

THE STANDARD ELECTROWEAK THEORY AND BEYOND

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1 Introduction

These lectures on electroweak (EW) interactions start with a short summary of the Glashow–Weinberg–Salam theory and then cover in detail some main subjects of present interest in phenomenology.

The modern EW theory inherits the phenomenological successes of the $(V - A) \otimes (V - A)$ four-fermion low-energy description of weak interactions, and provides a well-defined and consistent theoretical framework including weak interactions and quantum electrodynamics in a unified picture.

As an introduction, we recall some salient physical features of the weak interactions. The weak interactions derive their name from their intensity. At low energy the strength of the effective four-fermion interaction of charged currents is determined by the Fermi coupling constant G_F . For example, the effective interaction for muon decay is given by

$$\mathcal{L}_{\text{eff}} = (G_F/\sqrt{2}) [\bar{\nu}_\mu \gamma_\alpha (1 - \gamma_5) \mu] [\bar{e} \gamma^\alpha (1 - \gamma_5) \nu_e] , \quad (1)$$

with¹

$$G_F = 1.16639(1) \times 10^{-5} \text{ GeV}^{-2} . \quad (2)$$

In natural units $\hbar = c = 1$, G_F has dimensions of $(\text{mass})^{-2}$. As a result, the intensity of weak interactions at low energy is characterized by $G_F E^2$, where E is the energy scale for a given process ($E \approx m_\mu$ for muon decay). Since

$$G_F E^2 = G_F m_p^2 (E/m_p)^2 \simeq 10^{-5} (E/m_p)^2 , \quad (3)$$

where m_p is the proton mass, the weak interactions are indeed weak at low energies (energies of order m_p). Effective four fermion couplings for neutral current interactions have comparable intensity and energy behaviour. The quadratic increase with energy cannot continue for ever, because it would lead to a violation of unitarity. In fact, at large energies the propagator effects can no longer be neglected, and the current-current interaction is resolved into current- W gauge boson vertices connected by a W propagator. The strength of the weak interactions at high energies is then measured by g_W , the $W - \mu - \nu_\mu$ coupling, or, even better, by $\alpha_W = g_W^2/4\pi$ analogous to the fine-structure constant α of QED. In the standard EW theory, we have

$$\alpha_W = \sqrt{2} G_F \frac{m_W^2}{\pi} = \frac{\alpha}{\sin^2 \theta_W} \cong 1/30 . \quad (4)$$

That is, at high energies the weak interactions are no longer so weak.

The range r_W of weak interactions is very short: it is only with the experimental discovery of the W and Z gauge bosons that it could be demonstrated that r_W is non-vanishing. Now we know that

$$r_W = \frac{\hbar}{m_W c} \simeq 2.5 \times 10^{-16} \text{ cm} , \quad (5)$$

corresponding to $m_W \simeq 80$ GeV. This very large value for the W (or the Z) mass makes a drastic difference, compared with the massless photon and the infinite range of the QED force. The direct experimental limit on the photon mass is¹ $m_\gamma < 2 \cdot 10^{-16} \text{ eV}$. Thus, on the one hand, there is very good evidence that the photon is massless. On the other hand, the weak bosons are very heavy. A unified theory of EW interactions has to face this striking difference.

Another apparent obstacle in the way of EW unification is the chiral structure of weak interactions: in the massless limit for fermions, only left-handed quarks and leptons (and right-handed antiquarks and antileptons) are coupled to W 's. This clearly implies parity and charge-conjugation violation in weak interactions.

The universality of weak interactions and the algebraic properties of the electromagnetic and weak currents [the conservation of vector currents (CVC), the partial conservation of axial currents (PCAC), the algebra of currents, etc.] have been crucial in pointing to a symmetric role of electromagnetism and weak interactions at a more fundamental level. The old Cabibbo universality for the

weak charged current:

$$J_\alpha^{\text{weak}} = \bar{\nu}_\mu \gamma_\alpha (1 - \gamma_5) \mu + \bar{\nu}_e \gamma_\alpha (1 - \gamma_5) e + \cos \theta_c \bar{u} \gamma_\alpha (1 - \gamma_5) d + \sin \theta_c \bar{u} \gamma_\alpha (1 - \gamma_5) s + \dots, \quad (6)$$

suitably extended, is naturally implied by the standard EW theory. In this theory the weak gauge bosons couple to all particles with couplings that are proportional to their weak charges, in the same way as the photon couples to all particles in proportion to their electric charges [in Eq. (6), $d' = \cos \theta_c d + \sin \theta_c s$ is the weak-isospin partner of u in a doublet. The (u, d') doublet has the same couplings as the (ν_e, ℓ) and (ν_μ, μ) doublets].

Another crucial feature is that the charged weak interactions are the only known interactions that can change flavour: charged leptons into neutrinos or up-type quarks into down-type quarks. On the contrary, there are no flavour-changing neutral currents at tree level. This is a remarkable property of the weak neutral current, which is explained by the introduction of the Glashow-Iliopoulos-Maiani mechanism and has led to the successful prediction of charm.

The natural suppression of flavour-changing neutral currents, the separate conservation of e, μ and τ leptonic flavours, the mechanism of CP violation through the phase in the quark-mixing matrix, are all crucial features of the Standard Model. Many examples of new physics tend to break the selection rules of the standard theory. Thus the experimental study of rare flavour-changing transitions is an important window on possible new physics.

In the following sections we shall see how these properties of weak interactions fit into the standard EW theory.

2 Gauge Theories

In this section we summarize the definition and the structure of a gauge Yang–Mills theory. We will list here the general rules for constructing such a theory. Then in the next section these results will be applied to the EW theory.

Consider a Lagrangian density $\mathcal{L}[\phi, \partial_\mu \phi]$ which is invariant under a D dimensional continuous group of transformations:

$$\phi' = U(\theta^A) \phi \quad (A = 1, 2, \dots, D). \quad (7)$$

For θ^A infinitesimal, $U(\theta^A) = 1 + ig \sum_A \theta^A T^A$, where T^A are the generators of the group Γ of transformations in the (in general reducible) representation

of the fields ϕ . Here we restrict ourselves to the case of internal symmetries, so that T^A are matrices that are independent of the space–time coordinates. The generators T^A are normalized in such a way that for the lowest dimensional non-trivial representation of the group Γ (we use t^A to denote the generators in this particular representation) we have

$$\mathrm{tr}(t^A t^B) = \frac{1}{2} \delta^{AB} . \quad (8)$$

The generators satisfy the commutation relations

$$[T^A, T^B] = i C_{ABC} T^C . \quad (9)$$

In the following, for each quantity V^A we define

$$\mathbf{V} = \sum_A T^A V^A . \quad (10)$$

If we now make the parameters θ^A depend on the space–time coordinates $\theta^A = \theta^A(x_\mu)$, $\mathcal{L}[\phi, \partial_\mu \phi]$ is in general no longer invariant under the gauge transformations $U[\theta^A(x_\mu)]$, because of the derivative terms. Gauge invariance is recovered if the ordinary derivative is replaced by the covariant derivative:

$$D_\mu = \partial_\mu + ig \mathbf{V}_\mu , \quad (11)$$

where V_μ^A are a set of D gauge fields (in one-to-one correspondence with the group generators) with the transformation law

$$\mathbf{V}'_\mu = U \mathbf{V}_\mu U^{-1} - (1/ig)(\partial_\mu U) U^{-1} . \quad (12)$$

For constant θ^A , \mathbf{V} reduces to a tensor of the adjoint (or regular) representation of the group:

$$\mathbf{V}'_\mu = U \mathbf{V}_\mu U^{-1} \simeq \mathbf{V}_\mu + ig[\theta, \mathbf{V}_\mu] , \quad (13)$$

which implies that

$$V_\mu'^C = V_\mu^C - g C_{ABC} \theta^A V_\mu^B , \quad (14)$$

where repeated indices are summed up.

As a consequence of Eqs. (11) and (12), $D_\mu \phi$ has the same transformation properties as ϕ :

$$(D_\mu \phi)' = U(D_\mu \phi) . \quad (15)$$

Thus $\mathcal{L}[\phi, D_\mu\phi]$ is indeed invariant under gauge transformations. In order to construct a gauge-invariant kinetic energy term for the gauge fields V^A , we consider

$$[D_\mu, D_\nu]\phi = ig\{\partial_\mu\mathbf{V}_\nu - \partial_\nu\mathbf{V}_\mu + ig[\mathbf{V}_\mu, \mathbf{V}_\nu]\}\phi \equiv ig\mathbf{F}_{\mu\nu}\phi, \quad (16)$$

which is equivalent to

$$F_{\mu\nu}^A = \partial_\mu V_\nu^A - \partial_\nu V_\mu^A - gC_{ABC}V_\mu^B V_\nu^C. \quad (17)$$

From Eqs. (10), (15) and (16) it follows that the transformation properties of $F_{\mu\nu}^A$ are those of a tensor of the adjoint representation

$$\mathbf{F}'_{\mu\nu} = U\mathbf{F}_{\mu\nu}U^{-1}. \quad (18)$$

The complete Yang–Mills Lagrangian, which is invariant under gauge transformations, can be written in the form

$$\mathcal{L}_{\text{YM}} = -\frac{1}{4}\sum_A F_{\mu\nu}^A F^{A\mu\nu} + \mathcal{L}[\phi, D_\mu\phi]. \quad (19)$$

For an Abelian theory, as for example QED, the gauge transformation reduces to $U[\theta(x)] = \exp[ieQ\theta(x)]$, where Q is the charge generator. The associated gauge field (the photon), according to Eq. (12), transforms as

$$V'_\mu = V_\mu - \partial_\mu\theta(x). \quad (20)$$

In this case, the $F_{\mu\nu}$ tensor is linear in the gauge field V_μ so that in the absence of matter fields the theory is free. On the other hand, in the non-Abelian case the $F_{\mu\nu}^A$ tensor contains both linear and quadratic terms in V_μ^A , so that the theory is non-trivial even in the absence of matter fields.

3 The Standard Model of Electroweak Interactions

In this section, we summarize the structure of the standard EW Lagrangian and specify the couplings of W^\pm and Z , the intermediate vector bosons.

For this discussion we split the Lagrangian into two parts by separating the Higgs boson couplings:

$$\mathcal{L} = \mathcal{L}_{\text{symm}} + \mathcal{L}_{\text{Higgs}}. \quad (21)$$

We start by specifying $\mathcal{L}_{\text{symm}}$, which involves only gauge bosons and fermions:

$$\begin{aligned} \mathcal{L}_{\text{symm}} = & -\frac{1}{4} \sum_{A=1}^3 F_{\mu\nu}^A F^{A\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \bar{\psi}_L i\gamma^\mu D_\mu \psi_L \\ & + \bar{\psi}_R i\gamma^\mu D_\mu \psi_R . \end{aligned} \quad (22)$$

This is the Yang–Mills Lagrangian for the gauge group $SU(2) \otimes U(1)$ with fermion matter fields. Here

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \quad \text{and} \quad F_{\mu\nu}^A = \partial_\mu W_\nu^A - \partial_\nu W_\mu^A - g\epsilon_{ABC} W_\mu^B W_\nu^C \quad (23)$$

are the gauge antisymmetric tensors constructed out of the gauge field B_μ associated with $U(1)$, and W_μ^A corresponding to the three $SU(2)$ generators; ϵ_{ABC} are the group structure constants [see Eqs. (9)] which, for $SU(2)$, coincide with the totally antisymmetric Levi-Civita tensor (recall the familiar angular momentum commutators). The normalization of the $SU(2)$ gauge coupling g is therefore specified by Eq. (23).

The fermion fields are described through their left-hand and right-hand components:

$$\psi_{L,R} = [(1 \mp \gamma_5)/2]\psi, \quad \bar{\psi}_{L,R} = \bar{\psi}[(1 \pm \gamma_5)/2], \quad (24)$$

with γ_5 and other Dirac matrices defined as in the book by Bjorken–Drell. In particular, $\gamma_5^2 = 1$, $\gamma_5^\dagger = \gamma_5$. Note that, as given in Eq. (24),

$$\bar{\psi}_L = \psi_L^\dagger \gamma_0 = \psi^\dagger [(1 - \gamma_5)/2] \gamma_0 = \bar{\psi} [\gamma_0 (1 - \gamma_5)/2] \gamma_0 = \bar{\psi} [(1 + \gamma_5)/2].$$

The matrices $P_\pm = (1 \pm \gamma_5)/2$ are projectors. They satisfy the relations $P_\pm P_\pm = P_\pm$, $P_\pm P_\mp = 0$, $P_+ + P_- = 1$.

The sixteen linearly independent Dirac matrices can be divided into γ_5 -even and γ_5 -odd according to whether they commute or anticommute with γ_5 . For the γ_5 -even, we have

$$\bar{\psi} \Gamma_E \psi = \bar{\psi}_L \Gamma_E \psi_R + \bar{\psi}_R \Gamma_E \psi_L \quad (\Gamma_E \equiv 1, i\gamma_5, \sigma_{\mu\nu}), \quad (25)$$

whilst for the γ_5 -odd,

$$\bar{\psi} \Gamma_O \psi = \bar{\psi}_L \Gamma_O \psi_L + \bar{\psi}_R \Gamma_O \psi_R \quad (\Gamma_O \equiv \gamma_\mu, \gamma_\mu \gamma_5). \quad (26)$$

In the Standard Model (SM) the left and right fermions have different transformation properties under the gauge group. Thus, mass terms for fermions (of

the form $\bar{\psi}_L \psi_R + \text{h.c.}$) are forbidden in the symmetric limit. In particular, all ψ_R are singlets in the Minimal Standard Model (MSM). But for the moment, by ψ_R we mean a column vector, including all fermions in the theory that span a generic reducible representation of $SU(2) \otimes U(1)$. The standard EW theory is a chiral theory, in the sense that ψ_L and ψ_R behave differently under the gauge group. In the absence of mass terms, there are only vector and axial vector interactions in the Lagrangian that have the property of not mixing ψ_L and ψ_R . Fermion masses will be introduced, together with W^\pm and Z masses, by the mechanism of symmetry breaking. The covariant derivatives $D_\mu \psi_{L,R}$ are explicitly given by

$$D_\mu \psi_{L,R} = \left[\partial_\mu + ig \sum_{A=1}^3 t_{L,R}^A W_\mu^A + ig' \frac{1}{2} Y_{L,R} B_\mu \right] \psi_{L,R} , \quad (27)$$

where $t_{L,R}^A$ and $1/2 Y_{L,R}$ are the $SU(2)$ and $U(1)$ generators, respectively, in the reducible representations $\psi_{L,R}$. The commutation relations of the $SU(2)$ generators are given by

$$[t_L^A, t_L^B] = i \epsilon_{ABC} t_L^C \quad \text{and} \quad [t_R^A, t_R^B] = i \epsilon_{ABC} t_R^C . \quad (28)$$

We use the normalization (8) [in the fundamental representation of $SU(2)$]. The electric charge generator Q (in units of e , the positron charge) is given by

$$Q = t_L^3 + 1/2 Y_L = t_R^3 + 1/2 Y_R . \quad (29)$$

Note that the normalization of the $U(1)$ gauge coupling g' in (27) is now specified as a consequence of (29).

All fermion couplings to the gauge bosons can be derived directly from Eqs. (22) and (27). The charged-current (CC) couplings are the simplest. From

$$\begin{aligned} g(t^1 W_\mu^1 + t^2 W_\mu^2) &= g \left\{ [(t^1 + it^2)/\sqrt{2}](W_\mu^1 - iW_\mu^2)/\sqrt{2} + \text{h.c.} \right\} \\ &= g \left\{ [(t^+ W_\mu^-)/\sqrt{2}] + \text{h.c.} \right\} , \end{aligned} \quad (30)$$

where $t^\pm = t^1 \pm it^2$ and $W^\pm = (W^1 \pm iW^2)/\sqrt{2}$, we obtain the vertex

$$V_{\bar{\psi}\psi W} = g\bar{\psi}\gamma_\mu \left[(t_L^+/\sqrt{2})(1 - \gamma_5)/2 + (t_R^+/\sqrt{2})(1 + \gamma_5)/2 \right] \psi W_\mu^- + \text{h.c.} \quad (31)$$

In the neutral-current (NC) sector, the photon A_μ and the mediator Z_μ of the weak NC are orthogonal and normalized linear combinations of B_μ and

W_μ^3 :

$$\begin{aligned} A_\mu &= \cos \theta_W B_\mu + \sin \theta_W W_\mu^3 , \\ Z_\mu &= -\sin \theta_W B_\mu + \cos \theta_W W_\mu^3 . \end{aligned} \quad (32)$$

Equations (32) define the weak mixing angle θ_W . The photon is characterized by equal couplings to left and right fermions with a strength equal to the electric charge. Recalling Eq. (29) for the charge matrix Q , we immediately obtain

$$g \sin \theta_W = g' \cos \theta_W = e , \quad (33)$$

or equivalently,

$$\text{tg } \theta_W = g'/g \quad (34)$$

Once θ_W has been fixed by the photon couplings, it is a simple matter of algebra to derive the Z couplings, with the result

$$\Gamma_{\bar{\psi}\psi Z} = g/(2 \cos \theta_W) \bar{\psi} \gamma_\mu [t_L^3(1 - \gamma_5) + t_R^3(1 + \gamma_5) - 2Q \sin^2 \theta_W] \psi Z^\mu , \quad (35)$$

where $\Gamma_{\bar{\psi}\psi Z}$ is a notation for the vertex. In the MSM, $t_R^3 = 0$ and $t_L^3 = \pm 1/2$.

In order to derive the effective four-fermion interactions that are equivalent, at low energies, to the CC and NC couplings given in Eqs. (31) and (35), we anticipate that large masses, as experimentally observed, are provided for W^\pm and Z by $\mathcal{L}_{\text{Higgs}}$. For left-left CC couplings, when the momentum transfer squared can be neglected with respect to m_W^2 in the propagator of Born diagrams with single W exchange, from Eq. (31) we can write

$$\mathcal{L}_{\text{eff}}^{\text{CC}} \simeq (g^2/8m_W^2) [\bar{\psi} \gamma_\mu (1 - \gamma_5) t_L^+ \psi] [\bar{\psi} \gamma^\mu (1 - \gamma_5) t_L^- \psi] . \quad (36)$$

By specializing further in the case of doublet fields such as $\nu_e - e^-$ or $\nu_\mu - \mu^-$, we obtain the tree-level relation of g with the Fermi coupling constant G_F measured from μ decay [see Eq. (2)]:

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8m_W^2} . \quad (37)$$

By recalling that $g \sin \theta_W = e$, we can also cast this relation in the form

$$m_W = \frac{\mu_{\text{Born}}}{\sin \theta_W} , \quad (38)$$

with

$$\mu_{\text{Born}} = \left(\frac{\pi \alpha}{\sqrt{2} G_F} \right)^{1/2} \simeq 37.2802 \text{ GeV} , \quad (39)$$

where α is the fine-structure constant of QED ($\alpha \equiv e^2/4\pi = 1/137.036$).

In the same way, for neutral currents we obtain in Born approximation from Eq. (35) the effective four-fermion interaction given by

$$\mathcal{L}_{\text{eff}}^{\text{NC}} \simeq \sqrt{2} G_F \rho_0 \bar{\psi} \gamma_\mu [\dots] \psi \bar{\psi} \gamma^\mu [\dots] \psi , \quad (40)$$

where

$$[\dots] \equiv t_L^3(1 - \gamma_5) + t_R^3(1 + \gamma_5) - 2Q \sin^2 \theta_W \quad (41)$$

and

$$\rho_0 = m_W^2/m_Z^2 \cos^2 \theta_W . \quad (42)$$

All couplings given in this section are obtained at tree level and are modified in higher orders of perturbation theory. In particular, the relations between m_W and $\sin \theta_W$ [Eqs. (38) and (39)] and the observed values of ρ ($\rho = \rho_0$ at tree level) in different NC processes, are altered by computable EW radiative corrections, as discussed in Section 6.

