get such generators that have very small quanta of charge and for which the actual particles then turn out to have a huge number of quanta. Thus, w.r.t. such abelian charges, the Standard Model representations will be"very big " *i.e.* have lots of quanta for the realized representations ($\approx$ particles). In formulating a statement about whether the Standard Model has small or big abelian representations, it is therefore necessary to keep in mind that the different non-invariant charges we can think of have very different charge quanta sizes, in general, and thus are not at all represented on equally "small" representations.

These complicated linear combinations (let alone linear combinations with irrational coefficients) are not so attractive to study or use for the definition of the representations being small or large. So we prefer to define the question of whether a model—say the Standard Model—has small or large representations in terms of whether it is possible to define many, or only a few, charges which are realized with very low numbers of charge quanta. The trivial and smallest non-trivial charges you can hope for are 0 or 1 or $-1$ measured in charge quanta. So we should say that the abelian representations being small should, by definition, mean that they have relatively many charges (*i.e.* generators in the Lie algebra) with the property of having only the values 0, 1 or $-1$ realized.

So the question then is: will a reasonable "counting" of the charges (Lie algebra generators), that have this property of only having the three possibilities 0, 1 and $-1$ for the number of quanta on a particle, lead to the Standard Model having especially many such charges? The answer we want to suggest is that it is indeed so that the Standard Model has a relatively big family of such charges. Then one should be able to say that w.r.t. the abelian charges (and not only w.r.t. the non-abelian ones) the Standard Model is a model with "small" representations!

Let us, for simplicity, consider the Cartan algebra of the Standard Model gauge group and look for how many Lie algebra generators (or charges) we can find, inside this Cartan algebra, which have the property of having only eigenvalues $1, 0, -1$.

Actually we can present a family of ten such generators, several of which are already known in the literature as variations of the Han-Nambu charges [23]. Remember that the Han-Nambu charges were proposals for what the electric charge could be in a model for quarks etc., which lost support because of its disagreement with the results from deep inelastic scattering experiments. This model has the property that even the quarks have integer Millikan quanta of charges of this Han-Nambu type (which was suggested as a candidate for electric charge, but here is just considered a certain formal charge we can construct if we like). In fact, the idea is that one adds to the by now generally accepted form of the electric charge in the Standard Model, $Q = y/2 + t_3$, a colour dependent term $\lambda_8/\sqrt{3}$ which has eigenvalues 1/3, 1/3 and $-2/3$. If the quark which has the $-2/3$ eigenvalue is a red one, we can say we get the "red Han-Nambu charge".

It is clear that, even inside the Cartan algebra chosen by using diagonal $t_3$ and Gell-Mann's $\lambda_8$ and $\lambda_3$, we can find Han-Nambu charges of the two other colours–the blue and the yellow. This already makes up three Han-Nambu charges inside the Cartan algebra. In addition we can take "anti-Han-Nambu charges", which are obtained by replacing the electric charge $y/2 + t_3$ by the the charge $y/2 - t_3$, *i.e.* with a minus sign on the $t_3$-term. This gives us three more charges which, again, are easily seen to have no other eigenvalues than $0, 1, -1$. Now it is actually easily understood that if we have two charges, say $Q_A$ and $Q_B$, which have only the three eigenvalues $-1$, 0 and 1, then there is an enhanced chance that the sum or the difference would again be a charge with this property. It is, of course, by no means guaranteed, since we could easily risk that say the sum $Q_A + Q_B$ has also double charges. However it turns out that we can indeed construct sums and differences of some of the Han-Nambu and anti-Han-Nambu charges which again have this

$-1, 0, 1$ property. For instance subtraction of the Han-Nambu charges from each other leads to $\lambda_3$ type colour generators, of which we have three combinations in the Cartan algebra (counting as one a charge and its opposite). By subtracting analogous anti-Han-Nambu and Han-Nambu charges, we can also get $2t_3$ which is 1, or $-1$ on left-handed particles and 0 on the right-handed (in the conventional thinking of particles but ignoring of anti-particles).

