

Quasi-Particles in Non-Commutative Field Theory¹

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ABSTRACT

After a short introduction to the UV/IR mixing in non-commutative field theories we review the properties of scalar quasi-particles in non-commutative supersymmetric gauge theories at finite temperature. In particular we discuss the appearance of super-luminous wave propagation.

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1 Introduction

Given the experience of quantum mechanics it seems a rather natural idea that space-time at very small distance-scales might be described by non-commuting coordinates [1, 2]. Keeping the example of quantum mechanics in mind one is lead to write down a commutation relation for the coordinates such as

$$[x^m, x^n] = i\theta^{mn}. \quad (1.1)$$

In order to study quantum field theory on such non-commuting spaces it is useful to make some further simplifying assumptions, in particular we will take θ^{mn} to be an element of the centre of the algebra defined by (1.1).

A convenient way of thinking about non-commutativity is by deformation of the product on the space of ordinary function. Using θ^{mn} as deformation parameter we define the so-called Moyal product (or star-product) by

$$f(x) * g(x) := \lim_{y \rightarrow x} e^{\frac{i}{2}\theta^{mn}\partial_m^x \partial_n^y} f(x)g(y). \quad (1.2)$$

In momentum space it takes the form

$$f(x) * g(x) = \int \frac{d^n k}{(2\pi)^n} \int \frac{d^n q}{(2\pi)^n} \tilde{f}(k) \tilde{g}(q) e^{-i(k+q)x} e^{-\frac{i}{2}k_m \theta^{mn} q_n}. \quad (1.3)$$

An immediate consequence is that we can always delete one star under the integral because the additional terms by which the Moyal product differs from the usual product are total derivatives thanks to the antisymmetry of θ^{mn}

$$\int f(x) * g(x) d^n x = \int \left(f(x).g(x) + \frac{i}{2}\theta^{mn}\partial_m f(x)\partial_n g(x) + \dots \right) d^n x. \quad (1.4)$$

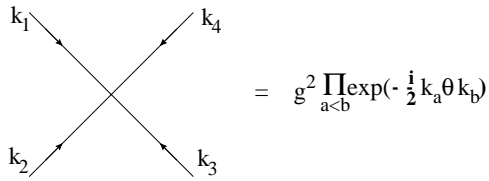
This furthermore implies cyclic symmetry under integral

$$\int f * g * h = \int f.g * h = \int g * h.f = \int g * h * f. \quad (1.5)$$

We have now all the ingredients do start discussing field theory. Before doing so we will introduce one further simplification, namely we will assume from that time is an ordinary commuting coordinate, i.e. $\theta^{m0} = 0$. This has the advantage that we are still dealing with a system with a finite number of time derivatives. Although a canonical formalism for theories with an infinite number of time derivatives can be developed [3] it turns out that quantum field theory on spaces with time-space non-commutativity are not unitary at the one-loop level [4]².

Non-commutative field theories can be viewed as non-local deformations of local field theories. For fields of spin zero or one-half we can take a Lagrangian of an

²This applies to the time-like case, i.e. in all coordinate systems with $\theta^{mn} = const$ the commutator (1.1) involves the time coordinate.



$$= g^2 \prod_{a < b} \exp(-\frac{i}{2} k_a \theta k_b)$$

Figure 1: Feynman rule for non-commutative Φ^4 vertex.

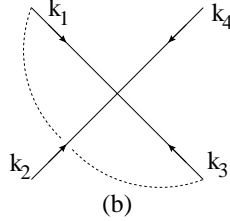


Figure 2: Corrections to the two-point function can be either planar as in (a) or non-planar as in (b)

ordinary field theory and deform the product of fields according to the Moyal product (1.4), i.e. we replace the ordinary product by the star product. For spin one-fields we also have to consider that the gauge symmetry is deformed, $\delta A_m = \partial_m \lambda + i\{A_m, \lambda\}_*$, where $\{.,.\}_*$ denotes the Moyal bracket $\{f, g\}_* = f * g - g * f$. The non-commutative field strength of a gauge field is defined accordingly as $F_{mn} = \partial_m A_n - \partial_n A_m + i\{A_m, A_n\}_*$ and the covariant derivative as $D_m \cdot = \partial_m \cdot + i\{A_m, \cdot\}_*$ [5].

