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Extra Dimensions: A View from the Top

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ABSTRACT:

In models with compact extra dimensions, where the Standard Model fields are confined to a 3+1 dimensional hyperplane, the $t\bar{t}$ production cross-section at a hadron collider can receive significant contributions from multiple exchange of KK modes of the graviton. These are carefully computed in the well-known ADD and RS scenarios, taking the energy dependence of the sum over graviton propagators into account. Using data from Run-I of the Tevatron, 95% C.L. bounds on the parameter space of both models are derived. For Run-II of the Tevatron and LHC, discovery limits are estimated.

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In recent years, there has been tremendous interest in higher dimensional theories where the extra dimensions are compactified in such a way as to have physically interesting consequences[1]. These have been inspired by recent developments in our understanding of some non-perturbative aspects of string theories. The weak-coupling spectrum of such a theory reveals only one-dimensional strings but, in the strong coupling regime, $p+1$ dimensional defects embedded in the higher-dimensional spacetime — called D_p-branes — arise. These are dynamical objects analogous to solitons in field theory and are particularly interesting because it is possible to localize gauge fields on them. We then have the exciting possibility that the observable universe is a D_3-brane (or 3-brane) with the Standard Model (SM) fields localized on it, leaving gravity free to propagate in the extra dimensions or bulk. In such a scenario, it becomes possible to lower the gravity/string scale to the order of a TeV, without running into contradiction with experiment. This new TeV-scale physics then provides fresh perspectives on the hierarchy problem arising in the standard electroweak theory.

The first realization of these ideas is the ADD scenario proposed by Arkani-Hamed, Dimopoulos and Dvali[2], where the SM fields are localized on a 3-brane embedded in a 10-dimensional spacetime consisting of 4 non-compact Minkowski dimensions, d dimensions compactified with a large radius R_c and the remaining $6-d$ dimensions compactified to the Planck length. The Planck scale M_P is then related[2] to the scale M_S of gravitational interactions by $M_P \sim R_c^d M_S^{d+2}$. Choosing the scale M_S to be of the order of a TeV (which eliminates the usual hierarchy problem) it follows that $R_c \sim 10^{32/d-19}$ m. This immediately precludes $d=1$, but for any value $d>1$, we find that M_S can be arranged to be a TeV for compactification radii as large as a millimetre, without confliciting with gravity experiments[3]. Effects of non-Newtonian (quantum) gravity can then become apparent at these surprisingly low values of energy. While it is possible[4] to construct a physically viable scenario with large R_c which can survive the existing astrophysical and cosmological constraints, such a scenario inevitably predicts large deviations from the SM in several low-energy effects [5]. Thus, laboratory data can be used to derive [5] bounds on M_S in the TeV range — though some astrophysical bounds can, in fact, be considerably stronger [6].

The main criticism of the ADD scenario is the reappearance of a large hierarchy between the string scale $M_S \sim 1$ TeV and the compactification radius $R_c \sim (10^{-16}\ \text{TeV})^{-1}$, which makes it difficult to stabilize the size of the large extra dimensions. This has inspired the RS scheme — proposed by Randall and Sundrum [7] — where only a single extra dimension is invoked. This fifth dimension ϕ is strongly curved and is compactified on a $\mathbf{S}^1/\mathbf{Z}^2$ orbifold with a radius R_c, which is assumed to be somewhat larger than the Planck length. Two 3-branes are located at the orbifold fixed points: a 'Planck brane' at $\phi = 0$ and a 'TeV brane' at $\phi = \pi$. The tension on the branes and the cosmological constant in the bulk are fine-tuned to give a flat (Minkowski) geometry on the 3-branes. The bulk metric that guarantees this fine-tuning is

$$ds^2 = e^{-\mathcal{K}R_c\phi}\eta_{\mu\nu}dx^\mu dx^\nu \ + \ R_c^2 d\phi^2. \tag{1}$$

