

## No-go for exactly degenerate neutrinos at high scale ?

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### Abstract

We show in a model independent manner that, if the magnitudes of Majorana masses of neutrinos are exactly equal at some high scale, the radiative corrections *cannot* reproduce the observed masses and mixing spectrum at the low scale, irrespective of the Majorana phases or the mixing angles at the high scale.

The data from the solar and atmospheric neutrino experiments can be explained through the mixing of three active neutrinos with nonzero masses. The atmospheric neutrino solution needs  $\nu_\mu - \nu_\tau$  mixing with the corresponding mass squared difference of  $\Delta m_{atm}^2 \approx 10^{-3} - 10^{-2}$  eV<sup>2</sup>, and a large mixing angle,  $\sin^2 2\theta_{atm} > 0.8$  [1]. The solar neutrino solution needs the mixing of  $\nu_e$  with a combination of  $\nu_\mu$  and  $\nu_\tau$  with a corresponding mass squared difference  $\Delta m_\odot^2 \lesssim 10^{-4}$  eV<sup>2</sup>, the mixing angle may be small or large, depending on the particular solution [2].

Since  $\Delta m_{atm}^2 \gg \Delta m_\odot^2$ , there are three distinct patterns for the neutrino masses from the point of view of mass hierarchy. We define the mass eigenstate  $\nu_3$  as the one such that  $|\Delta m_{31}^2| \approx |\Delta m_{32}^2| \approx \Delta m_{atm}^2$ , and the states  $\nu_1$  and  $\nu_2$  as the ones separated by  $|\Delta m_{21}^2| \approx \Delta m_\odot^2$ , with  $\Delta m_{31}^2$  and  $\Delta m_{21}^2$  having the same sign (here  $\Delta m_{ij}^2 \equiv |m_i|^2 - |m_j|^2$ ). With this convention, the possible patterns for the neutrino masses are (i) completely hierarchical:  $|m_1| \ll |m_2| \ll |m_3|$ , (ii) partially degenerate:  $|m_1| \sim |m_2| \gg |m_3|$ , also called as *inverted hierarchy* or (iii) completely degenerate:  $|m_1| \sim |m_2| \sim |m_3|$ . In contrast to the first two cases, the common mass in the third case can be near the direct limit on the electron neutrino mass as obtained from the Kurie plot [3]. Such a mass can have its own signature in the neutrinoless double beta decay [4]. In order for the neutrinos to contribute even a small fraction of the dark matter [5] of the universe, degenerate neutrino masses are essential.

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The pattern of mixing in the lepton sector (which definitely has at least one large mixing angle, as suggested by the atmospheric neutrino data) is very different from that in the quark sector (where all the mixing angles are small). The presence of large mixing angles is another indication that the neutrino masses may be partially or completely degenerate (cases *ii* and *iii* above). The origin of this degeneracy, as well as its breaking (which leads to the mass splittings among neutrinos) needs to be understood theoretically. Here, we concentrate on the latter.

The  $SU(2) \times U(1)$  interactions in the standard model (and its supersymmetric generalisations) break the symmetry among generations explicitly through different Yukawa couplings. The resulting radiative corrections modify the neutrino mass matrices while evolving from a high scale to a low scale. The effects of radiative evolutions on the masses and mixings have been studied in [6–8]. The most economical possibility would be to have *exactly* degenerate neutrinos at the high scale (the seesaw scale, or the GUT scale, for example), and let the radiative corrections lead to the required mass differences and mixing angles. The *exact* degeneracy of neutrino masses at the high scale may be obtained by a symmetry [9], which is broken later in the charged lepton sector.

The models with *exact* degeneracy at the high scale are constrained by the masses and mixings observed at the low scale (at the experiments). In the context of specific models, some constraints have been found. Refs. [10,11] have examined the consequences of the bimaximal mixing pattern at the high scale which was motivated to suppress the effect of large degenerate masses of neutrinos in the neutrinoless double beta decay. Taking the model of Georgi and Glashow [12] as an example, they show that the bimaximal pattern is unstable under radiative corrections. Moreover, one obtains the inverted hierarchy ( $m_1 > m_2 > m_3$ ) for the renormalized masses in case of the MSSM, which rules out the MSW solution for the solar neutrinos. Ref. [13] assumes  $U_{e3} = 0$  at the high scale, and claims that the mixing pattern is unstable under radiative corrections irrespective of the initial values of the angles.

