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Abstract

Head-tail damping rate and coherent tune shift depend on chromaticity and transverse wake field. Using this dependence, the transverse impedance of LEP can be estimated from coherently damped betatron oscillations measured at different chromaticities and beam currents. We compare measurements and analytical results.

1 INTRODUCTION

In LEP the three most important impedance sources are the copper and superconducting RF cavities and shielded bellows. The cavity impedances are known from earlier measurements [1]: The horizontal impedance Z_x/Q of 86 Cu cavities is about 1.0 MΩ/m and the resonant frequency 2 GHz. The impedance of the sc. cavities is taken to be 0.6 MΩ/m, and their resonant frequency is 0.7 GHz. The horizontal impedance Z_x of the bellows is more uncertain [1]. Their resonant frequency is high, about 12 GHz.

In 1997, damped coherent betatron oscillations in LEP were recorded turn-by-turn for various bunch currents and chromaticities. Betatron tune and the exponential damping for each data set can be obtained from a fit to a damped anharmonic oscillation [2, 3]. We will use these results to infer information on the effective impedance of the bellows.

2 IMPEDANCE MODELLING

The transverse impedance is commonly described by several broadband resonators of the form

$$Z_1^{\perp}(\omega) = \frac{c}{\omega}\frac{R_s}{1 + iQ\left(\frac{\omega_r}{\omega} - \frac{\omega}{\omega_r}\right)} \qquad (1)$$

where ω_r is the angular resonance frequency, and R_s has dimensions of $\Omega\mathrm{m}^{-2}$. The horizontal impedance Z_x is related to R_s by $R_s = Z_x\omega_r/c$. The complex frequency shift for the $l = 0$ head-tail mode of a Gaussian bunch is [4]

$$\Omega - \omega_\beta = -i\frac{Nec^2}{4\sqrt{\pi}(E/e)T_0\omega_\beta\sigma_z}\,(Z_1^{\perp})_{\mathrm{eff}} \qquad (2)$$

where $\omega_\beta = Q_\beta\omega_0$ is the angular betatron frequency (including integer part), Q_β the betatron tune, ω_0 the angular revolution frequency, N_b the bunch population, e the electron charge, c the speed of light, σ_z the rms bunch length, and E the beam energy. The frequency shift is proportional to the effective impedance [5] defined by

$$(Z_1^{\perp})_{\mathrm{eff}} = \frac{\sum_{p=-\infty}^{\infty} Z_1^{\perp}(\omega_p)h_0(\omega_p - \omega_\xi)}{\sum_{p=-\infty}^{\infty} h_0(\omega_p - \omega_\xi)} \qquad (3)$$

where, for a Gaussian beam, $h_0(\omega_p) = \exp(-\omega_p^2\sigma_z^2/c^2)$, $\omega_p = p\omega_0 + \omega_\beta$ and $\omega_\xi = \xi\omega_\beta/\alpha_c$ with $\xi = Q'/Q_\beta = \frac{\Delta Q_\beta}{Q_\beta}/\frac{\Delta p}{p}$ denoting the chromaticity and α_c the momentum compaction factor. The chromaticity enters in eq. (3) only through the frequency shift ω_ξ.

As an example, for $Q' = Q_\beta\xi = 10$, $\alpha_c = 1.8 \times 10^{-4}$ and $Q_{\beta,x} = 90$, the frequency shift is $f_\xi = \omega_\xi/(2\pi) = 0.1$ GHz. For an rms bunch length 5 mm, the beam spectrum extends roughly up to about 10 GHz.

