

## CURRENT TOPICS IN B PHYSICS<sup>a</sup>

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I discuss topics of particular interest in connection with the ongoing experiments at the B factories BaBar and Belle: B-mixing, QCD effects in nonleptonic decays and rare decays.

### 1 Introduction

We currently witness the dawn of a very exciting era for B physics: after years of planning and construction, the two B factories BaBar and Belle have finally started operating in 1999 and first thrilling results have been presented at this year's ICHEP 2000 conference<sup>1</sup>. The physics programme of the B factories has two clear focal points: the detailed exploration of CP violation in  $B_d$  decays on the one hand and the precise measurement of rare flavour-changing neutral current (FCNC) processes on the other hand, with the aim to find or at least constrain new physics. In the near and intermediate future, the investigation of B decays will also form an essential part of the physics programme of Tevatron Run II<sup>2</sup> and the LHC<sup>3</sup>.

Why these enormous efforts? — The answer to this question can maybe be formulated as: “Because B physics probes the scalar sector of the Standard Model.” Recall that the phenomena of both quark mixing, i.e. the CKM-matrix, and CP-violation are inseparably connected to the fact that quarks have mass; indeed, CP is a manifest and natural symmetry of massless gauge field theories, both chiral and vector-like:<sup>b</sup> CP-violation is hence intrinsically connected to the mechanism that gives mass to particles, i.e. the Higgs-mechanism in the SM. The information

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<sup>b</sup>As is well known, also strong CP-violation – the  $\theta$  term in the Lagrangian – is not observable if at least one quark is massless.

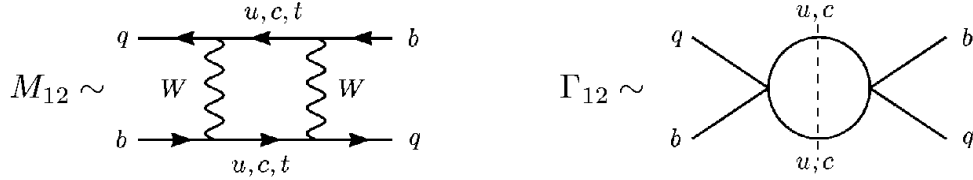


Figure 1: Typical diagrams contributing to B-mixing. The dashed line in the diagram to the right denotes that the imaginary part is to be taken. Figure taken from Ref.<sup>3</sup>.

on the scalar sector that can be obtained from B decays thus has to be viewed as complementary to that from direct Higgs-searches, and a complete picture of the mechanism of  $SU_L(2) \times U(1)$  symmetry-breaking and mass-generation can only be obtained by putting together the information from both direct searches and B physics experiments. The reason why it is just the B system that is so well suited for such studies can be traced back to the fact that the top is much heavier than the other quarks, which relaxes the GIM-suppression effective in box and penguin-diagrams with near-degenerate quarks. The relaxation of the GIM-mechanism also entails a large  $B_d$  mixing-phase, which gives rise to sizeable CP-violating effects in B decays, and yields typical branching ratios of “rare” FCNC  $b \rightarrow s$  transitions of the order  $10^{-6}$ , which is by orders of magnitude larger than in rare K and D decays.

Rare decays also offer the possibility to find or constrain new-physics effects – the experimental results for the penguin-mediated process  $b \rightarrow s\gamma$ , for instance, serve as severe constraints of sfermion masses and mixing in SUSY-models. Also here one observes a certain complementarity between direct searches and B physics: once new physics, e.g. SUSY, is found at the Tevatron or the LHC, it will entail simultaneous production of a plethora of new particles, making it hard to impossible to disentangle their decay chains and determine particle parameters in a model-independent way. Also here B physics will help in putting constraints from the observed indirect effects of such new particles and thus fill the gap between new-physics discovery at hadron machines and the takeoff of a linear collider at which the properties of new particles can be studied in detail.

The above considerations let it appear as appropriate to center this overview around theoretical challenges in CP-violating processes and rare decays; for the discussion of other, also very interesting topics in B physics, like e.g. semileptonic  $b$  decays, I refer to Bigi’s talk at ICHEP 2000.<sup>4</sup>