The gauge-boson self-interactions can be derived from the $F_{\mu\nu}$ term in $\mathcal{L}_{\text{symm}}$, by using Eq. (32) and $W^\pm = (W^1 \pm iW^2)/\sqrt{2}$. Defining the three-gauge-boson vertex as in Fig. 1, we obtain ($V \equiv \gamma, Z$)

$$\Gamma_{W^-W^+V} = ig_{W^-W^+V} [g_{\mu\nu}(q-p)_\lambda + g_{\mu\lambda}(p-r)_\nu + g_{\nu\lambda}(r-q)_\mu] , \quad (43)$$

with

$$g_{W^-W^+\gamma} = g \sin \theta_W = e \quad \text{and} \quad g_{W^-W^+Z} = g \cos \theta_W . \quad (44)$$

This form of the triple gauge vertex is very special: in general, there could be departures from the above SM expression, even restricting us to $SU(2) \otimes U(1)$ gauge symmetric and C and P invariant couplings. In fact some small corrections are already induced by the radiative corrections. But, in principle, more important could be the modifications induced by some new physics effect. The experimental testing of the triple gauge vertices is presently underway at LEP2 and limits on departures from the SM couplings have also been obtained at the Tevatron and elsewhere (see Section 12).

We now turn to the Higgs sector of the EW Lagrangian. Here we simply review the formalism of the Higgs mechanism applied to the EW theory. In the next section we shall make a more general and detailed discussion of the physics of the EW symmetry breaking. The Higgs Lagrangian is specified by the gauge principle and the requirement of renormalizability to be

$$\mathcal{L}_{\text{Higgs}} = (D_\mu \phi)^\dagger (D^\mu \phi) - V(\phi^\dagger \phi) - \bar{\psi}_L \Gamma \psi_R \phi - \bar{\psi}_R \Gamma^\dagger \psi_L \phi^\dagger , \quad (45)$$

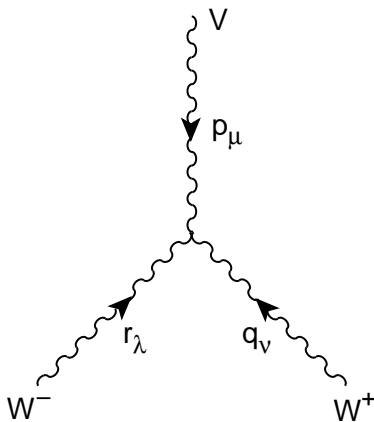


Figure 1: The three-gauge boson vertex: $V = \gamma, Z$

where ϕ is a column vector including all Higgs fields; it transforms as a reducible representation of the gauge group. The quantities Γ (which include all coupling constants) are matrices that make the Yukawa couplings invariant under the Lorentz and gauge groups. The potential $V(\phi^\dagger\phi)$, symmetric under $SU(2) \otimes U(1)$, contains, at most, quartic terms in ϕ so that the theory is renormalizable:

$$V(\phi^\dagger\phi) = -\frac{1}{2}\mu^2\phi^\dagger\phi + \frac{1}{4}\lambda(\phi^\dagger\phi)^2 \quad (46)$$

As discussed in the next section, spontaneous symmetry breaking is induced if the minimum of V which is the classical analogue of the quantum mechanical vacuum state (both are the states of minimum energy) is obtained for non-vanishing ϕ values. Precisely, we denote the vacuum expectation value (VEV) of ϕ , i.e. the position of the minimum, by v :

$$\langle 0|\phi(x)|0\rangle = v \neq 0 . \quad (47)$$

The fermion mass matrix is obtained from the Yukawa couplings by replacing $\phi(x)$ by v :

$$M = \bar{\psi}_L \mathcal{M}\psi_R + \bar{\psi}_R \mathcal{M}^\dagger\psi_L , \quad (48)$$

with

$$\mathcal{M} = \Gamma \cdot v . \quad (49)$$

In the MSM, where all left fermions ψ_L are doublets and all right fermions ψ_R are singlets, only Higgs doublets can contribute to fermion masses. There are enough free couplings in Γ , so that one single complex Higgs doublet is indeed sufficient to generate the most general fermion mass matrix. It is important to observe that by a suitable change of basis we can always make the matrix \mathcal{M} Hermitian, γ_5 -free, and diagonal. In fact, we can make separate unitary transformations on ψ_L and ψ_R according to

$$\psi'_L = U\psi_L, \quad \psi'_R = V\psi_R \quad (50)$$

and consequently

$$\mathcal{M} \rightarrow \mathcal{M}' = U^\dagger \mathcal{M} V . \quad (51)$$

This transformation does not alter the general structure of the fermion couplings in $\mathcal{L}_{\text{symm}}$.

If only one Higgs doublet is present, the change of basis that makes \mathcal{M} diagonal will at the same time diagonalize also the fermion–Higgs Yukawa couplings. Thus, in this case, no flavour-changing neutral Higgs exchanges are present. This is not true, in general, when there are several Higgs doublets. But one Higgs doublet for each electric charge sector i.e. one doublet coupled only to u -type quarks, one doublet to d -type quarks, one doublet to charged leptons would also be all right, because the mass matrices of fermions with different charges are diagonalized separately. For several Higgs doublets in a given charge sector it is also possible to generate CP violation by complex phases in the Higgs couplings. In the presence of six quark flavours, this CP-violation mechanism is not necessary. In fact, at the moment, the simplest model with only one Higgs doublet seems adequate for describing all observed phenomena.

We now consider the gauge-boson masses and their couplings to the Higgs. These effects are induced by the $(D_\mu\phi)^\dagger(D^\mu\phi)$ term in $\mathcal{L}_{\text{Higgs}}$ [Eq. (45)], where

$$D_\mu\phi = \left[\partial_\mu + ig \sum_{A=1}^3 t^A W_\mu^A + ig'(Y/2)B_\mu \right] \phi . \quad (52)$$

Here t^A and $1/2Y$ are the $SU(2) \otimes U(1)$ generators in the reducible representation spanned by ϕ . Not only doublets but all non-singlet Higgs representations can contribute to gauge-boson masses. The condition that the photon remains massless is equivalent to the condition that the vacuum is electrically neutral:

$$Q|v\rangle = (t^3 + \frac{1}{2}Y)|v\rangle = 0 . \quad (53)$$

The charged W mass is given by the quadratic terms in the W field arising from $\mathcal{L}_{\text{Higgs}}$, when $\phi(x)$ is replaced by v . We obtain

$$m_W^2 W_\mu^+ W^{-\mu} = g^2 |(t^+ v / \sqrt{2})|^2 W_\mu^+ W^{-\mu} , \quad (54)$$

whilst for the Z mass we get [recalling Eq. (32)]

$$\frac{1}{2} m_Z^2 Z_\mu Z^\mu = |[g \cos \theta_W t^3 - g' \sin \theta_W (Y/2)] v|^2 Z_\mu Z^\mu , \quad (55)$$

where the factor of $1/2$ on the left-hand side is the correct normalization for the definition of the mass of a neutral field. By using Eq. (53), relating the action of t^3 and $1/2Y$ on the vacuum v , and Eqs. (34), we obtain

$$\frac{1}{2} m_Z^2 = (g \cos \theta_W + g' \sin \theta_W)^2 |t^3 v|^2 = \left(\frac{g^2}{\cos^2 \theta_W} \right) |t^3 v|^2 . \quad (56)$$

For Higgs doublets

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \quad v = \begin{pmatrix} 0 \\ v \end{pmatrix} , \quad (57)$$

we have

$$|t^+ v|^2 = v^2, \quad |t^3 v|^2 = 1/4 v^2 , \quad (58)$$

so that

$$m_W^2 = \frac{1}{2} g^2 v^2, \quad m_Z^2 = \frac{1}{2} \frac{g^2 v^2}{\cos^2 \theta_W} . \quad (59)$$

Note that by using Eq. (37) we obtain

$$v = 2^{-3/4} G_F^{-1/2} = 174.1 \text{ GeV} . \quad (60)$$

It is also evident that for Higgs doublets

$$\rho_0 = m_W^2 / m_Z^2 \cos^2 \theta_W = 1 . \quad (61)$$

This relation is typical of one or more Higgs doublets and would be spoiled by the existence of Higgs triplets etc. In general,

$$\rho_0 = \frac{\sum_i ((t_i)^2 - (t_i^3)^2 + t_i) v_i^2}{\sum_i 2(t_i^3)^2 v_i^2} , \quad (62)$$

for several Higgses with VEVs v_i , weak isospin t_i , and z -component t_i^3 . These results are valid at the tree level and are modified by calculable EW radiative corrections, as discussed in Section 6.

In the minimal version of the SM only one Higgs doublet is present. Then the fermion–Higgs couplings are in proportion to the fermion masses. In fact, from the Yukawa couplings $g_{\phi\bar{f}f}(\bar{f}_L\phi f_R + h.c.)$, the mass m_f is obtained by replacing ϕ by v , so that $m_f = g_{\phi\bar{f}f}v$. In the minimal SM three out of the four Hermitian fields are removed from the physical spectrum by the Higgs mechanism and become the longitudinal modes of W^+ , W^- , and Z . The fourth neutral Higgs is physical and should be found. If more doublets are present, two more charged and two more neutral Higgs scalars should be around for each additional doublet.

The couplings of the physical Higgs H to the gauge bosons can be simply obtained from $\mathcal{L}_{\text{Higgs}}$, by the replacement

$$\phi(x) = \begin{pmatrix} \phi^+(x) \\ \phi^0(x) \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ v + (H/\sqrt{2}) \end{pmatrix}, \quad (63)$$

[so that $(D_\mu\phi)^\dagger(D^\mu\phi) = 1/2(\partial_\mu H)^2 + \dots$], with the result

$$\begin{aligned} \mathcal{L}[H, W, Z] = & g^2 \left(\frac{v}{\sqrt{2}} \right) W_\mu^+ W^{-\mu} H + \left(\frac{g^2}{4} \right) W_\mu^+ W^{-\mu} H^2 \\ & + \left[\frac{g^2 v Z_\mu Z^\mu}{2\sqrt{2} \cos^2 \theta_W} \right] H \\ & + \left[\frac{g^2}{8 \cos^2 \theta_W} \right] Z_\mu Z^\mu H^2. \end{aligned} \quad (64)$$

In the minimal SM the Higgs mass $m_H^2 \sim \lambda v^2$ is of order of the weak scale v . We will discuss in sect. 8 the direct experimental limit on m_H from LEP, which is $m_H \gtrsim m_Z$. We shall also see in sect.12, that, if there is no physics beyond the SM up to a large scale Λ , then, on theoretical grounds, m_H can only be within a narrow range between 135 and 180 GeV. But the interval is enlarged if there is new physics nearby. Also the lower limit depends critically on the assumption of only one doublet. The dominant decay mode of the Higgs is in the $b\bar{b}$ channel below the WW threshold, while the W^+W^- channel is dominant for sufficiently large m_H . The width is small below the WW threshold, not exceeding a few MeV, but increases steeply beyond the threshold, reaching the asymptotic value of $\Gamma \sim 1/2m_H^3$ at large m_H , where all energies are in TeV.

4 The Higgs Mechanism

The gauge symmetry of the Standard Model was difficult to discover because it is well hidden in nature. The only observed gauge boson that is massless is the photon. The gluons are presumed massless but are unobservable because of confinement, and the W and Z weak bosons carry a heavy mass. Actually a major difficulty in unifying weak and electromagnetic interactions was the fact that e.m. interactions have infinite range ($m_\gamma = 0$), whilst the weak forces have a very short range, owing to $m_{W,Z} \neq 0$.

The solution of this problem is in the concept of spontaneous symmetry breaking, which was borrowed from statistical mechanics.

Consider a ferromagnet at zero magnetic field in the Landau–Ginzburg approximation. The free energy in terms of the temperature T and the magnetization \mathbf{M} can be written as

$$F(\mathbf{M}, T) \simeq F_0(T) + 1/2 \mu^2(T) \mathbf{M}^2 + 1/4 \lambda(T) (\mathbf{M}^2)^2 + \dots . \quad (65)$$

This is an expansion which is valid at small magnetization. The neglect of terms of higher order in \mathbf{M}^2 is the analogue in this context of the renormalizability criterion. Also, $\lambda(T) > 0$ is assumed for stability; F is invariant under rotations, i.e. all directions of \mathbf{M} in space are equivalent. The minimum condition for F reads

$$\partial F / \partial \mathbf{M} = 0, \quad [\mu^2(T) + \lambda(T) \mathbf{M}^2] \mathbf{M} = 0 . \quad (66)$$

There are two cases. If $\mu^2 > 0$, then the only solution is $\mathbf{M} = 0$, there is no magnetization, and the rotation symmetry is respected. If $\mu^2 < 0$, then another solution appears, which is

$$|\mathbf{M}_0|^2 = -\mu^2 / \lambda . \quad (67)$$

The direction chosen by the vector \mathbf{M}_0 is a breaking of the rotation symmetry. The critical temperature T_{crit} is where $\mu^2(T)$ changes sign:

$$\mu^2(T_{\text{crit}}) = 0 . \quad (68)$$

It is simple to realize that the Goldstone theorem holds. It states that when spontaneous symmetry breaking takes place, there is always a zero-mass mode in the spectrum. In a classical context this can be proven as follows. Consider a Lagrangian

$$\mathcal{L} = |\partial_\mu \phi|^2 - V(\phi) \quad (69)$$

symmetric under the infinitesimal transformations

$$\phi \rightarrow \phi' = \phi + \delta\phi, \quad \delta\phi_i = i\delta\theta t_{ij}\phi_j . \quad (70)$$

The minimum condition on V that identifies the equilibrium position (or the ground state in quantum language) is

$$(\partial V/\partial\phi_i)(\phi_i = \phi_i^0) = 0 . \quad (71)$$

The symmetry of V implies that

$$\delta V = (\partial V/\partial\phi_i)\delta\phi_i = i\delta\theta(\partial V/\partial\phi_i)t_{ij}\phi_j = 0 . \quad (72)$$

By taking a second derivative at the minimum $\phi_i = \phi_i^0$ of the previous equation, we obtain

$$\partial^2 V/\partial\phi_k\partial\phi_i(\phi_i = \phi_i^0)t_{ij}\phi_j^0 + \frac{\partial V}{\partial\phi_i}(\phi_i = \phi_i^0)t_{ik} = 0 . \quad (73)$$

The second term vanishes owing to the minimum condition, Eq. (71). We then find

$$\partial^2 V/\partial\phi_k\partial\phi_i(\phi_i = \phi_i^0)t_{ij}\phi_j^0 = 0 . \quad (74)$$

The second derivatives $M_{ki}^2 = (\partial^2 V/\partial\phi_k\partial\phi_i)(\phi_i = \phi_i^0)$ define the squared mass matrix. Thus the above equation in matrix notation can be read as

$$M^2 t\phi^0 = 0 , \quad (75)$$

which shows that if the vector $(t\phi^0)$ is non-vanishing, i.e. there is some generator that shifts the ground state into some other state with the same energy, then $t\phi^0$ is an eigenstate of the squared mass matrix with zero eigenvalue. Therefore, a massless mode is associated with each broken generator.

When spontaneous symmetry breaking takes place in a gauge theory, the massless Goldstone mode exists, but it is unphysical and disappears from the spectrum. It becomes, in fact, the third helicity state of a gauge boson that takes mass. This is the Higgs mechanism. Consider, for example, the simplest Higgs model described by the Lagrangian

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^2 + |(\partial_\mu - ieA_\mu)\phi|^2 + \frac{1}{2}\mu^2\phi^*\phi - (\lambda/4)(\phi^*\phi)^2 . \quad (76)$$

Note the ‘wrong’ sign in front of the mass term for the scalar field ϕ , which is necessary for the spontaneous symmetry breaking to take place. The above Lagrangian is invariant under the $U(1)$ gauge symmetry

$$A_\mu \rightarrow A'_\mu = A_\mu - (1/e)\partial_\mu\theta(x), \quad \phi \rightarrow \phi' = \phi \exp[i\theta(x)] . \quad (77)$$

Let $\phi^0 = v \neq 0$, with v real, be the ground state that minimizes the potential and induces the spontaneous symmetry breaking. Making use of gauge invariance, we can make the change of variables

$$\begin{aligned}\phi(x) &\rightarrow (1/\sqrt{2})[\rho(x) + v] \exp[i\zeta(x)/v] , \\ A_\mu(x) &\rightarrow A_\mu - (1/ev)\partial_\mu\zeta(x).\end{aligned}\tag{78}$$

Then $\rho = 0$ is the position of the minimum, and the Lagrangian becomes

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}e^2v^2A_\mu^2 + \frac{1}{2}e^2\rho^2A_\mu^2 + e^2\rho vA_\mu^2 + \mathcal{L}(\rho) .\tag{79}$$

The field $\zeta(x)$, which corresponds to the would-be Goldstone boson, disappears, whilst the mass term $\frac{1}{2}e^2v^2A_\mu^2$ for A_μ is now present; ρ is the massive Higgs particle.

The Higgs mechanism is realized in well-known physical situations. For a superconductor in the Landau–Ginzburg approximation the free energy can be written as

$$F = F_0 + \frac{1}{2}\mathbf{B}^2 + |(\nabla - 2ie\mathbf{A})\phi|^2/4m - \alpha|\phi|^2 + \beta|\phi|^4 .\tag{80}$$

Here \mathbf{B} is the magnetic field, $|\phi|^2$ is the Cooper pair (e^-e^-) density, $2e$ and $2m$ are the charge and mass of the Cooper pair. The 'wrong' sign of α leads to $\phi \neq 0$ at the minimum. This is precisely the non-relativistic analogue of the Higgs model of the previous example. The Higgs mechanism implies the absence of propagation of massless phonons (states with dispersion relation $\omega = kv$ with constant v). Also the mass term for \mathbf{A} is manifested by the exponential decrease of \mathbf{B} inside the superconductor (Meissner effect).

5 The CKM Matrix

Weak charged currents are the only tree level interactions in the SM that change flavour: by emission of a W an up-type quark is turned into a down-type quark, or a ν_l neutrino is turned into a l^- charged lepton (all fermions are left-handed). If we start from an up quark that is a mass eigenstate, emission of a W turns it into a down-type quark state d' (the weak isospin partner of u) that in general is not a mass eigenstate. In general, the mass eigenstates and the weak eigenstates do not coincide and a unitary transformation connects

the two sets:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V \begin{pmatrix} d \\ s \\ b \end{pmatrix} \quad (81)$$

V is the Cabibbo-Kobayashi-Maskawa matrix. Thus in terms of mass eigenstates the charged weak current of quarks is of the form:

$$J_\mu^+ \propto \bar{u}\gamma_\mu(1 - \gamma_5)t^+Vd \quad (82)$$

Since V is unitary (i.e. $VV^\dagger = V^\dagger V = 1$) and commutes with T^2 , T_3 and Q (because all d-type quarks have the same isospin and charge) the neutral current couplings are diagonal both in the primed and unprimed basis (if the Z down-type quark current is abbreviated as $\bar{d}'\Gamma d'$ then by changing basis we get $\bar{d}V^\dagger\Gamma Vd$ and V and Γ commute because, as seen from eq.(41), Γ is made of Dirac matrices and T_3 and Q generator matrices). This is the GIM mechanism that ensures natural flavour conservation of the neutral current couplings at the tree level.

