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Electron Beam Distortions in Beam-Beam Compensation Setup

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Abstract

This article is devoted to electron beam distortions in the "electron compressor" setup for beam-beam compensation in the Tevatron collider. The distortions are due to interaction of the electron beam with impacting elliptical antiproton beam. We estimate of longitudinal magnetic field necessary to keep the distortions low.

1 INTRODUCTION

Compensation of the beam-beam effects in the Tevatron with use of high current, low energy electron beam is discussed in [1]. The electron beam travels in the direction opposite to the antiproton beam and interacts with an antiproton bunch via its space charge forces. The proton beam has to be separated from the electron and antiproton beams. A 10 kV electron beam about 2-m long, 2-mm diameter, with 1-2 Amperes of current is to be installed in a place with large beta-function($\sim$100m), away from the main interaction points. Experimental results from an "electron lens" prototype set-up are discussed in [2]. The electron beam is born on an electron gun cathode, transported through the interaction region, and absorbed in the collector. Strong longitudinal magnetic field plays a significant role in maintaining stability of both electron and antiproton beams. It also suppresses the electron beam current distribution distortions and, therefore, the electron space charge force distortions.

This article is focused on the time-dependent deviations of the electron beam shape due to interaction with antiproton beam in the "electron lens" [3].

2 ELECTRON BEAM DISTORTIONS

$ZBEAM$ code [4] is used for tracing electron trajectories. This is essentially two dimensional code which takes into account only transverse components of the electric and magnetic forces. It is a good approximation for the forces due to ultra-relativistic $\bar{p}$ bunch. The electrons are non-relativistic, but their space charge forces are mostly transverse, too. The code tracks number of particles by integrating their equations of motion over successive small time steps.

Interaction with round $\bar{p}$ bunch in strong magnetic field conserves axial symmetry and radial size of the electron beam. As the result, the electron beam space charge forces are the same for antiprotons at the head and at the tail of the $\bar{p}$ bunch. That is no longer true if electron or antiproton beam is not round. Roundness of the electron beam can be assured by using round cathode in the electron gun and by appropriate choice of the magnetic field in the transport section of the set-up. In opposite, the $\bar{p}$ beam roundness can be achieved in very few Tevatron locations where vertical and horizontal beta-functions are the same $\beta_x = \beta_y$ (vertical and horizontal emittances of 1000 GeV beams in the Tevatron are approximately equal $\varepsilon^{rms}_{x,y} \approx 3.3\pi\, mm\cdot mrad$). This condition can not be fulfilled *a priori*. E.g., at present stage we consider to install one of the "electron lens" devices at the Tevatron F48 location which is characterized by $\beta_x = 101.7$ m and $\beta_y = 30.9$ m, and, consequently, the rms bunch sizes are $\sigma_x = 0.61$mm and $\sigma_y = 0.31$mm [2].

Fig.1 shows what happens with round electron beam (radius $a_e = 0.31$mm) when it interacts with such an antiproton beam being in 2T solenoid field. $ZBEAM$ code is used for the electric field calculations and electron tracing. Top left plot shows an ellipse which corresponds to 1σ of the Gaussian $\bar{p}$ bunch and a circle of the electron beam cross section uniformly filled with electron macroparticles before the interaction with $\bar{p}$-s. Top right plot demonstrates traces of the electrons under impact of asymmetric electric field of the antiproton bunch. Resulted macroparticle positions and the shape of the electron beam in $x-y$ and $r-\phi$ planes are shown in lower left and right plots of Fig.1 respectively. One can see that the electron beam becomes a rotated ellipse to the moment when the tail of antiproton bunch passes through it, while the head of the bunch sees originally undisturbed round electron beam. This might be of concern because of two reasons: 1) there appears a "head-tail" interaction in the $\bar{p}$ bunch via higher than dipole wake fields propagating in the electron beam; 2) in addition to useful defocusing effect, electric fields of the elliptic electron beam produce effective $x-y$ coupling of vertical and horizontal betatron oscillations in the $\bar{p}$ beam.