The novel feature of this metric that it is *non-factorizable* or *warped* and this leads to interesting consequences, such as localised graviton fields. The real interest of this model, however, lies in the fact that the exponential warp factor in the metric helps to solve the hierarchy problem. Mass scales are determined by the location ϕ of the brane in the extra dimension, since the warp factor $e^{-\mathcal{K}R_c\phi}$ acts as a conformal factor for the brane fields, *i.e.*, all masses get rescaled by this factor. This exponential could be the source of the hierarchy between the electroweak scale and the Planck scale, because it is possible in this model to generate the enormous factor of $\frac{M_P}{M_{EW}} \sim 10^{15}$ by an argument $\pi\mathcal{K}R_c$ of order 30. Physically, the Planck scale is treated as the fundamental scale in the theory, but the overlap of the graviton wave-function with the TeV brane on which the gauge particles are localised is very small and this is what generates the small value of the electroweak scale. There still remains the less severe problem of stabilizing the compactification radius R_c against quantum fluctuations, but it has been shown that it is possible to achieve this, either by introducing an extra scalar field in the bulk [8], or by invoking supersymmetry [9]. The bulk scalar is expected to be light and may have interesting phenomenological consequences [10]. A deeper problem is to realize the RS scenario within the framework of a string theory, and some progress has been made in this direction [11].

To determine the consequences of the ADD and RS models for experiments, we need to extract the low-energy effective theory on the 3-brane where the SM fields live. We concentrate on the experimental signals of the Kaluza-Klein excitations of the graviton, which is massless in the bulk, but, on compactification of the extra dimensions, yields a tower of massive Kaluza-Klein (KK) excitations on the 3-brane. Using the linearised gravity approach, the curved metric can be approximated by small fluctuations $h_{\mu\nu}$ about its Minkowski value. The interactions of its KK modes $h_{\mu\nu}^{(\vec{n})}$ with the observable particles on the 3-brane are then given by:

$$\text{ADD}: \mathcal{L}_{int} = -\frac{1}{\overline{M}_P}\sum_{\vec{n}} T^{\mu\nu}(x)h_{\mu\nu}^{(\vec{n})}(x) \tag{2}$$

$$\text{RS}: \ \mathcal{L}_{int} = -\frac{1}{\overline{M}_P}T^{\mu\nu}(x)h_{\mu\nu}^{(0)}(x) - \frac{e^{\pi\mathcal{K}R_c}}{\overline{M}_P}\sum_{n}^{\infty} T^{\mu\nu}(x)h_{\mu\nu}^{(n)}(x) \ , \tag{3}$$

where $\overline{M}_P = M_P/\sqrt{8\pi}$ is the reduced Planck mass and $T^{\mu\nu}$ is the symmetric energy-momentum tensor for the observable particles on the 3-brane, computed using the flat space metric. The masses of the $h_{\mu\nu}^{(\vec{n})}$ are given by

$$\text{ADD}: M_{\vec{n}}^2 = \frac{1}{R_c}\left(n_1^2 + n_2^2 + \ldots + n_d^2\right)^2 \tag{4}$$

$$\text{RS}: \ M_n^2 = x_n\mathcal{K} \ e^{-\pi\mathcal{K}R_c} \tag{5}$$

where the x_n are the zeros of the Bessel function $J_1(x)$ of order unity [8]. It is important to note that the masses are evenly-spaced in the ADD case, but not so in the RS case. It is also obvious that in the ADD scenario, all the $h^{(\vec{n})}$ couple very weakly to matter (being suppressed by $\overline{M}_P^{-1}$), as does the zero-mode of the KK tower in the RS case. On the other hand the couplings of the massive RS gravitons are enhanced by the exponential $e^{\pi \mathcal{K} R_c}$ leading to interactions of electroweak strength. In the ADD model, the density of light KK gravitons is very high because of the large value of R_c, and their collective interaction again builds up to electroweak strength. The Feynman rules in either case are essentially the same as those worked out[12, 13] for the ADD case, except for the overall warp factor in the RS case.