For N generations of quarks, V is a $N \times N$ unitary matrix that depends on N^2 real numbers (N^2 complex entries with N^2 unitarity constraints). However, the $2N$ phases of up- and down-type quarks are not observable. Note that an overall phase drops away from the expression of the current in eq.(82), so that only $2N - 1$ phases can affect V . In total, V depends on $N^2 - 2N + 1 = (N - 1)^2$ real physical parameters. A similar counting gives $N(N - 1)/2$ as the number of independent parameters in an orthogonal $N \times N$ matrix. This implies that in V we have $N(N - 1)/2$ mixing angles and $(N - 1)^2 - N(N - 1)/2$ phases: for $N = 2$ one mixing angle (the Cabibbo angle) and no phase, for $N = 3$ three angles and one phase etc.

Given the experimental near diagonal structure of V a convenient parametrisation is the one proposed by Maiani. One starts from the definition:

$$|d'\rangle = c_{13}|d_C\rangle + s_{13}e^{-i\phi}|b\rangle \quad (83)$$

where $c_{13} \equiv \cos\theta_{13}$, $s_{13} \equiv \sin\theta_{13}$ (analogous shorthand notations will be used in the following), d_C is the Cabibbo down quark and $\theta_{12} \equiv \theta_C$ is the Cabibbo angle (experimentally $s_{12} \equiv \lambda \sim 0.22$).

$$|d_C\rangle = c_{12}|d\rangle + s_{12}|s\rangle \quad (84)$$

Note that in a four quark model the Cabibbo angle fixes both the ratio of couplings ($u \rightarrow d$)/($\nu_e \rightarrow e$) and the ratio of ($u \rightarrow d$)/($u \rightarrow s$). In a six quark

model one has to choose which to keep as a definition of the Cabibbo angle. Here the second definition is taken and, in fact the $u \rightarrow d$ coupling is given by $V_{ud} = c_{13}c_{12}$ so that it is no longer specified by θ_{12} only. Also note that we can certainly fix the phases of u, d, s so that a real coefficient appears in front of d_C in eq.(83). We now choose a basis of two orthonormal vectors, both orthogonal to $|d'\rangle$:

$$|s_C\rangle = -s_{12}|d\rangle + c_{12}|s\rangle, \quad |v\rangle = -s_{13}e^{i\phi}|d_C\rangle + c_{13}|b\rangle \quad (85)$$

Here $|s_C\rangle$ is the Cabibbo s quark. Clearly s' and b' must be orthonormal superpositions of the above base vectors defined in terms of an angle θ_{23} :

$$|s'\rangle = c_{23}|s_C\rangle + s_{23}|v\rangle, \quad |b'\rangle = -s_{23}|s_C\rangle + c_{23}|v\rangle \quad (86)$$

The general expression of V_{ij} can be obtained from the above equations. But a considerable notational simplification is gained if one takes into account that from experiment we know that $s_{12} \equiv \lambda$, $s_{23} \sim o(\lambda^2)$ and $s_{13} \sim o(\lambda^3)$ are increasingly small and of the indicated orders of magnitude. Thus, following Wolfenstein one can set:

$$s_{12} \equiv \lambda, \quad s_{23} = A\lambda^2, \quad s_{13}e^{-i\phi} = A\lambda^3(\rho - i\eta) \quad (87)$$

As a result, by neglecting terms of higher order in λ one can write down:

$$V = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \sim \begin{bmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{bmatrix}. \quad (88)$$

Indicative values of the CKM parameters as obtained from experiment are (a survey of the current status of the CKM parameters can be found in ref.¹):

$$\begin{aligned} \lambda &= 0.2196 \pm 0.0023 \\ A &= 0.82 \pm 0.04 \\ \sqrt{\rho^2 + \eta^2} &= 0.4 \pm 0.1; \quad \eta \sim 0.3 \pm 0.2 \end{aligned} \quad (89)$$

In the SM the non vanishing of the η parameter is the only source of CP violation. The most direct and solid evidence for η non vanishing is obtained from the measurement of ϵ in K decay. Unitarity of the CKM matrix V implies relations of the form $\sum_a V_{ba}V_{ca}^* = \delta_{bc}$. In most cases these relations do not imply particularly instructive constraints on the Wolfenstein parameters. But when the three terms in the sum are of comparable magnitude we get interesting information. The three numbers which must add to zero form a closed

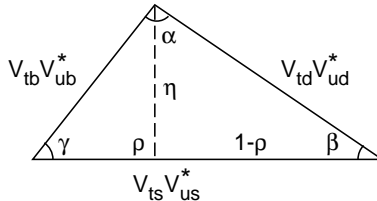


Figure 2: The Bjorken triangle corresponding to eq.(90)

triangle in the complex plane, with sides of comparable length. This is the case for the t-u triangle (Bjorken triangle) shown in fig.2:

$$V_{td}V_{ud}^* + V_{ts}V_{us}^* + V_{tb}V_{ub}^* = 0 \quad (90)$$

All terms are of order λ^3 . For $\eta=0$ the triangle would flatten down to vanishing area. In fact the area of the triangle, J of order $J \sim \eta A^2 \lambda^6$, is the Jarlskog invariant (its value is independent of the parametrization). In the SM all CP violating observables must be proportional to J .

We have only discussed flavour mixing for quarks. But, clearly, if neutrino masses exist, as indicated by neutrino oscillations, then a similar mixing matrix must also be introduced in the leptonic sector (see section 10.2).

6 Renormalisation and Higher Order Corrections

The Higgs mechanism gives masses to the Z , the W^\pm and to fermions while the Lagrangian density is still symmetric. In particular the gauge Ward identities and the conservation of the gauge currents are preserved. The validity of these relations is an essential ingredient for renormalisability. For example the massive gauge boson propagator would have a bad ultraviolet behaviour:

$$W_{\mu\nu} = \frac{-g_{\mu\nu} + \frac{q_\mu q_\nu}{m_W^2}}{q^2 - m_W^2} \quad (91)$$

But if the propagator is sandwiched between conserved currents J_μ the bad terms in $q_\mu q_\nu$ give a vanishing contribution because $q_\mu J^\mu = 0$ and the high energy behaviour is like for a scalar particle and compatible with renormalisation.

The fundamental theorem that in general a gauge theory with spontaneous symmetry breaking and the Higgs mechanism is renormalisable was proven by

't Hooft. For a chiral theory like the SM an additional complication arises from the existence of chiral anomalies. But this problem is avoided in the SM because the quantum numbers of the quarks and leptons in each generation imply a remarkable (and apparently miraculous) cancellation of the anomaly, as originally observed by Bouchiat, Iliopoulos and Meyer. In quantum field theory one encounters an anomaly when a symmetry of the classical lagrangian is broken by the process of quantisation, regularisation and renormalisation of the theory. For example, in massless QCD there is no mass scale in the classical lagrangian. Thus one would predict that dimensionless quantities in processes with only one large energy scale Q cannot depend on Q and must be constants. As well known this naive statement is false. The process of regularisation and renormalisation necessarily introduces an energy scale which is essentially the scale where renormalised quantities are defined. For example the renormalised coupling must be defined from the vertices at some scale. This scale μ cannot be zero because of infrared divergences. The scale μ destroys scale invariance because dimensionless quantities can now depend on Q/μ . The famous Λ_{QCD} parameter is a tradeoff of μ and leads to scale invariance breaking. Of direct relevance for the EW theory is the Adler-Bell-Jackiw chiral anomaly. The classical lagrangian of a theory with massless fermions is invariant under a $U(1)$ chiral transformations $\psi' = e^{i\gamma_5\theta}\psi$. The associated axial Noether current is conserved at the classical level. But, at the quantum level, chiral symmetry is broken due to the ABJ anomaly and the current is not conserved. The chiral breaking is introduced by a clash between chiral symmetry, gauge invariance and the regularisation procedure. The anomaly is generated by triangular fermion loops with one axial and two vector vertices (fig.3). For neutral currents (Z and γ) the axial coupling is proportional to the 3rd component of weak isospin t_3 , while vector couplings are proportional to a linear combination of t_3 and the electric charge Q . Thus in order for the chiral anomaly to vanish all traces of the form $tr\{t_3QQ\}$, $tr\{t_3t_3Q\}$, $tr\{t_3t_3t_3\}$ (and also $tr\{t_+t_-t_3\}$ etc., when charged currents are included) must vanish, where the trace is extended over all fermions in the theory that can circulate in the loop. Now all these traces happen to vanish for each fermion family separately. For example take $tr\{t_3QQ\}$. In one family there are, with $t_3 = +1/2$, three colours of up quarks with charge $Q = +2/3$ and one neutrino with $Q = 0$ and, with $t_3 = -1/2$, three colours of down quarks with charge $Q = -1/3$ and one l^- with $Q = -1$. Thus we obtain $tr\{t_3QQ\} = 1/2 \cdot 3 \cdot 4/9 - 1/2 \cdot 3 \cdot 1/9 - 1/2 \cdot 1 = 0$. This impressive cancellation suggests an interplay among weak isospin, charge and colour quantum numbers which appears as a miracle from the point of view of the low energy theory but is more understandable from the point of view of the high energy theory. For example in GUTs there are similar relations where charge

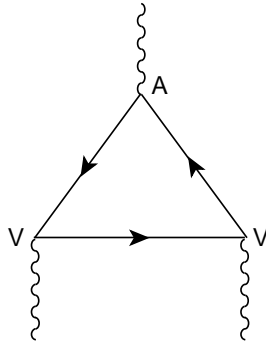


Figure 3: Triangle diagram that generates the ABJ anomaly

quantisation and colour are related: in the 5 of SU(5) we have the content $(d, d, d, e^+, \bar{\nu})$ and the charge generator has a vanishing trace in each SU(5) representation (the condition of unit determinant, represented by the letter S in the SU(5) group name, translates into zero trace for the generators). Thus the charge of d quarks is $-1/3$ of the positron charge because there are three colours.

Since the SM theory is renormalisable higher order perturbative corrections can be reliably computed. Radiative corrections are very important for precision EW tests. The SM inherits all successes of the old V-A theory of charged currents and of QED. Modern tests focus on neutral current processes, the W mass and the measurement of triple gauge vertices. For Z physics and the W mass the state of the art computation of radiative corrections include the complete one loop diagrams and selected dominant two loop corrections. In addition some resummation techniques are also implemented, like Dyson resummation of vacuum polarisation functions and important renormalisation group improvements for large QED and QCD logarithms. We now discuss in more detail sets of large radiative corrections which are particularly significant².

A set of important quantitative contributions to the radiative corrections arise from large logarithms [e.g. terms of the form $(\alpha/\pi \ln(m_Z/m_{f_\ell}))^n$ where f_ℓ is a light fermion]. The sequences of leading and close-to-leading logarithms are fixed by well-known and consolidated techniques (β functions, anomalous dimensions, penguin-like diagrams, etc.). For example, large logarithms dominate the running of α from m_e , the electron mass, up to m_Z . Similarly large

logarithms of the form $[\alpha/\pi \ln (m_Z/\mu)]^n$ also enter, for example, in the relation between $\sin^2 \theta_W$ at the scales m_Z (LEP, SLC) and μ (e.g. the scale of low-energy neutral-current experiments). Also, large logs from initial state radiation dramatically distort the line shape of the Z resonance as observed at LEP and SLC and must be accurately taken into account in the measure of the Z mass and total width.

For example, a considerable amount of work has deservedly been devoted to the theoretical study of the Z line-shape. The present experimental accuracy on m_Z obtained at LEP is $\delta m_Z = \pm 2.1$ MeV (see table 1, sect.7). This small error was obtained by a precise calibration of the LEP energy scale achieved by taking advantage of the transverse polarization of the beams and implementing a sophisticated resonant spin depolarization method. Similarly, a measurement of the total width to an accuracy $\delta \Gamma = \pm 2.4$ MeV has by now been achieved. The prediction of the Z line-shape in the SM to such an accuracy has posed a formidable challenge to theory, which has been successfully met. For the inclusive process $e^+e^- \rightarrow f\bar{f}X$, with $f \neq e$ (for simplicity, we leave Bhabha scattering aside) and X including γ 's and gluons, the physical cross-section can be written in the form of a convolution²:

$$\sigma(s) = \int_{z_0}^1 dz \hat{\sigma}(zs)G(z, s), \quad (92)$$

where $\hat{\sigma}$ is the reduced cross-section, and $G(z, s)$ is the radiator function that describes the effect of initial-state radiation; $\hat{\sigma}$ includes the purely weak corrections, the effect of final-state radiation (of both γ 's and gluons), and also non-factorizable terms (initial- and final-state radiation interferences, boxes, etc.) which, being small, can be treated in lowest order and effectively absorbed in a modified $\hat{\sigma}$. The radiator $G(z, s)$ has an expansion of the form

$$G(z, s) = \delta(1-z) + \alpha/\pi(a_{11}L + a_{10}) + (\alpha/\pi)^2(a_{22}L^2 + a_{11}L + a_{20}) \\ + \dots + (\alpha/\pi)^n \sum_{i=0}^n a_{ni}L^i, \quad (93)$$

where $L = \ln s/m_e^2 \simeq 24.2$ for $\sqrt{s} \simeq m_Z$. All first- and second-order terms are known exactly. The sequence of leading and next-to-leading logs can be exponentiated (closely following the formalism of structure functions in QCD). For $m_Z \approx 91$ GeV, the convolution displaces the peak by +110 MeV, and reduces it by a factor of about 0.74. The exponentiation is important in that it amounts to a shift of about 14 MeV in the peak position.

A very remarkable class of contributions among the one loop EW radiative corrections are those terms that increase quadratically with the top mass. The sensitivity of radiative corrections to m_t arises from the existence of these terms. The quadratic dependence on m_t (and on other possible widely broken isospin multiplets from new physics) arises because, in spontaneously broken gauge theories, heavy loops do not decouple. On the contrary, in QED or QCD, the running of α and α_s at a scale Q is not affected by heavy quarks with mass $M \gg Q$. According to an intuitive decoupling theorem³, diagrams with heavy virtual particles of mass M can be ignored at $Q \ll M$ provided that the couplings do not grow with M and that the theory with no heavy particles is still renormalizable. In the spontaneously broken EW gauge theories both requirements are violated. First, one important difference with respect to unbroken gauge theories is in the longitudinal modes of weak gauge bosons. These modes are generated by the Higgs mechanism, and their couplings grow with masses (as is also the case for the physical Higgs couplings). Second the theory without the top quark is no more renormalisable because the gauge symmetry is broken since the doublet (t,b) would not be complete (also the chiral anomaly would not be completely cancelled). With the observed value of m_t the quantitative importance of the terms of order $G_F m_t^2 / 4\pi^2 \sqrt{2}$ is substantial but not dominant (they are enhanced by a factor $m_t^2 / m_W^2 \sim 5$ with respect to ordinary terms). Both the large logarithms and the $G_F m_t^2$ terms have a simple structure and are to a large extent universal, i.e. common to a wide class of processes. In particular the $G_F m_t^2$ terms appear in vacuum polarisation diagrams which are universal and in the $Z \rightarrow b\bar{b}$ vertex which is not (this vertex is connected with the top quark which runs in the loop, while other types of heavy particles could in principle also contribute to vacuum polarisation diagrams). Their study is important for an understanding of the pattern of radiative corrections. One can also derive approximate formulae (e.g. improved Born approximations), which can be useful in cases where a limited precision may be adequate. More in general, another very important consequence of non decoupling is that precision tests of the electroweak theory may be sensitive to new physics even if the new particles are too heavy for their direct production.

While radiative corrections are quite sensitive to the top mass, they are unfortunately much less dependent on the Higgs mass. If they were sufficiently sensitive by now we would precisely know the mass of the Higgs. But the dependence of one loop diagrams on m_H is only logarithmic: $\sim G_F m_W^2 \log(m_H^2 / m_W^2)$. Quadratic terms $\sim G_F^2 m_H^2$ only appear at two loops and are too small to be important. The difference with the top case is that the

difference $m_t^2 - m_b^2$ is a direct breaking of the gauge symmetry that already affects the one loop corrections, while the Higgs couplings are "custodial" SU(2) symmetric in lowest order.

The basic tree level relations:

$$\frac{g^2}{8m_W^2} = \frac{G_F}{\sqrt{2}}, \quad g^2 \sin^2 \theta_W = e^2 = 4\pi\alpha \quad (94)$$

can be combined into

$$\sin^2 \theta_W = \frac{\pi\alpha}{\sqrt{2}G_F m_W^2} \quad (95)$$

A different definition of $\sin^2 \theta_W$ is from the gauge boson masses:

$$\frac{m_W^2}{m_Z^2 \cos^2 \theta_W} = \rho_0 = 1 \quad \implies \quad \sin^2 \theta_W = 1 - \frac{m_W^2}{m_Z^2} \quad (96)$$

where $\rho_0 = 1$ assuming that there are only Higgs doublets. The last two relations can be put into the convenient form

$$\left(1 - \frac{m_W^2}{m_Z^2}\right) \frac{m_W^2}{m_Z^2} = \frac{\pi\alpha}{\sqrt{2}G_F m_Z^2} \quad (97)$$

These relations are modified by radiative corrections:

$$\begin{aligned} \left(1 - \frac{m_W^2}{m_Z^2}\right) \frac{m_W^2}{m_Z^2} &= \frac{\pi\alpha(m_Z)}{\sqrt{2}G_F m_Z^2} \frac{1}{1 - \Delta r_W} \\ \frac{m_W^2}{m_Z^2 \cos^2 \theta_W} &= 1 + \rho_m \end{aligned} \quad (98)$$

In the first relation the replacement of α with the running coupling at the Z mass $\alpha(m_Z)$ makes Δr_W completely determined by the purely weak corrections. This relation defines Δr_W unambiguously, once the meaning of $\alpha(m_Z)$ is specified. On the contrary, in the second relation $\Delta\rho_m$ depends on the definition of $\sin^2 \theta_W$ beyond the tree level. For LEP physics $\sin^2 \theta_W$ is usually defined from the $Z \rightarrow \mu^+ \mu^-$ effective vertex. At the tree level we have:

$$Z \rightarrow f^+ f^- = \frac{g}{2 \cos \theta_W} \bar{f} \gamma_\mu (g_V^f - g_A^f \gamma_5) f \quad (99)$$

with $g_A^{f2} = 1/4$ and $g_V^f/g_A^f = 1 - 4|Q_f| \sin^2 \theta_W$. Beyond the tree level a corrected vertex can be written down in the same form of eq.(99) in terms of

modified effective couplings. Then $\sin^2 \theta_W \equiv \sin^2 \theta_{eff}$ is in general defined through the muon vertex:

$$\begin{aligned} g_V^\mu/g_A^\mu &= 1 - 4 \sin^2 \theta_{eff} \\ \sin^2 \theta_{eff} &= (1 + \Delta k) s_0^2, & s_0^2 c_0^2 &= \frac{\pi \alpha(m_Z)}{\sqrt{2} G_F m_Z^2} \\ g_A^{\mu 2} &= \frac{1}{4} (1 + \Delta \rho) \end{aligned} \tag{100}$$

Actually, since in the SM lepton universality is only broken by masses and is in agreement with experiment within the present accuracy, in practice the muon channel is replaced with the average over charged leptons.