Let us consider now a scalar Φ^4 theory on in four dimensions. Without further loss of generality we assume $\theta^{23} = -\theta^{32} = \theta$. Because one can drop one star-product in the Lagrangian the free theory is unchanged with respect to the one on ordinary \mathbb{R}^n . The tree level propagator is then the usual one

$$\langle \Phi(p) \Phi(-p) \rangle = \frac{i}{p^2 - m^2}. \quad (1.6)$$

The one-loop corrections to the two point function that arise from the Φ^4 vertex are shown in figure 2(a) and 2(b). Because of the cyclic symmetry of the vertex we have two distinct classes of graphs [6]. If we connect neighbouring lines of the vertex in figure (1) the dependence of the exponential on the internal momentum $k = k_1 = -k_2$ cancels. Thus the diagram 2(a) gives rise to a quadratic divergence in the same way as it happens in ordinary Φ^4 theory.

If we contract however non-neighbouring lines the dependence on the internal momentum of the exponent does not cancel. The distinct classes of Feynman diagrams in non-commutative field theories are called planar if they are of type 2(a) and non-planar if they are of type 2(b).

The divergence is regulated by the rapid oscillation of the exponential function at

large internal momentum and we find

$$4g^2 \int \frac{d^4k}{(2\pi)^4} \frac{e^{i\tilde{p}k}}{k^2 - m^2} = \frac{ig^2}{4\pi^2\tilde{p}^2} + \dots, \quad (1.7)$$

Where we introduced the notation $\tilde{p}^n = p_m \theta^{mn}$ and the dots indicate terms that are less singular for $\tilde{p} \rightarrow 0$. Resummation gives rise to a corrected two-point function on the one loop level of the form

$$\Gamma^2(p) = p^2 - m_R^2 + \frac{g^2}{\pi^2\tilde{p}^2}. \quad (1.8)$$

The quadratic divergence in the planar graph gives rise to a renormalization of the mass. The non-planar graph results in a dramatic change of the infrared behaviour of the theory. On a technical level the origin of this infrared divergence is easily understood. The non-planar diagram is regulated by the phase factor stemming from the star product. This phase is absent if the external momentum flowing into the diagram vanishes. Thus the ultraviolet divergence has been converted into an infrared divergence. This phenomenon UV/IR mixing has first been discussed in [7] and has been further investigated in [9]- [33]. Notice also that the IR-singularity is present even in the massive theory. Since it is induced by modes in the far UV circling in the loop it is insensitive to the presence of a mass term.

It should be emphasised that there are usually also sub-leading logarithmic infrared divergences. In the infrared these become important at momenta of the order of $p = \mathcal{O}(e^{-\frac{1}{g^2}})$. Down to these non-perturbatively small momenta the infrared behaviour is dominated by the effects stemming from the quadratic divergences. In the following we will always concentrate on the leading order IR-behaviour and thus neglect the contributions from the logarithms.

In supersymmetric theories quadratic divergences in four dimensions are absent. However at finite temperature supersymmetry is broken and the one-loop dispersion relation will again show effects from UV/IR mixing in non-planar graphs. Because temperature acts as a cutoff no IR-singularities are to be expected. The next section reviews these effects in the example of $\mathcal{N} = 4$ supersymmetric Yang-Mills theory.