This makes up the following 10 charges in the Cartan algebra with our $-1, 0, 1$ property:

$$Q_{\text{HN red}} = y/2 + t_3/2 + \lambda_{8(\text{red})}/\sqrt{3} \tag{60}$$
$$Q_{\text{HN blue}} = y/2 + t_3/2 + \lambda_{8(\text{blue})}/\sqrt{3} \tag{61}$$
$$Q_{\text{HN yellow}} = y/2 + t_3/2 + \lambda_{8(\text{yellow})}/\sqrt{3} \tag{62}$$
$$Q_{\overline{\text{HN}}\text{ red}} = y/2 - t_3/2 + \lambda_{8(\text{red})}/\sqrt{3} \tag{63}$$
$$Q_{\overline{\text{HN}}\text{ blue}} = y/2 - t_3/2 + \lambda_{8(\text{blue})}/\sqrt{3} \tag{64}$$
$$Q_{\overline{\text{HN}}\text{ yellow}} = y/2 - t_3/2 + \lambda_{8(\text{yellow})}/\sqrt{3} \tag{65}$$
$$\lambda_{3\,\text{red, blue}} \tag{66}$$
$$\lambda_{3\,\text{blue, yellow}} \tag{67}$$
$$\lambda_{3\,\text{yellow, red}} \tag{68}$$
$$2t_3 \tag{69}$$

If the sum of two of these 10 charges happens to be one of the other ones, the difference will not be one but will have double charge eigenvalues. They also form an algebra, together with 5 generators that have eigenvalues $\pm 2$ though, which is to be considered a modulo 2 algebra. This is to avoid having to distinguish between sums and differences, in order to really have the set of these charges closed under addition (because sometimes it is the sum sometimes the difference that belongs again to the set).

## XIV MODULO 2 CONSIDERATIONS

In the light of the fact that the algebra of the Han-Nambu-like charges tends to be a modulo 2 algebra, it is natural to consider the set of Han-Nambu-like charges as a $Z_2$-vector space. We may then first study the anomaly restrictions for the $Z_2$-vector space, hoping that this study is simpler but can nevertheless give interesting information.

Thus we shall study here the anomaly restrictions on a set of general Han-Nambu-like charges having, to as large an extent as possible, only the charges $-1, 0, 1$ on the particles. By such a study we hope to partially derive the Standard Model pattern of particles. So let us here imagine a series of "Han-Nambu-like charges" $Q_{HN\,i}$ ($i = 1, 2, 3, ..., n$), having the property that most of the charges take on the values $Q_{HN\,i} = -1, 0, 1$ for all the particles in the general model under consideration. In

$Z_2$ algebra language, we throw away the information of whether the charge is 1 or $-1$ but keep the information of whether it is even or odd.

We now consider the triangle anomaly cancellation conditions

$$\sum_{\text{the Weyl particles}} Q_{HN\ i} Q_{HN\ j} Q_{HN\ k} = 0 \tag{70}$$

and the mixed anomaly condition

$$\sum_{\text{the Weyl particles}} Q_{HN\ i} = 0 \tag{71}$$

for the model. These conditions can be interpreted as applying to the true gauge charges on the particles or as only applying to the charges modulo 2, so that the symbols $Q_{HN\ i}$ only take the values "even" or "odd". Then of course one shall use the algebra of the field $Z_2$. Considered this $Z_2$ way the mixed anomaly condition is really superfluous, as it is a special case of eq. (70) with $i = j = k$.

We now successively introduce a few assumptions that are supposed to be consequences or more precise forms of "the requirement of small representations". Firstly this requirement should mean that there exist many Han-Nambu-like charges. This in turn suggests that when we combine two Han-Nambu-like charges, by addition or subtraction, then we get another Han-Nambu-like charge (having only eigenvalues -1, 0, +1), in as many cases as possible. In the cases when the combination is not a Han-Nambu-like charge, the resulting charge has eigenvalues $\pm 2$ as well. Next we shall make this requirement of often finding the sum or difference to be a new Han-Nambu-like charge more specific as follows: Gauge field theory models with the "smallest possible representations" contain a group (algebra) of charges closed under $Z_2$-addition, of which as many as possible are Han-Nambu-like (having eigenvalues $\pm 1$, 0 only), while as few as possible may in addition have eigenvalues $\pm 2$.

From the point of view of the $Z_2$-algebra, the Weyl fermions of the model have their charges in the vector space dual to the charges $Q_{HN\ i}$: given a particle $p$ and a charge we get an inner product $\langle Q_{HN\ i} | p \rangle$ = "the (eigen)value of the charge $Q_{HN\ i}$ for the particle $p$" = "the charge of the particle p of kind $Q_{HN\ i}$". Again we may think of this algebra as being counted modulo 2 if we like to do so, and now we do just that.