The tune shift with bunch current is given by the real part of the complex frequency shift, *i.e.*, $\Delta Q_\beta/\Delta I_b \propto \mathrm{Re}(\Omega - \omega_\beta) \propto \mathrm{Im}(Z_1^{\perp})_{\mathrm{eff}}$, and the damping rate by the imaginary part $1/\tau \propto -\mathrm{Im}(\Omega - \omega_\beta) \propto \mathrm{Re}(Z_1^{\perp})_{\mathrm{eff}}$. The machine impedance can be reconstructed [6] from measurements of ΔQ_β and/or $1/\tau$ as a function of $Q\ '$ For long bunches, $\omega_r \gg c/\sigma_z$, we have $\Delta Q_\beta/\Delta I_b \propto Z_x/Q \propto R_s/(\omega_r Q)$, and $1/\tau \propto Z_x\omega/Q^2 \propto R_s/(\omega_r^2 Q^2)$. However, in our example, the bunch is not long enough to use this approximation.

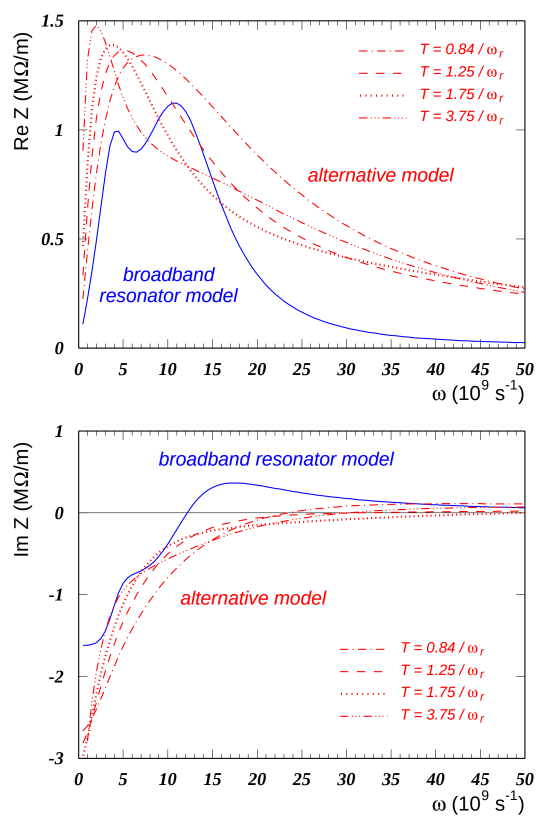

Figure 1: *Real (top) and imaginary (bottom) component of transverse LEP impedances as a function of angular frequency ω, comparing standard and improved broadband resonator model.*

Recently, an alternative impedance model has been proposed [7]:

$$Z_1^{\perp}(\omega) = -\frac{icL}{(1 - i\omega T)^{3/2}}. \tag{4}$$

Unlike the usual broadband resonator, this impedance allows for the correct behaviour at high frequencies (diffraction model [4]). It has two free parameters for each resonator (L,T), the same number as the conventional model, if, for the later, we choose Q equal to 1 as we will assume in the following. Identifying the frequency where the real part of the impedance is maximum, $\omega_{\max} \approx 0.73/T$, with the corresponding frequency in the broadband resonator model, $\omega_{\max} \sim 0.87\omega_r$, and the maximum impedance value itself with Z_x, the parameters of the two models can be related via $T \approx 0.84/\omega_r$ and $L \approx 5\mathrm{Re}Z_x/(3\omega_r)$ [7]. Alternatively, if we identify the frequency where the imaginary part of the impedance becomes zero with the resonant frequency ω_r, we obtain $T \approx 1.72/\omega_r$.

Figure 1 shows the real and imaginary components of the transverse impedance calculated for the broadband resonator model and several calculations for the alternative model with different identification of parameters T and L (see discussion in section 4).

3 MEASUREMENTS

All measurements discussed here were performed for the 90/60 optics at 45.625 GeV, with a momentum compaction factor $\alpha_c = 1.86 \times 10^{-4}$ (measured and computed), and betatron tunes of $Q_{\beta,x} \approx 90.28$ and $Q_{\beta,y} \approx 76.19$. The rms bunch length, inferred from synchrotron tune and previous streak-camera calibration measurements, was about 5 mm. At the time the data were taken, 86 Cu cavities and 240 s.c. cavities were installed in LEP.

