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We discuss the four-dimensional cosmological constant problem in a five-dimensional setting. A scalar field coupled to the SM forms dynamically a smooth brane with four-dimensional Poincaré invariance, independently of SM physics. In this respect, our solution may be regarded as a self-tuning solution, free of any singularities and fine-tuning problems.

Among the long-standing problems in physics, a central one, from a theoretical point of view, is the lack of any understanding of the smallness of the cosmological constant[1] . The cosmological constant predicted from a field theory calculation of the vacuum energy for a cutoff at the Planck mass M_P, is of an enormous $\Lambda \sim M_P^2$ size, more than 120 orders of magnitude from the observed bounds[2] . Supersymmetry does not provide a solution to the problem, although it would predict a vanishing cosmological constant in case it were an exact symmetry. However, after supersymmetry breaking, a non-zero cosmological constant $\sim M_{SUSY}^4/M_P^2$ has to arises, still far beyond any realistic value.

Recently, new proposals of relaxing the cosmological constant to zero or to a really small value have been put forward within the brane framework. In the brane set up our universe is modeled as a hypersurface embedded in a higher dimensional continuum. The idea of a *"wall-world"* is not new[3,4] and in modern language is realized by *D-branes*. These arise in String Theory and are extended stable objects on which open strings can end. Standard Model physics is confined on the brane whereas gravity propagates in the bulk. Nevertheless, bulk propagation of gravity is in contradiction with the observed fact of four-dimensional gravitation satisfying an inverse-square Newton's law. There have been a number of proposals in trying to isolate a four-dimensional graviton. After succeeding in localizing a massless graviton on the brane,[5] while massive modes introduce a small correction to Newton's law, the idea of a brane-resolution of the cosmological constant problem[6−18] has been put forward. One of the proposals is the *self-tuning* one, which involves sets of background solutions with 4D Poincaré invariance for arbitrary values of the tension of the Standard Model brane. However, these solutions, existing for a restricted form of bulk interactions, are singular. The existence of naked singularities and their connection to fine-tuning makes the whole proposal questionable[19−22] . In fact, regarding the bulk scalar employed in the self-tuning models as a KK scalar, the 5D solution can be lifted[23] to 6D. There, one observes that although the metric is regular, there is a global conical singularity and its resolution requires indeed fine-tuning.

The purpose of this work is to suggest a solution to the cosmological constant problem in the spirit of self-tuning proposal without however the drawbacks of singularities and/or fine-tunings. This is achieved by the dynamical formation of the 4D brane by a bulk five-dimensional scalar. The fact that bulk scalar fields with non-trivial potential can lead to smooth backgrounds with the desired localization properties has been illustrated in reference 24. In the present article we argue that by introducing the appropriate 5D coupling of the SM to the brane-forming scalar, not only localization of the SM on a brane is achieved, but in addition, any contribution to the 4D cosmological constant related to the SM physics can be neutralized leading to a 4D Poincaré invariant background.

Let us now consider the five-dimensional action

$$\mathcal{S} = \int d^5x\sqrt{-G}\left(2M^3R - \frac{1}{2}(\partial\phi)^2 - \mathcal{L}_m J(\phi)\right)\,, \qquad (1)$$

where $\mathcal{L}_m$ is the five-dimensional matter Lagrangian that gives rise to the four-dimensional Standard Model Lagragian after localization. It is in general a functional of gauge, Higgs and fermion fields (collectively denoted by χ_i) such that $\mathcal{L}_m = \mathcal{L}_m(\chi_i)$. The equations of motion resulting from the action (1) are

$$R_{MN} - \frac{1}{2}G_{MN}R = \frac{1}{4M^3}\left(\partial_M\phi\partial_N\phi + \frac{\delta\mathcal{L}_m}{\delta G_{MN}}J(\phi) - G_{MN}\left(\frac{1}{2}(\partial\phi)^2 + U(\phi)\right)\right)\,, \qquad (2)$$

$$\frac{1}{\sqrt{-G}}\partial_M\left\{\sqrt{-G}G^{MN}\partial_N\phi\right\} = \frac{\partial U}{\partial\phi}\,, \qquad (3)$$

$$\frac{\delta}{\delta\chi_i}\left(\sqrt{-G}\mathcal{L}_m J(\phi)\right) = 0\,, \qquad (4)$$

where

$$U(\phi) = J(\phi)\mathcal{L}_m\,, \qquad (5)$$

is the potential for the scalar ϕ. In what follows a capital index like M stands for $0,1,2,3,4$ while $\mu = 0,1,2,3$.

