DYNAMICAL RESUMMATION AND DAMPING IN THE O(N) MODEL *

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In this talk I summarize the one loop and higher loop calculations of the effective equations of motion of the $O(N)$ symmetric scalar model in the linear response approximation. At one loop one finds essential difference in long time behavior for the fields below and above a dynamically generated length scale. A partial resummation assuming quasi-particle propagation seems to cancel the relevance of this scale.

1 Introduction

The out of equilibrium behavior of the field theories can play important role in understanding many physical phenomena, as for example the cosmological inflation, reheating or some aspects of heavy ion physics. A possible treatment of these processes is to compute effective equations of motion (EOM) for the field expectation values and then solve these equations, most simply by applying one loop perturbation theory and linear response approximation. These approximations, however, may not give correct answers in certain dynamical regions, as calculations in gauge theories show, where linear response spoils gauge invariance[1], higher loop effects change the theory completely at the ultra-soft scale[2]. In this talk I would like to examine the effects of higher loops on the dynamical behavior of the $O(N)$ model in linear response approximation. For details and references c.f. Ref[3,4].

What new effects may we expect? In calculation of the imaginary part the cutting of a higher loop diagram provides more phase space, less constraint to the incoming momentum. This effect can be important, when the one loop contribution is small, as in the case of Goldstone damping in the O(N) model. Here the mass shell constraints send the internal Goldstone momentum into infinity at one loop resulting in exponentially suppressed Goldstone damping.

The other expected effect is that while at one loop the internal particles are stable, higher loops may provide imaginary part for their propagator. The stability of internal particles leads to long time memory of the system[5], their decay, on the other hand, leads to loss of memory.

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In the followings I first summarize the results of one loop linear response theory, then the results with resummed self energy propagators.

2 One loop linear response theory

The action of the theory

$$S=\int\left[\frac{1}{2}(\partial\hat{\Phi}_a)^2-\frac{m^2}{2}\hat{\Phi}_a^2-\frac{\lambda}{24}(\hat{\Phi}_a^2)^2\right]. \tag{1}$$

We want to calculate the EOM for the expectation value of the field $\Phi=\mathrm{Tr}\,\hat{\Phi}\equiv\langle\hat{\Phi}\rangle$, where ρ is some initial density matrix. We apply the operator EOM $\frac{\delta S}{\delta\hat{\Phi}(x)}=0$ to the decomposition $\hat{\Phi}=\Phi+\varphi$ (here, by construction, $\langle\varphi\rangle=0$). Then we take expectation value and obtain

$$0=(\partial^2+m^2+\frac{\lambda}{6}\Phi^2(x))\Phi(x)+J^{ind}(x), \tag{2}$$

where the quantum induced current is

$$J_a^{ind}(x)=\frac{\lambda}{6}\left[\Phi_a(x)\left\langle\varphi_b^2(x)\right\rangle+2\Phi_b(x)\left\langle\varphi_b(x)\varphi_a(x)\right\rangle+\left\langle\varphi_a(x)\varphi_b^2(x)\right\rangle\right]. \tag{3}$$

The expectation values are calculated using real time one loop perturbation theory (there $\left\langle\varphi_a\varphi_b^2\right\rangle=0$) in linear response approximation. We assume moreover that the fluctuations are in equilibrium. We concentrate on the broken phase where, with proper choice of the coordinate system, we write $\Phi_a\to\bar{\Phi}\delta_{a1}+\Phi_a$ with constant $\bar{\Phi}$. The EOM (2) determines the value of $\bar{\Phi}$.