The above arguments are not complete in ruling out the case of degenerate neutrinos for the following reasons. In the standard model, one does indeed obtain the *normal* mass hierarchy ( $m_1 < m_2 < m_3$ ) for the renormalized neutrino masses<sup>1</sup>. Even in the case of the inverted hierarchy (that one gets with MSSM), matter can play an important role in the solution to the solar neutrino problem, and this possibility is not experimentally ruled out [14]. Moreover, one need not insist on the bimaximal mixing pattern if the common degenerate mass is not much larger than the experimental limit coming from the neutrinoless double beta decay. Finally, while the arguments of [13] hinge on the assumption of  $U_{e3} = 0$ , nonzero values of  $U_{e3}$  are allowed by the experiments [15].

In this paper, we reinvestigate the viability of the *exact* degenerate spectrum in a model independent way. We show that (I) in any model with the magnitudes of the neutrino masses exactly equal at some high scale, the mixing at the high scale can always be defined in such a way that its structure is preserved in the process of evolution, so that the issue of the stability of mixing angles does not arise. (II) However, irrespective of the structure of the original mass matrix, the degenerate spectrum at a high scale *cannot* lead to the observed

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<sup>1</sup>Ref. [13] claim inverted hierarchy in case of the SM rather than the MSSM, but this can be traced to their having the wrong sign for the radiative correction parameter  $\epsilon$  to be defined later.

masses and mixings at the low scale, as long as the charged lepton masses are generated by only one Higgs particle. The results are valid irrespective of the model (in particular, they hold for the SM and the MSSM), the Majorana phases of the degenerate neutrinos or their mixing angles at the high scale.

The most general neutrino mass matrix  $M_\nu$  in the Majorana basis is a symmetric complex matrix and it can always be diagonalized by a unitary matrix  $U_0$  in the standard way:

$$U_0^T M_\nu U_0 = D \quad , \quad (1)$$

where  $D$  refers to a diagonal matrix with real and positive masses. In particular,  $D$  is proportional to the identity matrix for the exactly degenerate spectrum. It then follows that for degenerate neutrinos, the Majorana mass matrix  $\mathcal{M}_F(X)$  at the high scale  $X$  in the flavour basis (in which the charged lepton mass matrix is diagonal) can always be written as

$$\mathcal{M}_F(X) = m U_0^* U_0^\dagger \equiv m V \quad , \quad (2)$$

where  $m$  is a positive real number denoting the common mass of the degenerate system and  $V$  is a symmetric unitary matrix. The freedom in defining the phases of the charged leptons can be used to replace  $V \rightarrow KVK$  without loss of generality, where  $K$  is a diagonal phase matrix. We use this freedom to make the third row and the third column of  $V$  (and hence of  $\mathcal{M}_F(X)$ ) real and positive. The matrix  $V$  can be parametrised in terms of only two angles and a phase [16].

The radiative corrections modify  $\mathcal{M}_F$  as one evolves down to the low scale  $x$ . With only one Higgs giving masses to the charged leptons, the Yukawa couplings of the charged leptons are hierarchical ( $h_e : h_\mu : h_\tau = m_e : m_\mu : m_\tau \approx 3 \cdot 10^{-4} : 6 \cdot 10^{-2} : 1$ ). In the limit of neglecting the electron and muon Yukawa couplings, the radiative modification of the mass matrix is given by

$$\mathcal{M}_F(x) = \mathcal{I}_\tau \mathcal{M}_F(X) \mathcal{I}_\tau \quad , \quad (3)$$

where  $\mathcal{I}_\tau \equiv \text{Diag}(1, 1, \sqrt{I_\tau})$ . We define the radiative correction parameter  $\epsilon$  through  $\sqrt{I_\tau} \equiv 1 + \epsilon$ . The value of  $\epsilon$  is determined by the model. In particular, for the SM and the MSSM,  $\epsilon$  can be written in the form

$$\epsilon \approx \mathcal{C} \frac{h_\tau^2}{(4\pi)^2} \ln \frac{X}{x} \quad , \quad (4)$$

where  $h_\tau \equiv m_\tau/v$  is the tau Yukawa coupling in the standard model. The value of the constant  $\mathcal{C}$  is  $(1/2)$  and  $(-1/\cos^2 \beta)$  in the case of SM and MSSM respectively. Note that the sign of  $\epsilon$  is opposite in these two cases, and hence the mass shifts in these two cases are in opposite directions.