Figure 2 shows the damping rates of betatron oscillations for several currents and chromaticities. The analysis is based on the study of 1000-turn measurements after a single excitation. Damping rate and betatron tune can be extracted from a fit to a damped anharmonic oscillation taking into account the horizontal detuning with amplitude. Details on the fitting procedure can be found in [2, 3]. Since the damping of coherent bunch oscillations in LEP is composed of radiation and head-tail damping, we interpret the linear increase with bunch current in fig. 2 as due to head-tail damping and identify the offset for zero current with synchrotron radiation damping. The head-tail damping rates normalised by the bunch current obtained from linear fits to data shown in fig. 2 are depicted in fig. 4 as a function of Q'.

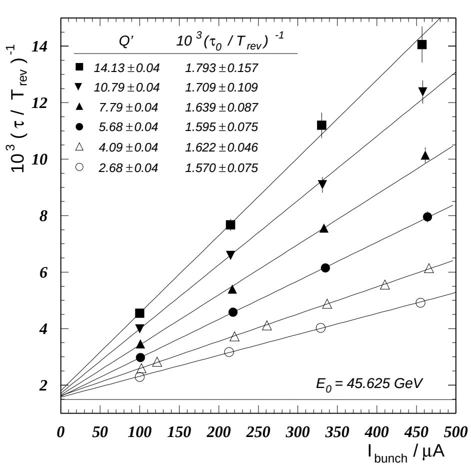

Figure 2: *Damping rate as function of bunch current for various chromaticities measured at a beam energy of 45.6 GeV [3]. The revolution period is 88.9 μs.*

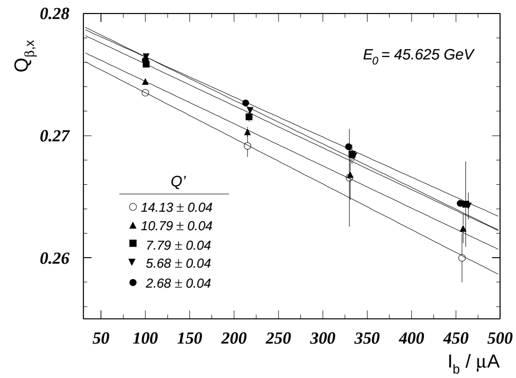

Figure 3: *Horizontal betatron tune as a function of bunch current for various chromaticities measured at a beam energy of 45.6 GeV. The revolution period is 88.9 μs.*

The machine broadband impedance does not only cause damping, but it also gives rise to a tune shift with current. Measurements of this coherent tune shift for various chromaticities are shown in fig. 3 as a function of bunch current, and the fitted slopes are summarised as a function of chromaticity in fig. 5.

4 RESULTS

We assume that the transverse impedance is just given by the sum of the three contributions discussed above. Also, for simplicity, we consider the impedance parameters for the cavities as known. From a fit of the measured head-tail damping rates to one of the two impedance models, we may then infer the horizontal impedance of the elliptical bellows. Since the measured head-tail damping rates do not cross zero at $Q' = 0$ (see fig. 4), we include a constant offset in Q'.

A two parameter fit for the conventional resonator model with a χ^2 of 2.3 per degree of freedom yields an offset $Q'_0 = 1.52 \pm 0.22$ and a bellows impedance $Z_x = 0.31 \pm 0.02$ MΩ/m, which is exactly equal to half the vertical impedance Z_y quoted in Ref. [1]. The value of the latter, $Z_y \approx 0.6$ MΩ/m, was deduced from vertical coherent tune shift measurements, and a two times smaller number for the horizontal bellows impedance was expected to reflect the vacuum-chamber aspect ratio [1]. A two-parameter fit to the improved broadband model gives about the same values of χ^2 and chromaticity offset, but the resulting impedance depends strongly on the exact relation between T and ω_r which we assume. For example, with $T \approx 0.84/\omega_r$ (equating the frequencies at the maximum real impedance) for all three resonating components we find a bellow impedance equal to zero, $Z_x = 0.00 \pm 0.02$ MΩ/m, and $\chi^2 = 13.3$ per degree of freedom; with $T \approx 1.72/\omega_r$ (equating the zero crossing of the imaginary impedance) we have $Z_x = 0.23 \pm 0.02$ MΩ/m and $\chi^2 \approx 2.3$. In order to obtain the same impedance $Z_x \approx 0.3$ MΩ/m as for the conventional model, we must choose $T \approx 2.1/\omega_r$.