## 2 B-Mixing

Why start this review with a section on B-mixing? — Because it is highly relevant both for understanding CP-violation and for the measurement of the CKM matrix elements  $|V_{tq}|$  with  $q = d, s$ . Let me shortly review the essentials: As is well known, the flavour-eigenstates of neutral B mesons,  $B_q^0 = (q\bar{b})$  and  $\bar{B}_q^0 = (b\bar{q})$ , mix on account of weak interactions. The mixing can be described, in the framework of quantum mechanics and in the basis of flavour-eigenstates, by the Hamiltonian

$$\mathbf{H} = \mathbf{M} - \frac{i}{2} \mathbf{\Gamma} = \begin{pmatrix} M & M_{12} \\ M_{12}^* & M \end{pmatrix} - \frac{i}{2} \begin{pmatrix} \Gamma & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma \end{pmatrix},$$

where  $\mathbf{M}$  and  $\mathbf{\Gamma}$  are Hermitian and their respective diagonal entries are equal by virtue of CPT invariance. Typical diagrams contributing to the off-diagonal elements are shown in Fig. 1. The induced mass and width differences of the mass eigenstates, i.e. the observed B mesons, are

given by

$$\Delta M_q = 2 \left| M_{12}^{(q)} \right|, \quad \Delta \Gamma_q = 2 \frac{\text{Re} (M_{12}^{(q)*} \Gamma_{12}^{(q)})}{|M_{12}^{(q)}|}.$$

A quantity especially relevant for CP measurements is the mixing-phase  $\phi_q = \arg M_{12}^{(q)}$ . In experiment,  $\Delta M_d$  has been measured with small error, but for  $\Delta M_s$  there exists only a lower bound to date; in the SM, the expected value is  $\sim (15 - 25) \text{ps}^{-1}$ , which induces  $B_s$  oscillations too fast to be resolved at present machines, but is well within the reach of Tevatron Run II and the LHC. The width-difference in the  $B_d$  system is expected to be too small to be measurable, but could be non-negligible for  $B_s$ ; present experimental results for the latter are too crude to be conclusive.

As the theoretical expression for  $\Delta M_d$  is directly proportional to  $|V_{td}|^2$ , the sum over box-diagrams being dominated by the  $t$  quark contribution, one would think that a precise measurement of  $|V_{td}|$  be possible. The achievable precision does, however, crucially depend on the accuracy to which the relevant hadronic matrix element,  $\langle B_d^0 | (\bar{d}b)_{V-A} (\bar{d}b)_{V-A} | \bar{B}_d^0 \rangle \sim f_B^2 \hat{B}_{B_d}$ , is known. Best available numbers come from lattice simulations; for a comprehensive review of the state of the art of lattice simulations of B meson matrix elements I refer to<sup>5</sup>; here, let it suffice to say that simulating the  $b$  quark with its mass  $\sim 5 \text{GeV}$ , i.e. a Compton-wavelength not large compared to typical lattice spacings,  $a \sim (2 - 4) \text{GeV}^{-1}$ , poses a severe challenge, which, together with the fact that most simulations are done in the quenched approximation, neglecting the feedback of quarks on the gauge-fields, entails a large quoted error of  $\sim 30\%$  on  $f_B^2 \hat{B}_{B_d}$ . Preliminary results from unquenched simulations have been presented by the CP-PACS collaboration at ICHEP 2000<sup>6</sup>; they indicate an increase of  $f_B$  by ca. 10% with respect to the quenched results.

A quantity in which much of lattice systematics and in particular quenching effects are expected to cancel, is the ratio of  $B_s$  and  $B_d$  matrix elements,  $(f_{B_s} \sqrt{\hat{B}_{B_s}}) / (f_{B_d} \sqrt{\hat{B}_{B_d}}) \equiv \xi$ , quoted as  $1.16 \pm 0.07$ <sup>7</sup>, which allows one to relate  $\Delta M_s / \Delta M_d$  to the ratio of CKM matrix elements  $|V_{ts} / V_{td}|$  as

$$\frac{\Delta M_s}{\Delta M_d} = \left| \frac{V_{ts}}{V_{td}} \right|^2 \frac{M_{B_s}}{M_{B_d}} \xi^2.$$

Thus, a determination of  $|V_{ts} / V_{td}|$  will be possible with small theoretical error once  $\Delta M_s$  has been measured. Note that the above formula is valid within in the SM only and can be upset by new-physics contributions to B-mixing; in this case, one expects this determination of  $|V_{ts} / V_{td}|$  to be at variance both with the one from FCNC  $b \rightarrow s$  and  $b \rightarrow d$  transitions and with constraints from the unitarity of the CKM matrix.