The ordinary four coordinates will be represented by x^μ while for the fifth coordinate we shall use the symbol y. In looking for four-dimensional Poincaré invariant solutions, the most general form of the metric respecting this symmetry is

$$ds^2 = e^{2A(y)}\left(-dt^2 + dx_1^2 + dx_2^2 + dx_3^2\right) + dy^2\,, \qquad (6)$$

while we shall consider the scalar ϕ to be only y-dependent. In addition, since we are looking for vacuum solutions, we put all fields in $\mathcal{L}_m$, as usual, to zero except the Higgs scalars H. In this case, $\mathcal{L}_m = -V_{\rm eff}(H)$ and eq.(4) is just the extremality condition $\partial V_{\rm eff}/\partial H = 0$ of $V_{\rm eff}$. We shall denote the value of $\mathcal{L}_m$ at the extremum as V_0. Then, the equations eqs.(2,3) become equivalent to the pair

$$\frac{1}{2}(\phi')^2 - U_0(\phi) = 24M^3(A')^2\,, \qquad (7)$$

$$\frac{1}{2}(\phi')^2 + U_0(\phi) = -12M^3A'' - 24M^3(A')^2\,, \qquad (8)$$

with

$$U_0(\phi) \equiv V_0 J(\phi)\,. \qquad (9)$$

All Standard Model physics is contained in the parameter V_0. The system of equations (7,8) will, presumably, produce as a solution the pair A, ϕ as functions of y. Equivalently, we can consider the pair of pair A, ϕ' *as functions of* ϕ. Then, since $A' = \phi'\frac{\partial A}{\partial\phi}$, we can always write

$$A' = -\frac{W(\phi)}{12M^3}\,, \qquad (10)$$

where $W(\phi)$ is a function of ϕ defined by this equation and called *superpotential*[25] . Subtracting equations (7,8) leads us to an expression of the potential in terms of the superpotential

$$U_0(\phi) = \frac{1}{2}\left(\frac{\partial W}{\partial\phi}\right)^2 - \frac{W^2}{6M^3}\,, \qquad (11)$$

The sum of eqs.(7,8), gives us

$$\phi' = \frac{\partial W}{\partial\phi}\,. \qquad (12)$$

Note that the above form of the potential is not an assumption but rather a necessary condition for smooth solutions to exist. This restriction ceases to hold for more than one fields. Thus, in our case, finding an appropriate solution is translated into choosing an appropriate superpotential W. Taking the trial choice

$$W = \gamma\sin(\beta\phi)\,,$$

we obtain the sine-Gordon form, also employed elswhere[26,27] of the potential[1]

$$U_0 = \frac{\gamma^2\beta^2}{2}\left(1 - g^2\sin^2(\beta\phi)\right)\,. \qquad (13)$$

We have introduced

$$g^2 = 1 + \frac{1}{3M^3\beta^2}\,. \qquad (14)$$

It can easily be seen that the potential (13) is of the form (9) with $V_0 = \frac{1}{2}\gamma^2\beta^2$ and $V_0 > 0$. We could alternatively have started with a potential $U_0(\phi) = V_0\left(1 - g^2\sin^2(\beta\phi)\right)$. Then, we would have the superpotential form (11) of the potential, and, therefore, a solution, only when β and g^2 would be restricted by (14). It should be stressed however that eq.(14) *is totally independent of the Standard Model Physics represented by* V_0*. It is not, therefore, a fine-tuning but a restriction in a subspace of the bulk-parameter space of* β*,* g^2*.*

Choosing V_0 and g^2 as our parameters, we can proceed to solve for $\phi(y)$ and $A(y)$. The scalar field solution can be obtained from eq.(12). It has the known kink-like form

$$\phi = \frac{2}{\beta}\arctan\Big(\tanh(ay/2)\Big)\,, \qquad (15)$$

where $a^2 \equiv 2V_0\beta^2 = \frac{2V_0}{3M^3(g^2-1)}$. Note the restriction $g^2 > 1$. Similarly, we can solve for $A(y)$ eq.(10) by using (15) and we obtain the *warp function*

$$A(y) = -\frac{(g^2-1)}{8}\ln\cosh^2(ay)\,. \qquad (16)$$

The background geometry is then described by the metric[2]

$$ds^2 = \{\cosh(ay)\}^{(1-g^2)/2}\,\eta_{\mu\nu}dx^\mu dx^\nu + dy^2\,. \qquad (17)$$

The calculation of all curvature invariants like R and $R_{MN}R^{MN}$, demonstrates that the geometry is nowhere singular for $g^2 > 1$. For every pair of values V_0, $g^2 > 1$ there exists a smooth geometry given by (16) and a smooth bounce-like field configuration given by (15). Quantum SM effects may change the value of V_0 but there will always be a solution corresponding to the modified value without any effect on the bulk parameter g^2.