It would be most elegant, and we could also claim that it would mean the biggest number of charges of this "Han-Nambu-like" type, if we had so many that we used up all modulo 2 independent charge assignments. Let us take this as an excuse for assuming that the mass protection of the particles we consider—we are after all interested in observable low energy physics particles which should be mass protected—is revealed in the modulo 2 counting. In order for a mass protected particle to exist with some combination of the charges $Q_{HN\ i}$ specified (*i.e.* in a certain vector of the dual vector space of the space of these charges), it is necessary to have an odd number of particle types (flavours) with these charges modulo 2.

Interpreting the no-anomaly condition (70) as a $Z_2$-charge condition, the term $Q_{HN\,i}Q_{HN\,j}Q_{HN\,k}$ in the sum is only odd (*i.e.* non-zero) when all three charges $Q_{HN\,i}$, $Q_{HN\,j}$ and $Q_{HN\,k}$ are odd for the Weyl particle considered. Expressed geometrically, this means that one only gets non-zero contributions from particles which are associated with the intersection of the three "displaced hyperplanes" $\{p \in V_p \mid Q_{HN\,i}(p) = \text{odd}\}$, $\{p \in V_p \mid Q_{HN\,j}(p) = \text{odd}\}$ and $\{p \in V_p \mid Q_{HN\,k}(p) = \text{odd}\}$ in the $Z_2$-vector space $V_p$ of particles. If the $Z_2$-vector spaces are of dimension less than or equal to three, this intersection can be made at just one point and if all the charges are gauged we can make it any point. So it is impossible to obtain a cancellation of anomalies between particles and all the charges must be even, giving no particles mass-protected modulo 2, if the charge and particle space is of dimension less than or equal to 3.

Thus the smallest allowed dimension for the space of particles, as well as that of the charges $Q_{HN\,i}$, should be four. In this case one can easily show that, if just one charge combination is assumed to have an odd number of particles, then all non-trivial charge combinations (we now only work modulo 2) have an odd number of particles. Of course a particle with all charges 0 is of no help in solving the anomaly conditions and can be left out. This means that we have to fill $2^4 - 1 = 15$ possible charge assignments modulo 2 with an odd number of particles.

The number 15 is interesting: there are just 15 Weyl particles in a Standard Model family, and the 10 (good with only $-1, 0, 1$) + 5 (bad, with double charges) = 15 Han-Nambu-like charges in the Standard Model can be seen to be assigned to the 15 particles in a family. These 15 Weyl particles are distributed over all the 15 non-zero vectors in the particle-vector-space dual to that of the charges!

In other words, with a few "smallest representation assumptions", the Standard Model structure modulo 2 for the Han-Nambu-like charges is obtained as the smallest dimensional $Z_2$-space that can cancel the anomalies but still be mass protected! Crudely speaking this means we can claim that indeed a Standard Model family is having the smallest representation, as far as the charges in the Cartan algebra goes.

We shall now take the above argumentation to suggest more generally a $Z_2$-vector space structure, for both a set of Han-Nambu-like charges and the set of Weyl particles. It is then suggested that we should find, in the true model of Nature, a number of Weyl particles equal to a power of 2 perhaps minus 1. The latter subtraction is expected, because we can in no way mass protect a super-sterile particle having all its quantum numbers zero. So the particles that are totally "even" in the $Z_2$-formulation may be totally sterile and impossible to mass protect. Thus we would count them as only belonging to the "garbage at the Planck scale", where everything is to be found in our philosophy.

The number of already observed Weyl fermions is, of course, 45 corresponding to the three Standard Model families. This number could be increased to 48 if one took seriously the indirect evidence for three see-saw scale right-handed neutrinos. The next power of 2 minus 1, greater than 48, is 63. So, if the above idea were to be upheld, there would be a need to postulate the existence of an extra set of 15 Weyl particles. This extra set of particles would have to be so weakly mass protected that the particles have become so heavy that we do not see them. There is an extra scale in our Anti-GUT model a single order of magnitude under the Planck scale. So it is, of course, not unexpected that there could be Weyl particles in the Anti-GUT model mass protected only down to this scale—counting the Planck scale as the *a priori* mass scale. Indeed one could easily imagine constructing a system of 15 particles, in analogy with a usual Standard Model family, just using some other gauge fields with a more strongly broken/Higgsed gauge symmetry. If this extra set of particles have masses of the order of 1/10 of the Planck mass, they will stay safely in the fantasy sector for a long time to come!