### 3 Challenges I: Nonleptonic Decays & CP-Violation

Measuring CP-violating time-dependent asymmetries is at the heart of BaBar's and Belle's physics programme. CP-asymmetries are in general non-vanishing only because of the presence of strong phases — and it is just the same strong phases that often complicate the aim of such measurements, i.e. the extraction of CP-violating weak phases. To illustrate the issue, let us consider the decay of  $B_d$  into a final state  $F$  which is an eigenstate under CP,  $\text{CP} | F \rangle = \pm | F \rangle$ , such as  $J/\psi K_S$  or  $\pi\pi$ . In this case, the time-dependent CP-asymmetry can be written as

$$\begin{aligned} a_{\text{CP}}(t) &= \frac{\Gamma(B_d^0(t) \rightarrow F) - \Gamma(\bar{B}_d^0(t) \rightarrow F)}{\Gamma(B_d^0(t) \rightarrow F) + \Gamma(\bar{B}_d^0(t) \rightarrow F)} \\ &= \mathcal{A}_{\text{CP}}^{\text{dir}}(B_d \rightarrow F) \cos(\Delta M_d t) + \mathcal{A}_{\text{CP}}^{\text{mix}}(B_d \rightarrow F) \sin(\Delta M_d t), \end{aligned} \quad (1)$$

where the observables  $\mathcal{A}_{\text{CP}}^{\text{dir,mix}}$  can be expressed in terms of the hadronic quantity

$$\xi_F = \mp e^{-i\phi_d} \frac{\langle F | H_{\text{weak}}^{\text{eff}} | \bar{B}_d^0 \rangle}{\langle F | H_{\text{weak}}^{\text{eff}} | B_d^0 \rangle} \quad (2)$$

as

$$\mathcal{A}_{\text{CP}}^{\text{dir}} = \frac{1 - |\xi_F|^2}{1 + |\xi_F|^2}, \quad \mathcal{A}_{\text{CP}}^{\text{mix}} = \frac{2\text{Im} \xi_F}{1 + |\xi_F|^2}; \quad (3)$$

$\phi_d = \arg M_{12}$  is the  $B_d$  mixing-phase introduced in the previous section and

$$H_{\text{weak}}^{\text{eff}} = \sum_{j=u,c,t;r=s,d} V_{j^*}^* V_{jb} Q^{jr} + \text{c.c.}$$

the effective weak Hamiltonian, written as sum over weak amplitudes. As  $\xi_F$  in general depends on hadronic matrix elements, its exact calculation is possible only for the special case of one single weak amplitude dominating, such that the hadronic matrix elements in  $\xi_F$  cancel; these are the so-called “gold-plated” decays<sup>8</sup>, e.g.  $B_d \rightarrow J/\psi K_S$ , where to good approximation  $\xi_F$  is a pure phase.  $B_d \rightarrow J/\psi K_S$  is also to date the only B decay channel where CP-violation has been observed. The results of BaBar and Belle may give first hints at a non-standard mechanism of CP-violation, with — amusingly — numerical results not too far away from predictions in a model where CP-violation is understood as spontaneous breakdown of a manifest CP-symmetry of the underlying Lagrangian.<sup>9</sup> However, large experimental uncertainties do not yet allow a definite conclusion.

The majority of data, however, will be collected in “brazen” channels, where the extraction of weak phases requires precise knowledge of hadronic matrix elements, i.e. nonperturbative QCD. As is well known, in the SM, CP-violation is due to one single complex phase in the CKM-matrix; the discussion in the literature, however, often refers to three “weak phases”, labelled  $\alpha$ ,  $\beta$ ,  $\gamma$ , which are the three angles of the “unitarity triangle”, i.e. the graphic representation of the unitarity relation  $\sum_{q=u,c,t} V_{qb} V_{qd}^* = 0$  in the complex plane; with the usual phase-conventions of the CKM-matrix one has e.g. for the B-mixing phases  $\phi_d = 2\beta$  and  $\phi_s \approx 0$ . The aim of CP-measurements is then to overconstrain the unitarity triangle by measuring its sides and angles in as many ways as possible in order to verify or falsify the CKM-picture of CP-violation.

I would like to stress here that the problem of how to calculate nonleptonic B hadronic matrix elements with phenomenologically acceptable precision is indeed very important and that despite recent progress to be reported below it is premature to consider the problem as sufficiently well understood. Much work is still needed before we can extract weak phases from “brazen” channels with a similar accuracy as from the “gold-plated” ones.<sup>c</sup>