Distortion of electron density. The continuity equation for the electron charge density $\rho(x,y,z,t)$ can be written as $\frac{\partial\rho}{\partial t} + \vec{v}_d \cdot \vec{\bigtriangledown}\rho_0 = 0$, where ρ_0 stands for initial charge density, and drift velocity of an electron in crossed electric and magnetic fields $\vec{E}$ and $\vec{B}$ is equal to $\vec{v}_d = c\frac{[\vec{E}\times\vec{B}]}{B^2}$. The resulted distortion due to elliptic Gaussian relativistic $\bar{p}$ beam is given by [3]:

$$\delta\rho(x,y,t = \frac{z}{(1+\beta_e)c}) = \Big(\int_{-\infty}^{z}\lambda(z')dz'\Big)\times$$
$$\times\frac{2eN_{\bar{p}}}{B}\frac{d\rho_0(r^2)}{d(r^2)}\cdot\frac{xyI(x,y)(\sigma_x^2-\sigma_y^2)}{\sigma_x^2\sigma_y^2}, \qquad (1)$$

where now z is the coordinate inside the $\bar{p}$ bunch [1] and

[1] i.e. $z=-\infty$ is for the bunch head and $\int_{-\infty}^{z}\lambda(z')dz'$ is proportional to the antiproton charge which passed through the given part of the electron beam.

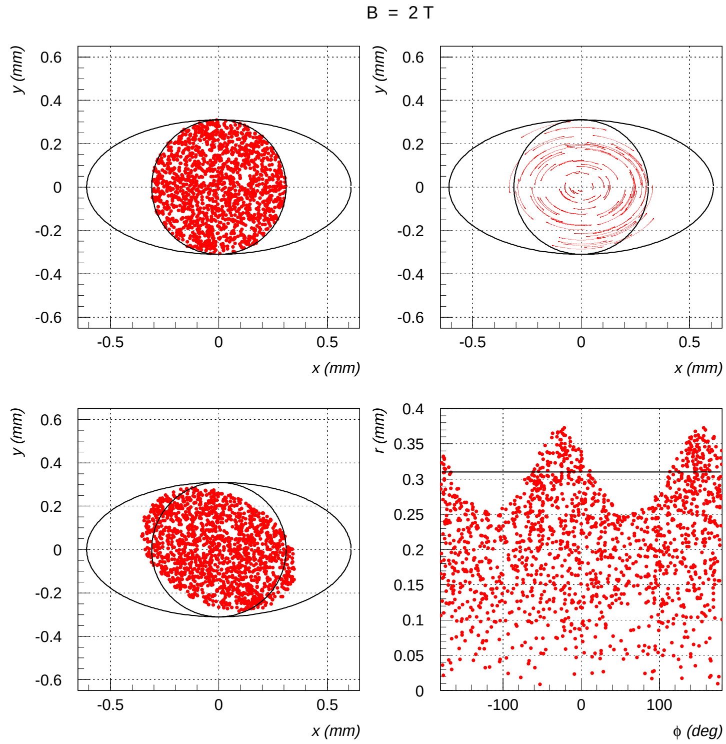

Figure 1: Narrow electron beam distortion due to $\bar{p}$ bunch.

$$I(x,y) = \int_0^\infty dq \frac{e^{-\frac{x^2}{2\sigma_x^2(1+qR)} - \frac{y^2}{2\sigma_y^2(1+q/R)}}}{(1+qR)^{3/2}(1+q/R)^{3/2}},$$

$$I(0,0) = \frac{2R}{(1+R)^2}, \quad R = \frac{\sigma_y}{\sigma_x}. \qquad (2)$$

Now we can see major features of the resulted distortion: a) it is absent in the case of round $\bar{p}$ beam when $\sigma_x = \sigma_y$; b) it performs two variations over azimuth $\delta\rho \propto xy \sim sin(2\theta)$; 3) it vanishes with the solenoid field B increase, or with decrease of antiproton intensity $N_{\bar{p}}$; 4) most of the distortion takes place at the radial edge of the electron beam, and, since $d\rho_0(r^2)/d(r^2) \simeq \rho_0^{max}/a_e^2$, then wider electron beam gets smaller density distortions during the interaction. Finally, the scaling of the maximum distortion strength is:

