arXiv:hep-ph/0011017v1 1 Nov 2000

CERN-TH/2000-327

WEAK DECAYS OF PSEUDO GOLDSTONE BOSONS*

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A short overview of recent progress in describing kaon and pion decays within Chiral Perturbation Theory is presented. Particular attention is devoted to the issues of final-state interactions and isospin breaking in $K \to 2\pi$, as well as to the estimate of long-distance contributions in $K \to \ell^+\ell^-(\pi)$.

1 Introduction

The weak decays of light pseudoscalar mesons, namely the decays of kaons and charged pions, represent the widest domain of applicability of Chiral Perturbation Theory (CHPT). The field is certainly dominated by kaon decays, with about 50 different channels observed so far, which constitute by themselves one of the richest sources of information about the Standard Model as a whole.

The interest and the variety of such decays is definitely too large to be covered in one talk. I will therefore concentrate on few selected topics, not covered by other plenary speakers, among those where interesting work has recently been done. In the next section I will review the development in the construction of chiral Lagrangians without baryon fields. Two *hot topics* in $K \to 2\pi$ decays, namely the issues of final-state interactions and isospin breaking, will be discussed in section 3. Section 4 is devoted to the rare processes $K \to \pi\ell^+\ell^-$ and $K_L \to \ell^+\ell^-$, and the last section contains some concluding remarks.

2 Developments in the construction of chiral Lagrangians

Kaon and charged-pion decays are usually classified into two large categories: (semi-)leptonic (including all pion decays) and non-leptonic. As long as elec-

*To appear in the proceedings of *Chiral Dynamics 2000*, Newport News, USA, 17-22 July 2000.
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tromagnetic interactions are neglected, the two categories have very different properties from the point of view of chiral dynamics:

- In semileptonic transitions, where the lepton pair interacts with the mesons only via the W boson, the latter can be considered as an external field. These transitions can therefore be simply described in terms of the *strong* generating functional of CHPT, with an appropriate identification of the external sources.[1]

- In non-leptonic decays, where quarks lines are attached to both vertices of the W propagator, the latter cannot be considered as an external field. These processes can be described within CHPT introducing a new generating functional, that transforms linearly under chiral rotations as the four-quark $|\Delta S| = 1$ effective Hamiltonian.[2]

The structure of both functionals has been known up to $O(p^4)$, i.e. to the next-to-leading order, for several years, allowing a systematic inclusion of meson–meson interactions up to one loop in both types of processes.[1,2] The number of low-energy couplings (LECs) appearing at $O(p^4)$ is apparently large: 12 in the strong sector and 37 in the octet part of the weak non-leptonic functional. Nonetheless the theory still has a good predictive power since only few LEC combinations appear in physical processes.[3] More recently, also the $O(p^6)$ structure of the strong generating functional has been determined,[4] leading to a systematic inclusion of mesonic two-loop effects in semileptonic processes. The total number of $O(p^6)$ strong LECs is definitely large (about one hundred), but a few accurate predictions can still be made concerning specific observables.[5]

The distinction between semileptonic and non-leptonic processes becomes less clear when electromagnetic interactions are taken into account. Indeed the W boson can no longer be considered as an external source if the charged lepton interacts electromagnetically with the pseudoscalar fields. On the other hand electromagnetic corrections to the leading $O(p^2)$ terms cannot be neglected with respect to the purely mesonic two-loop effects. As we shall discuss in section 4, electromagnetic interactions play an even more important rôle in non-leptonic transitions, where they could compete already with the $O(p^2)$ terms in $\Delta I = 3/2$ transitions.

The first step toward a systematic inclusion of electromagnetic effects has been undertaken almost five years ago,[6] with the determination of the $O(e^2p^2)$ local terms needed to regularize the one-loop divergences induced by virtual photons in the strong sector. This step, however, was not sufficient for a complete treatment of $O(e^2p^2)$ effects, be it in semileptonic or in non-leptonic decays. Interestingly, the missing ingredients for this program have recently become available: Knecht *et al.*[7] have determined the additional divergences (and associated counterterms) generated at one loop in the strong sector by the presence of virtual leptons. Finally the effect of virtual photons in non-leptonic $|\Delta S| = 1$ transitions has been taken into account in a systematic way up to $O(e^2 p^2)$.[8,9]

Given this situation, we can state that the structure of chiral Lagrangians is known, at present, to a very good accuracy for almost all decays of phenomenological interest. Nonetheless this is only one side of the problem, the other being the determination of the LECs. As I will discuss in the following, substantial improvements are still needed in the latter direction in order to match the potential degree of precision allowed by the present knowledge of chiral Lagrangians.