### 3.1 “Diagrammatics” and U-Spin Flavour-Symmetry

One rather pragmatic possibility to treat the unknown QCD matrix elements is to exploit dynamical symmetries of QCD in order to reduce the number of independent matrix elements and actually *measure* them in experiment rather than calculate them from first principles. A representative example for this approach, discussed in Ref.<sup>10</sup>, is provided by the pair of decay channels  $B_d \rightarrow \pi^+ \pi^-$  and  $B_s \rightarrow K^+ K^-$  which are related by U-spin symmetry, i.e. the exchange of  $d$  and  $s$  quarks, under which QCD is assumed to be invariant. The decay amplitudes can be written as

$$\begin{aligned} A(B_d \rightarrow \pi^+ \pi^-) &\propto e^{i\gamma} [1 - d e^{i\theta} e^{-i\gamma}], \\ A(B_s \rightarrow K^+ K^-) &\propto e^{i\gamma} [1 + (\text{CKM factor}) \times d' e^{i\theta'} e^{-i\gamma}], \end{aligned} \quad (4)$$

<sup>c</sup>Note that in contrast to B-mixing, lattice calculations are not likely to provide substantial help in the foreseeable future, as the simulation of a two-particle final state with sharp momentum requires a lattice with spatial extension by orders of magnitude larger than what is within present reach.

where the terms in  $de^{i\theta}$  and  $d'e^{i\theta'}$  denote the disturbing “penguin-pollution” contributions. In the limit of U-spin symmetry be realized exactly, one has  $d = d'$  and  $\theta = \theta'$ , and the four CP-observables accessible in the two channels can be expressed in terms of only three unknowns, the weak phase  $\gamma$  and the two penguin-parameters  $d$  and  $\theta$ , provided the mixing-phases  $\phi_d$  and  $\phi_s$  have been measured in gold-plated channels before. The validity of this approach is, however, limited by U-spin breaking effects, which need not necessarily be small. We have already encountered an example in the previous section:

$$\frac{\langle B_s^0 | (\bar{s}b)_{V-A}(\bar{s}b)_{V-A} | \bar{B}_s^0 \rangle}{\langle B_d^0 | (\bar{d}b)_{V-A}(\bar{d}b)_{V-A} | \bar{B}_d^0 \rangle} = 1.37 \pm 0.16 \neq 1.$$

Also the ratio of the two amplitudes given in (4) does, in factorization approximation, and setting all CKM factors equal, deviate from 1:<sup>11</sup>

$$\frac{A(B_s \rightarrow K^+K^-)}{A(B_d \rightarrow \pi^+\pi^-)} = \frac{f_K}{f_\pi} \frac{F_0^{B_s \rightarrow K}(M_K^2)}{F_0^{B_d \rightarrow \pi}(M_\pi^2)} \left( \frac{M_{B_s}^2 - M_K^2}{M_{B_d}^2 - M_\pi^2} \right) \approx 1.35.$$

Once experimental data are available, the accuracy to which U-spin symmetry is realized in (4) can partially be tested by relaxing one of the two conditions  $d = d'$  and  $\theta = \theta'$ , so that one extracts four unknowns from four observables. Further theoretical studies of U-spin breaking effects are, however, definitely needed before one can use U-spin relations with confidence as a precision tool to extract weak phases.

### 3.2 Hard Perturbative QCD and the Heavy Quark Limit

In the limit  $m_b \rightarrow \infty$ , nonleptonic B decays share some crucial features with hard exclusive QCD reactions at large momentum transfer, such as e.g. hadron electromagnetic form factors. The theoretical description of such processes was pioneered by Brodsky and Lepage (see e.g.<sup>12</sup>) who showed that, to leading order in a light-cone expansion in terms of contributions of increasing twist, i.e. in terms of contributions of increasing inverse power of momentum-transfer, factorization is possible and the amplitude can be written as convolution of a process-dependent, perturbative hard-scattering kernel with universal, nonperturbative functions describing the parton momentum-distribution inside the hadron; the factorization is such that only parton-momenta in direction of the particle momentum do contribute and transverse degrees of freedom are suppressed – hence the term “collinear factorization”. Recently, Beneke et al. have shown that a conceptually similar approach can also be applied to nonleptonic B meson decays, where, loosely speaking, the  $b$  quark mass plays the rôle of the large parameter and serves to suppress higher-twist contributions. To  $O(\alpha_s)$ , the amplitude of the process  $B \rightarrow \pi\pi$  can be written as<sup>13</sup>

$$\begin{aligned} \langle \pi\pi | H_{\text{weak}}^{\text{eff}} | B \rangle &= f_+^{B \rightarrow \pi}(0) \int_0^1 dx T^I(x) \phi_\pi(x) \\ &+ \int_0^1 d\xi dx dy T^{II}(\xi, x, y) \phi_B(\xi) \phi_\pi(x) \phi_\pi(y) + O(1/m_b). \end{aligned} \quad (5)$$