Once you work with $Z_2$ vector spaces, you can imagine that somehow or another the known Standard Model families fit into a $Z_2$ vector space with four elements, *i.e.* of dimension 2. It should then be remarked that there is a permutation symmetry of the $Z_2$ vector field structure among the three non-zero elements while the zero-vector is, of course, special w.r.t. the $Z_2$-algebra. This could be interpreted as support for the existence of 3 families.

# XV CONCLUSION

In this article we have made an extension of the Anti-GUT model to neutrinos, by including see-saw $\nu_R$ particles at a scale of mass around $10^{12}$ GeV. By this extension we introduced two more parameters, namely the vacuum expectation values of two additional Higgs fields, $\phi_{B-L}$ and $\chi$. But one extracts two mixing angles, $\theta_\odot$ and $\theta_{\rm atm}$, and two mass squared differences, $\Delta m^2_\odot$, and $\Delta m^2_{\rm atm}$ from the neutrino oscillation data. So in this sense we have two predictions:

$$\sin^2 2\theta_\odot \approx 3 \times 10^{-2} \tag{72}$$
$$\frac{\Delta m^2_\odot}{\Delta m^2_{\rm atm}} \approx 6 \times 10^{-4} \tag{73}$$

These results are *only order of magnitude* estimates, and we shall count something like an uncertainty of 50% for mixing angles and masses. Thus for the square, $\sin^2 2\theta$, we estimate an uncertainty of 100% *i.e.* a factor of 2 up or down and for the ratio, $\Delta m^2_\odot/\Delta m^2_{\rm atm}$, an uncertainty of $\sqrt{2} \cdot 100\%$ meaning roughly a factor of 3 up or down:

$$\sin^2 2\theta_\odot = (3\,^{+3}_{-2}) \times 10^{-2} \tag{74}$$
$$\frac{\Delta m^2_\odot}{\Delta m^2_{\rm atm}} = (6\,^{+11}_{-4}) \times 10^{-4} \ . \tag{75}$$

These two small numbers both come from the parameter $\xi$—the VEV in "fundamental units" of one of the 7 Higgs fields in our model—which has already been fitted to the charged fermions in earlier works [9]. Its value, eq. (10), is essentially the Cabibbo angle measuring the strange to up-quark weak transitions ($\xi \simeq 0.1$ essentially giving $\sin\theta_c \simeq 0.22$). More precisely we find from our fit [17]:

$$\sin^2 2\theta_\odot = 3\,\xi^2 \tag{76}$$

$$\sin\theta_c = 1.8\,\xi \tag{77}$$

$$\frac{\Delta m^2_\odot}{\Delta m^2_{\rm atm}} = 6\,\xi^4 \tag{78}$$

The numerical factors in front of equations (76), (77) and (78) are the result of our rather arbitrary averaging over random order unity factors and the inclusion of diagram counting square root factors (factorial corrections), as put forward in reference [19]. But in principle these factors are just of order unity. It is also important for the success of our model that there has been room to put in the $\chi$ field, with which we could fix the atmospheric mixing angle to be of order unity (by taking $\chi \sim T$), as well as a parameter $\phi_{B-L}$. The latter parameter is the Higgs field VEV for breaking the gauged $B-L$ charge and is used to fit the overall scale of the observed neutrino masses.

We want to emphasise here that our model—extended Anti-GUT as well as "old" Anti-GUT—is itself a good model in the sense that all coupling constants are of order of unity, except for Higgs fields VEVs and thereby also the Higgs masses giving rise to these VEVs. In the Standard Model the most remarkable unnatural feature is the tremendously small value of the Weinberg-Salam Higgs VEV compared to the Planck or realistic GUT scales. If somehow we have to accept that there must be a mechanism in nature for making the Weinberg-Salam Higgs VEV very small, we also should admit that the other Higgs VEVs could be very small. In our model we manage to interpret the second unnatural feature of most Standard Model Yukawa couplings, namely that they are very small, to be also due to small Higgs field VEVs. In this way all small numbers come from VEVs in our model; the rest is put to unity in Planck units. In this sense it is "natural", meaning that it has only one source of small numbers—the VEVs. Even the gauge couplings can be interpreted as being of order of unity, if we follow our MPP assumption, which goes together extremely well with the present model.