3 $K \to 2\pi$

Neglecting isospin-breaking (and electromagnetic) effects, $K \to 2\pi$ amplitudes can be decomposed as follows

$$
\begin{aligned}
A(K^0 \to \pi^+\pi^-) &= A_0 e^{i\delta_0} + \frac{1}{\sqrt{2}} A_2 e^{i\delta_2} \ , \\
A(K^0 \to \pi^0\pi^0) &= A_0 e^{i\delta_0} - \sqrt{2} A_2 e^{i\delta_2} \ , \\
A(K^+ \to \pi^+\pi^0) &= \frac{3}{2} A_2 e^{i\delta_2} \ ,
\end{aligned} \tag{1}
$$

where A_I are the weak amplitudes (real in the absence of CP violation) and δ_I denote the S-wave strong phases of the $\pi\pi$ system in isospin I. As is well known, the ratio $\omega = |A_2/A_0|$ is found experimentally to be very small ($\omega \sim 1/22$), and a precise dynamical explanation of this result (the $\Delta I = 1/2$ rule) is one of the most challenging open problems in understanding non-leptonic weak decays.

The effective $|\Delta S| = 1$ four-quark Hamiltonian contributing to these processes contains operators that transform as $(8_L, 1_R)$ and $(27_L, 1_R)$ under chiral rotations.[a] Both these structures have a unique chiral realization at $O(p^2)$, which leads to the following simple Lagrangian:[b]

$$
\mathcal{L}_W^{(2)} = G_8 F^4 \left[D_\mu U^\dagger D^\mu U \right]_{23}
$$

[a] Consistently with the assumption of neglecting isospin-breaking effects, at this level we can neglect the $(8_L, 8_R)$ term induced by electromagnetic interactions.
[b] Notation is as in D'Ambrosio *et al.*[3]

$$+ G_{27}F^4 \left[(U^\dagger \partial_\mu U)_{23}(\partial_\mu U^\dagger U)_{11} + \frac{2}{3}(U^\dagger \partial_\mu U)_{21}(\partial_\mu U^\dagger U)_{13} \right] + \text{h.c.} \quad (2)$$

The dimensional couplings G_8 and G_{27} are free parameters from the point of view of pure CHPT, which by itself is not expected (at least at the lowest order) to shed light on the origin of the $\Delta I = 1/2$ rule. The latter is phenomenologically implemented by fitting G_8 and G_{27} from $K \to 2\pi$ amplitudes:

$$|G_8|^{\text{exp}}_{\pi\pi} = 9.1 \times 10^{-6}\ \text{GeV}^{-2}\ , \qquad |G_{27}/G_8|^{\text{exp}}_{\pi\pi} \simeq 1/18\ . \quad (3)$$

Once G_8 and G_{27} have been fixed, CHPT becomes predictive and efficient in describing other processes, such as $K \to 3\pi$ decays.

The puzzle of the $\Delta I = 1/2$ rule arises in the attempt to derive (3) from the underlying SM dynamics. Indeed, in the strict $N_c \to \infty$ limit of QCD, when the dominant operators in the effective four-quark Hamiltonian have no anomalous dimension and their hadronic matrix elements can be factorized, one finds:

$$|G_8|^{\text{th}}_{N_c\to\infty} = |G_{27}|^{\text{th}}_{N_c\to\infty} = 1.1 \times 10^{-6}\ \text{GeV}^{-2}\ . \quad (4)$$

The situation slightly improves when QCD corrections to the Wilson coefficients of the effective four-quark Hamiltonian are taken into account. Evaluating the latter the leading order with a renormalization scale $\mu \sim 1$ GeV, and still employing factorized matrix elements, leads to an enhancement factor ~ 2 for G_8 and a suppression of around 0.7 for G_{27}. This estimate is not very precise, since the matrix elements of the four-quark operators ($\langle Q_i \rangle$) do not have, at this level, the correct scale dependence needed to match the one of the Wilson coefficients; nonetheless it shows that the bulk of the effect is still missing. In particular an enhancement factor of about 4 is still needed in the $\Delta I = 1/2$ amplitude. If the Wilson coefficients are evaluated at a scale $\mu \gtrsim 1$ GeV, condition necessary to trust their perturbative estimate, this enhancement can only be addressed to matrix elements that differ substantially from their factorized values.