[1]Note the symmetry of the potential $\phi \to \phi + 2\pi n/\beta$ for integer n. A potential of this form is expected to arise in theories of antisymmetric tensor fields.[28]

[2]The four-dimensional Planck mass has a finite value

$$M_P^2 = 4M^3\xi^{-1}\sqrt{\pi}\frac{\Gamma\left((g^2+3)/4\right)}{\Gamma\left((g^2+1)/4\right)}$$

We give below in fig.1 a graph of the solution for some special values of the parameters.

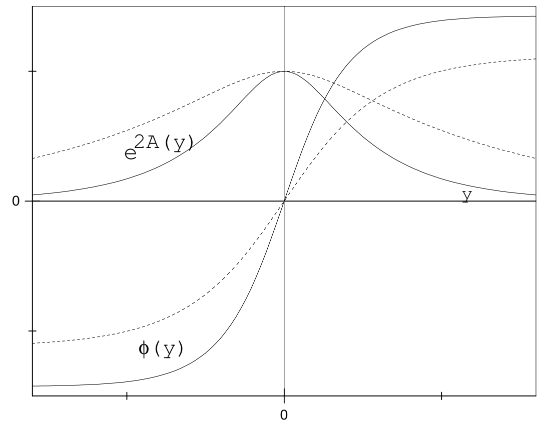

Fig. 1: The functions $e^{A(y)}$, $\phi(y)$ for $V_0 = 1\,, \beta = 1.1$ (solid lines) and $V = 0.25\,, \beta = 1.4$ (dashed lines).

There is a *brane-limit* to the smooth solution found above defined as $g^2 \to 1 \;\; , \;\; 2V_0(g^2-1)/3M^3 \equiv \xi^2 < \infty$. In this limit the warp function takes the Randall Sundrum[5] form $A(y) \to -\frac{\xi}{4}|y|$. If we substitute our solution to the bulk matter action in this limit, we obtain

$$\frac{1}{2}(\phi')^2 + U_0(\phi) \to -\frac{3}{2}M^3\xi^2 + 12M^3\xi\delta(y)\,.$$

ξ is by definition positive. Thus, in this limit the sine-Gordon bounce behaves as a brane of tension $12M^3\xi$ placed at $y=0$. It should be remarked that both the bulk kinetic term $(\phi')^2/2$ and the potential $U_0(\phi)$ contribute equally to the brane term. Localization of SM fields will not be automatic. Massless scalars and chiral fermions in the SM Lagrangian are going to be localized by the e^{2A} warp factor, while for gauge bosons a separate localization mechanism will be required.[24]

The coupling function $J(\phi)$ employed above, changes sign away from the *brane* at $y=0$. Although this is not something that one should necessarily worry about, since SM fields will somehow get to be localized, it is important to show that we can replace the coupling function with one of constant sign. We can consider the superpotential

$$W = \frac{\beta}{4\alpha^2}\left(2\alpha\phi + \sin(2\alpha\phi)\right)\,, \tag{18}$$

that leads to $\phi = \frac{1}{\alpha}\arctan(\beta y) + 2\pi n/\alpha$, where n is an arbitrary integer, and to the perfectly localizing warp function

$$A(y) = -\mu y\arctan(\beta y)\,, \tag{19}$$

with $\mu = \beta/24M^3\alpha^2$. The potential $U_0(\phi)$ resulting from $W(\phi)$ is

$$\frac{\beta^2}{8\alpha^2}\left\{\left(1+\cos(2\alpha\phi)\right)^2 - \frac{1}{12M^3\alpha^2}\left(2\alpha\phi + \sin(2\alpha\phi)\right)^2\right\}\,. \tag{20}$$

Note that this potential does not posses the symmetry $\phi \to \phi + n\pi/\alpha$ present in the previous example. Expressing U_0 as a function of y shows that for a sufficiently large value of n the potential will be always negative and thus, the SM Lagrangian has the correct sign. A detailed study of the model with the potential of eq.(20) will be given elsewhere.[29]

Although we have constructed a flat solution with four-dimensional Poincaré invariance, we have not finished yet. We have to make sure that a localized four-dimensional massless graviton exists. After all, the cosmological constant is a problem as long as gravity is present. For this, let us consider a perturbation around the previously described solution of the form

$$\delta G_{MN} = \delta^\mu_M \delta^\nu_N h_{\mu\nu}(x,y) \quad , \quad \delta\phi = 0\,. \tag{21}$$

