# RANDOM WALK MODEL FOR CELL-TO-CELL MISALIGNMENTS IN ACCELERATOR STRUCTURES*

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*Abstract*

Due to manufacturing and construction errors, cells in accelerator structures can be misaligned relative to each other. As a consequence, the beam generates a transverse wakefield even when it passes through the structure on axis. The most important effect is the long-range transverse wakefield that deflects the bunches and causes growth of the bunch train projected emittance. In this paper, the effect of the cell-to-cell misalignments is evaluated using a random walk model that assumes that each cell is shifted by a random step relative to the previous one. The model is compared with measurements of a few accelerator structures.

## 1 INTRODUCTION

The Next Linear Collider (NLC) is a proposed $e^+e^-$ facility capable of achieving a luminosity in excess of $10^{34}\mathrm{cm}^{-2}\mathrm{sec}^{-1}$ at a center-of-mass energy of 1 TeV [1]. To achieve the desired luminosity with good rf-to-beam efficiency, a train of roughly 100 bunches is accelerated on each rf pulse in the X-Band (11.424 GHz) linacs. With the long bunch trains, the long-range transverse wakefield must be carefully controlled to prevent beam-breakup (BBU) and/or dilution of the projected transverse phase space.

To prevent BBU, the transverse wakefield is detuned, causing a rapid decoherence of the wakefunction, and then weakly damped to prevent re-coherence at a later time [2]. Because of the detuning, the projected emittance of the bunch train is relatively insensitive to rigid offsets or long wavelength misalignments of the accelerator structures, however, the internal misalignments of the accelerator structures break the detuning scheme causing multi-bunch emittance dilution.

The sensitivity to misalignments is usually calculated as a function of the misalignment wavelength as illustrated in Fig. 1. However, without knowledge of the typical misalignment wavelengths, such a description does not provide an easy method of quantifying the tolerances. This is particularly true for the manufacturing and construction tolerances which are short wavelength and thus cause multi-bunch dilution. In this paper, we describe a random walk model for the structure misalignments which yields simple tolerances that are straightforward to apply and reasonably models measurements.

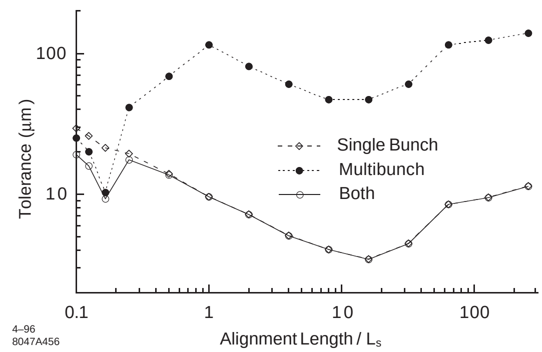


Figure 1: Example of tolerance vs. misalignment wavelength in units of the structure length from Ref. [1]; note that at long wavelength the tolerances are dominated by the short-range wakefield and single bunch dilutions while, at short wavelengths, the long-range wakefield and multi-bunch dilutions are most important.

Based on the matrix wakefield calculated for the NLC accelerator structure [2], and using analytical estimation for the emittance growth in the train [3], we find the emittance growth as a function of the step size in the random walk. This allows us to estimate the tolerances for the misalignment.

## 2 EMITTANCE GROWTH DUE TO LONG RANGE WAKEFIELD

In order to estimate the growth of the projected emittance $\Delta\epsilon$ of a train of bunches caused by randomly misaligned structure in the linac we will use the following formula for the expectation value of $\Delta\epsilon$ [3]

$$\langle\Delta\epsilon\rangle = r_e^2 N^2 \bar{\beta}_0 N_\mathrm{s} L_\mathrm{s}^2 \langle\Delta S_k^2\rangle \frac{1-(\gamma_0/\gamma_f)^{1/2}}{\gamma_0^{1/2}\gamma_f^{3/2}}\,, \quad (1)$$

where $r_e$ is the classical electron radius, $N$ is the number of particles in the bunch, $\bar{\beta}_0$ is the average value of the beta function at the beginning of the linac, $N_\mathrm{s}$ is the number of structures in the linac, $L_\mathrm{s}$ is the length of the structure, $\gamma_0$ and $\gamma_f$ are the initial and final relativistic factors of the beam, and $S_k$ is the sum wake. The quantity $S_k$ is defined as a sum of the transverse wakes $w_i$ generated by all bunches preceding the bunch number $k$ (with