In linear response approximation $J_a^{ind}(k)=\Pi_{ab}^R(k)\Phi_b$, where Π^R is the retarded self energy. In the present case it turns out that the self energy is diagonal $\Pi_{ab}^R=\Pi_a^R\delta_{ab}$, and

$$\begin{aligned}\Pi_1^R(k)&=\frac{\lambda}{2}\left[S_1+\bar{\Phi}^2S_{11}(k)\right]+\frac{(N-1)\lambda}{6}\left[S_i+\frac{\bar{\Phi}^2}{3}S_{ii}(k)\right]\\ \Pi_i^R(k)&=\frac{\lambda}{6}\left[S_1+(N+1)S_i\right]+\frac{\lambda}{9}\bar{\Phi}^2S_{1i}(k),\end{aligned} \tag{4}$$

where

$$S_a=\int\frac{d^4q}{(2\pi)^4}\,n(q_0)\varrho_a(q),\qquad iS_{ab}(x)=\Theta(x_0)\rho_{ab}(x), \tag{5}$$

and

$$\rho_{ab}(k)=\int\frac{d^4q}{(2\pi)^4}\,\varrho_a(q)\varrho_b(k-q)(1+n(q_0)+n(k_0-q_0)). \tag{6}$$

Here $\varrho_a(k) = (2\pi)\varepsilon(k_0)\delta(k^2 - m_a^2)$ free spectral function and n is the Bose-Einstein distribution.

To avoid IR divergences we have to perform (mass) resummation. Here it is done by using $m_H^2 = \frac{\lambda}{3}\bar{\Phi}$ and $m_G^2 = 0$ in the propagators with the one-loop value of $\bar{\Phi}$.

For a detailed computation see Ref. [3], here I just quote the main results: *1.)* Different methods (kinetic theory, perturbation theory, equations for mode functions) give the same result in one loop linear response approximation. *2.)* Goldstone theorem is fulfilled in the present approximation scheme taking into account the quantum corrections to $\bar{\Phi}$ (i.e. $\Pi_i^R(k=0) = 0$). *3.)* The calculations can be done analytically. The damping rates can be read off from the imaginary part at the mass shell. At high temperatures

$$\gamma_1 = (N-1)\frac{\lambda T}{48\pi}, \qquad \gamma_i = \frac{\lambda m_H^2}{96\pi|\mathbf{k}|}\, n(\frac{m_H^2}{4|\mathbf{k}|}). \tag{7}$$

The leading term in damping rate of the Higgs mode is classical (can be obtained using classical statistical field theory), but for the Goldstone mode it is classical only for large momenta $|\mathbf{k}| > M \equiv \frac{m_H^2}{4T}$. For small momenta $|\mathbf{k}| < M$ the Goldstone damping is exponentially suppressed $\gamma_i \sim e^{-M/|\mathbf{k}|}$ (c.f. also Ref[6].).

3 Beyond one loop

Already in the plain one loop case it was necessary to apply some resummation in order to avoid IR divergences. Similar ideas can be used to resum self energy diagrams. We add a term to and subtract the same term from the original Lagrangian

$$\mathcal{L} = \mathcal{L} - \frac{1}{2}\int d^4y\, \hat{\Phi}_a(x)P_{ab}(x,y)\hat{\Phi}_b(y) + \frac{1}{2}\int d^4y\, \hat{\Phi}_a(x)P_{ab}(x,y)\hat{\Phi}_b(y), \tag{8}$$

where P depends also on $\bar{\Phi}$ in the broken phase. We treat the first term as part of the propagator, the second one as counterterm. In this way we did not change the physics, at infinite loop order P is irrelevant. At finite loop order, however, the results are sensitive to the choice of P, which sensitivity can be used to optimize the perturbation theory. We can demand, for example, that the one loop correction to the self energy (propagator) be zero. There are two contributions, one comes from a direct calculation with the new propagator, the other is the counterterm. Their cancellation leads to a gap equation

$$\Pi^R(P, \bar{\Phi}) = P, \tag{9}$$

where we have denoted the explicit dependence of the self energy on P (through the propagator) and on the background. In the later calculations we shall use the resulting $P = \bar{P}(\bar{\Phi})$ function. Since the Lagrangian was symmetric under $O(N)$ rotation where P was transformed as a tensor, the $\bar{P}$ solution transforms also as a tensor under the rotation of the background. Using this function instead of P we maintain the $O(N)$ symmetry of the Lagrangian. We assume in the sequel that we have chosen the coordinate system properly and $\bar{P}(\bar{\Phi})$ is diagonal.