We end this discussion by writing a symbolic equation that summarises the status of what has been computed up to now for the radiative corrections Δr_W , $\Delta \rho$ and Δk :

$$\Delta r_W, \Delta \rho, \Delta k = g^2 \frac{m_t^2}{m_W^2} (1 + \alpha_s + \alpha_s^2) + g^2 (1 + \alpha_s + \sim \alpha_s^2) + g^4 \frac{m_t^4}{m_W^4} + g^4 \frac{m_t^2}{m_W^2} + \dots \tag{101}$$

The meaning of this relation is that the one loop terms of order g^2 are completely known, together with their first order QCD corrections (the second order QCD corrections are only estimated for the g^2 terms not enhanced by m_t^2/m_W^2), and the terms of order g^4 enhanced by the ratios m_t^4/m_W^4 or m_t^2/m_W^2 are also known.

In recent years new powerful tests of the SM have been performed mainly at LEP but also at SLC and at the Tevatron. The running of LEP1 was terminated in 1995 and close-to-final results of the data analysis are now available. The SLC is still running. The experiments at the Z resonance have enormously improved the accuracy in the electroweak neutral current sector. The top quark has been at last found at the Tevatron and the errors on m_Z and $\sin^2 \theta_{eff}$ went down by two and one orders of magnitude respectively since the start of LEP in 1989. The LEP2 programme is in progress. The validity of the SM has been confirmed to a level that we can say was unexpected at the beginning. In the present data there is no significant evidence for departures from the SM, no convincing hint of new physics (also including the first results from LEP2). The impressive success of the SM poses strong limitations on the possible forms of new physics. Favoured are models of the Higgs sector and of new physics that preserve the SM structure and only very delicately improve it, as is the case for fundamental Higgs(es) and Supersymmetry. Disfavoured are

models with a nearby strong non perturbative regime that almost inevitably would affect the radiative corrections, as for composite Higgs(es) or technicolor and its variants.

7 Status of the Data

The relevant electro-weak data together with their SM values are presented in table 1^{4,5,6}. The SM predictions correspond to a fit of all the available data (including the directly measured values of m_t and m_W) in terms of m_t , m_H and $\alpha_s(m_Z)$, described later in sect.8, table 4.

Other important derived quantities are, for example, N_ν the number of light neutrinos, obtained from the invisible width: $N_\nu = 2.994(11)$, which shows that only three fermion generations exist with $m_\nu < 45 \text{ GeV}$. This is one of the most important results of LEP. Other important quantities are the leptonic width Γ_l , averaged over e, μ and τ : $\Gamma_l = 83.90(10) \text{ MeV}$ and the hadronic width $\Gamma_h = 1742.3(2.3) \text{ MeV}$.

For indicative purposes, in table the "pulls" are also shown, defined as: pull = (data point- fit value)/(error on data point). At a glance we see that the agreement with the SM is quite good. The distribution of the pulls is statistically normal. The presence of a few $\sim 2\sigma$ deviations is what is to be expected. However it is maybe worthwhile to give a closer look at these small discrepancies.

One persistent feature of the data is the difference between the values of $\sin^2\theta_{eff}$ measured at LEP and at SLC (although the discrepancy is going down in the most recent data). The value of $\sin^2\theta_{eff}$ is obtained from a set of combined asymmetries. From asymmetries one derives the ratio $x = g_V^l/g_A^l$ of the vector and axial vector couplings of the Z, averaged over the charged leptons. In turn $\sin^2\theta_{eff}$ is defined by $x = 1 - 4\sin^2\theta_{eff}$. SLD obtains x from the single measurement of A_{LR} , the left-right asymmetry, which requires longitudinally polarized beams. The distribution of the present measurements of $\sin^2\theta_{eff}$ is shown in fig.4. The LEP average, $\sin^2\theta_{eff} = 0.23187(24)$, differs by 2.2σ from the SLD value $\sin^2\theta_{eff} = 0.23101(31)$. The most precise individual measurement at LEP is from A_b^{FB} : the combined LEP error on this quantity is comparable to the SLD error, but the two values are 2.5σ 's away. One might attribute this to the fact that the b measurement is more delicate and affected by a complicated systematics. In fact one notices from fig.4 that the value obtained at LEP from A_l^{FB} , the average for l=e, μ and τ ,

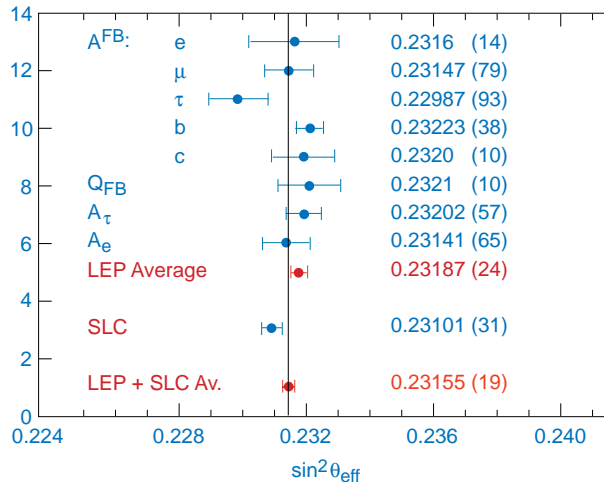


Figure 4: A summary of $\sin^2 \theta_{eff}$ measurements)

is somewhat low (indeed quite in agreement with the SLD value). However the statement that LEP and SLD agree on leptons while they only disagree when the b quark is considered is not quite right. First, the low value of $\sin^2 \theta_{eff}$ found at LEP from A_l^{FB} turns out to be entirely due to the τ lepton channel which leads to a central value different than that of e and μ . The e and μ asymmetries, which are experimentally simpler, are perfectly on top of the SM fit. Second, if we take only e and μ asymmetries at LEP and disregard the b and τ measurements the LEP average becomes $\sin^2 \theta_{eff} = 0.23168(36)$, which is still 1.4σ away from the SLD value. Thus it is difficult to find a simple explanation for the SLD-LEP discrepancy on $\sin^2 \theta_{eff}$. In the following we will tentatively use the official average

$$\sin^2 \theta_{eff} = 0.23155 \pm 0.00019 \quad (102)$$

obtained by a simple combination of the LEP-SLC data. One could be more conservative and enlarge the error because of the larger dispersion, but the difference would not be too large. Also, this dispersion has decreased in the most recent data. The data-taking by the SLD experiment is still in progress and also at LEP sizeable improvements on A_τ and A_b^{FB} are foreseen as soon as the corresponding analyses will be completed. We hope to see the difference to be further reduced in the end.

Table 1: Data on precision electroweak tests

Quantity	Data (August'98)	Pull
m_Z (GeV)	91.1867(21)	0.1
Γ_Z (GeV)	2.4939(24)	-0.8
σ_h (nb)	41.491(58)	0.3
R_h	20.765(26)	0.7
R_b	0.21656(74)	0.9
R_c	0.1735(44)	0.3
A_{FB}^l	0.01683(96)	0.7
A_τ	0.1431(45)	-0.8
A_e	0.1479(51)	0.25
A_{FB}^b	0.0990(21)	-1.8
A_{FB}^c	0.0709(44)	-0.6
A_b (SLD direct)	0.867(35)	-1.9
A_c (SLD direct)	0.647(40)	-0.5
$\sin^2 \theta_{eff}$ (LEP-combined)	0.23187(24)	1.3
$A_{LR} \rightarrow \sin^2 \theta_{eff}$	0.23101(31)	-1.8
m_W (GeV) (LEP2+p \bar{p})	80.39(60)	-0.4
$1 - \frac{m_W^2}{m_Z^2}$ (νN)	0.2253(21)	1.1
Q_W (Atomic PV in Cs)	-72.11(93)	1.2
m_t (GeV)	173.8(5.0)	0.5

From the above discussion one may wonder if there is evidence for something special in the τ channel, or equivalently if lepton universality is really supported by the data. Indeed this is the case: the hint of a difference in A_τ^{FB} with respect to the corresponding e and μ asymmetries is not confirmed by the measurements of A_τ and Γ_τ which appear normal. In principle the fact that an anomaly shows up in A_τ^{FB} and not in A_τ and Γ_τ is not unconceivable because the FB lepton asymmetries are very small and very precisely measured. For example, the extraction of A_τ^{FB} from the data on the angular distribution of τ 's could be biased if the imaginary part of the continuum was altered by some non universal new physics effect. But a more trivial experimental problem is at the moment the most plausible option.

A similar question can be asked for the b couplings. We have seen that the measured value of A_b^{FB} is 1.8σ 's below the SM fit. At the same time R_b which used to show a major discrepancy is now only about 1σ 's away from the SM fit (as a result of the more sophisticated second generation experimental techniques). There is a -2.5σ deviation on the measured value of A_b vs the SM expectation. That somewhat depends on how the data are combined. Let us discuss this point in detail. A_b can be measured directly at SLC by taking

advantage of the beam longitudinal polarization. The SLC value (see Table 1) is 2.2σ 's below the SM value. At LEP one measures $A_b^{FB} = 3/4 A_e A_b$. One can then derive A_b by inserting a value for A_e . The question is what to use for A_e : the LEP value obtained, using lepton universality, from the measurements of A_l^{FB} , A_τ , A_e : $A_e = 0.1470(27)$, or the combination of LEP and SLD etc. Since we are here concerned with the b couplings it is perhaps wiser to obtain A_b from LEP by using the SM value for A_e (that is the pull-zero value of table 1): $A_e^{SM} = 0.1467(15)$. With the value of A_b derived in this way from LEP (which is 1.7σ 's below the SM value) we finally obtain

$$A_b = 0.890 \pm 0.018 \quad (\text{LEP} + \text{SLD}, A_e = A_e^{\text{SM}} : -2.5\sigma) \quad (103)$$

In the SM A_b is so close to 1 because the b quark is almost purely left-handed. A_b only depends on the ratio $r = (g_R/g_L)^2$ which in the SM is small: $r \sim 0.033$. To adequately decrease A_b from its SM value one must increase r by a factor of about 1.6, which appears large for a new physics effect. Also such a large change in r must be compensated by decreasing g_L^2 by a small but fine-tuned amount in order to counterbalance the corresponding large positive shift in R_b . In view of this the most likely way out is that A_b^{FB} and A_b have been a bit underestimated at LEP and actually there is no anomaly in the b couplings. Then the LEP value of $\sin^2 \theta_{eff}$ would slightly move down, in the direction of decreasing the SLD-LEP discrepancy.

8 Precision Electroweak Data and the Standard Model

For the analysis of electroweak data in the SM one starts from the input parameters: some of them, α , G_F and m_Z , are very well measured, some other ones, m_{light} , m_t and $\alpha_s(m_Z)$ are only approximately determined while m_H is largely unknown. With respect to m_t the situation has much improved since the CDF/D0 direct measurement of the top quark mass. From the input parameters one computes the radiative corrections to a sufficient precision to match the experimental capabilities. Then one compares the theoretical predictions and the data for the numerous observables which have been measured, checks the consistency of the theory and derives constraints on m_t , $\alpha_s(m_Z)$ and hopefully also on m_H .

Some comments on the least known of the input parameters are now in order. The only practically relevant terms where precise values of the light quark masses, m_{light} , are needed are those related to the hadronic contribution to the photon vacuum polarisation diagrams, that determine $\alpha(m_Z)$. This

correction is of order 6%, much larger than the accuracy of a few per mille of the precision tests. Fortunately, one can use the actual data to in principle solve the related ambiguity. But we shall see that the left over uncertainty is still one of the main sources of theoretical error. As is well known², the QED running coupling is given by:

$$\alpha(s) = \frac{\alpha}{1 - \Delta\alpha(s)} \quad (104)$$

$$\Delta\alpha(s) = \Pi(s) = \Pi_\gamma(0) - \text{Re}\Pi_\gamma(s) \quad (105)$$

where $\Pi(s)$ is proportional to the sum of all 1-particle irreducible vacuum polarization diagrams. In perturbation theory $\Delta\alpha(s)$ is given by:

$$\Delta\alpha(s) = \frac{\alpha}{3\pi} \sum_f Q_f^2 N_{Cf} \left(\log \frac{2}{m_f^2} - \frac{5}{3} \right) \quad (106)$$

where $N_{Cf} = 3$ for quarks and 1 for leptons. However, the perturbative formula is only reliable for leptons, not for quarks (because of the unknown values of the effective quark masses). Separating the leptonic, the light quark and the top quark contributions to $\Delta\alpha(s)$ we have:

$$\Delta\alpha(s) = \Delta\alpha(s)_\ell + \Delta\alpha(s)_h + \Delta\alpha(s)_t \quad (107)$$

with:

$$\Delta\alpha(s)_\ell = 0.0331421 ; \quad \Delta\alpha(s)_t = \frac{\alpha}{3\pi} \frac{4}{15} \frac{m_Z^2}{m_t^2} = -0.000061 \quad (108)$$

Note that in QED there is decoupling so that the top quark contribution approaches zero in the large m_t limit. For $\Delta\alpha(s)_h$ one can use eq.(105) and the Cauchy theorem to obtain the representation:

$$\Delta\alpha(m_Z^2)_h = -\frac{\alpha m_Z^2}{3\pi} \text{Re} \int_{4m_\pi^2}^{\infty} \frac{ds}{s} \frac{R(s)}{s - m_Z^2 - i\epsilon} \quad (109)$$

where $R(s)$ is the familiar ratio of the hadronic to the pointlike $\ell^+\ell^-$ cross-section from photon exchange in e^+e^- annihilation. At s large one can use the perturbative expansion for $R(s)$ while at small s one can use the actual data. In recent years there has been a lot of activity on this subject and a number of independent new estimates of $\alpha(m_Z)$ have appeared in the literature⁷. A consensus has been established and the value used at present is

$$\alpha(m_Z)^{-1} = 128.90 \pm 0.09 \quad (110)$$

Table 2: Measurements of $\alpha_s(m_Z)$. In parenthesis we indicate if the dominant source of errors is theoretical or experimental. For theoretical ambiguities our personal figure of merit is given.

Measurements	$\alpha_s(m_Z)$
R_τ	0.122 ± 0.006 (Th)
Deep Inelastic Scattering	0.116 ± 0.005 (Th)
Y_{decay}	0.112 ± 0.010 (Th)
Lattice QCD	0.117 ± 0.007 (Th)
$Re^+e^-(\sqrt{s} < 62 \text{ GeV})$	0.124 ± 0.021 (Exp)
Fragmentation functions in e^+e^-	0.124 ± 0.012 (Th)
Jets in e^+e^- at and below the Z	0.121 ± 0.008 (Th)
Z line shape (Assuming SM)	0.120 ± 0.004 (Exp)

As I said, for the derivation of this result the QCD theoretical prediction is actually used for large values of s where the data do not exist. But the sensitivity of the dispersive integral to this region is strongly suppressed, so that no important model dependence is introduced. More recently some analyses have appeared where one studied by how much the error on $\alpha_s(m_Z)$ is reduced by using the QCD prediction down to $\sqrt{s} = m_\tau$, with the possible exception of the regions around the charm and beauty thresholds⁸. These attempts were motivated by the apparent success of QCD predictions in τ decays, despite the low τ mass (note however that the relevant currents are V-A in τ decay but V in the present case). One finds that the central value is not much changed while the error in eq.(110) is reduced from 0.09 down to something like 0.03-0.04, but, of course, at the price of more model dependence. For this reason, in the following, we shall use the more conservative value in eq.(110).

As for the strong coupling $\alpha_s(m_Z)$ the world average central value is by now quite stable. The error is going down because the dispersion among the different measurements is much smaller in the most recent set of data. The most important determinations of $\alpha_s(m_Z)$ are summarised in table 2⁹. For all entries, the main sources of error are the theoretical ambiguities which are larger than the experimental errors. The only exception is the measurement from the electroweak precision tests, but only if one assumes that the SM electroweak sector is correct. My personal views on the theoretical errors are reflected in the table 2. The error on the final average is taken by all authors between ± 0.003 and ± 0.005 depending on how conservative one is. Thus in the following our reference value will be

$$\alpha_s(m_Z) = 0.119 \pm 0.004 \tag{111}$$

Finally a few words on the current status of the direct measurement of m_t . The present combined CDF/D0 result is

$$m_t = 173.8 \pm 5.0 \text{ GeV} \quad (112)$$

The error is so small by now that one is approaching a level where a more careful investigation of the effects of colour rearrangement on the determination of m_t is needed. One wants to determine the top quark mass, defined as the invariant mass of its decay products (i.e. $b+W+$ gluons + γ 's). However, due to the need of colour rearrangement, the top quark and its decay products cannot be really isolated from the rest of the event. Some smearing of the mass distribution is induced by this colour crosstalk which involves the decay products of the top, those of the antitop and also the fragments of the incoming (anti)protons. A reliable quantitative computation of the smearing effect on the m_t determination is difficult because of the importance of non perturbative effects. An induced error of the order of 1 GeV on m_t could reasonably be expected. Thus further progress on the m_t determination demands tackling this problem in more depth.

In order to appreciate the relative importance of the different sources of theoretical errors for precision tests of the SM, we report in table 3 a comparison for the most relevant observables. What is important to stress is that the ambiguity from m_t , once by far the largest one, is by now smaller than the error from m_H . We also see from table 3 that the error from $\Delta\alpha(m_Z)$ is especially important for $\sin^2\theta_{eff}$ and, to a lesser extent, is also sizeable for Γ_Z and ϵ_3 .

The most important recent advance in the theory of radiative corrections is the calculation of the $o(g^4 m_t^2/m_W^2)$ terms in $\sin^2\theta_{eff}$, m_W and, more recently in $\delta\rho$ ¹⁰. The result implies a small but visible correction to the predicted values but especially a sizeable decrease of the ambiguity from scheme dependence (a typical effect of truncation). These calculations are now implemented in the fitting codes used in the analysis of LEP data. The fitted value of the Higgs mass is lowered by about 30 GeV due to this effect.

We now discuss fitting the data in the SM. As the mass of the top quark is now rather precisely known from CDF and D0 one must distinguish two different types of fits. In one type one wants to answer the question: is m_t from radiative corrections in agreement with the direct measurement at the Tevatron? Similarly how does m_W inferred from radiative corrections compare with the direct measurements at the Tevatron and LEP2? For answering these interesting but somewhat limited questions, one must clearly exclude the

Table 3: Errors from different sources: Δ_{now}^{exp} is the present experimental error; $\Delta\alpha^{-1}$ is the impact of $\Delta\alpha^{-1} = \pm 0.09$; Δ_{th} is the estimated theoretical error from higher orders; Δm_t is from $\Delta m_t = \pm 6\text{GeV}$; Δm_H is from $\Delta m_H = 60\text{--}1000\text{ GeV}$; $\Delta\alpha_s$ corresponds to $\Delta\alpha_s = \pm 0.003$. The epsilon parameters are defined in sect.9.1.

Parameter	Δ_{now}^{exp}	$\Delta\alpha^{-1}$	Δ_{th}	Δm_t	Δm_H	$\Delta\alpha_s$
Γ_Z (MeV)	± 2.4	± 0.7	± 0.8	± 1.4	± 4.6	± 1.7
σ_h (pb)	58	1	4.3	3.3	4	17
$R_h \cdot 10^3$	26	4.3	5	2	13.5	20
Γ_l (keV)	100	11	15	55	120	3.5
$A_{FB}^l \cdot 10^4$	9.6	4.2	1.3	3.3	13	0.18
$\sin^2\theta \cdot 10^4$	19	2.3	0.8	1.9	7.5	0.1
m_W (MeV)	60	12	9	37	100	2.2
$R_b \cdot 10^4$	7.4	0.1	1	2.1	0.25	0
$\epsilon_1 \cdot 10^3$	1.2		~ 0.1			0.2
$\epsilon_3 \cdot 10^3$	1.2	0.5	~ 0.1			0.12
$\epsilon_b \cdot 10^3$	2.1		~ 0.1			1

Table 4: Standard Model fits of electroweak data.

