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LOCALIZING GRAVITY ON A 3-BRANE IN HIGHER DIMENSIONS

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We present metric solutions in six and higher dimensions with a bulk cosmological constant, where gravity is localized on a 3-brane. The corrections to four-dimensional gravity from the bulk continuum modes are power-law suppressed. Furthermore, the introduction of a bulk "hedgehog" magnetic field leads to a regular geometry, and can localize gravity on the 3-brane with either positive, zero or negative bulk cosmological constant.

The idea that we live inside a domain wall in a five-dimensional (5d) universe,[1] has recently received new impetus. A solution of the 5d Einstein equations with a bulk cosmological constant has been found where gravity is localized on a 3-brane.[2] The resulting four-dimensional (4d) Kaluza-Klein spectrum consists of a massless zero-mode localized at the origin, and a gapless continuous spectrum for the non-zero modes. Remarkably, one still recovers the usual 4d Newton's law at the origin.

This idea can be generalized to six and higher dimensions where gravity is localized on a 3-brane which is a topological local defect of the higher-dimensional theory.[3,4] In D-dimensions the Einstein equations with a bulk cosmological constant Λ_D and stress-energy tensor T_{AB} are

$$R_{AB} - \frac{1}{2}g_{AB}R = \frac{1}{M_D^{n+2}}\left(\Lambda_D g_{AB} + T_{AB}\right) , \tag{1}$$

where M_D is the reduced D-dimensional Planck scale. A D-dimensional metric ansatz that repects 4d Poincare invariance with n transverse spherical coordinates satisfying $0 \leq \rho < \infty$, $0 \leq \{\theta_{n-1}, \ldots, \theta_2\} < \pi$ and $0 \leq \theta_1 < 2\pi$, is

$$ds^2 = \sigma(\rho) g_{\mu\nu} dx^\mu dx^\nu - d\rho^2 - \gamma(\rho) d\Omega_{n-1}^2 \, , \tag{2}$$

where the metric signature of $g_{\mu\nu}$ is $(+,-,-,-)$ and $d\Omega_{n-1}^2$ is the surface area element. At the origin $\rho = 0$ we will assume that there is a core defect of radius $\rho < \epsilon$, whose source is parameterised by the brane tension components, μ_0, μ_ρ, and μ_θ. In order to have a regular geometry at the origin we will require that the solution satisfies

$$\sigma'\big|_{\rho=0} = 0 \; , \quad (\sqrt{\gamma})'\big|_{\rho=0} = 1, \quad \sigma\big|_{\rho=0} = A \; , \quad \text{and} \quad \gamma\big|_{\rho=0} = 0 \; , \tag{3}$$

where A is a constant. This leads to the following set of boundary conditions

$$\sigma\sigma'\sqrt{\gamma^{n-1}}\Big|_0^\epsilon = \frac{2}{(n+2)}\frac{1}{M_D^{n+2}}\Big((n-2)\mu_0 - \mu_\rho - (n-1)\mu_\theta\Big) \, , \tag{4}$$

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and

$$\sigma^2 \sqrt{\gamma^{n-2}}(\sqrt{\gamma})'\big|_0^\epsilon = -\frac{1}{(n+2)}\frac{1}{M_D^{n+2}}\left(4\mu_0 + \mu_\rho - 3\mu_\theta\right) \ , \tag{5}$$

where it is understood that the limit $\epsilon \to 0$ is taken. The equations (4) and (5) are the general conditions relating the brane tension components to the metric solution of the Einstein equations (1), and lead to nontrivial relationships between the components of the brane tension per unit length.

For example, in six dimensions the solution outside the core defect is given by[3]

$$\sigma(\rho) = e^{-c\rho} \qquad \text{and} \qquad \gamma(\rho) = R_0^2 e^{-c\rho} \ , \tag{6}$$

where $c = \sqrt{\frac{2}{5}\frac{(-\Lambda_6)}{M_6^4}}$, and the arbitrary integration constant (which corresponds to an overall rescaling of the coordinates x^μ), is chosen such that $\lim_{\epsilon\to 0}\sigma(\epsilon) = 1$. Clearly the negative exponential solution requires that $\Lambda_6 < 0$. Thus, the geometry of the solution outside the core is simply AdS_6/Γ where Γ corresponds to a periodic identification of one of the coordinates. If we now demand that the solution (6) is consistent with the boundary conditions (4) and (5), the brane tension components must satisfy

$$\mu_0 = \mu_\theta + A^2 M_6^4 \ , \tag{7}$$

where μ_ρ remains undetermined. Even though the condition (7) amounts to a fine-tuning, one can imagine that it results from an underlying supersymmetry. This is precisely what happens in the supersymmetric version of the 5d Randall-Sundrum solution.[5] Thus, it is encouraging to note that the bosonic background of the 6d solution (6) can also be supersymmetrized.[6]