2 Quasi-particles in non-commutative $\mathcal{N} = 4$ SYM

We limit ourselves to the study of a non-commutative $U(1)$ $\mathcal{N} = 4$ gauge theory. The spectrum of the theory consists of six scalars, four Majorana Fermions and a vector field. The Lagrangian takes the form

$$\mathcal{L} = \frac{1}{g^2} \int \left(-\frac{1}{4} F_{mn} F^{mn} + \frac{1}{2} D_m \Phi^{ab} D^m \Phi_{ab} + \frac{1}{4} \{ \Phi^{ab}, \Phi^{cd} \}_* \{ \Phi_{ab}, \Phi_{cd} \}_* + \right. \quad (2.1) \\ \left. + i \lambda_a \sigma^m D_m \bar{\lambda}^a + i \{ \lambda_a, \lambda_b \}_* \Phi^{ab} + i \{ \bar{\lambda}^a, \bar{\lambda}^b \}_* \Phi_{ab} \right).$$

The theory has a global $SU(4)$ symmetry under which the fermions transform under the $4, \bar{4}$. The 6 scalars transform in the antisymmetric. This symmetry is indicated by indices a, b .

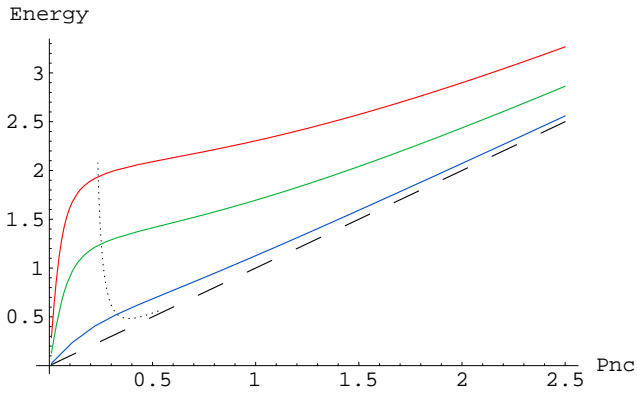


Figure 1: Dispersion relation for scalars in $\mathcal{N} = 4$ Yang-Mills for different temperatures. The momentum p is taken to lie entirely in the non-commutative directions. The dashed line shows the light cone $\omega = p$. The dotted line shows the momentum p_c below which the group velocity $\partial\omega/\partial p$ is bigger than one.

We will study the dispersion relation of the $\mathcal{N} = 4$ scalars at finite temperature and one loop level. Finite temperature is implemented in the Matsubara formalism by considering the theory on $S^1 \times \mathbb{R} \times \mathbb{R}_{nc}^2$. The last factor indicates the two-dimensional non-commutative plane. The fermions are taken to have anti-periodic boundary conditions on the S^1 factor. Non-commutative field theories at finite temperature have been investigated in [34]-[37]

The scalar self-energy is given by

$$\Sigma_T = 32g^2 \int d^3k (2\pi)^3 \sin^2 \tilde{p} \cdot k 2k (n_B(k) + n_F(k)) + 4g^2 P^2 \bar{\Sigma}, \quad (2.2)$$

$n_B(k)$ and $n_F(k)$ denote Bose-Einstein and Fermi-Dirac distributions. Four momentum is denoted by $P^2 = p_0^2 - p^2$, lowercase denotes three-momentum. Momenta along the non-commutative directions as will be called transverse.

The first term in (2.2) vanishes at $T = 0$ because of supersymmetry. The second term contributes to the finite temperature wave-function renormalization of the scalar field. It affects the position of the pole only to $\mathcal{O}(g^4)$ and we will drop it in the sequel.

Using the relation $\sin^2 \tilde{p} \cdot k = 12(1 - \cos \tilde{p} \cdot k)$ we can separate the planar and non-planar contributions to the self-energy. The dispersion relation becomes

$$\omega^2 = p^2 + 2g^2 T^2 - 4g^2 T \pi |\tilde{p}| \tanh \pi |\tilde{p}| T. \quad (2.3)$$

A plot of the dispersion relation is shown in figure (3). The hyperbolic tangent arises solely from the non-planar contribution to the dispersion relation.