$$\frac{\delta\rho^{max}}{\rho_0^{max}} \sim \frac{0.2eN_{\bar{p}}}{a_e^2 B} \approx \frac{0.6[N_{\bar{p}}/6 \cdot 10^{10}]}{a_e^2[mm]B[kG]}, \qquad (3)$$

and value of 0.2 comes from geometrical factor $\propto xy \cdot I(x,y)$. For example, the distortion is about 3% for 1 mm radius electron beam in $B = 20$kG=2T solenoid field. As soon as the elliptic distortion is excited, it starts the rotation drift in the crossed fields of the electron space charge and the solenoid field. Under conditions of the Tevatron beam-beam compensation setup the rotation is small. E.g., one gets that over time of the $\bar{p}$ bunch passage $\pm 2\sigma_z/c =$ 2ns, the angle in the field of $B = 20$kG is about $\theta_d \approx 4j\sigma_z a_e/\beta_e B \approx 0.1$rad.

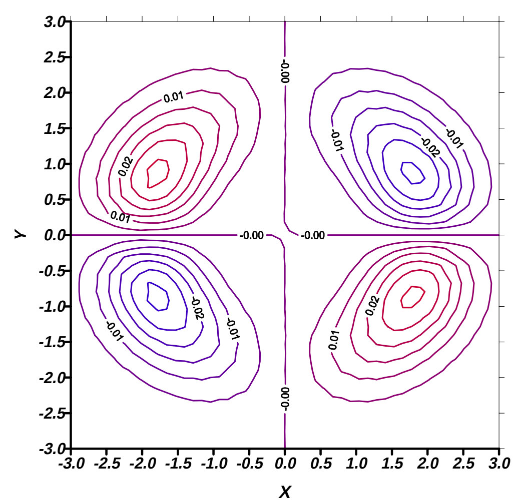

Figure 2: Distortions $\delta\rho/\rho$ of an electron beam wider than antiproton beam $a_e = 2.5 \cdot \sigma_x$. x and y coordinates are given in units of σ_x.

Distortion of other than constant electron density can be calculated analytically with use of Eq.(1). For example, Fig.2 shows contour lines of the electron beam distortion $\delta\rho/\rho$ in the case when initial density is $\rho_0(r) = \frac{1}{1+(r/a_e)^{2\mu}}$, $\mu = 3$, with $a_e = 2.5$ mm, $\sigma_x = 0.61$ mm, $\sigma_y = 0.31$ mm), the antiproton bunch population $N_{\bar{p}} = 6 \cdot 10^{10}$, the magnetic field $B = 4$kG. Resulted distortion $\delta\rho/\rho$ is now less than 0.05.

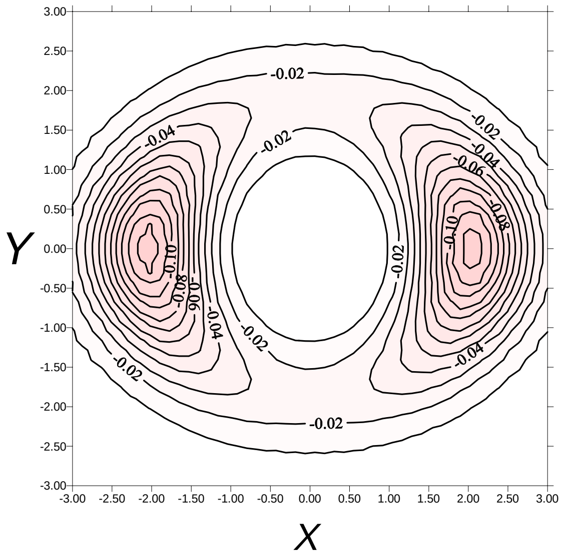

Figure 3: Coupling function $S(x, y)$ for antiproton betatron oscillations with a wide electron beams.