The diagrams contributing to the hard-scattering kernels are shown in Fig. 2.  $\phi_\pi$ , technically speaking the twist-2 distribution amplitude of the  $\pi$ , is a rather well-studied object whose functional dependence on  $x$  can be understood, exploiting conformal symmetry of massless QCD (symmetry-group  $SL(2, R)$ ), in terms of a partial wave expansion in terms of contributions of increasing conformal spin.<sup>12,14</sup> Much less, however, is known about the B meson’s distribution amplitude  $\phi_B$ , for whose parametrization one has to rely on models. The numerical analysis of (5) reveals that the “penguin-pollution” term  $d \exp(i\theta)$  of the last section is small and that the branching ratio reads

$$B(\bar{B}_d \rightarrow \pi^+\pi^-) = 6.5 \cdot 10^{-6} \times \left| e^{-i\gamma} + 0.09 e^{i \cdot 12.7^\circ} \right|^2, \quad (6)$$

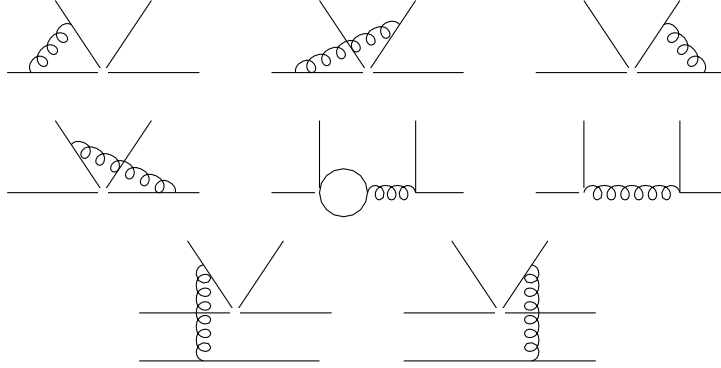


Figure 2:  $O(\alpha_s)$  corrections to the hard-scattering kernels  $T^I$  (first two rows) and  $T^{II}$  (last row). The two lines directed upwards represent the two quarks forming the emitted pion. Figure taken from Ref.<sup>13</sup>.

which indicates that corrections to the time-honoured “vacuum-saturation” picture are small.

The crucial question is now about the size of power-suppressed  $1/m_b$  corrections to (5).<sup>d</sup> As discussed in<sup>15</sup>, there are two problems associated with them: first, there are formally power-suppressed, but chirally enhanced twist-3 terms in  $2m_\pi^2/(m_u + m_d)/m_b \approx 0.7$ , which is not a small parameter. Second, at  $O(\alpha_s)$  these terms induce logarithmic divergences which violate the QCD factorization formula (5). On a more technical level, these logarithmic divergences are due to soft contributions similar to those that, in principle, also appear at leading order in  $1/m_b$ , but in (5) are subsumed in the form factor  $f_+^{B \rightarrow \pi}$ . Soft terms actually spoil the calculation of B decay form factors by conventional hard perturbative QCD methods,<sup>e</sup> and it is probably the main achievement of<sup>13</sup> as compared to competing approaches<sup>17</sup>, also based on hard perturbative QCD, to show that, for nonleptonic B decays, at leading order in  $1/m_b$ , all soft terms can be included into the experimental observable  $f_+^{B \rightarrow \pi}(0)$ . Nevertheless it is somewhat disturbing that soft effects show up again at order  $1/m_b$  and are chirally enhanced.

The emerging picture thus seems to be that the formal limit  $m_b \rightarrow \infty$  can be treated on theoretically safe grounds, but that power-suppressed terms in  $1/m_b$ , whose treatment in a systematic way is not yet understood, are phenomenologically relevant. I note in passing that this appears to be a general feature of calculations in the heavy quark limit: unless terms linear in  $1/m_b$  are absent due to a protecting symmetry, they yield sizeable contributions.<sup>f</sup> It thus appears presently very difficult to attach any meaningful theoretical uncertainty to (5) and (6).

## 4 Challenges II: Rare Decays

Flavour-changing neutral current decays involving  $b \rightarrow s$  or  $b \rightarrow d$  transitions occur only at loop-level in the SM, come with small exclusive branching ratios  $\sim O(10^{-6})$  or smaller and thus provide an excellent probe of indirect effects of new physics and information on the masses and couplings of the virtual SM or beyond-the-SM particles participating. Within the SM, these decays are sensitive to the CKM matrix elements  $|V_{ts}|$  and  $|V_{td}|$ , respectively; a measurement of these parameters or their ratio would be complementary to their determination from B-mixing.