For large transverse external momenta the non-planar contribution is sub-leading with respect to the planar one,

$$\omega^2 \approx p^2 + 2g^2 T^2 - 4g^2 T \pi |\tilde{p}|, \quad T \tilde{p} \gg 1. \quad (2.4)$$

The second term comes from the planar diagrams and gives a mass to the scalar excitations. The sub-dominant term linear in T arises solely from soft bosons in non-planar diagrams. These are modes with characteristic momentum $k \ll T$ and large occupation number $n_B \approx T/k \gg 1$,

$$\Sigma_{np} \sim \int d^3k k \cos \tilde{p}k T k \sim T \tilde{p}. \quad (2.5)$$

In usual space-time the approximation $n_B \approx T/k \gg 1$ results in the well known ultraviolet catastrophe of classical field theory. In the non-planar sector of non-commutative space this does not happen as long as \tilde{p} is different from zero. This is yet another manifestation of the UV/IR mixing of non-commutative field theories: to leading order at high temperature, the non-planar contribution is effectively purely classical [36].

At low transverse external momenta, the non-planar contribution tends to cancel the planar one. For zero external transverse momentum the interaction switches off. The theory becomes a free, gap-less $U(1)$ gauge theory with $\omega^2 \approx p_3^2$.

Let us consider now the case where the momentum lies along the non-commutative directions. Since $\omega(0) = 0$ and for large p , $\omega(p) \approx \sqrt{p^2 + 2g^2T^2}$, which lies above the lightcone, there is a region in between with $\partial\omega(p)\partial p > 1$. Thus the group velocity must exceed the speed of light for small transverse momenta!

$$\omega^2 \approx (1 + g^2\pi^2T^4\theta^2) p^2. \quad (2.6)$$

The low momentum excitations are massless, but propagate with an index of refraction $n = p/\omega$ smaller than one. Because the interactions switch off at low momenta, we expect these modes to be long-lived. In figure (3) the momentum p_c below which the group velocity exceeds one is depicted by a dotted line. The dashed line shows the light cone $\omega = p$.

Let us emphasise that these qualitative features should be quite general and not an artifact of our one loop approximation, as they simply arise from the fact that the theory is non-interacting at zero transverse momentum and develops a mass gap otherwise³.

We now investigate the consequences of the dispersion relation (2.6) for wave propagation. Imagine that some disturbance of the scalar field is created in the thermal bath at time $t = 0$. To simplify matters we will consider only a one dimensional problem with momentum pointing in a non-commutative direction. The fastest moving modes are the ones with longest wavelength. These are also the modes which are long lived in the thermal bath. For these it is possible to obtain the exact asymptotic behaviour by noting that the dispersion relation around $k = 0$ is

$$\omega(k) = c_0k - \gamma k^3 + O(k^5), \quad (2.7)$$

³One might also be worried if these effects are gauge dependent. A model without gauge symmetry can be obtained if one sets the gauge field and one fermion (the field content of an $\mathcal{N} = 1$ vector multiplet) to zero. This would result in a $\mathcal{N} = 1$ Wess-Zumino model with Moyal bracket interactions. It would only change the overall factor in (2.2).

with $c_0 = \sqrt{1 + g^2\pi^2 T^4 \theta^2 6}$ and $\gamma = g^2\pi^4\theta^4 T^6 120c_0$. This is the dispersion relation of the linearised Korteweg-deVries equation whose solution is expressed in terms of the Airy function $Ai(z)$. We can express the solution for the head of a wavetrain by [38]

$$\Phi = A2(3\gamma t)^{13} Ai(x - c_0 t(3\gamma t)^{13}) . \quad (2.8)$$

The Airy function has oscillatory behaviour for negative argument and decays exponentially for positive argument. Thus the wavetrain decays exponentially ahead of $x = c_0 t$. Behind the wave becomes oscillatory. In this region one can match the Airy function with the asymptotics obtained from a stationary wave approximation. In between the oscillatory region and the exponential decay there is a transition region of width proportional to $(\gamma t)^{13}$ around $x = c_0 t$. In this region the wavetrain has its first crest which therefore is moving with a velocity approximately given by c_0 .