3.1 Final-state interactions

The evaluation of hadronic matrix elements is a non-perturbative problem related mainly to low-energy dynamics. It is therefore natural to address the question whether CHPT can help to clarify it, once effects beyond $O(p^2)$ are taken into account.

An interesting suggestion in this direction has recently been proposed by Pallante and Pich,[10] following an older work by Truong.[11] Their proposal is based on the observation that the large $\pi\pi$ scattering phase in $I = 0$ signals a large final-state interaction (FSI) effect, not well described by CHPT at lowest order, where there is no absorptive contribution to the decay amplitudes. This observation is certainly correct, as it can be checked by the explicit $O(p^4)$ CHPT calculation of Kambor *et al.*.[12] There, it has been shown that pion loops provide a sizeable renormalization of G_8. The amount of this effect, however, cannot be unambiguously determined within CHPT alone, owing to the presence of $O(p^4)$ local operators with unknown couplings.[c] The further assumptions employed by Pallante and Pich in order to obtain a quantitative information about the $\langle Q_i \rangle$ are that i) FSI effects can be unambiguously resummed to all orders in CHPT using an Omnès factor,[11] ii) in some cases (most notably in the case of the operator Q_6 relevant to ε'/ε) this FSI effect constitutes the dominant correction with respect to the large N_c estimate of the matrix element.

The use of dispersion relations (and the corresponding Omnès solution) provides, in some cases, an efficient tool to resum FSI effects. However this is hardly justified in the context of $K \to \pi\pi$ amplitudes.[14] In order to apply this tool to $K \to \pi\pi$ one could try to treat m_K and $s = (p_{\pi_1} + p_{\pi_2})^2$ as two independent variables, but in this case one would have to deal with the (unknown) effect of operators that are eliminated by the equations of motion. Otherwise, identifying s and m_K^2, one should be worried by the impact induced on $\pi\pi$ phases by a variation of the kaon mass.

Besides the justification of this approach, an even more serious problem is posed by the fact that the Omnès solution does not completely determine the amplitudes. In the most optimistic case it can be used to relate the physical amplitudes to those obtained at a different value of s.[14] Not surprisingly, this problem is somehow equivalent to the issue of determining the local $O(p^4)$ terms within CHPT. Thus we are back to the starting point, unless we make the further assumption that large N_c provides a good estimate of some $\langle Q_i \rangle$ at $s = 0$. Again it is hard to find a justification of this hypothesis, in particular it is not clear at all whether large N_c should work better at $s = 0$ than at large s. The argument that chiral corrections are small at $s = 0$ does not help since one should rather be worried by the size of $1/N_c$ corrections. Moreover, if $s \neq m_K^2$, one could still expect sizeable chiral corrections driven by m_K rather than s that are potentially large at $s = 0$. Finally, we note that applying this procedure to all the $\Delta I = 1/2$ operators *does not* lead to reproducing the

[c] Assuming for instance that the effect of the local operators is negligible at a given CHPT renormalization scale μ_χ (not to be confused with the renormalization scale of the four-quark Hamiltonian), and varying this scale between 0.5 and 1 GeV, one finds that pion loops provide an enhancement of $A^{(4)}$ over $A^{(2)}$ that ranges between 20% and 80%.[13]

observed $|A_0|$. Indeed the FSI enhancement factor computed in this way[10] is only ~ 1.5, to be compared with the factor 4 needed by the data. It is thus clear that, at least in some cases, this procedure is incomplete.

Having pointed out these problems, it should also be stressed that the work of Pallante and Pich[10] had the merit of focusing the attention on a potentially large effect, sometimes ignored in the literature, which needs to be taken into account when trying to evaluate $K \to \pi\pi$ matrix elements of four-quark operators. I believe that a satisfactory solution to this problem can only be achieved within a self-consistent calculation of the full dispersive contribution to the $\langle Q_i \rangle$. Several attempts in this direction exist in the literature, using both analytical tools and lattice QCD. In the first category there have been encouraging results (obtained by means of different strategies) concerning the $\Delta I = 1/2$ rule[15] and the matrix elements of electroweak operators.[16] Nonetheless all the analytical methods are still far from having reached the precision needed to push the theoretical error on ε'/ε below 10^{-3} (assuming this is estimated in a conservative way…). In the long term, lattice QCD seems to be much more promising, especially in view of some recent theoretical developments in this field.[17]

3.2 Isospin-breaking effects

Violations of isospin symmetry are generated by the mass difference between up and down quarks and by electromagnetic interactions. The two types of effects are comparable in size and generally small in the K system ($\sim 1\%$). In $K \to 2\pi$ decays, however, these could be enhanced by a factor $1/\omega$ if an isospin-breaking (IB) correction to A_0 leads to a $\Delta I > 1/2$ transition. Thus, whereas we can safely neglect IB corrections proportional to G_{27}, and effects quadratic in $(m_d - m_u)$ or α, it is important to treat in a systematic way terms of $O[G_8(m_d - m_u)p^n]$ and $O(G_8 e^2 p^n)$.