The effective field theory for  $b \rightarrow s(d)$  transitions is universal for all channels; due to space-restrictions, I cannot review all important features of that effective theory; for a quick overview

<sup>d</sup>Note also that even in the strict limit  $m_b \rightarrow \infty$ , collinear factorization may fail in the second term on the right-hand side of (5) at  $O(\alpha_s^2)$ .<sup>15</sup>

<sup>e</sup>They are, however, included in a systematic way in a conceptionally similar, alternative approach for calculating heavy-to-light form factors: QCD sum rules on the light-cone.<sup>16</sup>

<sup>f</sup>In this connection, I recall the long-standing discussion of the (comparatively) simple case of  $1/m_b$  corrections to the leptonic decay constant of the B meson,  $f_B$ ; cf. Ref.<sup>18</sup> and references therein.

I refer to Chapter 9 of the BaBar Physics Book<sup>19</sup>, where also references to more detailed reviews can be found. Here, let it suffice to say that the effective Hamiltonian governing rare decays can be obtained from the SM Hamiltonian by performing an operator product expansion yielding

$$\mathcal{H}_{\text{eff}}^q = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{tq}^* \sum_{i=1}^{11} C_i(\mu) \mathcal{O}_i^q(\mu), \quad (7)$$

where the  $\mathcal{O}_i^q$  are local renormalized operators. The Wilson-coefficients  $C_i$  can be calculated in perturbation theory and encode the relevant short-distance physics, in particular any potential new-physics effects; they are known to NLO in QCD.<sup>20</sup> The renormalization-scale  $\mu$  can be viewed as separating the long- and short-distance regimes. For calculating decay rates with the help of (7), the value of  $\mu$  has to be chosen as  $\mu \sim m_b$  in a truncated perturbative expansion. The Hamiltonian (7) is suitable to describe physics in the SM as well as in a number of its extensions, for instance the minimal supersymmetric model. The operator basis in (7) is, however, not always complete, and in some models, for instance those exhibiting left-right symmetry, new physics also shows up in the form of new operators. This proviso should be kept in mind when analysing rare B decays for new-physics effects by measuring Wilson-coefficients.

The challenge in rare decays is to correctly assess the size of long-distance QCD contributions. Contrary to naive expectations, such contributions do not only affect exclusive, but also inclusive decays; for  $b \rightarrow s$  transitions, such long-distance effects come from short-distance  $b \rightarrow c\bar{c}s$  transitions, where the  $c\bar{c}$  pair (plus soft and/or hard gluons) forms an intermediate state that at large distances couples to a photon or a lepton-pair.

Let me discuss the issue in more detail for the simplest case of the exclusive decay  $B \rightarrow K^* \gamma$ , concentrating on non-perturbative QCD effects. For the treatment of perturbative issues, in particular the reduction of renormalization-scale dependence and remaining uncertainties, I refer to<sup>21</sup>.

The theoretical description of the  $B \rightarrow K^* \gamma$  decay is quite involved with regard to both long- and short-distance contributions. In terms of the effective Hamiltonian (7), the decay amplitude can be written as

$$\mathcal{A}(\bar{B} \rightarrow \bar{K}^* \gamma) = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \langle \bar{K}^* \gamma | C_7 O_7 + i\epsilon^\mu \sum_{i \neq 7} C_i \int d^4x e^{iqx} T \{ j_\mu^{em}(x) O_i(0) \} | \bar{B} \rangle, \quad (8)$$

where  $j_\mu^{em}$  is the electromagnetic current and  $\epsilon_\mu$  the polarization vector of the photon.  $O_7$  is the only operator containing the photon field at tree-level:

$$O_7 = \frac{e}{16\pi^2} m_b \bar{s} \sigma_{\mu\nu} R b F^{\mu\nu} \quad (9)$$

with  $R = (1 + \gamma_5)/2$ . Other operators, the second term in (8), contribute mainly closed fermion loops. The first complication is now that the first term in (8) depends on the regularization- and renormalization-scheme. For this reason, one usually introduces a scheme-independent linear combination of coefficients, called “effective coefficient” (see<sup>21</sup> and references therein):

$$C_7^{\text{eff}}(\mu) = C_7(\mu) + \sum_{i=3}^6 y_i C_i(\mu),$$

where the numerical coefficients  $y_i$  are given in<sup>21</sup>.