When P is fixed, the calculation goes like in the symmetric phase, but the propagator changes. We can read off the propagators at finite temperature from the spectral function as

$$\begin{array}{ll} iG_a^<(k) = n(k_0)\varrho_a(k), & iG_a^>(k) = (1+n(k_0))\varrho_a(k), \\ iG_a^c(t,\mathbf{k}) = \Theta(t)\varrho_a(t,\mathbf{k}) + iG_a^<(k), & iG_a^a(t,\mathbf{k}) = iG_a^>(k) - \Theta(t)\varrho_a(t,\mathbf{k}), \end{array} \tag{10}$$

and the spectral function can be expressed in the present case as

$$\varrho_a(k) = \frac{-2\,\mathrm{Im}\,\bar{P}_a}{(p^2 - m_a^2 - \,\mathrm{Re}\,\bar{P}_a)^2 + \,\mathrm{Im}\,\bar{P}^2}. \tag{11}$$

These relations make the gap equation (9) explicit.

To have an analytical solution we have to make some assumptions. We use Breit-Wigner approximation (assuming pole dominance), i.e. we approximate the true spectral function as

$$\varrho(k) \approx \frac{\pi}{\omega_{\mathbf{k}}}\left(\delta_{\gamma_{\mathbf{k}}}(k_0 - \omega_{\mathbf{k}}) - \delta_{\gamma_{\mathbf{k}}}(k_0 + \omega_{\mathbf{k}})\right), \tag{12}$$

where $\delta_\gamma(\omega) = \frac{1}{\pi}\frac{\gamma}{\omega^2+\gamma^2}$ smeared delta-function. Passing by the calculations (c.f. [4]) I summarize the changes compared to the $\gamma = 0$ case: *1.)* Instead of Landau prescription $k_0 \to k_0 + i\varepsilon$ we find $k_0 \to k_0 + i\hat{\gamma}$ in S_{ab}, where $\hat{\gamma} = \gamma_{a\mathbf{k}} + \gamma_{b\mathbf{k}}$. *2.)* Instead of strict energy conservation the energy is conserved by $\delta_{\hat{\gamma}}$ in calculation of the imaginary part. *3.)* For low momenta ($|\mathbf{k}| < \gamma_1$ for Higgs and $|\mathbf{k}| < m_H$ for Goldstones) both the Higgs and Goldstone fields have imaginary part proportional to k_0

$$\mathrm{Im}\,\Pi_a^R(k) = -\eta_a k_0, \tag{13}$$

where $\eta_1 \sim T\lambda^2 \log\lambda$ and $\eta_i \sim m_H$.

This latter point yields finite on-shell damping rate for the Goldstone modes, showing that the one loop result was not reliable as expected in the Introduction. It also means that in the effective equation of motion a term $\sim \dot{\Phi}$ appears instead of the integral over the past. That is the loss-of-memory effect indicated in the Introduction.

4 Conclusions

We have computed the effective EOM for the $O(N)$ model in the linear response approximation at one loop level and with self energy resummation. At one loop we find that the Higgs dynamics in the leading temperature order is consistent with the classical expectations, while for the Goldstone we obtain exponentially small damping rate for momenta $|\mathbf{k}| < M = m_H^2/4T$

$$\gamma_i^{1-loop} \sim e^{-M/|\mathbf{k}|}. \tag{14}$$

To go beyond one loop we have performed self energy resummation formulated in gap equations. For the solution we have used Breit-Wigner approximation, which have modified in the result the Landau prescription (now $k_0 \to k_0 + i\hat{\gamma}$) and have resulted in a broadened mass shell for the intermediate particles. As a consequence we have found that for low momenta

$$\Pi_a^R \sim -\eta_a \partial_t \tag{15}$$

for both the Goldstone and Higgs fields. Therefore the Goldstone on-shell damping rate is finite, and we can describe the dynamics of low momentum fields with a differential equation without long time memory kernels.

There can be also other effects which can modify this statement, first of all the ones coming from the running of the coupling constant. On the other hand similar considerations may be applicable for other theories (e.g. gauge theories) as well.

References

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