Once IB effects are included, the amplitude decomposition (1) is no longer valid. In this case a useful parametrization is provided by[8]

$$
\begin{aligned}
A(K^0 \to \pi^+\pi^-) &= A_{1/2} e^{i(\delta_0+\gamma_0)} + \frac{1}{\sqrt{2}}(A_{3/2} + A_{5/2}) e^{i(\delta_2+\gamma_2)} \ , \\
A(K^0 \to \pi^0\pi^0) &= A_{1/2} e^{i(\delta_0+\gamma_0)} - \sqrt{2}(A_{3/2} + A_{5/2}) e^{i(\delta_2+\gamma_2)} \ , \\
A(K^+ \to \pi^+\pi^0) &= \left[\frac{3}{2} A_{3/2} - A_{5/2}\right] e^{i(\delta_2+\gamma_2')} \ ,
\end{aligned}
\tag{5}
$$

where again the A_i are real in the absence of CP violation.[d]

[d] In the CP-conserving case (5) provides the most general parametrization of three complex

As long as $O[G_8(m_d - m_u)p^n]$ effects are concerned, it is easy to show that both γ_i and $A_{5/2}$ are zero to all orders. Thus the main effect induced by $m_d \neq m_u$ is only a shift in $A_{3/2}$, usually parametrized by the ratio

$$\Omega_{IB} = \delta A_{3/2}/(\omega A_{1/2}). \tag{6}$$

At the lowest order, $O[G_8(m_d - m_u)p^0]$, only the π^0–η mixing in the strong Lagrangian ($\mathcal{L}_S^{(2)}$) contributes to Ω_{IB}, leading to the unambiguous result $\Omega_{IB}^{(2)} = 0.13$. Interestingly at this level Ω_{IB} is a universal correction factor, i.e. it applies independently to all the $\Delta I = 1/2$ matrix elements of four-quark operators. Also at $O[G_8(m_d - m_u)p^2]$ there is a contribution coming from the mixing on the external legs that is universal and calculable unambiguously from the strong Lagrangian. This has recently been evaluated[18] and, summed to the lowest-order result, leads to $\Omega_{IB}^{(4-mix)} = 0.16 \pm 0.03$. However this is not the full story at $O[G_8(m_d - m_u)p^2]$, since at that order there appear also contributions from $O(p^4)$ weak counterterms which are not universal and not known from data. At the moment these can only be estimated by means of model-dependent assumptions, and some recent analyses[19,20] indicate sizeable effects, comparable in size to the one of the leading-order term. It should also be noted that a positive Ω_{IB} worsens the problem of the $\Delta I = 1/2$ rule, indicating that in the isospin limit the ratio $|A_2/A_0|$ should be smaller than ω. It is thus more likely that the remaining $O[G_8(m_d - m_u)p^2]$ terms will decrease Ω_{IB}, rather than enhance it, in agreement with the recent findings.[19,20] At the moment, in the absence of precise estimates, what can be considered a conservative approach toward phenomenological analyses is the use of two independent Ω_{IB} for CP-conserving and CP-violating parts of the amplitudes, with a central value close $\Omega_{IB}^{(2)}$ and a $\sim 100\%$ error in both cases.

Given the absence of $O[G_8(m_d - m_u)p^n]$ contributions in $A_{5/2}$ and γ_i, these terms are expected to be mainly of electromagnetic origin. Present data show some evidence for these effects. Indeed fixing the phase difference $(\delta_0 - \delta_2)$ from $\pi\pi$ scattering, setting $\gamma_i = 0$ and fitting $K \to \pi\pi$ widths leads to extract a non-vanishing $\Delta I = 5/2$ amplitude:[21] $\Re(A_{5/2}/A_{1/2}) = -(7 \pm 2) \times 10^{-3}$. Interestingly this is of the correct order of magnitude, being $O(\alpha A_{1/2})$. At this point, however, an important warning should be made concerning the unclear treatment of soft radiation in the $K \to \pi\pi$ data available at present. This issue is very important in this context,[22] and it is highly desirable to have a clearer experimental information in this respect, together with more precise measurements of $K \to \pi\pi$ widths. Hopefully these should become available in the very near future from KLOE.[23]

numbers.