The current-current operators  $O_1 = (\bar{s}\gamma^\mu Lb)(\bar{c}\gamma_\mu Lc)$  and  $O_2 = (\bar{s}\gamma_\mu Lc)(\bar{c}\gamma^\mu Lb)$  give vanishing contribution to the perturbative  $b \rightarrow s\gamma$  amplitude at one loop. Thus, to leading logarithmic accuracy (LLA) in QCD and neglecting long-distance contributions from  $O_{1,2}$  to the  $b\bar{s}\gamma X$  Green’s functions, the  $\bar{B} \rightarrow \bar{K}^* \gamma$  amplitude is given by

$$\mathcal{A}_{O_7}^{\text{LLA}}(\bar{B} \rightarrow \bar{K}^* \gamma) = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* C_7^{(0)\text{eff}} \langle \bar{K}^* \gamma | O_7 | \bar{B} \rangle. \quad (10)$$

Here,  $C_7^{(0)\text{eff}}$  denotes the leading logarithmic approximation to  $C_7^{\text{eff}}$ . The above expression is, however, not the end of the story, as the second term in (8) also contains long-distance contributions. Some of them can be viewed as the effect of virtual intermediate resonances  $\bar{B} \rightarrow \bar{K}^* V^* \rightarrow \bar{K}^* \gamma$ . The main effect comes from  $c\bar{c}$  resonances and is contributed by the operators  $O_1$  and  $O_2$  in (8). It is governed by the virtuality of  $V^*$ , which, for a real photon, is just  $-1/m_{V^*}^2 \sim -1/4m_c^2$ . Such power-suppressed long-distance effects  $\sim 1/m_c^2$  are also present in inclusive decays, which is actually the process for which they were discussed first.<sup>22</sup> The first, and to date only, study for exclusive decays was done in<sup>23</sup>. Technically, one performs an operator product expansion of the correlation function in (8), with a soft non-perturbative gluon being attached to the charm loop, resulting in terms being parametrically suppressed by inverse powers of the charm quark mass. As pointed out in<sup>21</sup>, although the power increases for additional soft gluons, it is possible that contributions of additional external hard gluons could remove the power-suppression. This question is also relevant for inclusive decays and deserves further study.

After inclusion of the power-suppressed terms  $\sim 1/m_c^2$ , the  $\bar{B} \rightarrow \bar{K}^* \gamma$  amplitude reads

$$\mathcal{A}^{\text{LLA}}(B \rightarrow \bar{K}^* \gamma) = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \langle \bar{K}^* \gamma | C_7^{(0)\text{eff}} O_7 + \frac{1}{4m_c^2} C_2^{(0)} O_F | \bar{B} \rangle. \quad (11)$$

Here,  $O_F$  is the effective quark-quark-gluon operator obtained in<sup>23</sup>, which describes the leading non-perturbative corrections. The two hadronic matrix elements can be described in terms of three form factors,  $T_1$ ,  $L$  and  $\tilde{L}$ :

$$\begin{aligned} \langle \bar{K}^*(p) \gamma | \bar{s} \sigma_{\mu\nu} q^\nu b | \bar{B}(p_B) \rangle &= i \epsilon_{\mu\nu\rho\sigma} \epsilon_\gamma^{*\mu} \epsilon_{K^*}^{*\nu} p_B^\rho p^\sigma 2T_1(0), \\ \langle \bar{K}^*(p) \gamma | O_F | \bar{B}(p_B) \rangle &= \frac{e}{36\pi^2} \left[ L(0) \epsilon_{\mu\nu\rho\sigma} \epsilon_\gamma^{*\mu} \epsilon_{K^*}^{*\nu} p_B^\rho p^\sigma \right. \\ &\quad \left. + i \tilde{L}(0) \left\{ (\epsilon_{K^*}^* p_B)(\epsilon_\gamma^* p_B) - \frac{1}{2} (\epsilon_{K^*}^* \epsilon_\gamma^*) (m_B^2 - m_{K^*}^2) \right\} \right]. \quad (12) \end{aligned}$$

The calculation of the above form factors requires genuinely non-perturbative input. Available methods include, but do not exhaust, lattice calculations and QCD sum rules. Again, a discussion of the respective strengths and weaknesses of these approaches is beyond the scope of this talk. Let it suffice to say that — at least at present — lattice cannot reach the point  $(p_B - p)^2 = 0$  relevant for  $B \rightarrow K^* \gamma$ , and that QCD sum rules predict the relevant form factors with an estimated 20% uncertainty.<sup>23,24</sup> Numerically, these corrections increase the decay rate by about 5 to 10%. After their inclusion, one obtains

$$B(B \rightarrow K^* \gamma) = 4.4 \times 10^{-5} \times (1 + 8\%)$$

for the central values of the QCD sum rule results, where the second term in brackets denotes the shift of the result induced by  $1/m_c^2$  terms.