Using (2), (3) and the unitarity of  $V$ , we get

$$\mathcal{M}_F(x) \mathcal{M}_F(x)^\dagger = m^2 I + (I_\tau - 1) \begin{pmatrix} V_{13}^2 & V_{13} V_{23} & V_{13} V_{33} \sqrt{I_\tau} \\ V_{13} V_{23} & V_{23}^2 & V_{23} V_{33} \sqrt{I_\tau} \\ V_{13} V_{33} \sqrt{I_\tau} & V_{23} V_{33} \sqrt{I_\tau} & 1 + V_{33}^2 I_\tau \end{pmatrix} \quad , \quad (5)$$

where  $I$  is an identity matrix. All the entries in the above matrix are real and positive due to the phase convention we have chosen (the third column of  $V$  has real and positive elements). The matrix appearing in the second term has one zero eigenvalue, indicating that one of the originally degenerate eigenvalues remains unchanged in magnitude after the radiative corrections. The other two masses are affected by the radiative corrections so that the eigenvalues of (5) are given to leading order in  $\epsilon$  by

$$\begin{aligned} |m_a|^2 &= m^2 \\ |m_b|^2 &= m^2 + 4\epsilon \sin^2 \tilde{\Psi} + O(\epsilon^2) \\ |m_c|^2 &= m^2 + 4\epsilon \cos^2 \tilde{\Psi} + O(\epsilon^2) \ , \end{aligned} \quad (6)$$

where the angle  $\tilde{\Psi}$  is defined such that

$$V_{33} = -\cos 2\tilde{\Psi} \ . \quad (7)$$

The corresponding eigenvectors are given by the columns of the following matrix:

$$U_x = R_{12}(\Omega)R_{23}(\Psi) = \begin{pmatrix} c_\Omega & s_\Omega c_\Psi & s_\Omega s_\Psi \\ -s_\Omega & c_\Omega c_\Psi & c_\Omega s_\Psi \\ 0 & -s_\Psi & c_\Psi \end{pmatrix} \ , \quad (8)$$

where  $c_\theta \equiv \cos \theta$ ,  $s_\theta \equiv \sin \theta$ , and the matrices  $R_{ij}$  are the rotation matrices in the corresponding planes. The angles  $\Omega$  and  $\Psi$  are given by

$$\tan \Omega = \frac{V_{13}}{V_{23}} \quad ; \quad \tan \Psi = (1 + 2\epsilon) \tan \tilde{\Psi} + O(\epsilon^2) \quad (9)$$

The matrix  $U_x$  (8) is the mixing matrix at the low scale  $x$ . The corresponding matrix  $U_X$  at the high scale is defined only up to  $U_X \rightarrow U_X OK$  due to the exact degeneracy in masses (here  $O$  is an orthogonal matrix, and  $K$  is a diagonal phase matrix), however a ‘‘natural’’ choice of  $U_X$  can be made. Indeed, following the arguments in [16], one can always write  $V$  in the form

$$V = \begin{pmatrix} c_{\Omega'} & s_{\Omega'} & 0 \\ -s_{\Omega'} & c_{\Omega'} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{2i\alpha} & 0 & 0 \\ 0 & c_{2\Psi'} & -s_{2\Psi'} \\ 0 & -s_{2\Psi'} & -c_{2\Psi'} \end{pmatrix} \begin{pmatrix} c_{\Omega'} & -s_{\Omega'} & 0 \\ s_{\Omega'} & c_{\Omega'} & 0 \\ 0 & 0 & 1 \end{pmatrix} \ , \quad (10)$$

$$= R_{12}(\Omega') \text{Diag}(e^{i\alpha}, 1, 1) R_{23}(\Psi') \text{Diag}(1, 1, -1) R_{23}^T(\Psi') \text{Diag}(e^{i\alpha}, 1, 1) R_{12}^T(\Omega') \ , \quad (11)$$

which conforms to our phase convention of a real third row and third column. The matrix  $U_X$  that diagonalizes this  $V$  (and hence,  $\mathcal{M}_F(X)$ ) can be chosen to be

$$U_X = R_{12}(\Omega') R_{23}(\Psi') = \begin{pmatrix} c_{\Omega'} & s_{\Omega'} c_{\Psi'} & s_{\Omega'} s_{\Psi'} \\ -s_{\Omega'} & c_{\Omega'} c_{\Psi'} & c_{\Omega'} s_{\Psi'} \\ 0 & -s_{\Psi'} & c_{\Psi'} \end{pmatrix} \ . \quad (12)$$