Parameter	LEP(incl. m_W)	All but m_W, m_t	All Data
m_t (GeV)	160+13 - 10	158+9 - 8	171.3 \pm 4.9
m_H (GeV)	66+142 - 38	34+45 - 16	84+91 - 51
$\log[m_H(\text{GeV})]$	1.82+0.50 - 0.37	1.53+0.37 - 0.28	1.92+0.32 - 0.41
$\alpha_s(m_Z)$	0.121 \pm 0.003	0.120 \pm 0.003	0.119 \pm 0.003
χ^2/dof	4.2/9	14/12	16.4/15

measurements of m_t and m_W from the input set of data. Fitting all other data in terms of m_t , m_H and $\alpha_s(m_Z)$ one finds the results shown in the second column of table 4⁵. The extracted value of m_t is typically a bit too low. For example, as shown in the table 4, from all the electroweak data except the direct production results on m_t and m_W , one finds $m_t = 158 \pm 9\text{GeV}$. There is a strong correlation between m_t and m_H . $\sin^2\theta_{eff}$ and m_W drive the fit to small values of m_H . Then, at small m_H the widths, in particular the leptonic width (whose prediction is nearly independent of α_s) drive the fit to small m_t . In a more general type of fit, e.g. for determining the overall consistency of the SM or the best present estimate for some quantity, say m_W , one should of course not ignore the existing direct determinations of m_t and m_W . Then, from all the available data, by fitting m_t , m_H and $\alpha_s(m_Z)$ one finds the values shown in the last column of table 4.

This is the fit also referred to in table 1. The corresponding fitted values

of $\sin^2 \theta_{eff}$ and m_W are:

$$\sin^2 \theta_{eff} = 0.23156 \pm 0.00019; \quad m_W = 80.370 \pm 0.027 \text{ GeV} \quad (113)$$

The fitted value of $\sin^2 \theta_{eff}$ is practically identical to the LEP+SLD average. The error of 27 MeV on m_W clearly sets up a goal for the direct measurement of m_W at LEP2 and the Tevatron.

As a final comment we want to recall that the radiative corrections are functions of $\log(m_H)$. It is truly remarkable that the fitted value of $\log(m_H)$ is found to fall right into the very narrow allowed window around the value 2 specified by the lower limit from direct searches, $m_H > \sim 90 \text{ GeV}$, and the theoretical upper limit in the SM $m_H < 600 - 800 \text{ GeV}$ (see later). Note that if the Higgs is removed from the theory, $\log m_H \rightarrow \log \Lambda + \text{constant}$, where Λ is a cutoff or the scale of the new physics that replaces the Higgs. The control of the finite terms is lost. Thus the fact that from experiment, one finds $\log m_H \sim 2$ is a strong argument in favour of the precise form of the Higgs mechanism as in the SM. The fulfilment of this very stringent consistency check is a beautiful argument in favour of a fundamental Higgs (or one with a compositeness scale much above the weak scale).

9 A More General Analysis of Electroweak Data

We now discuss an update of the epsilon analysis⁶ which is a method to look at the data in a more general context than the SM. The starting point is to isolate from the data that part which is due to the purely weak radiative corrections. In fact the epsilon variables are defined in such a way that they are zero in the approximation when only effects from the SM Λ at the tree level plus pure QED and pure QCD corrections are taken into account. This very simple version of improved Born approximation is a good first approximation according to the data and is independent of m_t and m_H . In fact the whole m_t and m_H dependence arises from weak loop corrections and therefore is only contained in the epsilon variables. Thus the epsilons are extracted from the data without need of specifying m_t and m_H . But their predicted value in the SM or in any extension of it depend on m_t and m_H . This is to be compared with the competitor method based on the S, T, U variables. The latter cannot be obtained from the data without specifying m_t and m_H because they are defined as deviations from the complete SM prediction for specified m_t and m_H . Of course there are very many variables that vanish if pure weak loop corrections are neglected, at least one for each relevant observable. Thus for

a useful definition we choose a set of representative observables that are used to parametrize those hot spots of the radiative corrections where new physics effects are most likely to show up. These sensitive weak correction terms include vacuum polarization diagrams which being potentially quadratically divergent are likely to contain all possible non decoupling effects (like the quadratic top quark mass dependence in the SM). There are three independent vacuum polarization contributions. In the same spirit, one must add the $Z \rightarrow b\bar{b}$ vertex which also includes a large top mass dependence. Thus altogether we consider four defining observables: one asymmetry, for example A_{FB}^l , (as representative of the set of measurements that lead to the determination of $\sin^2 \theta_{eff}$), one width (the leptonic width Γ_l is particularly suitable because it is practically independent of α_s), m_W and R_b . Here lepton universality has been taken for granted, because the data show that it is verified within the present accuracy. The four variables, ϵ_1 , ϵ_2 , ϵ_3 and ϵ_b are defined in one to one correspondence with the set of observables A_{FB}^l , Γ_l , m_W , and R_b . The definition is so chosen that the quadratic top mass dependence is only present in ϵ_1 and ϵ_b , while the m_t dependence of ϵ_2 and ϵ_3 is logarithmic. The definition of ϵ_1 and ϵ_3 is specified in terms of A_{FB}^l and Γ_l only. Then adding m_W or R_b one obtains ϵ_2 or ϵ_b . We now specify the relevant definitions in detail.

9.1 Basic Definitions and Results

We start from the basic observables m_W/m_Z , Γ_l and A_{FB}^l and Γ_b . From these four quantities one can isolate the corresponding dynamically significant corrections Δr_W , $\Delta\rho$, Δk and ϵ_b , which contain the small effects one is trying to disentangle and are defined in the following. First we introduce Δr_W as obtained from m_W/m_Z by the relation:

$$\left(1 - \frac{m_W^2}{m_Z^2}\right) \frac{m_W^2}{m_Z^2} = \frac{\pi\alpha(m_Z)}{\sqrt{2}G_F m_Z^2 (1 - \Delta r_W)} \quad (114)$$

Here $\alpha(m_Z) = \alpha/(1 - \Delta\alpha)$ is fixed to the central value $1/128.90$ so that the effect of the running of α due to known physics is extracted from $1 - \Delta r = (1 - \Delta\alpha)(1 - \Delta r_W)$. In fact, the error on $1/\alpha(m_Z)$, as given in eq.(110) would then affect Δr_W . In order to define $\Delta\rho$ and Δk we consider the effective vector and axial-vector couplings g_V and g_A of the on-shell Z to charged leptons, given by the formulae:

$$\Gamma_l = \frac{G_F m_Z^3}{6\pi\sqrt{2}} (g_V^2 + g_A^2) \left(1 + \frac{3\alpha}{4\pi}\right),$$

$$A_l^{FB}(\sqrt{s} = m_Z) = \frac{3g_V^2 g_A^2}{(g_V^2 + g_A^2)^2} = \frac{3x^2}{(1+x^2)^2}. \quad (115)$$

Note that Γ_l stands for the inclusive partial width $\Gamma(Z \rightarrow l\bar{l} + \text{photons})$. We stress the following points. First, we have extracted from $(g_V^2 + g_A^2)$ the factor $(1 + 3\alpha/4\pi)$ which is induced in Γ_l from final state radiation. Second, by the asymmetry at the peak in eq.(115) we mean the quantity which is commonly referred to by the LEP experiments (denoted as A_{FB}^0 in ref.⁵), which is corrected for all QED effects, including initial and final state radiation and also for the effect of the imaginary part of the γ vacuum polarization diagram. In terms of g_A and $x = g_V/g_A$, the quantities $\Delta\rho$ and Δk are given by:

$$g_A = -\frac{\sqrt{\rho}}{2} \sim -\frac{1}{2}\left(1 + \frac{\Delta\rho}{2}\right),$$

$$x = \frac{g_V}{g_A} = 1 - 4\sin^2\theta_{eff} = 1 - 4(1 + \Delta k)s_0^2. \quad (116)$$

Here s_0^2 is $\sin^2\theta_{eff}$ before non pure-QED corrections, given by:

$$s_0^2 c_0^2 = \frac{\pi\alpha(m_Z)}{\sqrt{2}G_F m_Z^2} \quad (117)$$

with $c_0^2 = 1 - s_0^2$ ($s_0^2 = 0.231095$ for $m_Z = 91.188$ GeV).

We now define ϵ_b from Γ_b , the inclusive partial width for $Z \rightarrow b\bar{b}$ according to the relation

$$\Gamma_b = \frac{G_F m_Z^3}{6\pi\sqrt{2}} \beta \left(\frac{3 - \beta^2}{2} g_{bV}^2 + \beta^2 g_{bA}^2 \right) N_C R_{QCD} \left(1 + \frac{\alpha}{12\pi} \right) \quad (118)$$

where $N_C = 3$ is the number of colours, $\beta = \sqrt{1 - 4m_b^2/m_Z^2}$, with $m_b = 4.7$ GeV, R_{QCD} is the QCD correction factor given by

$$R_{QCD} = 1 + 1.2a - 1.1a^2 - 13a^3; \quad a = \frac{\alpha_s(m_Z)}{\pi} \quad (119)$$

and g_{bV} and g_{bA} are specified as follows

$$g_{bA} = -\frac{1}{2}\left(1 + \frac{\Delta\rho}{2}\right)(1 + \epsilon_b),$$

$$\frac{g_{bV}}{g_{bA}} = \frac{1 - 4/3\sin^2\theta_{eff} + \epsilon_b}{1 + \epsilon_b}. \quad (120)$$

This is clearly not the most general deviation from the SM in the $Z \rightarrow b\bar{b}$ but ϵ_b is closely related to the quantity $-Re(\delta_{b\text{-vertex}})$ where the large m_t corrections are located in the SM.

As is well known, in the SM the quantities Δr_W , $\Delta\rho$, Δk and ϵ_b , for sufficiently large m_t , are all dominated by quadratic terms in m_t of order $G_F m_t^2$. As new physics can more easily be disentangled if not masked by large conventional m_t effects, it is convenient to keep $\Delta\rho$ and ϵ_b while trading Δr_W and Δk for two quantities with no contributions of order $G_F m_t^2$. We thus introduce the following linear combinations:

$$\begin{aligned}\epsilon_1 &= \Delta\rho, \\ \epsilon_2 &= c_0^2 \Delta\rho + \frac{s_0^2 \Delta r_W}{c_0^2 - s_0^2} - 2s_0^2 \Delta k, \\ \epsilon_3 &= c_0^2 \Delta\rho + (c_0^2 - s_0^2) \Delta k.\end{aligned}\tag{121}$$

The quantities ϵ_2 and ϵ_3 no longer contain terms of order $G_F m_t^2$ but only logarithmic terms in m_t . The leading terms for large Higgs mass, which are logarithmic, are contained in ϵ_1 and ϵ_3 . In the Standard Model one has the following "large" asymptotic contributions:

$$\begin{aligned}\epsilon_1 &= \frac{3G_F m_t^2}{8\pi^2 \sqrt{2}} - \frac{3G_F m_W^2}{4\pi^2 \sqrt{2}} \tan^2 \theta_W \ln \frac{m_H}{m_Z} + \dots, \\ \epsilon_2 &= -\frac{G_F m_W^2}{2\pi^2 \sqrt{2}} \ln \frac{m_t}{m_Z} + \dots, \\ \epsilon_3 &= \frac{G_F m_W^2}{12\pi^2 \sqrt{2}} \ln \frac{m_H}{m_Z} - \frac{G_F m_W^2}{6\pi^2 \sqrt{2}} \ln \frac{m_t}{m_Z} \dots, \\ \epsilon_b &= -\frac{G_F m_t^2}{4\pi^2 \sqrt{2}} + \dots\end{aligned}\tag{122}$$

The relations between the basic observables and the epsilons can be linearised, leading to the approximate formulae

$$\begin{aligned}\frac{m_W^2}{m_Z^2} &= \frac{m_W^2}{m_Z^2}|_B (1 + 1.43\epsilon_1 - 1.00\epsilon_2 - 0.86\epsilon_3), \\ \Gamma_l &= \Gamma_l|_B (1 + 1.20\epsilon_1 - 0.26\epsilon_3), \\ A_l^{FB} &= A_l^{FB}|_B (1 + 34.72\epsilon_1 - 45.15\epsilon_3), \\ \Gamma_b &= \Gamma_b|_B (1 + 1.42\epsilon_1 - 0.54\epsilon_3 + 2.29\epsilon_b).\end{aligned}\tag{123}$$

Table 5: Values of the epsilons in the SM as functions of m_t and m_H as obtained from recent versions of ZFITTER and TOPAZ0. These values (in 10^{-3} units) are obtained for $\alpha_s(m_Z) = 0.119$, $\alpha(m_Z) = 1/128.90$, but the theoretical predictions are essentially independent of $\alpha_s(m_Z)$ and $\alpha(m_Z)$.

m_t (GeV)	ϵ_1 (GeV) =			ϵ_2 (GeV) =			ϵ_3 (GeV) =			ϵ_b All m_H
	m_H 70	m_H 300	m_H 1000	m_H 70	m_H 300	m_H 1000	m_H 70	m_H 300	m_H 1000	
150	3.55	2.86	1.72	-6.85	-6.46	-5.95	4.98	6.22	6.81	-4.50
160	4.37	3.66	2.50	-7.12	-6.72	-6.20	4.96	6.18	6.75	-5.31
170	5.26	4.52	3.32	-7.43	-7.01	-6.49	4.94	6.14	6.69	-6.17
180	6.19	5.42	4.18	-7.77	-7.35	-6.82	4.91	6.09	6.61	-7.08
190	7.18	6.35	5.09	-8.15	-7.75	-7.20	4.89	6.03	6.52	-8.03
200	8.22	7.34	6.04	-8.59	-8.18	-7.63	4.87	5.97	6.43	-9.01

The Born approximations, as defined above, depend on $\alpha_s(m_Z)$ and also on $\alpha(m_Z)$. Defining

$$\delta\alpha_s = \frac{\alpha_s(m_Z) - 0.119}{\pi}; \quad \delta\alpha = \frac{\alpha(m_Z) - \frac{1}{128.90}}{\alpha}, \quad (124)$$

we have

$$\begin{aligned} \frac{m_W^2}{m_Z^2}|_B &= 0.768905(1 - 0.40\delta\alpha), \\ \Gamma_l|_B &= 83.563(1 - 0.19\delta\alpha)\text{MeV}, \\ A_l^{FB}|_B &= 0.01696(1 - 34\delta\alpha), \\ \Gamma_b|_B &= 379.8(1 + 1.0\delta\alpha_s - 0.42\delta\alpha). \end{aligned} \quad (125)$$

Note that the dependence on $\delta\alpha_s$ for $\Gamma_b|_B$, shown in eq.(125), is not simply the one loop result for $m_b = 0$ but a combined effective shift which takes into account both finite mass effects and the contribution of the known higher order terms.

The important property of the epsilons is that, in the Standard Model, for all observables at the Z pole, the whole dependence on m_t (and m_H) arising from one-loop diagrams only enters through the epsilons. The same is actually true, at the relevant level of precision, for all higher order m_t -dependent corrections. Actually, the only residual m_t dependence of the various observables not included in the epsilons is in the terms of order $\alpha_s^2(m_Z)$ in the pure QCD correction factors to the hadronic widths. But this one is quantitatively irrelevant, especially in view of the errors connected to the uncertainty on the

value of $\alpha_s(m_Z)$. The theoretical values of the epsilons in the SM from state of the art radiative corrections, also including the recent development of ref.¹⁰, are given in table 5. It is important to remark that the theoretical values of the epsilons in the SM, as given in table 5, are not affected, at the percent level or so, by reasonable variations of $\alpha_s(m_Z)$ and/or $\alpha(m_Z)$ around their central values. By our definitions, in fact, no terms of order $\alpha_s^n(m_Z)$ or $\alpha \ln m_Z/m$ contribute to the epsilons. In terms of the epsilons, the following expressions hold, within the SM, for the various precision observables

$$\begin{aligned}
\Gamma_T &= \Gamma_{T0}(1 + 1.35\epsilon_1 - 0.46\epsilon_3 + 0.35\epsilon_b), \\
R &= R_0(1 + 0.28\epsilon_1 - 0.36\epsilon_3 + 0.50\epsilon_b), \\
\sigma_h &= \sigma_{h0}(1 - 0.03\epsilon_1 + 0.04\epsilon_3 - 0.20\epsilon_b), \\
x &= x_0(1 + 17.6\epsilon_1 - 22.9\epsilon_3), \\
R_b &= R_{b0}(1 - 0.06\epsilon_1 + 0.07\epsilon_3 + 1.79\epsilon_b).
\end{aligned} \tag{126}$$

where $x=g_V/g_A$ as obtained from A_l^{FB} . The quantities in eqs.(123–126) are clearly not independent and the redundant information is reported for convenience. By comparison with the computed radiative corrections we obtain

$$\begin{aligned}
\Gamma_{T0} &= 2489.46(1 + 0.73\delta\alpha_s - 0.35\delta\alpha) \text{ MeV}, \\
R_0 &= 20.8228(1 + 1.05\delta\alpha_s - 0.28\delta\alpha), \\
\sigma_{h0} &= 41.420(1 - 0.41\delta\alpha_s + 0.03\delta\alpha) \text{ nb}, \\
x_0 &= 0.075619 - 1.32\delta\alpha, \\
R_{b0} &= 0.2182355.
\end{aligned} \tag{127}$$

Note that the quantities in eqs.(127) should not be confused, at least in principle, with the corresponding Born approximations, due to small "non universal" electroweak corrections. In practice, at the relevant level of approximation, the difference between the two corresponding quantities is in any case significantly smaller than the present experimental error.

In principle, any four observables could have been picked up as defining variables. In practice we choose those that have a more clear physical significance and are more effective in the determination of the epsilons. In fact, since Γ_b is actually measured by R_b (which is nearly insensitive to α_s), it is preferable to use directly R_b itself as defining variable, as we shall do hereafter. In practice, since the value in eq.(127) is practically indistinguishable from the Born approximation of R_b , this determines no change in any of the equations given above but simply requires the corresponding replacement among the defining relations of the epsilons.

Table 6: Experimental values of the epsilons in the SM from different sets of data. These values (in 10^{-3} units) are obtained for $\alpha_s(m_Z) = 0.119 \pm 0.003$, $\alpha(m_Z) = 1/128.90 \pm 0.09$, the corresponding uncertainties being included in the quoted errors.

ϵ	10^3	Only def. quantities	All asymmetries	All High Energy	All Data
ϵ_1	10^3	4.1 ± 1.2	4.3 ± 1.2	4.0 ± 1.1	3.7 ± 1.1
ϵ_2	10^3	-8.2 ± 2.1	-8.8 ± 1.9	-9.0 ± 2.0	-9.3 ± 2.0
ϵ_3	10^3	3.3 ± 1.9	4.4 ± 1.2	4.2 ± 1.1	3.9 ± 1.1
ϵ_b	10^3	-4.3 ± 1.9	-4.4 ± 1.9	-4.8 ± 1.9	-4.6 ± 1.9

9.2 Experimental Determination of the Epsilon Variables

The values of the epsilons as obtained, following the specifications in the previous sect.9.1, from the defining variables m_W , Γ_l , A_l^{FB} and R_b are shown in the first column of table 6.