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$S_1 = w_1 = 0$),

$$S_k = \sum_{i=1}^{k} w_i \,, \qquad (2)$$

and $\Delta S_k$ is the difference between $S_k$ and the average value $\langle S\rangle$, $\Delta S_k = S_k - \langle S\rangle$, with

$$\langle S\rangle = \frac{1}{N_b}\sum_{k=1}^{N_b} S_k \,, \qquad (3)$$

where $N_b$ is the number of bunches. Eq. (1) is derived assuming a lattice with the beta function smoothly increasing along the linac as $\bar\beta \sim E^{1/2}$.

Usually, when the transverse wake is excited by an off axis beam passing through the structure, it is divided by the beam offset, and has dimension V/pC/m/mm. In our problem, however, the beam travels along the axis, and the wakefield arises due to the internal misalignment of the cells within the structure. Hence $w_i$ has dimension V/pC/m. It is convenient, however, to introduce a quantity $\Sigma = \sqrt{\langle \Delta S_k^2\rangle}/\Delta$, where $\Delta$ is the rms offset of cells in the structure

$$\Delta^2 = \frac{1}{N_c}\sum_{k=1}^{N_c} x_k^2 \,, \qquad (4)$$

where $x_k$ is the offset of $k$th cell ($\langle x\rangle = 0$), and $N_c$ is the number of cells in the structure. With these definitions, Eq. (1) takes the form

$$\langle \Delta\epsilon\rangle = r_e^2 N^2 \bar\beta_0 N_{\rm s} L_{\rm s}^2 \Delta^2 \Sigma^2 \frac{1-(\gamma_0/\gamma_f)^{1/2}}{\gamma_0^{1/2}\gamma_f^{3/2}} \,. \qquad (5)$$

Now, specifying an allowable fraction of the emittance growth in the linac $f$, for a given pattern of cell misalignment, we can find a tolerance on the allowable $\Delta$ from the equation $\langle \Delta\epsilon\rangle = f\epsilon$, where $\epsilon$ is the normalized vertical beam emittance:

$$\Delta_{\rm tol} = \frac{1}{r_e N L_{\rm s}\Sigma}\left[\frac{f\epsilon\gamma_0^{1/2}\gamma_f^{3/2}}{\bar\beta_0 N_{\rm s}\left(1-(\gamma_0/\gamma_f)^{1/2}\right)}\right]^{1/2} \,. \qquad (6)$$

## 3 TOLERANCE

From Eqs. (2) and (3) and definition of $\Delta S_k$ one can express $\langle \Delta S_k^2\rangle$ in terms of the wake $w_i$

$$\langle \Delta S_k^2\rangle = \frac{1}{N_b^2}\sum_{i,k=1}^{N_b} D_{ik} w_i w_k \,, \qquad (7)$$

where, in the limit $N_b \gg 1$,

$$D_{ik} = N_b(N_b - \max(i,k)) - (N_b - i)(N_b - k) \,. \qquad (8)$$

For small misalignments, $w_i$ is a linear function of cell offsets,

$$w_i = \sum_{k=1}^{N_c} W_{is} x_s \,, \qquad (9)$$

which can be found from the solution of Maxwell's equations for the structure. The matrix $W_{is}$ for the NLC structure RDDS1 with 206 cells has been calculated in Ref. [4]. It has a dimension of $N_b \times 206$. In our calculation we used $N_b = 90$ for bunch spacing 2.8 ns.