**TABLE 6.** Number of parameters.

| | "Yukawa" | "Neutrino" | # of parameters | # of predictions |
|---|---|---|---|---|
| Standard Model | 13 | 4 | 17 | — |
| "Old" Anti-GUT | 4 | —* | 4† | 9 |
| "New" Anti-GUT | 6 | | 6 | 11 |

* The "old"Anti-GUT cannot predict the neutrino oscillation.

† Here we have not counted the neutrino oscillation parameters.

Concerning this question of the Higgs fields often having numerically small values compared to the fundamental (Planck) scale, we presented an idea for how these different scales could come in order of magnitudewise: The proposal was that there is a principle working in nature, which finetunes the coupling constants so as to arrange for several degenerate vacuum states. We argued that that there is a good chance that exponentially small VEV-scales could come out from this just postulated finetuning principle (called MPP = multiple point principle). Such a picture pre-supposes some dynamics, so to speak, in the coupling constants much like in baby universe theory [24].

So we may say that we did not really solve the hierarchy problem in the sense of getting the scale out *without* finetuning, but rather proposed a model for finetuning, namely the MPP. We have earlier claimed some success with such a principle w.r.t. getting finestructure constants and predicting the top-mass to be $173 \pm 6$ GeV. Also there is a true <u>prediction</u> for the Higgs mass of $135 \pm 9$ GeV (so watch out in the future).

For the working of our scheme to get scales of highly different orders of magnitude, it is important to have a fermion, like the top quark in the Standard Model, with a large unsuppressed coupling to the relevant Higgs field—*e.g.* to finetune the Weinberg-Salam Higgs field VEV to the electroweak scale rather than the Planck scale. Assuming that we need a similar unsuppressed fermion for the other cases of Higgs fields in the Anti-GUT model, some heavy fermions at *e.g.* 1/10th the Planck scale mass are very speculatively required to exist. For this purpose, we proposed a set of 15 Weyl particles, with anomaly cancellation and carrying family $(B-L)_i$ charges and weak hypercharges only. Since they are not to be mass protected by the Standard Model, these 15 very heavy particles must have zero diagonal quantum numbers, *i.e.* with zero total $(B-L)$ and zero total $y/2$.

In this connection we had some weak indication that the number 3 for the number of families had some special significance in these constructions. We also suggested that the Standard Model could be considered as having very small, one could say minimal, representations w.r.t. (non-invariant) abelian (sub)groups, as well as for the non-abelian groups.

At the end we should emphasize that our Anti-GUT model mass matrices have managed to fit order of magnitudewise about 17 quantities (11 observed fermion masses or mass squared differences, 5 mixing angles and the CP-violating phase of quarks) with 6 parameters—the Higgs field VEVs. As we can see from Table 6, in order to fit the four quantities measured in the neutrino oscillation data, we have introduced two more parameters and gained two predictions (the solar mixing angle and the ratio of the neutrino oscillation masses). Also it should be mentioned that we really developed this Anti-GUT model, by seeking to understand the fermion spectrum in a model that D.L. Bennett and I. Picek used with one of us (H.B.N.) to predict the values of the fine structure constants. The postulates of this earlier model were later replaced by the the above-mentioned MPP. These predictions of the finestructure constants actually agree with experiment, with an uncertainty of $\pm 6$ in the inverse fine structure constants [12]. When you remember that this same

MPP is promising for some finetuning problems, we can claim that the combined MPP and (extended) Anti-GUT model provides a fit to a very large number of the Standard Model parameters!

Note that our model is very successful in describing neutrino oscillations and their mixing angles, but this model does not have any good candidate for dark matter; the monopoles could be such a candidate. We will study this problem in a forthcoming article.