To proceed further and include other measured observables in the analysis we need to make some dynamical assumptions. The minimum amount of model dependence is introduced by including other purely leptonic quantities at the Z pole such as A_τ , A_e (measured from the angular dependence of the τ polarization) and A_{LR} (measured by SLD). For this step, one is simply assuming that the different leptonic asymmetries are equivalent measurements of $\sin^2 \theta_{eff}$. We add, as usual, the measure of A_b^{FB} because this observable is dominantly sensitive to the leptonic vertex. We then use the combined value of $\sin^2 \theta_{eff}$ obtained from the whole set of asymmetries measured at LEP and SLC given in eq.(102). At this stage the best values of the epsilons are shown in the second column of table 6. In figs. 5-8 we report the 1σ ellipses in the indicated ϵ_i - ϵ_j planes that correspond to this set of input data.

All observables measured on the Z peak at LEP can be included in the analysis provided that we assume that all deviations from the SM are only contained in vacuum polarization diagrams (without demanding a truncation of the q^2 dependence of the corresponding functions) and/or the $Z \rightarrow b\bar{b}$ vertex. From a global fit of the data on m_W , Γ_T , R_h , σ_h , R_b and $\sin^2 \theta_{eff}$ (for LEP data, we have taken the correlation matrix for Γ_T , R_h and σ_h given by the LEP experiments⁵, while we have considered the additional information on R_b and $\sin^2 \theta_{eff}$ as independent) we obtain the values shown in the third column of table 6. The comparison of theory and experiment at this stage is also shown in figs. 5-8. More detailed information is shown in fig. 9, which refers to the level when also hadronic data are taken into account. But in fig.9 we compare

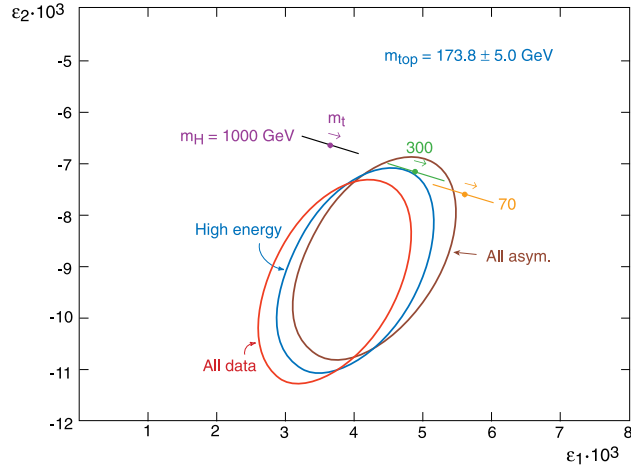


Figure 5: Data vs theory in the ϵ_2 - ϵ_1 plane. The origin point corresponds to the "Born" approximation obtained from the SM at tree level plus pure QED and pure QCD corrections. The predictions of the full SM (also including the improvements of ref.¹⁰) are shown for $m_H = 70, 300$ and 1000 GeV and $m_t = 175.6 \pm 5.5$ GeV (a segment for each m_H with the arrow showing the direction of m_t increasing from -1σ to $+1\sigma$). The three $1 - \sigma$ probability contours (38% probability) are obtained from a) "All Asymm.": Γ_l, m_W and $\sin^2 \theta_{eff}$ as obtained from the combined asymmetries (the value in eq. (102)); b) "All High En.": the same as in a) plus all the hadronic variables at the Z; c) "All Data": the same as in b) plus the low energy data.

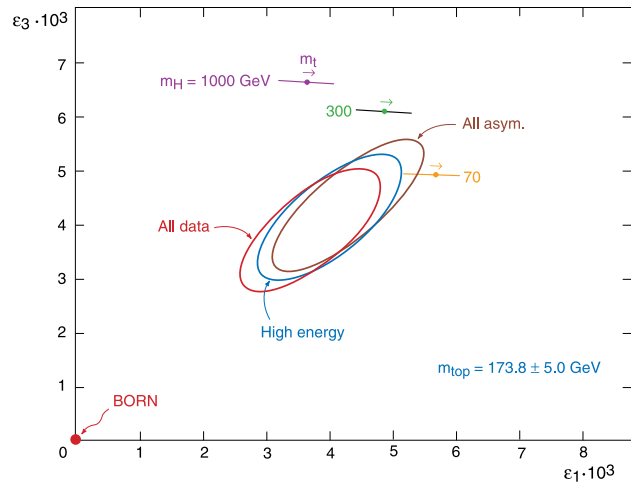


Figure 6: Data vs theory in the ϵ_3 - ϵ_1 plane (notations as in fig.5)

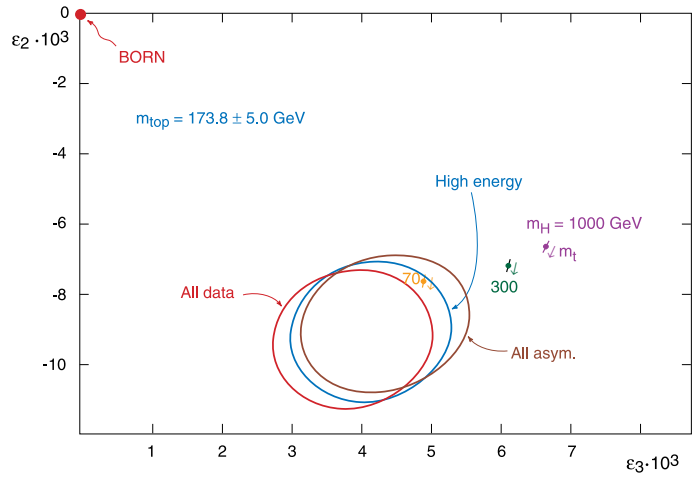


Figure 7: Data vs theory in the ϵ_2 - ϵ_3 plane (notations as in fig.5)

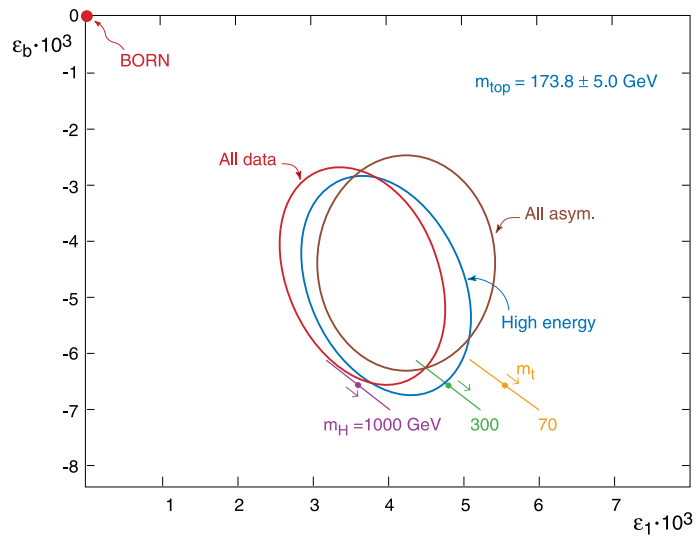


Figure 8: Data vs theory in the ϵ_b - ϵ_1 plane (notations as in fig.5)

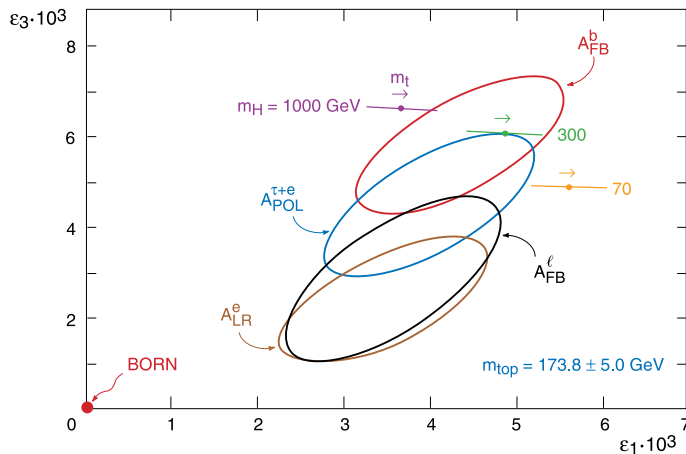


Figure 9: Data vs theory in the ϵ_3 - ϵ_1 plane (notations as in fig.5). The ellipse indicated with "Average" corresponds to the case "All high en" of fig.6 and is obtained from the combined value of $\sin^2\theta_{eff}$. The other ellipses are obtained by replacing the combined $\sin^2\theta_{eff}$ with the values obtained in turn from each individual asymmetry as shown by the labels.

the results obtained if $\sin^2\theta_{eff}$ is extracted in turn from different asymmetries among those listed in fig.4. The ellipse marked "average" is the same as the one labelled "All high en." in fig.6 and corresponds to the value of $\sin^2\theta_{eff}$ which is shown on the figure (and in eq.(102)). We confirm that the value from A_{LR} is far away from the SM given the experimental value of m_t and the bounds on m_H and would correspond to very small values of ϵ_3 and of ϵ_1 . We see also that while the τ FB asymmetry is also on the low side, the combined e and μ FB asymmetry are right on top of the average. Finally the b FB asymmetry is on the high side.

To include in our analysis lower energy observables as well, a stronger hypothesis needs to be made: vacuum polarization diagrams are allowed to vary from the SM only in their constant and first derivative terms in a q^2 expansion. In such a case, one can, for example, add to the analysis the ratio R_ν of neutral to charged current processes in deep inelastic neutrino scattering on nucle^{11,12} the "weak charge" Q_W measured in atomic parity violation experiments on Cs¹³ and the measurement of g_V/g_A from $\nu_\mu e$ scattering¹⁴. In this way one obtains the global fit given in the fourth column of table 6 and shown in figs. 5-8. In fig. 10 we see the ellipse in the ϵ_1 - ϵ_3 plane that is obtained from the low energy data by themselves. It is interesting that the tendency

towards low values of ϵ_1 and ϵ_3 is present in the low energy data as in the high energy ones. Note that the low energy data by themselves are actually compatible with the "Born" approximation. With the progress of LEP the low energy data, while important as a check that no deviations from the expected q^2 dependence arise, play a lesser role in the global fit. This does not mean that they are not important. For example, the measured parity violation in atomic physics provides the best limits on possible new physics in the electron-quark sector. When HERA suggested the presence of leptoquarks, the limits from atomic parity violation practically excluded all possible parity violating four fermion electron-quark contact terms. So low energy data are no more powerful enough to improve the determination of the parameters if the SM is assumed, but they are a very powerful constraint on new physics models. The best values of the ϵ 's from all the data are at present:

$$\begin{aligned}
\epsilon_1 10^3 &= 3.7 \pm 1.1 \\
\epsilon_2 10^3 &= -9.3 \pm 2.0 \\
\epsilon_3 10^3 &= 3.9 \pm 1.1 \\
\epsilon_b 10^3 &= -4.6 \pm 1.9.
\end{aligned}
\tag{128}$$

Note that the present ambiguity on the value of $\delta\alpha^{-1}(m_Z) = \pm 0.09$ corresponds to an uncertainty on ϵ_3 (the other epsilons are not much affected) given by $\Delta\epsilon_3 10^3 = \pm 0.6$. Thus the theoretical error is still comfortably less than the experimental error. In fig.11 we present a summary of the experimental values of the epsilons as compared to the SM predictions as functions of m_t and m_H , which shows agreement within 1σ , but the central value of ϵ_1 , ϵ_2 and ϵ_3 are all low, while the central value of ϵ_b is shifted upward with respect to the SM as a consequence of the still imperfect matching of R_b . A number of interesting features are clearly visible from figs.5-11. First, the good agreement with the SM and the evidence for weak corrections, measured by the distance of the data from the improved Born approximation point (based on tree level SM plus pure QED or QCD corrections). There is by now a solid evidence for departures from the improved Born approximation where all the epsilons vanish. In other words a clear evidence for the pure weak radiative corrections has been obtained and LEP/SLC are now measuring the various components of these radiative corrections. For example, some authors¹⁵ have studied the sensitivity of the data to a particularly interesting subset of the weak radiative corrections, i.e. the purely bosonic part. These terms arise from virtual exchange of gauge bosons and Higgses. The result is that indeed the measurements are sufficiently precise to require the presence of these contributions in order to fit the data. Second, the general results of the SM fits are reobtained

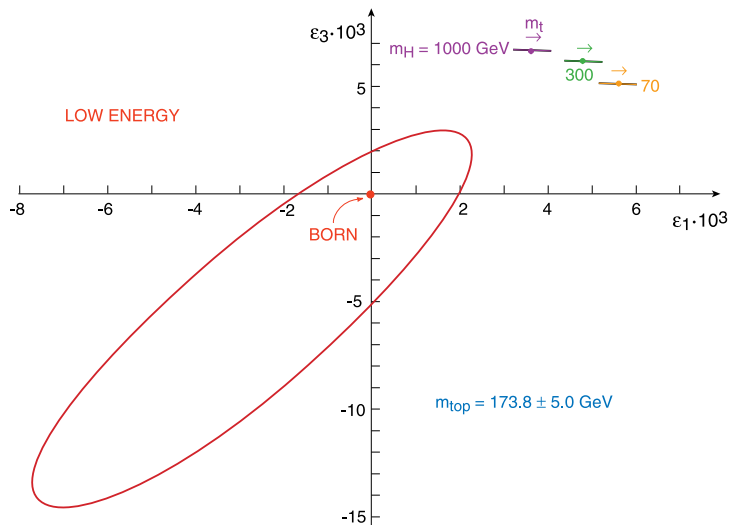


Figure 10: Data vs theory in the ϵ_3 - ϵ_1 plane (notations as in fig.5). Here the ellipse from the low energy data by themselves is plotted (deep inelastic neutrino scattering, atomic parity violation and $\nu_\mu - e$ scattering).

from a different perspective. We see the preference for light Higgs manifested by the tendency for ϵ_3 to be rather on the low side. Since ϵ_3 is practically independent of m_t , its low value demands m_H small. If the Higgs is light then the preferred value of m_t is somewhat lower than the Tevatron result (which in the epsilon analysis is not included among the input data). This is because also the value of $\epsilon_1 \equiv \delta\rho$, which is determined by the widths, in particular by the leptonic width, is somewhat low. In fact ϵ_1 increases with m_t and, at fixed m_t , decreases with m_H , so that for small m_H the low central value of ϵ_1 pushes m_t down. Note that also the central value of ϵ_2 is on the low side, because the experimental value of m_W is a little bit too large. Finally, we see that adding the hadronic quantities or the low energy observables hardly makes a difference in the ϵ_i - ϵ_j plots with respect to the case with only the leptonic variables being included (the ellipse denoted by "All Asymm."). But, for example for the ϵ_1 - ϵ_3 plot, while the leptonic ellipse contains the same information as one could obtain from a $\sin^2 \theta_{eff}$ vs Γ_l plot, the content of the other two ellipses is much larger because it shows that the hadronic as well as the low energy quantities match the leptonic variables without need of any new physics. Note that the experimental values of ϵ_1 and ϵ_3 when the hadronic quantities are included also depend on the input value of α_s given in eq.(111).

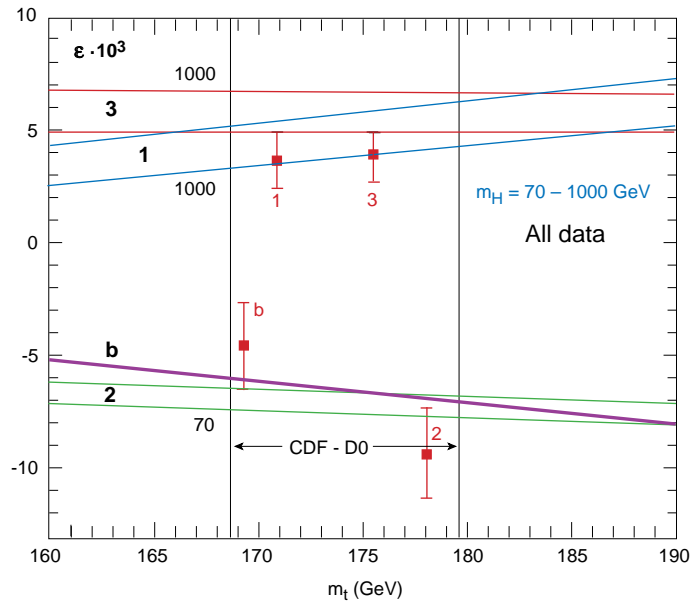


Figure 11: The bands (labeled by the ϵ index) are the predicted values of the epsilons in the SM as functions of m_t for $m_H = 70 - 1000$ GeV (the m_H value corresponding to one edge of the band is indicated). The CDF/D0 experimental $1\text{-}\sigma$ range of m_t is shown. The experimental results for the epsilons from all data are displayed (from the last column of table 6). The position of the data on the m_t axis has been arbitrarily chosen and has no particular meaning.

The good agreement of the fitted epsilon values with the SM impose strong constraints on possible forms of new physics. Consider, for example, new quarks or leptons. Mass splitted multiplets contribute to $\Delta\epsilon_1$, in analogy to the t-b quark doublet. Recall that $\Delta\epsilon_1 \sim +9.5 \cdot 10^{-3}$ for the t-b doublet, which is about eight σ 's in terms of the present error¹⁶. Even mass degenerate multiplets are strongly constrained. They contribute to $\Delta\epsilon_3$ according to¹⁷

$$\Delta\epsilon_3 \sim N_C \frac{G_F m_W^2}{8\pi^2 \sqrt{2}} \frac{4}{3} (T_{3L} - T_{3R})^2 \quad (129)$$

For example a new left-handed quark doublet, degenerate in mass, would contribute $\Delta\epsilon_3 \sim +1.3 \cdot 10^{-3}$, that is about one σ , but in the wrong direction, in the sense that the experimental value of ϵ_3 favours a displacement, if any, with negative sign. Only vector fermions ($T_{3L} = T_{3R}$) are not constrained. In particular, naive technicolour models, that introduce several new technifermions, are strongly disfavoured because they tend to produce large corrections with the wrong sign to ϵ_1 , ϵ_3 and also to ϵ_b ¹⁸.

10 Why Beyond the Standard Model?

Given the striking success of the SM why are we not satisfied with that theory? Why not just find the Higgs particle, for completeness, and declare that particle physics is closed? The main reason is that there are strong conceptual indications for physics beyond the SM. There are also some phenomenological hints.