The Kaluza-Klein graviton spectrum of the 6d solution contains a zero-mode which is localized at the origin $\rho = 0$. The nonzero modes consist of radial modes which form a gapless continuum, together with the discrete angular modes with mass scale R_0^{-1}. At distances $r \gg R_0$, the correction from the bulk continuum radial modes to the 4d gravitational potential is

$$V(r) = G_N \frac{m_1 m_2}{r}\left[1 + \frac{32}{3\pi}\frac{1}{(cr)^3}\right] \ . \tag{8}$$

This compares with the result in 5d where the corrections from the continuum modes are $\mathcal{O}(1/r^2)$.

The spherically symmetric setup (2) can only be generalized to dimensions greater than six, provided that there are additional energy-momentum sources in the bulk. A particularly interesting possibility is that due to p-form fields.[4] Consider the D-dimensional action

$$S = \int d^D x \sqrt{|g|}\left(\frac{1}{2}M_D^{n+2}R - \frac{\Lambda_D}{M_D^{n+2}} + (-1)^p \frac{1}{4}F_{\mu_1\ldots\mu_{p+1}}F^{\mu_1\ldots\mu_{p+1}}\right) \ . \tag{9}$$

When $p = n-2$, a solution to the equation of motion for the p-form field is

$$F_{\theta_1\ldots\theta_{n-1}} = Q(\sin\theta_{n-1})^{(n-2)}\ldots\sin\theta_2 \ , \tag{10}$$

where Q is the charge of the field configuration, and all other components of F are equal to zero. In fact, this "hedgehog" field configuration is the generalization of the magnetic field of a monopole, and consequently the stability of these configurations is ensured by magnetic flux conservation. Outside the core ($\rho > \epsilon$) we will assume a solution of the form

$$\sigma(\rho) = e^{-c\rho} \quad \text{and} \quad \gamma(\rho) = \text{constant} \ , \tag{11}$$

where the arbitrary integration constant is again chosen such that $\lim_{\epsilon \to 0} \sigma(\epsilon) = 1$. With this ansatz and including the contribution of the p-form bulk field to the stress-energy tensor, the Einstein equations (1), are reduced to the following two equations for the metric factors outside the 3-brane source

$$(n-1)! \frac{Q^2}{\gamma^{n-1}} - \frac{1}{2\gamma}(n-2)(n+2) + \frac{\Lambda_D}{M_D^{n+2}} = 0 \ , \tag{12}$$

$$c^2 = -\frac{1}{2}\frac{\Lambda_D}{M_D^{n+2}} + \frac{1}{4\gamma}(n-2)^2 \ . \tag{13}$$

We are interested in the solutions of these two algebraic equations which lead to an exponential, $c^2 > 0$ and do not change the metric signature, $\gamma > 0$. Remarkably, solutions to these equations exist for which these conditions can be simultaneously satisfied. In particular, for the $n = 3$ case, there are solutions not only for $\Lambda_7 < 0$, but also for $\Lambda_7 \geq 0$, provided that $Q^2 \Lambda_7 / M_7^5 < 1/2$. Thus, the bulk cosmological constant does not need to be negative in order to localize gravity.[4] Similar solutions exist for all $n \geq 3$.

The equation of motion for the spin-2 radial modes using the solution (11) is qualitatively similar to the 5d case. The constant γ factor effectively plays no role in the localization of gravity. Thus, the corrections to Newton's law will be suppressed by $1/r^2$ for all solutions $n \geq 3$. This is easy to understand since the geometry outside the core defect is simply $AdS_5 \times S^{n-1}$, and there is just one non-compact dimension for all $n \geq 3$.

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