From the theoretical point of view, electromagnetic effects have a rather trivial structure at $O(G_8 e^2 p^0)$, where there are no contributions to $A_{5/2}$ and γ_i. At this level there are two contributions to Ω_{IB}, one from the $\pi^+ - \pi^0$ mass difference, the other from the lowest-order realization of a $|\Delta S| = 1$ operator transforming as $(8_L, 8_R)$. Only the former is calculable unambiguously, but the cancellation of quadratic divergences in the photon loops is a strong indication that the two contributions tend to cancel each other, leading to an overall small effect. This expectation is supported by a detailed analysis of Cirigliano *et al.*[24] and seems to indicate that Ω_{IB} is largely dominated by the $m_d \neq m_u$ effects discussed before.

The interesting aspects of electromagnetic effects appear at $O(G_8 e^2 p^2)$, with a non-vanishing $\Delta I = 5/2$ amplitude and also with the bremsstrahlung of the leading $O(G_8 e^0 p^2)$ terms.[22] As already mentioned, the structure of the local $O(G_8 e^2 p^2)$ operators and their anomalous dimensions has recently been analysed,[9] but a precise evaluation of their couplings is not available yet. Despite this uncertainty some interesting conclusion can still be drawn. For instance Cirigliano *et al.*[8] have been able to show that also at $O(G_8 e^2 p^2)$ electromagnetic contributions to Ω_{IB} are rather small (at the per cent level). On the other hand more work is needed to understand the size of the $\Delta I = 5/2$ amplitude, and new precision data on $K \to \pi\pi$ widths could be of great help in this direction.

4 Rare $K \to \ell^+\ell^-(\pi)$ decays

The rare processes $K \to \pi\ell^+\ell^-$ and $K_L \to \ell^+\ell^-$ are particularly interesting since they offer, at the same time, a new laboratory for understanding chiral dynamics and a rather clean window on the short-distance mechanism of flavour-changing neutral currents.

4.1 $K \to \pi\ell^+\ell^-$

The single-photon exchange amplitude ($K \to \pi\gamma^* \to \pi\ell^+\ell^-$) is largely dominant in these channels when allowed by CP invariance. This occurs in the charged modes, experimentally observed for both $\ell = e$ and $\ell = \mu$, and in the decays of the K_S. This long-distance amplitude can be described in a model-independent way in terms of two form factors, $W_+(z)$ and $W_S(z)$, defined by[25]

$$i \int d^4x e^{iqx} \langle \pi(p) | T \left\{ J^\mu_{\rm elm}(x) \mathcal{L}_{\Delta S=1}(0) \right\} | K_i(k) \rangle =$$

$$\frac{W_i(z)}{(4\pi)^2}\left[z(k+p)^\mu - (1-r_\pi^2)q^\mu\right] \ , \tag{7}$$

where $q = k - p$, $z = q^2/M_K^2$ and $r_\pi = M_\pi/M_K$. The two form factors are non-singular at $z = 0$ and, because of gauge invariance, vanish to $O(p^2)$.[26] Beyond lowest order one can identify two separate contributions to the $W_i(z)$: a non-local term, $W_i^{\pi\pi}(z)$, due to the $K \to 3\pi \to \pi\gamma^*$ scattering, and a local term, $W_i^{\rm pol}(z)$, that encodes contributions of the LECs. At $O(p^4)$ the local term is simply a constant, whereas at $O(p^6)$ also a term linear in z arises. Already at $O(p^4)$ chiral symmetry alone does not help to relate W_S and W_+, or K_S and K^+ decays.[26]

Recent results[27] on $K^+ \to \pi^+ e^+ e^-$ and $K^+ \to \pi^+ \mu^+ \mu^-$ by E865 indicate very clearly that, owing to a large linear slope, the $O(p^4)$ expression of $W_+(z)$ is not sufficient to describe experimental data. This should not be considered as a failure of CHPT, rather as an indication that large $O(p^6)$ contributions are present in this channel. Indeed the $O(p^6)$ expression of $W_+(z)$ fit the data very well; this is not only due to the presence of a new free parameter, but also to the presence of the non-local term. The evidence of the latter provides a real significant test of the chiral approach. It should also be stressed that the appearance of a large $O(p^6)$ correction in this channel can be qualitatively understood in terms of vector-meson exchange. A quantitative test of this hypothesis could be performed with the observation of the $K_S \to \pi^0 e^+ e^-$ decay, whose branching ratio is expected in the 10^{-9}–10^{-8} range.[e]