10.1 Conceptual Problems with the Standard Model

It is considered highly unplausable that the origin of the electro-weak symmetry breaking can be explained by the standard Higgs mechanism, without accompanying new phenomena. New physics should be manifest at energies in the TeV domain. This conclusion follows from an extrapolation of the SM at very high energies. The computed behaviour of the $SU(3) \otimes SU(2) \otimes U(1)$ couplings with energy clearly points towards the unification of the electro-weak and strong forces (Grand Unified Theories: GUTs) at scales of energy $M_{GUT} \sim 10^{14} - 10^{16} \text{ GeV}$ which are close to the scale of quantum gravity, $M_{Pl} \sim 10^{19} \text{ GeV}$ ¹⁹. One can also imagine a unified theory of all interactions also including gravity (at present superstrings provide the best attempt at such a theory). Thus GUTs and the realm of quantum gravity set a very

distant energy horizon that modern particle theory cannot anymore ignore. Can the SM without new physics be valid up to such large energies? This appears unlikely because the structure of the SM could not naturally explain the relative smallness of the weak scale of mass, set by the Higgs mechanism at $m \sim 1/\sqrt{G_F} \sim 250 \text{ GeV}$ with G_F being the Fermi coupling constant. The weak scale m is $\sim 10^{17}$ times smaller than M_{Pl} . Even if the weak scale is set near 250 GeV at the classical level, quantum fluctuations would naturally shift it up to where new physics starts to apply, in particular up to M_{Pl} if there was no new physics up to gravity. This so-called hierarchy problem²⁰ is related to the presence of fundamental scalar fields in the theory with quadratic mass divergences and no protective extra symmetry at $m=0$. For fermions, first, the divergences are logarithmic and, second, at $m=0$ an additional symmetry, i.e. chiral symmetry, is restored. Here, when talking of divergences we are not worried of actual infinities. The theory is renormalisable and finite once the dependence on the cut off is absorbed in a redefinition of masses and couplings. Rather the hierarchy problem is one of naturalness. If we consider the cut off as a manifestation of new physics that will modify the theory at large energy scales, then it is relevant to look at the dependence of physical quantities on the cut off and to demand that no unexplained enormously accurate cancellation arise.

According to the above argument the observed value of $m \sim 250 \text{ GeV}$ is indicative of the existence of new physics nearby. There are two main possibilities. Either there exist fundamental scalar Higgses but the theory is stabilised by supersymmetry, the boson-fermion symmetry that would downgrade the degree of divergence from quadratic to logarithmic. For approximate supersymmetry the cut off is replaced by the splitting between the normal particles and their supersymmetric partners. Then naturalness demands that this splitting (times the size of the weak gauge coupling) is of the order of the weak scale of mass, i.e. the separation within supermultiplets should be of the order of no more than a few TeV. In this case the masses of most supersymmetric partners of the known particles, a very large number of states, would fall, at least in part, in the discovery reach of the LHC. There are consistent, fully formulated field theories constructed on the basis of this idea, the simplest one being the MSSM²¹. Note that all normal observed states are those whose masses are forbidden in the limit of exact $SU(2) \otimes U(1)$. Instead for all SUSY partners the masses are allowed in that limit. Thus when supersymmetry is broken in the TeV range but $SU(2) \otimes U(1)$ is intact only s-partners take mass while all normal particles remain massless. Only at the lower weak scale the masses of ordinary particles are generated. Thus a simple criterium exists to

understand the difference between particles and s-particles.

The other main avenue is compositeness of some sort. The Higgs boson is not elementary but either a bound state of fermions or a condensate, due to a new strong force, much stronger than the usual strong interactions, responsible for the attraction. A plethora of new "hadrons", bound by the new strong force would exist in the LHC range. A serious problem for this idea is that nobody so far has been able to build up a realistic model along these lines, but that could eventually be explained by a lack of ingenuity on the theorists side. The most appealing examples are technicolor theories¹⁸. These models were inspired by the breaking of chiral symmetry in massless QCD induced by quark condensates. In the case of the electroweak breaking new heavy techniquarks must be introduced and the scale analogous to Λ_{QCD} must be about three orders of magnitude larger. The presence of such a large force relatively nearby has a strong tendency to clash with the results of the electroweak precision tests. Another interesting idea is to replace the Higgs by a $t\bar{t}$ condensate²². The Yukawa coupling of the Higgs to the $t\bar{t}$ pair becomes a four fermion $t\bar{t}t\bar{t}$ coupling with the corresponding strength. The strong force is in this case provided by the large top mass. At first sight this idea looks great: no fundamental scalars, no new states. But, looking closely, the advantages are largely illusory. First, in the SM the required value of m_t is too large $m_t \geq 220 \text{ GeV}$ or so. Also a tremendous fine tuning is required, because m_t would naturally be of the order of M_{GUT} or M_{Pl} if no new physics is present (the hierarchy problem in a different form!). Supersymmetry could come to the rescue in this case also. In a minimal SUSY version the required value of the top mass is lowered²³, $m_t \sim 205 \sin \beta \text{ GeV}$. But the resulting theory is physically indistinguishable from the MSSM with small $\tan \beta$, at least at low energies²⁴. This is because a strongly coupled Higgs looks the same as a $t\bar{t}$ pair.

The hierarchy problem is certainly not the only conceptual problem of the SM. There are many more: the proliferation of parameters, the mysterious pattern of fermion masses and so on. But while most of these problems can be postponed to the final theory that will take over at very large energies, of order M_{GUT} or M_{Pl} , the hierarchy problem arises from the instability of the low energy theory and requires a solution at relatively low energies. A supersymmetric extension of the SM provides a way out which is well defined, computable and that preserves all virtues of the SM. The necessary SUSY breaking can be introduced through soft terms that do not spoil the stability of scalar masses. Precisely those terms arise from supergravity when it is spontaneously broken in a hidden sector²⁵. But alternative mechanisms of SUSY breaking are also being considered²⁶. In the most familiar approach SUSY is

broken in a hidden sector and the scale of SUSY breaking is very large of order $\Lambda \sim \sqrt{G_F^{-1/2} M_P}$ where M_P is the Planck mass. But since the hidden sector only communicates with the visible sector through gravitational interactions the splitting of the SUSY multiplets is much smaller, in the TeV energy domain, and the Goldstino is practically decoupled. In an alternative scenario the (not so much) hidden sector is connected to the visible one by ordinary gauge interactions. As these are much stronger than the gravitational interactions, Λ can be much smaller, as low as 10-100 TeV. It follows that the Goldstino is very light in these models (with mass of order or below 1 eV typically) and is the lightest, stable SUSY particle, but its couplings are observably large. The radiative decay of the lightest neutralino into the Goldstino leads to detectable photons. The signature of photons comes out naturally in this SUSY breaking pattern: with respect to the MSSM, in the gauge mediated model there are typically more photons and less missing energy. Gravitational and gauge mediation are extreme alternatives: a spectrum of intermediate cases is conceivable. The main appeal of gauge mediated models is a better protection against flavour changing neutral currents. In the gravitational version even if we accept that gravity leads to degenerate scalar masses at a scale near M_{Pl} the running of the masses down to the weak scale can generate mixing induced by the large masses of the third generation fermions²⁷.

10.2 Hints from Experiment

Unification of Couplings

At present the most direct phenomenological evidence in favour of supersymmetry is obtained from the unification of couplings in GUTs. Precise LEP data on $\alpha_s(m_Z)$ and $\sin^2 \theta_W$ confirm what was already known with less accuracy: standard one-scale GUTs fail in predicting $\sin^2 \theta_W$ given $\alpha_s(m_Z)$ (and $\alpha(m_Z)$) while SUSY GUTS²⁸ are in agreement with the present, very precise, experimental results. According to the recent analysis of ref.²⁹, if one starts from the known values of $\sin^2 \theta_W$ and $\alpha(m_Z)$, one finds for $\alpha_s(m_Z)$ the results:

$$\begin{aligned} \alpha_s(m_Z) &= 0.073 \pm 0.002 && \text{(Standard GUTS)} \\ \alpha_s(m_Z) &= 0.129 \pm 0.010 && \text{(SUSY GUTS)} \end{aligned} \tag{130}$$

to be compared with the world average experimental value $\alpha_s(m_Z) = 0.119(4)$.

Dark Matter

There is solid astrophysical and cosmological evidence^{30, 31} that most of the matter in the universe does not emit electromagnetic radiation, hence is "dark". Some of the dark matter must be baryonic but most of it must be non baryonic. Non baryonic dark matter can be cold or hot. Cold means non relativistic at freeze out, while hot is relativistic. There is general consensus that most of the non baryonic dark matter must be cold dark matter. A couple of years ago the most likely composition was quoted to be around 80% cold and 20% hot. At present it appears to me that the need of a sizeable hot dark matter component is more uncertain. In fact, recent experiments have indicated the presence of a previously disfavoured cosmological constant component in $\Omega = \Omega_m + \Omega_\Lambda$ ³⁰. Here Ω is the total matter-energy density in units of the critical density, Ω_m is the matter component (dominated by cold dark matter) and Ω_Λ is the cosmological component. Inflationary theories almost inevitably predict $\Omega = 1$ which is consistent with present data. At present, still within large uncertainties, the approximate composition is indicated to be $\Omega_m \sim 0.4$ and $\Omega_\Lambda \sim 0.6$ (baryonic dark matter gives $\Omega_b \sim 0.05$).

The implications for particle physics is that certainly there must exist a source of cold dark matter. By far the most appealing candidate is the neutralino, the lowest supersymmetric particle, in general a superposition of photino, Z-ino and higgsinos. This is stable in supersymmetric models with R parity conservation, which are the most standard variety for this class of models (including the Minimal Supersymmetric Standard Model:MSSM). A neutralino with mass of order 100 GeV would fit perfectly as a cold dark matter candidate. Another common candidate for cold dark matter is the axion, the elusive particle associated to a possible solution of the strong CP problem along the line of a spontaneously broken Peccei-Quinn symmetry. To my knowledge and taste this option is less plausible than the neutralino. One favours supersymmetry for very diverse conceptual and phenomenological reasons, as described in the previous sections, so that neutralinos are sort of standard by now. For hot dark matter, the self imposing candidates are neutrinos. If we demand a density fraction $\Omega_\nu \sim 0.1$ from neutrinos, then it turns out that the sum of stable neutrino masses should be around 5 eV.

Baryogenesis

Baryogenesis is interesting because it could occur at the weak scale³² but not in the SM. For baryogenesis one needs the three famous Sakharov conditions³³: B violation, CP violation and no thermal equilibrium. In principle these conditions could be verified in the SM. B is violated by instantons when kT is of the order of the weak scale (but B-L is conserved). CP is violated by the CKM phase and out of equilibrium conditions could be verified during the electroweak phase transition. So the conditions for baryogenesis appear superficially to be present for it to occur at the weak scale in the SM. However, a more quantitative analysis^{34, 35} shows that baryogenesis is not possible in the SM because there is not enough CP violation and the phase transition is not sufficiently strong first order, unless $m_H < 80 \text{ GeV}$, which is by now excluded by LEP. Certainly baryogenesis could also occur below the GUT scale, after inflation. But only that part with $|B - L| > 0$ would survive and not be erased at the weak scale by instanton effects. Thus baryogenesis at $kT \sim 10^{12} - 10^{15} \text{ GeV}$ needs B-L violation at some stage like for m_ν . The two effects could be related if baryogenesis arises from leptogenesis³⁶ then converted into baryogenesis by instantons. While baryogenesis at a large energy scale is thus not excluded it is interesting that recent studies have shown that baryogenesis at the weak scale could be possible in the MSSM³⁵. In fact, in this model there are additional sources of CP violations and the bound on m_h is modified by a sufficient amount by the presence of scalars with large couplings to the Higgs sector, typically the s-top. What is required is that $m_h \sim 80 - 110 \text{ GeV}$ (in the LEP2 range!), a s-top not heavier than the top quark and, preferentially, a small $\tan\beta$.

Neutrino Masses

Recent data from Superkamiokande³⁷ (and also MACRO³⁸) have provided a more solid experimental basis for neutrino oscillations as an explanation of the atmospheric neutrino anomaly. In addition the solar neutrino deficit is also probably an indication of a different sort of neutrino oscillations. Results from the laboratory experiment by the LNSD collaboration³⁹ can also be considered as a possible indication of yet another type of neutrino oscillation. But the preliminary data from Karmen⁴⁰ have failed to reproduce this evidence. The case of LNSD oscillations is far from closed but one can tentatively assume, pending the results of continuing experiments, that the signal will not persist. Then solar and atmospheric neutrino oscillations can possibly be explained in

terms of the three known flavours of neutrinos without invoking extra sterile species. Neutrino oscillations for atmospheric neutrinos require $\nu_\mu \rightarrow \nu_\tau$ with $\Delta m_{atm}^2 \sim 2 \cdot 10^{-3} \text{ eV}^2$ and a nearly maximal mixing angle $\sin^2 2\theta_{atm} \geq 0.8$. In most of the Superkamiokande allowed region the bound by Chooz⁴¹ essentially excludes $\nu_e \rightarrow \nu_\mu$ oscillations for atmospheric neutrino oscillations. Furthermore the last results from Superkamiokande allow a solution of the solar neutrino deficit in terms of ν_e disappearance vacuum oscillations (as opposed to MSW⁴² oscillations within the sun) with $\Delta m_{sol}^2 \sim 10^{-10} \text{ eV}^2$ and again nearly maximal mixing angles. Among the large and small angle MSW solutions the small angle one is perhaps more likely at the moment (with⁴³ $\Delta m_{sol}^2 \sim 0.5 \cdot 10^{-5} \text{ eV}^2$ and $\sin^2 2\theta_{sol} \sim 5.5 \cdot 10^{-3}$) than the large angle MSW solution. Of course experimental uncertainties are still large and the numbers given here are presumably only indicative. But by now it is very unlikely that all this evidence for neutrino oscillations will disappear or be explained away by astrophysics or other solutions. The consequence is that we have a substantial evidence that neutrinos are massive.

In a strict minimal standard model point of view neutrino masses could vanish if no right handed neutrinos existed (no Dirac mass) and lepton number was conserved (no Majorana mass). In Grand Unified theories both these assumptions are violated. The right handed neutrino is required in all unifying groups larger than SU(5). In SO(10) the 16 fermion fields in each family, including the right handed neutrino, exactly fit into the 16 dimensional representation of this group. This is really telling us that there is something in SO(10)! The SU(5) alternative in terms of $\bar{5} + 10$, without a right handed neutrino, is certainly less elegant. The breaking of $|B - L|$, B and L is also a generic feature of Grand Unification. In fact, the see-saw mechanism⁴⁴ explains the smallness of neutrino masses in terms of the large mass scale where $|B - L|$ and L are violated. Thus, neutrino masses, as would be proton decay, are important as a probe into the physics at the GUT scale.

Oscillations only determine squared mass differences and not masses. The case of three nearly degenerate neutrinos is the only one that could in principle accomodate neutrinos as hot dark matter together with solar and atmospheric neutrino oscillations. According to our previous discussion, the common mass should be around 1-3 eV. The solar frequency could be given by a small 1-2 splitting, while the atmospheric frequency could be given by a still small but much larger 1,2-3 splitting. A strong constraint arises in the degenerate case from neutrinoless double beta decay which requires that the ee entry of m_ν must obey $|(m_\nu)_{11}| \leq 0.46 \text{ eV}$. As observed in ref.⁴⁵, this bound can only be satisfied if double maximal mixing is realized, i.e. if also solar neutrino

oscillations occur with nearly maximal mixing. We have mentioned that it is not at all clear at the moment that a hot dark matter component is really needed³⁰. However the only reason to consider the fully degenerate solution is that it is compatible with hot dark matter. Note that for degenerate masses with $m \sim 1 - 3$ eV we need a relative splitting $\Delta m/m \sim \Delta m_{atm}^2/2m^2 \sim 10^{-3} - 10^{-4}$ and an even smaller one for solar neutrinos. It is difficult to imagine a natural mechanism compatible with unification and the see-saw mechanism to arrange such a precise near symmetry.

If neutrino masses are smaller than for cosmological relevance, we can have the hierarchies $|m_3| \gg |m_{2,1}|$ or $|m_1| \sim |m_2| \gg |m_3|$. Note that we are assuming only two frequencies, given by $\Delta_{sun} \propto m_2^2 - m_1^2$ and $\Delta_{atm} \propto m_3^2 - m_{1,2}^2$. We prefer the first case, because for quarks and leptons one mass eigenvalue, the third generation one, is largely dominant. Thus the dominance of m_3 for neutrinos corresponds to what we observe for the other fermions. In this case, m_3 is determined by the atmospheric neutrino oscillation frequency to be around $m_3 \sim 0.05$ eV. By the see-saw mechanism m_3 is related to some large mass M , by $m_3 \sim m^2/M$. If we identify m with either the Higgs vacuum expectation value or the top mass (which are of the same order), as suggested for third generation neutrinos by Grand Unification in simple SO(10) models, then M turns out to be around $M \sim 10^{15}$ GeV, which is consistent with the connection with GUT's. If solar neutrino oscillations are determined by vacuum oscillations, then $m_2 \sim 10^{-5}$ eV and we have that the ratio m_2/m_3 is well consistent with $(m_c/m_t)^2$.

A lot of attention is being devoted to the problem of a natural explanation of the observed nearly maximal mixing angle for atmospheric neutrino oscillations and possibly also for solar neutrino oscillations, if explained by vacuum oscillations⁴⁶. Large mixing angles are somewhat unexpected because the observed quark mixings are small and the quark, charged lepton and neutrino mass matrices are to some extent related in GUT's. There must be some special interplay between the neutrino Dirac and Majorana matrices in the see-saw mechanism in order to generate maximal mixing. It is hoped that looking for a natural explanation of large neutrino mixings can lead us to deciphering some interesting message on the physics at the GUT scale.

11 Comparing the Data with the Minimal Supersymmetric Standard Model

The MSSM ²¹ is a completely specified, consistent and computable theory. There are too many parameters to attempt a direct fit of the data to the most general framework. So we consider two significant limiting cases: the "heavy" and the "light" MSSM.

The "heavy" limit corresponds to all s-particles being sufficiently massive, still within the limits of a natural explanation of the weak scale of mass. In this limit a very important result holds ⁴⁷: for what concerns the precision electroweak tests, the MSSM predictions tend to reproduce the results of the SM with a light Higgs, say $m_H \sim 100$ GeV. So if the masses of SUSY partners are pushed at sufficiently large values the same quality of fit as for the SM is guaranteed. Note that for $m_t \sim 175$ GeV and $m_H \sim 70$ GeV the values of the four epsilons computed in the SM lead to a fit of the corresponding experimental values with $\chi^2 \sim 4$, which is reasonable for *d.o.f* = 4. This value corresponds to the fact that the central values of $\epsilon_1, \epsilon_2, \epsilon_3$ and $-\epsilon_b$ are all below the SM value by about 1σ , as can be seen from fig.11.

In the "light" MSSM option some of the superpartners have a relatively small mass, close to their experimental lower bounds. In this case the pattern of radiative corrections may sizeably deviate from that of the SM ⁵¹. The potentially largest effects occur in vacuum polarisation amplitudes and/or the $Z \rightarrow b\bar{b}$ vertex. In particular we recall the following contributions :

i) a threshold effect in the Z wave function renormalisation ⁴⁷ mostly due to the vector coupling of charginos and (off-diagonal) neutralinos to the Z itself. Defining the vacuum polarisation functions by $\Pi_{\mu\nu}(q^2) = -ig_{\mu\nu}[A(0) + q^2 F(q^2)] + q_\mu q_\nu$ terms, this is a positive contribution to $\epsilon_5 = m_Z^2 F'_{ZZ}(m_Z^2)$, the prime denoting a derivative with respect to q^2 (i.e. a contribution to a higher derivative term not included in the usual epsilon formalism). The ϵ_5 correction shifts ϵ_1, ϵ_2 and ϵ_3 by $-\epsilon_5, -c^2\epsilon_5$ and $-c^2\epsilon_5$ respectively, where $c^2 = \cos^2 \theta_W$, so that all of them are reduced by a comparable amount. Correspondingly all the Z widths are reduced without affecting the asymmetries. This effect falls down particularly fast when the lightest chargino mass increases from a value close to $m_Z/2$. Now that we know, from the LEP2 runs, that the chargino mass is not smaller than m_Z its possible impact is drastically reduced.

ii) a positive contribution to ϵ_1 from the virtual exchange of split multiplets of SUSY partners, for example of the scalar top and bottom superpartners ⁴⁸,

analogous to the contribution of the top-bottom left-handed quark doublet. From the experimental value of m_t not much space is left for this possibility, and the experimental value of ϵ_1 is an important constraint on the spectrum. This is especially true now that the rather large lower limits on the chargino mass reduce the size of a possible compensation from ϵ_5 . For example, if the stop is light then it must be mainly a right-handed stop. Also large values of $\tan\beta$ are disfavoured because they tend to enhance the splittings among SUSY partner multiplets. In general it is simpler to decrease the predicted values of ϵ_2 and ϵ_3 by taking advantage of ϵ_5 than to decrease ϵ_1 , because the negative shift from ϵ_5 is most often counterbalanced by the increase from the effect of split SUSY multiplets.

iii) a negative contribution to ϵ_b due to the virtual exchange of a charged Higgs⁴⁹. If one defines, as customary, $\tan\beta = v_2/v_1$ (v_1 and v_2 being the vacuum expectation values of the Higgs doublets giving masses to the down and up quarks, respectively), then, for negligible bottom Yukawa coupling or $\tan\beta \ll m_t/m_b$, this contribution is proportional to $m_t^2/\tan^2\beta$.

iv) a positive contribution to ϵ_b due to virtual chargino-s-top exchange⁵⁰ which in this case is proportional to $m_t^2/\sin^2\beta$ and prefers small $\tan\beta$. This effect again requires the chargino and the s-top to be light in order to be sizeable.