We assume that the quantities $x_s$ are random numbers that vary from one structure to another subject to a statistical distribution that will be specified below. One can then average $\langle \Delta S_k^2\rangle$ over random variation of $x_s$, and from Eqs. (7) and (9) find

$$\Sigma^2 = \frac{1}{N_b^2}\sum_{i,k=1}^{N_b}\sum_{s,l=1}^{N_c} D_{ik} W_{is} W_{kl}\, \overline{x_s x_l} \,, \qquad (10)$$

where the bar indicates the statistical averaging.

We considered two cases of the linac with different final energies. For NLC-I we used: $E = 250$ GeV, $f = 10\%$, $N = 0.9\cdot 10^{10}$, $L_{\rm s} = 1.8$ m, $N_{\rm s} = 2240$, $\bar\beta_0 = 7$ m, $E_0 = 10$ GeV, $E_f = 250$ GeV, $\epsilon = 4\cdot 10^{-8}$ m. For NLC-II we assumed the same parameters except $E = 500$ GeV, $N_{\rm s} = 4720$, $E_f = 500$ GeV, and $N = 1.1\cdot 10^{10}$. Then Eq. (6) gives the tolerance in terms of the quantity $\Sigma$,

$$\Delta_{\rm tol}\,[\rm micron] = \frac{35}{\Sigma\,[\rm V/pC/m/mm]} \,, \qquad {\rm NLC-I} \,, \qquad (11)$$

and

$$\Delta_{\rm tol}\,[\rm micron] = \frac{22}{\Sigma\,[\rm V/pC/m/mm]} \,, \qquad {\rm NLC-II} \,. \qquad (12)$$

## 4 UNCORRELATED RANDOM CELL MISALIGNMENTS

As a simplest conceivable model for cell misalignments, we consider uncorrelated random cell offsets with an equal rms value $\Delta$,

$$\overline{x_s x_l} = \Delta^2 \delta_{sl} \,. \qquad (13)$$

Calculation of $\Sigma$ in this case gives $\Sigma = 2.87\rm V/pC/m/mm$ with the tolerance equal to $\Delta_{\rm tol}$ =12.2 microns for NLC-1, and $\Delta_{\rm tol}$ =8.1 microns for NLC-2. However, this tolerance is unrealistic in that it does not model measured structure misalignments.

## 5 RANDOM WALK MODEL

A more reasonable model, the random walk model, assumes that each cell is randomly offset *relative to the previous one* (beginning from $x_1 = 0$) so that

$$x_i = \sum_{s=1}^{i-1} \xi_s \,, \qquad (i > 1) \,, \qquad (14)$$

where $\xi_i$ are uncorrelated random steps,

$$\overline{\xi_s \xi_k} = h^2 \delta_{sk} \,. \qquad (15)$$

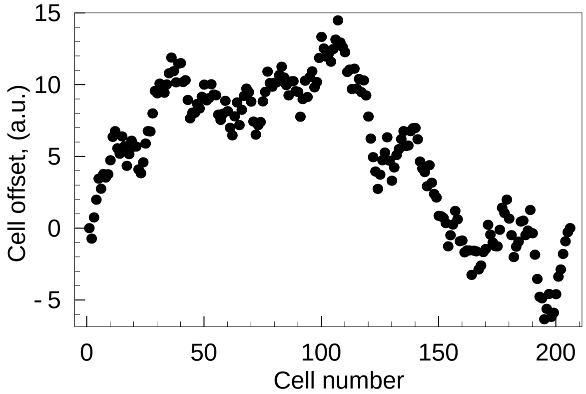


Figure 2: Example of random walk misalignment with fixed structure ends.