In $K_L \to \pi^0 e^+ e^-$ the long-distance part of the single-photon exchange amplitude is forbidden by CP invariance, but it contributes to the processes via K_L–K_S mixing, leading to

$$B(K_L \to \pi^0 e^+ e^-)_{\rm CPV-ind} \ = \ 3 \times 10^{-3} \ B(K_S \to \pi^0 e^+ e^-) \ . \tag{8}$$

On the other hand, there is also a sizeable direct-CP-violating contribution to this channel that is dominated by short-distance dynamics.[29,30] Within the SM, this theoretically clean part of the amplitude leads to[30]

$$B(K_L \to \pi^0 e^+ e^-)^{\rm SM}_{\rm CPV-dir} \ = \ (2.5 \pm 0.2) \times 10^{-12} \ \left[\frac{\Im(V_{ts}^* V_{td})}{10^{-4}}\right]^2 , \tag{9}$$

where V_{ij} denote the elements of the Cabibbo–Kobayashi–Maskawa matrix. The two CP-violating components of the $K_L \to \pi^0 e^+ e^-$ amplitude will in general interfere with a relative phase that is known (the phase of ε). Thus if $B(K_S \to \pi^0 e^+ e^-)$ will be measured, it will be possible to determine the interference between direct and indirect CP-violating components of $B(K_L \to$

[e] An experimental upper limit of 1.6×10^{-7} has recently been obtained by NA48.[28]

$\pi^0 e^+ e^-$) up to a sign ambiguity. Given the present uncertainty on $B(K_S \to \pi^0 e^+ e^-)$ we can only, for the moment, set a rough upper limit of few$\times 10^{-11}$ on the sum of all CP-violating contributions to this mode,[25] which is almost one order of magnitude below the recent experimental upper bound by KTeV.[31]

An additional contribution to $K_L \to \pi^0 \ell^+ \ell^-$ decays is generated by the CP-conserving amplitude $K_L \to \pi^0 \gamma\gamma \to \pi^0 \ell^+ \ell^-$.[32] Here again chiral dynamics, together with data, helps us to put bounds on this long-distance effect. The recent results[28,33] on $K_L \to \pi^0 \gamma\gamma$ at small $M_{\gamma\gamma}$ indicate that this contribution is small in the electron channel ($\lesssim\ 2 \times 10^{-12}$). In addition the CP-conserving amplitude does not interfere with the CP-violating one in the total width and, in principle, it could be experimentally constrained by means of a Dalitz plot analysis. In view of these arguments, the CP-conserving contamination should not represent a serious problem for the extraction of the interesting direct-CP-violating component of $B(K_L \to \pi^0 e^+ e^-)$.

4.2 $K_L \to l^+ l^-$

The dominant contribution to these transitions, for both $\ell = e$ and $\ell = \mu$, is generated by the intermediate two-photon state. This leads to an absorptive amplitude computed unambiguously by means of $\Gamma(K_L \to \gamma\gamma)$ and a dispersive one that is more difficult to estimate, depending on *a priori* unknown LECs.

In $K_L \to e^+ e^-$ the dispersive integral of the $K_L \to \gamma\gamma \to l^+ l^-$ loop is dominated by a large logarithm ($\log(m_K^2/m_e^2)$), and the relative contribution of the local term is small. This implies that in this case also the dispersive amplitude can be estimated with a relatively good accuracy in terms of $\Gamma(K_L \to \gamma\gamma)$, yielding the prediction[34] $B(K_L \to e^+ e^-) \sim 9 \times 10^{-12}$, which has been confirmed by E871.[35]

More interesting from the short-distance point of view is the case of $K_L \to \mu^+ \mu^-$. Here the absorptive two-photon long-distance amplitude is not enhanced by large logs and is comparable in size with the short-distance contribution of Z-penguin and W-box diagrams. The latter is particularly interesting, since it is sensitive to $\Re V_{td}$ and calculable with high accuracy.[36] On the other hand, the dispersive part of the two-photon contribution is more difficult to be estimated in this case, as it is more sensitive to the local counterterm.