For the ADD model, then, the only unknown parameter in the theory is the string scale M_S, which is related to the compactification radius R_c by $M_P \sim R_c^d M_S^{d+2}$. For exchange of virtual gravitons, M_S also acts as a cutoff for the sum over KK states. In the RS case, it is expedient to define the free parameters

$$\begin{aligned} m_0 &= \mathcal{K} e^{-\pi \mathcal{K} R_c} \\ c_0 &= \mathcal{K}/M_P \end{aligned} \tag{6}$$

where m_0 sets the scale for the masses of the excitations, and c_0 is an effective coupling, since the interaction of massive KK gravitons with matter can be written as

$$\mathcal{L}_{int} = -\sqrt{8\pi} \frac{c_0}{m_0} \sum_n^\infty T^{\mu\nu}(x) h_{\mu\nu}^{(n)}(x) \ . \tag{7}$$

Typical values of the parameter c_0 lie in the range [0.01, 0.1]. This estimate results from the requirement that the scale $\mathcal{K}$ (related to the curvature of the fifth dimension) is small compared to $\overline{M}_P$, but not too small, since that would introduce a new hierarchy. Values of m_0 are determined in terms of $\mathcal{K} R_c \sim 10$, so that m_0 ranging from about 50 or 60 GeV to a TeV are consistently obtained.

Before discussing detailed phenomenological consequences of these models, we take note of some of the generic features. The KK graviton modes in the ADD model can be very light, with masses ranging from 10^{-13} GeV to 0.1 GeV, for $M_S \sim 1$ TeV, depending on the number d of extra dimensions. For the current generation of high-energy experiments, performed at energies of order 100 GeV, huge numbers of KK gravitons can contribute to the same process, leading to effective interactions of electroweak strength. These interactions have been extensively studied in the literature [5]. In the RS model, the individual KK modes are heavier — $\mathcal{O}$(TeV) — and couple individually with electroweak strength. One has then, the interesting possibility that they may be produced on resonance. As one increases the centre-of-mass energy, one may hope to probe a multi-resonance effect. Moreover, instead of escaping the detector, as will be the case for ADD gravitons, the massive gravitons will decay, mostly within the detectors, into pairs of SM particles. Thus, the experimental manifestations of the RS model are expected to be somewhat different from those in the ADD model. Some work on the collider phenomenology of this model has already appeared in the literature. These include the resonant production of the KK excitations and the virtual effects in processes like dilepton production at hadron colliders [14], in deep-inelastic scattering at HERA [15] and in e^-e^-[16] and $e\gamma$ colliders.

In this letter, we study, within the framework of the ADD and RS models, the virtual effects of the exchange of spin-2 KK modes in the production of $t\bar{t}$ pairs at hadron colliders, in particular, the Tevatron and LHC. Hadroproduction of $t\bar{t}$ pairs has already been used to constrain the ADD model earlier[18], using a low-energy approximation. We improve upon the earlier results for the ADD model by using a better approximation, and go on to derive constraints from the $t\bar{t}$ cross-section on the parameter space of the RS model, which has not been done before.

To estimate the cross-section for $t\bar{t}$-hadroproduction, in addition to the SM processes, we consider production mechanisms with s-channel exchange of KK gravitons in $q\bar{q}$- or gg-initiated sub-processes. The graviton contributions to the sub-process cross-section in the two models can be written in the common form, as

$$\begin{aligned}\frac{d\hat{\sigma}}{d\hat{t}}(q\bar{q}\to t\bar{t}) &= \frac{d\sigma}{d\hat{t}}_{\rm SM}(q\bar{q}\to t\bar{t}) \\ &+\frac{\pi|C(x_s)|^2}{64\hat{s}^2}\left[5\hat{s}^2(\hat{t}-\hat{u})^2+4(\hat{t}-\hat{u})^4+8\hat{s}(\hat{t}-\hat{u})^2(\hat{t}+\hat{u})-2\hat{s}^3(\hat{t}+\hat{u})-\hat{s}^4\right],\end{aligned} \tag{8}$$