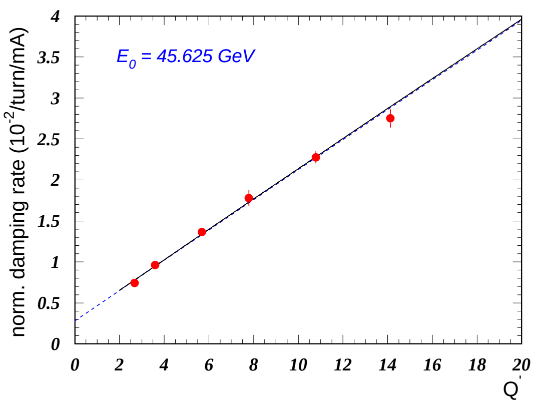

Figure 4: *Head-tail damping rate normalised to the bunch current as function of measured chromaticity for a beam energy of 45.6 GeV [3]. The solid and dashed lines represent broadband and alternative impedance model respectively fitted to experimental data.*

Figure 4 compares the damping rates predicted by the two models, including the fitted contributions from the bellows and horizontal offset, with the measurements. There is no notable difference between the curves for the two models. In particular, neither model is able to reproduce the apparent small curvature in the data, even when additional resonators at much lower frequency are included.

The coherent tune shift calculated from the alternative model depends on the choice of the parameter T. Figure 5 shows the tune shift with current as function of chromaticity for the broadband resonator model, for the alternative model with several choices for the parameter T and the measured tune shifts. The two models give the same tune shift for $T \approx 1.25/\omega_r$, whereas the agreement between alternative model and the measured tune shift is best for $T \approx 1.00/\omega_r$. Yet in either case the bellows impedances fitted from the damping rate would differ significantly for the two models.

5 SUMMARY

Horizontal head-tail damping rates and coherent tune shifts with current were measured as a function of chromaticity in LEP at 46 GeV. In order to parametrise the transverse impedance, we fitted the experimental data to a conventional broadband resonator model and to an alternative model with correct high-frequency behaviour, in either case taking into account the contributions from bellows, s.c. cavities and Copper cavities. The two impedance models describe the data equally well if the parameters in the alternative model are properly chosen. The fit to the conventional model yields a horizontal bellows impedance which is a factor 2 smaller than the computed vertical impedance. This difference was expected and reflects the vacuum-chamber aspect ratio at the bellows. A constant offset in the horizontal chromaticity of $\Delta Q' \approx 1.5$ or a corresponding offset in the normalised damping rate remains unexplained.

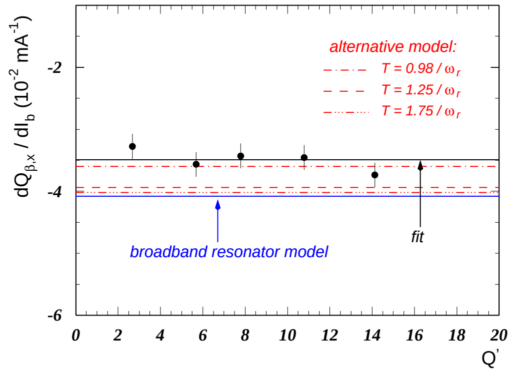

Figure 5: *Horizontal tune shift with bunch current (in μA) as function of chromaticity for a beam energy of 45.6 GeV [3]. The solid line represents a straight line fit to the data, the dotted and dashed lines depict the tune shifts as calculated using the two types of impedance models.*

6 ACKNOWLEDGEMENTS

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7 REFERENCES

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