The counterterm appearing in the two-photon amplitude of $K_L \to \mu^+ \mu^-$ is related to the behaviour of the $K_L \to \gamma^* \gamma^*$ form factor and it can be constrained by means of theoretical and experimental information on the latter.[34,37–39] To this purpose it should be stressed how important it is to have precise experimental data on $K_L \to \gamma \ell^+ \ell^-$ and $K_L \to e^+e^-\mu^+\mu^-$ decays; these are also sensitive to the $K_L \to \gamma^*\gamma^*$ form factor,[38,40] although only in a limited kinematical region. For instance the recent KTeV data[41] suggest an inconsistency of the so-called BMS ansatz[37] in fitting the $K_L \to \gamma e^+e^-$ and $K_L \to \gamma\mu^+\mu^-$ modes at the same time. Using the very precise experimental determination of $B(K_L \to \mu^+\mu^-)$ reported by E871,[42] together with the analysis of the two-photon dispersive contribution discussed in Ref. 38, we can obtain stringent constraints on possible extensions of the SM and also an interesting lower bound on $\Re V_{td}$.[42] Two experimental tests that could strengthen (or weaken) the validity of this information could be obtained by means of the quadratic slope in the $K_L \to \gamma \ell^+ \ell^-$ form factor (i.e. the dependence on $M_{\ell\ell}^4$) and, especially, by means of the mixed slope in $K_L \to e^+e^-\mu^+\mu^-$ (i.e. the dependence on $M_{ee}^2 \times M_{\mu\mu}^2$).[38]

5 Concluding remarks

In the last few years there has been major progress in the study of kaon decays from the experimental point of view. Just to mention a few examples, I wish to recall the observation of direct CP violation in $K \to 2\pi$ decays, the evidence of the rare modes $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to e^+e^-$, the high-precision measurements of $K^+ \to \pi^+\pi^- e\nu_e$ and $K^+ \to \pi^+e^+e^-$ form factors. Most of these results are of the utmost importance and have triggered a large amount of theoretical work, only a small fraction of which has been mentioned in this talk.

On the other hand, I would like to emphasize that this field is far from being exhausted. There are still fundamental problems that require a detailed answer, such as the nature of the underlying mechanism of CP violation or the rôle of resonances in non-leptonic weak transitions. These questions could be addressed by means of new experimental studies on K decays, which have just entered a new exciting era of precision measurements.

Acknowledgements

I am grateful to G. Buchalla, G. Colangelo, G. Martinelli and T. Pich for useful discussions. I am also in debt with the co-authors of Ref. 9 for having let me join them after their collaboration was already started. Finally I wish to thank the organizers of Chiral Dynamics 2000 for the invitation to this fruitful workshop.

References

1. J. Gasser and H. Leutwyler, *Nucl. Phys.* B **250**, 465 (1985); *Ann. Phys.* **158**, 142 (1984).
2. J. Kambor, J. Missimer and D. Wyler, *Nucl. Phys.* B **346**, 17 (1990); G. Ecker, J. Kambor and D. Wyler, *ibid.* **394**, 101 (1993).
3. See e.g. J. Bijnens *et al.* [hep-ph/9411311] and G. D'Ambrosio *et al.* [hep-ph/9411439] in *The Second DAPHNE Handbook*, Eds. L. Maiani, G. Pancheri, N. Paver (SIS-Frascati, 1995); G. Isidori, *Nucl. Phys.* A **623**, 202c (1997) [hep-ph/9612429].
4. J. Bijnens, G. Colangelo and G. Ecker, *JHEP* **02**, 020 (1999); *Ann. Phys.* **280**, 100 (2000); H.W. Fearing and S. Scherer, *Phys. Rev.* D **53**, 315 (1996).
5. G. Colangelo, these proceedings.
6. R. Urech, *Nucl. Phys.* B **433**, 234 (1995); H. Neufeld and H. Rupertsberger, *Z. Phys.* C **71**, 131 (1996).
7. M. Knecht, H. Neufeld, H. Rupertsberger and P. Talavera, *Eur. Phys. J.* C **12**, 469 (2000)
8. V. Cirigliano, J.F. Donoghue and E. Golowich, *Phys. Rev.* D **61**, 093001 (2000); *ibid.* 093002 (2000).
9. G. Ecker, G. Isidori, G. Müller, H. Neufeld and A. Pich, hep-ph/0006172, to appear in *Nucl. Phys.* B.
10. E. Pallante and A. Pich, *Phys. Rev. Lett.* **84**, 2568 (2000); hep-ph/0007208.
11. T.N. Truong, *Phys. Lett.* B **207**, 495 (1988).
12. J. Kambor, J. Missimer and D. Wyler, *Phys. Lett.* B **261**, 496 (1991).
13. G. Isidori and A. Pugliese, *Nucl. Phys.* B **385**, 437 (1992).
14. A.J. Buras *et al.*, *Phys. Lett.* B **480**, 80 (2000).
15. S. Bertolini, J.O. Eeg, M. Fabbrichesi and E.I. Lashin, *Nucl. Phys.* B **514**, 63 (1998); J. Bijnens and J. Prades, *JHEP* **01**, 023 (1999); T. Hambye, G.O. Kohler and P.H. Soldan, *Eur. Phys. J.* C **10**, 271 (1999).
16. M. Knecht, S. Peris and E. de Rafael, *Phys. Lett.* B **457**, 227 (1999); J.F. Donoghue and E. Golowich, *Phys. Lett.* B **478**, 172 (2000).
17. M. Golterman, these proceedings.
18. G. Ecker, G. Müller, H. Neufeld and A. Pich, *Phys. Lett.* B **477**, 88 (2000).
19. S. Gardner and G. Valencia, *Phys. Lett.* B **466**, 355 (1999).
20. C.E. Wolfe and K. Maltman, *Phys. Lett.* B **482**, 77 (2000); hep-ph/0007319.
21. S. Gardner and G. Valencia, *Phys. Rev.* D **62**, 094024 (2000).