Coupling due to distorted electron beam. Electric and magnetic fields of the elliptic electron beam lead to effective $x - y$ coupling of vertical and horizontal betatron oscillations in the $\bar{p}$ beam. Since originally the electron beam is round, the head of the $\bar{p}$ bunch experiences no coupling force. But, as the electron density distortion grows as $\int^z \lambda(z')dz'$, the coupling grows proportionally. Particles in the head and in the tail of the bunch change their positions while performing synchrotron oscillations, thus, an average coupling effect is half of the maximum coupling spread. The average coupling can be corrected in the Tevatron, while there are no tools to compensate the spread in coupling. Therefore, the spread has to be small enough in order not to affect $\bar{p}$ beam dynamics.

The tunes of a small amplitude particle can be written as

$$\nu_{\pm} = \frac{\Big[(\nu_x + \Delta\nu_x) + (\nu_y + \Delta\nu_y)\Big]}{2} \pm$$

$$\pm\sqrt{\frac{\Big[(\nu_x + \Delta\nu_x) - (\nu_y + \Delta\nu_y)\Big]^2}{4} + |\kappa + \Delta\kappa|^2}, \quad (4)$$

where ν_x and ν_y are the unperturbed horizontal and vertical tunes, in the current Tevatron lattice they are 0.585 and 0.575 correspondingly. κ is a complex number describing the coupling. For a satisfactory operation of the Tevatron collider, the global coupling is corrected down to value of $|\kappa| \approx 0.001$ [5]. Δ's in Eq.(4) represent the changes of these quantities that arise from the interaction with the electron beam. E.g., the horizontal tune shift due to electron lens is about $\Delta\nu_x = -0.01$. The maximum coupling shift can be calculated from Eq.(1):

$$|\kappa| \approx |\Delta\nu_x| \frac{eN_{\bar{p}}}{2\sqrt{3}\sigma_x^2 B} \cdot < S(x, y) > \approx$$

$$\approx \frac{0.84[N_{\bar{p}}/6 \cdot 10^{10}]}{\sigma_x^2[mm]B[kG]} \cdot < S(x, y) > . \quad (5)$$

Brackets $< \dots >$ denote averaging over antiproton betatron oscillations. In the above equation we used an approximate relation $\beta_x \approx 3\beta_y$. Fig.3 shows numerical factor $S(x, y)$ for electron distributions $\rho_0(r) = \frac{1}{1+(r/a_e)^{2\mu}}$, $\mu =$ 3,with $a_e = 2.5\sigma_x$. The maximum value of this factor of $S^{max}(x, y)$ =0.13. The coupling vanishes for small betatron amplitude particles and at very large amplitudes. The effect is larger in the plane of the longer antiproton ellipse axis (horizontal in our case).

For example, having parameters $\sigma_x = 0.61$mm, $N_{\bar{p}} = 6 \cdot 10^{10}$, $\Delta\nu_x \simeq 0.01$ we get maximum numerical factor is about $< S(x, y) >^{max} \approx 0.5 \cdot S^{max}(x, y)$, i.e. 0.065 for $a_e = 2.5\sigma_x$. Now, with solenoid field of $B = 2$T, one gets the maximum coupling spread $|\kappa| \simeq 7 \cdot 10^{-5}$ for the electron beam. The value is rather small with respect to the Tevatron global coupling correction goal of about 0.001.

3 CONCLUSION.

We have considered distortions of the electron beam density due to interaction with an antiproton beam in the beam-beam compensation set-up. For a given number of antiprotons per bunch, both the electron density distortion and the coupling due to the distortion are inversely proportional to the magnetic field B in the set-up. In the case when the transverse sizes of the electron and antiproton beams are about the same (as it should be in the nonlinear electron lens for compensation of nonlinear beam-beam effects in the Tevatron), one needs more than 2T longitudinal field to keep the electron charge distribution distortions less than 3% with respect to original axisymmetric distribution and the spread of the coupling $|\kappa|$ less than $4 \cdot 10^{-4}$. Electron beam in the linear electron lens (for compensation of the bunch-to-bunch tune spread) has to be 2-3 times wider than the antiproton beam, and because of that in the same field of $B = 2$T the distortion $\delta\rho/\rho$ is smaller, less then 1%, and the coupling spread in the antiproton bunch becomes negligible $|\kappa| < 7 \cdot 10^{-5}$.

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4 REFERENCES

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