with zero average and rms value equal to $h$. In this model, the far end of the structure will be offset from the axis by the amount equal to the sum of all steps, and on the average, the structure will be tilted. In reality, this tilt can be easily corrected by rotating the structure by some angle. To take this correction into account, we add a linear slope to the offsets given by Eq. (14) such that the last cell has a zero offset, $x_{N_c} = 0$,

$$x_i = \sum_{s=1}^{i-1} \xi_s - \frac{i}{N_c} \sum_{s=1}^{N_c-1} \xi_s \,. \tag{16}$$

Using Eqs. (15) and (16), one can find the correlation function (in the limit $N_c \gg 1$)

$$\overline{x_i x_k} = h^2 (\min(i,k) - \frac{ik}{N_c}) \,, \tag{17}$$

and the rms offset for $k$th cell

$$\sqrt{\overline{x_k^2}} = h \left[ \frac{k(N_c - k)}{N_c} \right]^{1/2} \,, \tag{18}$$

and the rms offset for the whole structure

$$\Delta^2 = \frac{1}{N_c} \sum_{k=1}^{N_c} \overline{x_k^2} = \frac{h^2}{6} N_c \,. \tag{19}$$

For 90 bunches we have

$$\Sigma = 1.1 \text{ V/pC/m/mm} \tag{20}$$

which gives the tolerance $\Delta_{\text{tol}} = 32$ microns for 250 GeV, and $\Delta_{\text{tol}} = 20$ microns for 500 GeV. Using Eq. (19) we can convert these values into a tolerable step $h$, $h = 5.5\,\mu$m for NLC-1, and $h = 3.4\,\mu$m for NLC-2, respectively.

## 6 STRUCTURE MEASUREMENTS

Cell misalignments for NLC structures DDS1 and DDS3 measured, as described in in Refs. [5, 6], are shown in

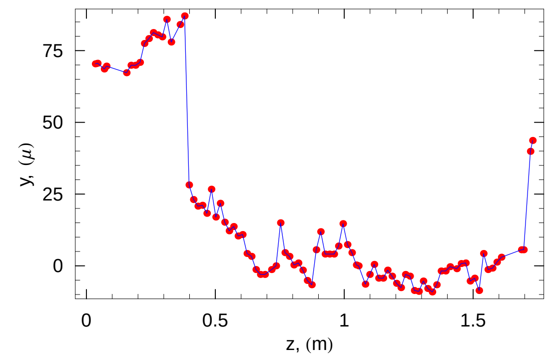


Figure 3: Measurements of the DDS1 structure alignment from Ref. [5]; note the large kink at roughly 35 cm arose when constructing the structure in segments and then bonding the segements together.

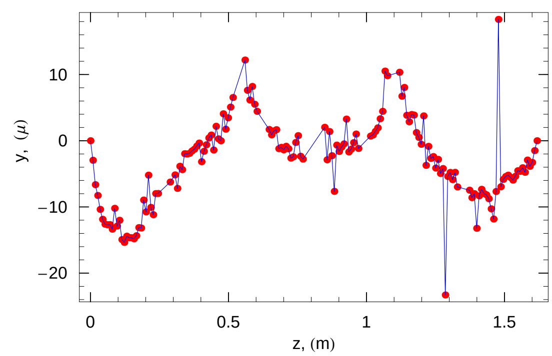


Figure 4: Measurements of the DDS3 structure alignment from Ref. [6].

Figs. 3 and 4 respectively. Using the measurement data for DDS3 and theoretical wakefields from [4], we calculated the quantity (10) (without averaging indicated by the bar) and found the emittance growth using Eq. (5). Such a calculation assumes that the misalignment errors in different structures in the linac have the same statistical value of $\Sigma$ as the measured one. Our result for the emittance growth expectation corresponding to the misalignments shown in Fig. 4 is $\langle \Delta\epsilon \rangle = 1.7 \times 10^{-9}$ m, or about 4% of the nominal vertical emittance of the beam.

## 7 REFERENCES


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