Group velocities faster than the speed of light do also appear in conventional physics, e.g. it is well-known that this happens for light propagation in media close to an absorption line. Since the dispersive effects are however large, the group velocity loses its meaning as the velocity of signal transportation. In our case, it is interesting to notice that as the temperature increases, not only c_0 but also γ grows. This implies that at high temperatures the soft transverse momenta become very dispersive. In such situations it is useful to introduce the concept of a front velocity which is the velocity of the head of the wavetrain. For the propagation of light in a medium it can be shown that this front velocity never exceeds the speed of light even if the group velocity can be faster than the speed of light [39]. In our case the front velocity can be defined as the velocity of the first crest of the wavetrain. According to (2.6) and (2.7) this is always bigger than the speed of light. The advance of the first crest with respect to an imagined light front is $(c_0 - 1)t$. Since its spread grows as $(\gamma t)^{13}$, the first crest is well defined outside the lightcone for large enough time, $t > t_0$ where $t_0 = \sqrt{\gamma(c_0 - 1)^3}$.

The question arises if this super-luminosity implies a violation of causality. This is not necessarily the case. Violation of causality needs both ingredients: super-luminosity and the relativity principle. Imagine an observer A emitting some signal with super-luminous velocity c_0 . If the relativity principle is valid another observer B in a boosted frame relative to A could then catch the signal. B could send an answer also with super-luminous speed c_0 . The answer would reach observer A before he sent out the original signal. The crucial point is of course that in the non-commutative space-time we are considering boosts are not anymore symmetries. In particular only in the frame of observer A time is ordinary, commuting time. Any other frame involving a boost in a non-commutative direction implies that also time is non-commutative. To obtain an answer if causality is violated one would have to calculate the dispersion relation also in such a frame and study wave propagation then. Finite temperature field theory with non-commutative time is however difficult to formulate due to the infinite number of time derivatives appearing in the star product. This is an open question though progress could possibly be achieved along the lines in [3].

3 Discussion and Outlook

We have concentrated on reviewing the properties of scalar quasi-particles at finite temperature in non-commutative $\mathcal{N} = 4$ gauge theory. Another system that has been studied in [37] is the non-commutative Wess-Zumino model with star-product interactions instead of Moyal-brackets. The one-loop self-energy is given by a similar expression as (2.2) except that $\sin \frac{\vec{p}k}{2}$ is substituted by $\cos \frac{\vec{p}k}{2}$. It turns out that this has the effect that for temperatures $T > T_0 \approx \frac{1}{\sqrt{g\theta}}$ the minimum of the dispersion relation is displaced from $p = 0$! It has been argued that this makes Bose-condensation of scalar modes impossible for temperatures higher than T_0 [37]⁴.

Another system that has been studied in [37] was $\mathcal{N} = 2$ gauge theory at finite density. The results are qualitatively analogous to the case with temperature. The role of the temperature is then played by the chemical potential.

Non-commutative field theories in the setup discussed here appear also in string theory. In [41] it was shown that the physics of D-branes in a B -field background in a particular scaling limit with $\alpha' \rightarrow 0$ is described by non-commutative supersymmetric gauge theories. It has been suggested that the effects of UV/IR mixing could be understood from a string perspective [7]. The UV/IR mixing in this stringy context has been considered in [43]-[49]. Since the model considered here arises as the scaling limit of a D-3-brane in a B -field background it would be very interesting to reconsider the one-loop dispersion relations from a string theory perspective.

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⁴The phase structure of a non-commutative scalar field model in four dimensions has been investigated in [40] where it has been argued that condensation to stripe phases occurs.

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