This mixing matrix  $U_X$  at the high scale has the same form as the matrix  $U_x$  at the low scale (8) resulting after the radiative corrections. Expanding the right hand side of (11), we get  $V_{33} = -\cos 2\Psi'$ , so that using (7), we can identify  $\tilde{\Psi} = \Psi'$ . This implies that

$\tan \Psi = \tan \Psi' [1 + O(\epsilon)]$ . Also,  $\tan \Omega' = \frac{V_{13}}{V_{23}}$ , so that from (9), we get  $\Omega = \Omega'$ . Thus, the radiative corrections leave  $\Omega'$  unchanged<sup>2</sup>, and modify the angle  $\Psi'$  only at  $O(\epsilon)$ . The specific choice for  $U_X$  made in (12) is thus a “natural” choice, such that the mixing is not perturbed in the process of evolution.

The specific structure of the mixing matrix  $U_x$  (eq. (8)) is imposed in a model independent way just by the requirement that the neutrinos be exactly degenerate at some high scale  $X$ . This structure does not allow large angle solutions to the solar neutrino problem without conflicting either with the atmospheric neutrino data [1] or the CHOOZ constraints [15]. The small angle solution is still allowed (see [17] for the detailed allowed region in angles  $\Omega, \Psi$  in this case). However, even in that case it is not possible to obtain the required mass squared differences, as we show below.

Note that the rows of  $U_x$  in (8) are labelled by the flavour indices  $e, \mu, \tau$ , while its columns are labelled by the mass eigenstates  $a, b, c$ . Reordering of the mass eigenvalues amounts to the interchange of columns of  $U_x$ . We have denoted by  $a$  the eigenvalue which remains unchanged and the corresponding eigenvector at low scale (which does not have any  $\tau$  flavour component) is given by the first column of  $U_x$ . The other columns correspond to the eigenvalues  $|m_b|^2$  and  $|m_c|^2$  respectively. The neutrino mass squared differences are given from eq.(6) by

$$\Delta m_{cb}^2 \approx 4\epsilon \cos 2\tilde{\Psi} \quad , \quad \Delta m_{ca}^2 \approx 4\epsilon \cos^2 \tilde{\Psi} \quad , \quad \Delta m_{ba}^2 \approx 4\epsilon \sin^2 \tilde{\Psi} \quad . \quad (13)$$

It is seen from (13) that two hierarchical mass squared differences are possible if (i)  $\tilde{\Psi} \sim \pi/4$ , (ii)  $\tilde{\Psi} \sim \pi/2$ , (iii)  $\tilde{\Psi} \sim 0$ . These three cases correspond to the identification of the mass eigenstate  $\nu_3$  with  $a, b, c$  respectively. We shall show below that all the three identifications lead to phenomenological problems. Note that since the arguments below depend only on the ratios of the mass squared differences, they are independent of the magnitude or sign of  $\epsilon$ , and hence are valid for all the models.

In case (i), the first column of  $U_x$  should be identified with the third column of the leptonic mixing matrix  $U$ , leading to the prediction

$$|U_{e3}|^2 + |U_{\mu 3}|^2 = 1 \quad .$$

This is clearly in contradiction with the required values for the atmospheric neutrino mixing and the bound on  $|U_{e3}|$  from CHOOZ [15].

In case (ii), one has

$$\frac{\Delta m_{\odot}^2}{\Delta m_{atm}^2} \approx \cot^2 \tilde{\Psi} \quad , \quad (14)$$

$$\sin^2 2\theta_{atm} = 4c_{\Omega}^2 c_{\Psi}^2 (1 - c_{\Omega}^2 c_{\Psi}^2) \quad . \quad (15)$$

Since  $\Delta m_{\odot}^2 / \Delta m_{atm}^2 < 0.1$ , from (14) we have  $\cos^2 \Psi \approx \cos^2 \tilde{\Psi} < \cot^2 \tilde{\Psi} < 0.1$ . Then eq. (15) gives  $\sin^2 2\theta_{atm} < 0.4$ , which is clearly inconsistent with the atmospheric neutrino data.