22. V. Cirigliano, J.F. Donoghue and E. Golowich, hep-ph/0008290.
23. P. Franzini, these proceedings.
24. V. Cirigliano, J.F. Donoghue and E. Golowich, *Phys. Lett.* B **450**, 241 (1999).
25. G. D'Ambrosio, G. Ecker, G. Isidori and J. Portolés, *JHEP* **08**, 004 (1998)
26. G. Ecker, A. Pich and E. de Rafael, *Nucl. Phys.* B **291**, 692 (1987).
27. R. Appel *at al.* (E865 Collab.), *Phys. Rev. Lett.* **83**, 4482 (1999); J.A. Thompson *et al.* (E865 Collab.), hep-ex/9904026.
28. V.D. Kekelidze (NA48 Collab.), talk presented at ICHEP2000, Osaka, Japan, 2000.
29. F.J. Gilman and M.B. Wise, *Phys. Rev.* D **21**, 3150 (1980).
30. A.J. Buras, M.E. Lautenbacher, M. Misiak and M. Münz, *Nucl. Phys.* B **423**, 349 (1994).
31. A. Alavi-Harati *et al.* (KTeV Collab.), hep-ex/0010059.
32. L.M. Sehgal, *Phys. Rev.* D **38**, 808 (1988); G. Ecker, A. Pich and E. de Rafael, *Phys. Lett.* B **237**, 481 (1990); L. Cappiello, G. D'Ambrosio and M. Miragliuolo, *Phys. Lett.* B **298**, 423 (1993); P. Heiliger and L.M. Sehgal, *Phys. Rev.* D **47**, 4920 (1993); A.G. Cohen, G. Ecker and A. Pich, *Phys. Lett.* B **304**, 347 (1993); J.F. Donoghue and F. Gabbiani, *Phys. Rev.* D **51**, 2187 (1995); G. D'Ambrosio and J. Portolés, *Nucl. Phys.* B **492**, 417 (1997).
33. A. Alavi-Harati *et al.* (KTeV Collab.), *Phys. Rev. Lett.* **83**, 917 (1999);
34. G. Valencia, *Nucl. Phys.* B **517**, 339 (1998); G. Dumm and A. Pich, *Phys. Rev. Lett.* **80**, 4633 (1998).
35. D. Ambrose *et al.* (E871 Collab.), *Phys. Rev. Lett.* **81**, 4309 (1998).
36. G. Buchalla and A.J. Buras, *Nucl. Phys.* B **412**, 106 (1994); *ibid.* **400**, 225 (1993); *ibid.* **548**, 309 (1999); M. Misiak and J. Urban, *Phys. Lett.* B **451**, 161 (1999).
37. L. Bergstrom, E. Masso and P. Singer, *Phys. Lett.* B **249**, 141 (1990).
38. G. D'Ambrosio, G. Isidori and J. Portolés, *Phys. Lett.* B **423**, 385 (1998).
39. M. Knecht, S. Peris, M. Perrottet and E. de Rafael, *Phys. Rev. Lett.* **83**, 5230 (1999).
40. J.L. Goity and L. Zhang, *Phys. Lett.* B **398**, 387 (1997); *Phys. Rev.* D **57**, 7031 (1998).
41. Y.W. Wah (KTeV Collab.), talk presented at ICHEP2000, Osaka, Japan, 2000.
42. D. Ambrose *et al.* (E871 Collab.), *Phys. Rev. Lett.* **84**, 1389 (2000).