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## REFERENCES

1. L. O'Raifeartaigh, *Group structure of gauge theories* (Cambridge University Press, 1986).
2. C. D. Froggatt, G. Lowe and H. B. Nielsen, Phys. Lett. **B311** (1993) 163.
3. C. D. Froggatt, M. Gibson, H. B. Nielsen and D. J. Smith, Int. J. Mod. Phys. **A13** (1998) 5037;
   C. D. Froggatt, M. Gibson, H. B. Nielsen and D. J. Smith, *Proc. of the 29th Int. Conf. on High-Energy Physics*, (Vancouver, 1998), eds. A. Astbury, D. Axen and J. Robinson (World Scientific,1999) Vol. 2 p. 1661.
4. S. Rajpoot, Phys. Rev. **D24** (1981) 1890.
5. H. B. Nielsen and N. Brene, NBI-HE-84-47, *Elaborated version of lecture given at Symp. on High Energy Physics, Ahrenshoop, GDR, October 21-26, 1984.*
6. C.D. Froggatt and H.B. Nielsen, *Origin of Symmetries* (World Scientific, 1991).
7. D.L. Bennett, C.D. Froggatt and H.B. Nielsen, *Recent Developments in Nonperturbative Quantum Field Theory* (Seoul, 1997), eds. Y.M. Cho and M. Virasoro, (World Scientific, 1998) p. 362; hep-ph/9710407;
   D.L. Bennett, Ph.D. thesis, Niels Bohr Institute, 1996; hep–ph/9607341.
8. D.L. Bennett and H.B. Nielsen, Int. J. Mod. Phys. **A14** (1999) 3313.
9. C.D. Froggatt, M. Gibson, H.B. Nielsen and D.J. Smith, Int. J. Mod. Phys. **A13** (1998) 5037.
10. C.D. Froggatt, M. Gibson and H.B. Nielsen, Phys. Lett. **B446** (1999) 256.

11. C.D. Froggatt and H.B. Nielsen, Nucl. Phys. **B147** (1979) 277.
12. D.L. Bennett, C.D. Froggatt and H.B. Nielsen, *Proc. of the 27th Int. Conf. on High Energy Physics* (Glasgow, 1994), eds. P. Bussey and I. Knowles, (IOP Publishing Ltd, 1995) p. 557;
    D.L. Bennett, C.D. Froggatt and H.B. Nielsen, *Perspectives in Particle Physics '94*, eds. D. Klabučar, I. Picek and D. Tadić, (World Scientific, 1995) p. 255; hep–ph/9504294.
13. C.D. Froggatt and H.B. Nielsen, Phys. Lett. **B368** (1996) 96.
14. C.D. Froggatt, H.B. Nielsen and D.J. Smith, Phys. Lett. **B385** (1996) 150.
15. M.B. Green and J. Schwarz, Phys. Lett. **B149** (1984) 117.
16. C.D. Froggatt and H.B. Nielsen, *Lepton and Baryon Number Violation in Particle physics, Astrophysics and Cosmology* (Trento 1998), eds. H.V. Klapdor-Kleingrothaus and I. Krivosheina, (IOP Publishing Ltd, 1999); hep-ph/9810388.
17. H. B. Nielsen and Y. Takanishi, Nucl. Phys. **B588** (2000) 281.
18. T. Yanagida, *Proc. of the Workshop on Unified Theories and Baryon Number in the Universe* (Tsukuba 1979), eds. O. Sawada and A. Sugamoto, (KEK Report No. 79-18) p. 95;
    M. Gell-Mann, P. Ramond and R. Slansky, *Supergravity* (Stony Brook, 1979), eds. P. van Nieuwenhuizen and D. Freedman (North-Holland, Amsterdam, 1979) p. 315.
19. C.D. Froggatt, H.B. Nielsen and D.J. Smith, in progress.
20. L.V. Laperashvili and H.B. Nielsen, "Phase Transition in Higgs Model of Scalar Fields with Electric and Magnetic Charges " in progress.
21. C. D. Froggatt and H. B. Nielsen, Phys. Lett. **B368** (1996) 96.
22. M. Veltman, Acta Phys. Polon. **B12** (1981) 437;
    B. Stech, *Proc. of the Int. Workshop on What Comes Beyond the Standard Model* (Bled, 1998), eds. N.M. Mankoć Borśtnik, C.D. Froggatt and H.B. Nielsen, (DMFA - zalożniśtvo, Ljubljana, 1999) p. 40; hep-ph/9905357.
23. M. Y. Han and Y. Nambu, Phys. Rev. **139** (1965) B1006.
24. S. W. Hawking, Phys. Rev. **D37** (1988) 904;
    S. Coleman, Nucl. Phys. **B307** (1988) 867; Nucl. Phys. **B310** (1988) 643;
    T. Banks, Nucl. Phys. **B309** (1988) 493.