With the recent limits set by LEP2 on the masses of SUSY partners the above effects are small enough that other contributions from vertex diagrams could be comparable. Thus in the following we will only consider the experimental values of the epsilons obtained at the level denoted by "All Asymmetries" which only assumes lepton universality.

We have analysed the problem of what configurations of masses in the "light" MSSM are favoured or disfavoured by the present data⁶, (updating ref.⁵²). We find that no lower limits on the masses of SUSY partners are obtained which are better than the direct limits. One exception is the case of s-top and s-bottom masses, which are severely constrained by the ϵ_1 value and also, at small $\tan\beta$, by the increase at LEP2 of the direct limit on the Higgs mass. Charged higgs masses are also rather severely constrained. Since the central values of ϵ_1, ϵ_2 and ϵ_3 are all below the SM it is convenient to make ϵ_5 as large as possible. For this purpose light gaugino and s-lepton masses are favoured. We find that for $m_{\chi_1^+} \sim 90 - 120 \text{ GeV}$ the effect is still sizeable. Also favoured are small values of $\tan\beta$ that allow to put s-lepton masses relatively low, say, in the range 100-500 GeV, without making the

split in the isospin doublets too large for ϵ_1 . Charged Higgses must be heavy because they contribute to ϵ_b with the wrong sign. A light right-handed s-top could help on R_b for a higgsino-like chargino. But one needs small mixing (the right-handed s-top must be close to the mass eigenstate) and beware of the higgs mass constraint at small $\tan\beta$ (a higgs mass above 83 GeV, the range of LEP2 at $\sqrt{s} = 183$ GeV, starts being a strong constraint at small $\tan\beta$). So we prefer in the following to keep the s-top mass large. The limits on $b \rightarrow s\gamma$ also prefer heavy charged higgs and s-top⁵³.

12 The LEP2 Programme and the Search for the Higgs and New Physics

The LEP2 programme has started in the second part of 1996. At first the energy has been fixed at 161 GeV, which is the most favourable energy for the measurement of m_W from the cross-section for $e^+e^- \rightarrow W^+W^-$ at threshold. Then gradually the energy was brought up to 172, 183, 189 GeV. It will be increased up to a maximum of about 200 GeV to be reached in mid '99. An integrated luminosity of about 150 pb^{-1} per year is now achievable (in fact more was achieved in 1998). LEP2 has been approved to run until the end of 2000, before the shutdown for the installation of the LHC. The main goals of LEP2 are the search for the Higgs and for new particles, the measurement of m_W and the investigation of the triple gauge vertices WWZ and $WW\gamma$. A complete updated survey of the LEP2 physics is collected in the two volumes of ref.⁵⁴.

An important competitor of LEP2 is the Tevatron collider. In mid 2000 the Tevatron will start RunII with the purpose of collecting a few fb^{-1} of integrated luminosity at 2 TeV. The competition is especially on the search of new particles, but also on m_W and the triple gauge vertices. For example, for supersymmetry while the Tevatron is superior for gluinos and squarks, LEP2 is strong on Higgses, charginos, neutralinos and sleptons. There are plans for RunIII to start in 2002 or so with the purpose of collecting of the order 5 fb^{-1} of integrated luminosity per year. Then the Tevatron could also hope to find the Higgs before the LHC starts if the Higgs mass is close to the LEP2 range.

Concerning the Higgs it is interesting to recall that the large value of m_t has important implications on m_H both in the minimal SM⁵⁵⁻⁵⁷ and in its minimal supersymmetric extension^{58,59}. I will now discuss the restrictions on m_H that follow from the observed value of m_t .

It is well known^{55–57} that in the SM with only one Higgs doublet a lower limit on m_H can be derived from the requirement of vacuum stability. The limit is a function of m_t and of the energy scale Λ where the model breaks down and new physics appears. Similarly an upper bound on m_H (with mild dependence on m_t) is obtained⁶¹ from the requirement that up to the scale Λ no Landau pole appears. The lower limit on m_H is particularly important in view of the search for the Higgs at LEP2. Indeed the issue is whether one can reach the conclusion that if a Higgs is found at LEP2, i.e. with $m_H \leq m_Z$, then the SM must break down at some scale $\Lambda > 1$ TeV.

The possible instability of the Higgs potential $V[\phi]$ is generated by the quantum loop corrections to the classical expression of $V[\phi]$. At large ϕ the derivative $V'[\phi]$ could become negative and the potential would become unbound from below. The one-loop corrections to $V[\phi]$ in the SM are well known and change the dominant term at large ϕ according to $\lambda\phi^4 \rightarrow (\lambda + \gamma \log \phi^2/\Lambda^2)\phi^4$. The one-loop approximation is not enough for our purposes, because it fails at large enough ϕ , when $\gamma \log \phi^2/\Lambda^2$ becomes of order 1. The renormalization group improved version of the corrected potential leads to the replacement $\lambda\phi^4 \rightarrow \lambda(\Lambda)\phi'^4(\Lambda)$ where $\lambda(\Lambda)$ is the running coupling and $\phi'(\mu) = \exp \int^t \gamma(t') dt' \phi$, with $\gamma(t)$ being an anomalous dimension function and $t = \log \Lambda/v$ (v is the vacuum expectation value $v = (2\sqrt{2}G_F)^{-1/2}$). As a result, the positivity condition for the potential amounts to the requirement that the running coupling $\lambda(\Lambda)$ never becomes negative. A more precise calculation, which also takes into account the quadratic term in the potential, confirms that the requirements of positive $\lambda(\Lambda)$ leads to the correct bound down to scales Λ as low as ~ 1 TeV. The running of $\lambda(\Lambda)$ at one loop is given by:

$$\frac{d\lambda}{dt} = \frac{3}{4\pi^2} [\lambda^2 + 3\lambda h_t^2 - 9h_t^4 + \text{gauge terms}] , \quad (131)$$

with the normalization such that at $t = 0, \lambda = \lambda_0 = m_H^2/2v^2$ and the top Yukawa coupling $h_t^0 = m_t/v$. We see that, for m_H small and m_t large, λ decreases with t and can become negative. If one requires that λ remains positive up to $\Lambda = 10^{15}\text{--}10^{19}$ GeV, then the resulting bound on m_H in the SM with only one Higgs doublet is given by⁵⁶:

$$m_H > 134 + 2.1 [m_t - 173.8] - 4.5 \frac{\alpha_s(m_Z) - 0.119}{0.006} . \quad (132)$$

Summarizing, we see that the discovery of a Higgs particle at LEP2, or $m_H \lesssim 100$ GeV, would imply that the SM breaks down at a scale Λ of the order

of a few TeV. It can be shown⁵⁵ that the lower limit is not much relaxed even if strict vacuum stability is replaced by some sufficiently long metastability.

The upper limit on the Higgs mass in the SM is important for assessing the chances of success of the LHC as an accelerator designed to solve the Higgs problem. The upper limit⁶¹ arises from the requirement that the Landau pole associated with the non asymptotically free behaviour of the $\lambda\phi^4$ theory does not occur below the scale Λ . The initial value of λ at the weak scale increases with m_H and the derivative is positive at large m_H . Thus if m_H is too large the Landau pole occurs at too low an energy. The upper limit on m_H has been recently reevaluated⁶⁰. For $m_t \sim 175 \text{ GeV}$ one finds $m_H \lesssim 180 \text{ GeV}$ for $\Lambda \sim M_{GUT} - M_{Pl}$ and $m_H \lesssim 0.5 - 0.8 \text{ TeV}$ for $\Lambda \sim 1 \text{ TeV}$. Actually, for $m_t \sim 174 \text{ GeV}$, only a small range of values for m_H is allowed, $130 < m_H < \sim 200 \text{ GeV}$, if the SM holds up to $\Lambda \sim M_{GUT}$ or M_{Pl} .

A particularly important example of theory where the above bounds do not apply and in particular the lower bound is violated is the MSSM, which we now discuss. As is well known²¹, in the MSSM there are two Higgs doublets, which implies three neutral physical Higgs particles and a pair of charged Higgses. The lightest neutral Higgs, called h , should be lighter than m_Z at tree-level approximation. However, radiative corrections⁶² increase the h mass by a term proportional to m_t^4 and logarithmically dependent on the stop mass. Once the radiative corrections are taken into account the h mass still remains rather small: for $m_t = 174 \text{ GeV}$ one finds the limit (for all values of $\tan\beta$) $m_h < 130 \text{ GeV}$ ⁵⁹. Actually there are reasons to expect that m_h is well below the bound. In fact, if h_t is large at the GUT scale, which is suggested by the large observed value of m_t and by a natural onset of the electroweak symmetry breaking induced by m_t , then at low energy a fixed point is reached in the evolution of m_t . The fixed point corresponds to $m_t \sim 205 \sin\beta \text{ GeV}$ (a good approximate relation for $\tan\beta = v_{up}/v_{down} < 10$). If the fixed point situation is realized, then m_h is considerably below the bound, as shown in ref.⁵⁹.

In conclusion, for $m_t \sim 174 \text{ GeV}$, we have seen that, on the one hand, if a Higgs is found at LEP the SM cannot be valid up to M_{Pl} . On the other hand, if a Higgs is found at LEP, then the MSSM has good chances, because this model would be excluded for $m_h > 130 \text{ GeV}$.

For the SM Higgs, which plays the role of a benchmark also important for a more general context, the LEP2 reach has been studied in detail⁵⁴. At 200 GeV with about 150 pb^{-1} per experiment one can discover or exclude a SM Higgs up to about 105 GeV of mass. In the MSSM a more complicated

discussion is needed because there are several Higgses and the parameter space is multidimensional. Only the lightest MSSM Higgs h is accessible at LEP2. The dominant production processes are $e^+e^- \rightarrow hZ$ and $e^+e^- \rightarrow hA$, where A is the CP odd MSSM Higgs particle. They are nicely complementary. At given m_h within the range of interest, at large $\tan\beta$ the first process is the relevant one, while the second determines the bound at small $\tan\beta$. The absolute lower limit on m_h for a given beam energy and integrated luminosity is always below the limit on the SM Higgs, because the crosssections are smaller. For example the present limit is around 90 GeV for the SM Higgs and around 80 GeV for the MSSM Higgs. It is interesting that by the end of LEP2 one will have completely explored the region at small $\tan\beta$ (below a value of about 5), which is a particularly likely region.

A main goal of LEP2 is the search for supersymmetry. For charginos the discovery range at LEP2 is only limited by the beam energy for practically all values of the parameters. In fact the typical limit is at present about 90 GeV . Thus every increase of the beam energy is directly translated into the upper limit in chargino mass for discovery or exclusion. For the Tevatron the discovery range is much more dependent on the position in parameter space. For some limited regions of this space, with 1 fb^{-1} of integrated luminosity, the discovery range for charginos at the Tevatron goes well beyond $m_\chi = 100 \text{ GeV}$, i.e. the boundary of LEP2, but in much of the parameter space LEP2 at the maximum energy would be sensitive to larger chargino masses..

The stop is probably the lightest squark. For a light stop the most likely decay modes are $\tilde{t} \rightarrow b\chi^+$ if kinematically allowed, otherwise $\tilde{t} \rightarrow c\chi$. At LEP2 the discovery range is up to about $(E_{beam} - 10) \text{ GeV}$. At the Tevatron there is some difference between the two possible decay modes and some dependence on the position in the $\tilde{t} - \chi$ or the $\tilde{t} - \chi^+$ planes, but in general the Tevatron is very powerful for s-quarks and gluinos and much of the LEP2 range is already excluded by the Tevatron.

By now most of the discovery potential of LEP2 for supersymmetry has been already deployed. For example, the limit on the chargino mass was about 45 GeV after LEP1 and is now about 90 GeV and can only improve up to 100 GeV . For the Higgs the experimental task is more demanding and so one is only a bit more than half way through: the lower limit on the SM Higgs was around 67 GeV after LEP1, is now about 90 GeV and could go up to 105 GeV or so. So there are still fair chances for LEP2 to find the Higgs, especially because the attainable range of masses is particularly likely in the MSSM.

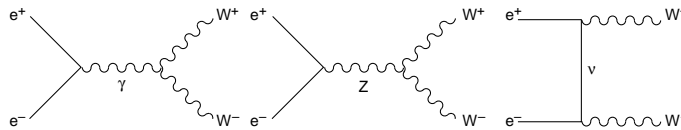


Figure 12: Lowest order Feynman diagrams for $e^+e^- \rightarrow W^+W^-$.

The study of the W^+W^- crosssection is a very important chapter of LEP2 physics⁵⁴. In the Born approximation three Feynman diagrams contribute to the crosssection, as shown in fig.12. In the two s-channel exchange diagrams the triple gauge vertices $WW\gamma$ and WWZ appear, while the third is the t-channel neutrino exchange that only involves well established charged current couplings. One loop radiative corrections have also been computed. It is interesting that if we take neutrino exchange alone or neutrino plus γ exchanges alone the crosssection increases much faster with energy than the complete result. This corresponds to the good convergence properties of the SM which in fact is renormalisable. Indeed, the WW crosssection is related to the imaginary part of the WW loop contribution to the amplitude for $e^+e^- \rightarrow e^+e^-$. The good large energy behavior of the former crosssection is related to the convergence of the latter loop correction. The data neatly confirm the SM prediction as shown in fig.13. Thus the WW crosssection supports the specific form of the triple gauge vertices as predicted by the SM. More detailed studies with large statistics are useful to set bounds on possible departures from the exact SM predicted couplings. In fact the study of triple gauge vertices is another major task of LEP2. The capabilities of LEP2 in this domain are comparable to those of the LHC. LEP2 can push down the existing direct limits considerably down. For given anomalous couplings the departures from the SM are expected to increase with energy. For the energy and the luminosity available at LEP2, given the accuracy of the SM established at LEP1, it is however not very likely, to find signals of new physics in the triple gauge vertices. The measurement of m_W is been done at LEP2 from the W^+W^- cross-section at threshold and from direct reconstruction of the W mass from the final state after W decay. At present m_W is known with an error of ± 60 MeV from the combined LEP2 and Tevatron direct measurements (see table 1), with the same error of ± 90 MeV at LEP2 and at the Tevatron. From the fit to all electroweak data one finds $m_W = 80370 \pm 27$ MeV (see eq.(113)), in agreement with the direct measurement. As a consequence the goal for LEP2 is to measure m_W with an accuracy $\delta m_W \leq \pm(30 - 40)$ MeV, in order to provide an additional significant check of the theory.

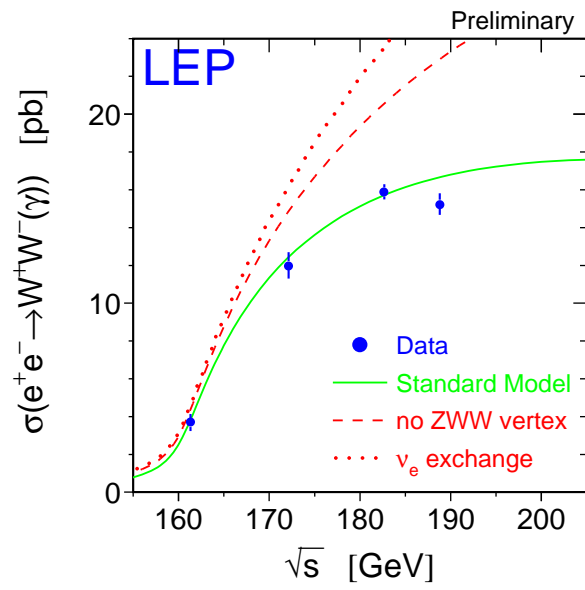


Figure 13: Data vs theory for the WW cross-section measured at LEP2. The solid line is the SM prediction. The dashed and dotted lines refer to only a subset diagrams as indicated.

For the threshold method⁵⁴ the minimum of the statistical error is obtained for $\sqrt{s} = 2m_W + 0.5 \text{ GeV} = 161 \text{ GeV}$, which in fact was the initial operating energy of LEP2. At threshold the WW crosssection is dominated by the neutrino t-channel exchange which is quite model independent. The total error of this method is dominated by the statistics. With the collected luminosity at 161 GeV of $\sim 10 \text{ pb}^{-1}$ per experiment, the present combined result is $m_W = (80.4 \pm 0.2 \pm 0.03) \text{ GeV}$ ⁵. Thus with the available data at threshold this method is not sufficient by itself.

In principle the direct reconstruction method can use the totally hadronic or the semileptonic final states $e^+e^- \rightarrow W^+W^- \rightarrow jjjj$ or $jjl\nu$. The total branching ratio of the hadronic modes is 49%, while that of the $\ell = e, \mu$ semileptonic channels is 28%. The hadronic channel has more statistics but could be severely affected by non-perturbative strong interaction effects: colour recombination among the jets from different W's and Bose correlations among mesons in the final state from WW overlap. Colour recombination is perturbatively small. But gluons with $E < \Gamma_W$ are important and non-perturbative effects could be relatively large, of the order of 10–100 MeV. Similarly for Bose correlations. One is not in a position to really quantify the associated uncertainties. Fortunately the direct reconstruction from the semi-leptonic channels can, by itself, lead to a total error $\delta m_W = \pm 44 \text{ MeV}$, for the combined four experiments, each with 500 pb^{-1} of luminosity collected at $\sqrt{s} \geq 175 \text{ GeV}$. Thus the goal of measuring m_W with an accuracy below $\delta m_W = \pm 50 \text{ MeV}$ can be fulfilled, and it is possible to do better by learning from the data how to limit the error from colour recombination and Bose correlations.

13 Conclusion

Today in particle physics we follow a double approach: from above and from below. From above there are, on the theory side, quantum gravity (that is superstrings), GUT theories and cosmological scenarios. On the experimental side there are underground experiments (e.g. searches for neutrino oscillations and proton decay), cosmic ray observations, satellite experiments (like COBE, IRAS etc) and so on. From below, the main objectives of theory and experiment are the search of the Higgs and of signals of particles beyond the Standard Model (typically supersymmetric particles). Another important direction of research is aimed at the exploration of the flavour problem: study of CP violation and rare decays. The general expectation is that new physics is close by and that should be found very soon if not for the complexity of the

necessary experimental technology that makes the involved time scale painfully long.

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