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<sup>2</sup>This general result has been proved in [17] in the case of the mixing matrix of the form  $U = R_{12} \cdot R_{23}$ . Here we have shown that  $U$  can always be brought in this form in the present context.

In case (iii),

$$\frac{\Delta m_{\odot}^2}{\Delta m_{atm}^2} \approx \tan^2 \tilde{\Psi} \quad , \quad (16)$$

$$\sin^2 2\theta_{atm} = 4c_{\Omega}^2 s_{\Psi}^2 (1 - c_{\Omega}^2 s_{\Psi}^2) \quad . \quad (17)$$

Since  $\Delta m_{\odot}^2/\Delta m_{atm}^2 < 0.1$ , from (16) we have  $\sin^2 \Psi \approx \sin^2 \tilde{\Psi} < \tan^2 \tilde{\Psi} < 0.1$ . Then eq. (17) gives  $\sin^2 2\theta_{atm} < 0.4$ , which is clearly inconsistent with the atmospheric neutrino data, similar to the case (ii) above.

The above reasoning holds both in case of the small angle as well as the large angle solution to the solar neutrino problem. As already remarked, the latter can be independently ruled out from the structure of the mixing matrix  $U_x$ . Thus, it is not possible to generate the observed pattern of neutrino masses and mixings solely through radiative corrections, starting from exactly degenerate Majorana neutrinos at some high scale  $X$ . This conclusion is valid irrespective of the Majorana phases or the mixing angles at the high scale, or the value of the high scale itself as long as there is no new physics between the high and the low scale.

When the masses at the high scale are not exactly equal, but the mass splittings are small compared to the magnitude of radiative corrections ( $\Delta m_{ij}^2(X) \ll \epsilon |m_i|^2$ ), the radiative corrections determine the mixing matrix at the low scale, and hence the mixing matrix is  $U_x$  as given by eq.(8). The mass squared differences at the low scale are still dominantly given by eq.(13). (In the case where the original Majorana masses of the neutrinos are  $m_i = \pm m$ , this is shown explicitly in [18].) This implies that the required masses and mixings at the low scale still cannot be reproduced even with the introduction of small mass splittings ( $\Delta m_{ij}^2(X) \ll \epsilon |m_i|^2$ ) at the high scale. On the other hand, when  $\Delta m_{ij}^2(X) \gg \epsilon |m_i|^2$ , the radiative corrections clearly fail to have any significant impact on the values of  $\Delta m_{ij}^2$  and the mixing angles. The net result is that at least one of the mass splittings at the high scale ( $\Delta m_{ij}^2(X)$ ) needs to be approximately equal to the one observed at the low scale ( $\Delta m_{ij}^2(x)$ ), and hence needs to be generated through some other mechanism. The radiative corrections can help only in generating the other (most likely smaller) mass splitting. Such masses and mixing patterns at the high scale have been shown to lead to correct masses and mixings at the low scale in several models [19].

The other possibility is that the degeneracy is completely broken at the high scale explicitly by a source other than the Yukawa couplings. Specific models have been investigated in [13,20]. These may give rise to quasi-degenerate neutrino masses at the high scale. One possible origin of such splitting is the running of the degenerate right handed neutrino masses from the Planck scale to the GUT or the scale of the right handed neutrino masses [11]. The radiative corrections can then modify the mass splittings as well as the mixing angles. The change in the mass splittings will be proportional to the magnitude of the radiative corrections, but the mixing angles can get modified drastically – they can get magnified or can be driven to zero, depending on the magnitude and sign of the radiative corrections as compared to the original mass splittings at the high scale [6,7,18]. It may even be possible to obtain large angles at the low scale irrespective of the mixing angles at the high scale [8].

Throughout this analysis, we have assumed that the charged lepton Yukawa couplings involve only one Higgs. In the models in which two or more Higgs contribute to the charged

lepton masses, the possibility of exactly degenerate neutrinos at the high scale reproducing the mass and mixing spectrum at the low scale is still open.

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