Let me also spend a few words on the decay  $B \rightarrow \rho \gamma$ . Although at first glance it might seem that its structure is the same as for  $B \rightarrow K^* \gamma$ , this is actually not the case: there are additional long-distance contributions to  $B \rightarrow \rho \gamma$ , which are CKM-suppressed for  $B \rightarrow K^* \gamma$  and have been neglected in the previous discussion; these contributions comprise

- weak annihilation mediated by  $O_{1,2}^u$  with non-perturbative photon-emission from light quarks; these contributions are discussed in<sup>25</sup> and found to be of order 10% at the amplitude level;
- effects of virtual  $u\bar{u}$  resonances ( $\rho, \omega, \dots$ ); they are often said to be small, but actually have not been studied yet in a genuinely non-perturbative framework.



From the above open questions it is evident that further theoretical work is needed before one can aim at an accurate experimental determination of  $|V_{ts}/V_{td}|$  from  $B(B \rightarrow \rho\gamma)$  and  $B(B \rightarrow K^*\gamma)$ .

I would like to conclude the discussion of rare decays with  $B \rightarrow K^*\ell^+\ell^-$ . The motivation for studying this decay is either, assuming the SM to be correct, the measurement of  $|V_{ts}|$ , or the search for manifestations of new physics in non-standard values of the Wilson-coefficients. A very suitable observable for the latter purpose is the forward-backward asymmetry which is independent of CKM matrix elements, but, due to extremely small event numbers, only accessible at the LHC.  $B \rightarrow K^*\mu^+\mu^-$  has the potential of high impact both on SM physics and beyond (see e.g. Ref.<sup>26</sup> for a recent discussion of potentially large SUSY-effects).

In  $b \rightarrow s\ell^+\ell^-$  transitions, the issue of intermediate  $c\bar{c}$  resonances is even more relevant than it is for  $b \rightarrow s\gamma$ , as they show up as observable peaks in the dilepton-mass spectrum and completely obscure the interesting underlying short-distance physics at intermediate masses. It is evident that appropriate cuts have to be applied in the mass-spectrum, but they cannot completely eliminate the resonances' tails that will show up in the measured value of the Wilson-coefficient  $C_9$ . One thus defines a (process- and momentum-dependent) "effective" Wilson-coefficient,  $C_9^{\text{eff}}(s) = C_9 + Y(s)$ , where  $s$  is the dilepton invariant mass and  $Y$  describes long-distance effects associated with the  $c\bar{c}$  loop. Sometimes  $Y(s)$  is written as sum of "perturbative" and "resonance" contributions, where the former comprise just the perturbative loop-diagrams (to leading order in  $\alpha_s$ ; explicit  $O(\alpha_s)$  corrections are not yet known), and the latter are expressed as sum over Breit-Wigner amplitudes. Actually, however, there is no such clear-cut separation of resonance and perturbative contributions; they are, on the contrary, dual to each other in the sense that the perturbative result, valid well below the threshold of  $J/\psi$  production, should match the dispersion integral over the resonance region; summing up both contributions leads to double-counting. A clear discussion of this issue can be found in<sup>27</sup>, where  $Y(s)$  was calculated from all available information on resonances using the factorization approximation, i.e. neglecting gluon-exchange between the  $c\bar{c}$  pair and the  $b$  and  $s$  quark. A calculation of the dominant nonperturbative (soft-gluon) terms from operator product expansion proves feasible at nonzero  $s$ , well below the  $J/\psi$  threshold, and yields, analogous to  $b \rightarrow s\gamma$ , contributions suppressed as  $1/m_c^2$ .<sup>28</sup> A corresponding calculation for the exclusive case is still missing.

In summary, the measurement of Wilson-coefficients in  $b \rightarrow s\ell^+\ell^-$  transitions is, even well below the  $c\bar{c}$  resonances in the dilepton-mass spectrum, affected by long-distance contributions that still need to be assessed in more detail.

## 5 Summary & Conclusions

Summarizing, I have given a status report of currently much heeded topics in B physics related to B-mixing and QCD effects in CP-asymmetries and rare decays, whose understanding is essential for a theoretically clean extraction of CP-violating phases and new-physics effects from experimental results. I am confident that by the time of the next Rencontres de Vietnam the fruitful mutual interaction of experimental and theoretical developments will have resulted in a much better comprehension of the mechanism underlying observable CP-violating effects and – maybe – even have led to unequivocal evidence for new physics.

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