$h_{\mu\nu}$ represents the graviton in the axial gauge defined by the constraint $h_{5M} = 0$. We are interested on the transverse modes and, therefore, we shall assume $h^\mu_\mu = \partial_\mu h^{\mu\nu} = 0$. The equations give, to first order in $h_{\mu\nu}$,

$$\left\{-\frac{1}{2}\frac{\partial^2}{\partial y^2} + \left(A'' + 2(A')^2\right) - \frac{1}{2}e^{-2A(y)}\partial^2\right\} h_{\mu\nu}(x,y) = 0\,,$$

where $\partial^2 \equiv \eta^{\mu\nu}\partial_\mu\partial_\nu$. Introducing a trial solution in the form of a product of an ordinary-space plane wave times a *bulk wave function* $h_{\mu\nu} = e^{ip\cdot x}\psi_{\mu\nu}$, we get the *Schröedinger-like* equation

$$\left\{-\frac{1}{2}\frac{d^2}{dy^2} + \left(A'' + 2(A')^2\right)\right\}\psi(y) = \frac{m^2}{2}e^{-2A(y)}\psi(y)\,.$$

We have dropped the spacetime indices and introduced the mass $m^2 = -p^2$. The existence of localized graviton in ordinary space amounts to the existence of a normalizable localized bound state of this equation at zero energy $m^2 = 0$ (zero-mode). It is not difficult to see that indeed such a zero mode exists. It has the wave-function

$$\psi_0(y) = e^{2A(y)} = \left\{\cosh\left(\frac{\xi y}{g^2-1}\right)\right\}^{-(g^2-1)/2}\,. \tag{22}$$

In order to study the massive spectrum we must transform the above equation into a conventional Schröedinger equation. This can be done with the help of the transformation $y \to z = f(y) \;\; , \;\; \psi(y) = \Lambda\overline{\psi}$. Demanding the absence of first derivative terms and a standard constant coefficient m^2 in the right hand side, we arrive at $\Lambda = e^{A/2} \;\; , \;\; f' = e^{-A}$. The resulting Schröedinger equation is

$$\left\{-\frac{1}{2}\frac{d^2}{dz^2}+\overline{U}(z)\right\}\overline{\psi}(z)=m^2\overline{\psi}(z)\,, \tag{23}$$

while the potential $\overline{U}(z)$ is

$$\overline{U}(z)=\frac{3}{8}e^{2A}\left(2A^{''}+5(A')^2\right)=\frac{3}{4}\left(\ddot{A}+\frac{3}{2}(\dot{A})^2\right)\,, \tag{24}$$

and the dots denote derivatives with respect to z. Note that the Schröedinger equation has the form corresponding to supersymmetric Quantum Mechanics

$$\mathcal{Q}^\dagger\mathcal{Q}\overline{\psi}=\left\{-\frac{d}{dz}-\frac{3}{2}\dot{A}\right\}\left\{\frac{d}{dz}-\frac{3}{2}\dot{A}\right\}\overline{\psi}=2m^2\overline{\psi}\,. \tag{25}$$

This form clearly excludes the possibility of tachyonic states. Nevertheless, in order to know whether there is a gap in the continuum spectrum we need to know the asymptotic behaviour of the potential. It is possible to argue that since $\lim_{y\to\infty}\{z\}\propto e^{\xi|y|/4}$, $\lim_{z\to\infty}\left\{\overline{U}(z)\right\}=\lim_{y\to\infty}\left\{\overline{U}(z(y))\right\}\propto e^{2A}\to 0$ and, as expected, there is no gap and the continuous spectrum starts from zero energy. The situation is entirely analogous to the Randall-Sundrum case and one could repeat the same arguments with respect to massive graviton excitations. As a result the corrections to Newton's law have the same, adequately suppressed, form $V(r)\propto\frac{1}{r}(1+O(r^{-2}))$.

Summarising our results, we have considered a five-dimensional theory of a scalar coupled to gravitation with a restricted coupling to the Standard Model. We have obtained classical Poincaré-invariant solutions describing a localized smooth geometry which is everywhere finite. There is a limit in which the warp function of the metric tends to the Randall-Sundrum form and the scalar field duplicates a brane of positive tension. The solutions depend on two parameters, a dimensionless one g^2 and the dimensionfull parameter V_0 that is related to Standard Model physics. For any quantum change in V_0 (f.e. quantum vacuum energy corrections induced by a symmetry-breaking phase transition) there is always a solution corresponding to the new value of V_0. Thus, there is no fine-tuning involved in order to obtain a flat four-dimensional solution. In that aspect, the solutions found display self-tuning and resolve the four-dimensional cosmological constant problem. However, in contrast to the recently proposed self-tuning solutions, they have no singularities. Nevertheless, they do involve restrictions on the bulk interactions. These restrictions, however, are totally independent of Standard Model physics. Another crucial difference to the singular self-tuning solutions is also the fact that the scalar field involved is not an extra bulk scalar field but the field that forms the brane itself.

K.T. acknowledges partial support by the European Union TMR program with contract No. ERB FMRX-CT96-0090. A. K. acknowledges partial support by the RTN programmes HPRN-CT-2000-00122 and HPRN-CT-2000-00131 and the ΓΓΕΤ grant ΕΛ/71.

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