assuming massless quarks in the initial state, and

$$\begin{aligned}\frac{d\hat{\sigma}}{d\hat{t}}(gg\to t\bar{t}) &= \frac{d\sigma}{d\hat{t}}_{\rm SM}(gg\to t\bar{t}) \\ &-\frac{\pi}{16\hat{s}^2}\left[3|C(x_s)|^2-2\alpha_s\frac{{\rm Re}[C(x_s)]}{(M_t^2-\hat{t})(M_t^2-\hat{u})}\right] \\ &\times\left[6M_t^8-4M_t^6(\hat{t}+\hat{u})+4M_t^2\hat{t}\hat{u}(\hat{t}+\hat{u})-\hat{t}\hat{u}(\hat{t}^2+\hat{u}^2)+M_t^4(\hat{t}^2-6\hat{t}\hat{u}+\hat{u}^2)\right],\end{aligned} \tag{9}$$

where $x_s\equiv\frac{\sqrt{\hat{s}}}{M_S}$ in the ADD model, and $x_s\equiv\frac{\sqrt{\hat{s}}}{m_0}$ in the RS model. The remaining model-dependence is absorbed into the function $C(x)$, which is defined as

$$\begin{aligned}{\rm ADD}:\ C_{ADD}(x) &= \frac{16\pi}{M_S^4}\lambda_{ADD}(x) \\ {\rm RS}:\ C_{RS}(x) &= \frac{32\pi c_0^2}{m_0^4}\lambda_{RS}(x)\end{aligned} \tag{10}$$

where

$$\begin{aligned}\lambda_{ADD}(x_s) &= \kappa^2\sum_{\vec{n}}\frac{1}{\hat{s}-M_{\vec{n}}^2+i\epsilon} \\ \lambda_{RS}(x_s) &= m_0^2\sum_n\frac{1}{\hat{s}-M_n^2+iM_n\Gamma_n}\ .\end{aligned} \tag{11}$$

where the M_n are the masses of the individual resonances and the Γ_n are the corresponding widths. In the ADD model, the indiviadual graviton KK modes decay only through the feeble gravitational constant $\kappa = \frac{2}{M_P}$ and hence they are assumed to be stable, at least for the purpose of collider studies. In the RS model, the graviton widths are obtained by calculating their decays into final states involving SM particles. This gives

$$\Gamma_n = m_0 c_0^2 x_n^3 \Delta_n \tag{12}$$

where

$$\Delta_n = \Delta_n^{\gamma\gamma} + \Delta_n^{gg} + \Delta_n^{WW} + \Delta_n^{ZZ} + \sum_\nu \Delta_n^{\nu\nu} + \sum_l \Delta_n^{ll} + \sum_q \Delta_n^{qq} + \Delta_n^{HH} \tag{13}$$

and each Δ_n^{aa} is a numerical coefficient arising in the decay $G_n \to a\bar{a}$. For the partial width Δ_n^{HH}, we have fixed $M_H = 250$ GeV in our numerical studies. However, we have checked that variation of M_H in the region allowed by fits to electroweak precision data affects the results presented here only very weakly.

In defining $C(x)$ and $\lambda(x)$ in the above fashion, we are motivated by the connection with earlier studies of virtual effects of graviton exchange in the ADD model. In these studies of the ADD model, $C(x) \sim \frac{\lambda}{M_S^4}$ (where M_S is the string-scale) and it has been customary to assume λ to be an energy-independent constant of ($\mathcal{O}(1)$). However, even in the ADD model, $\lambda(x)$ is constant only in the limit $x \ll 1$ for $d > 2$ and is slowly varying for $d = 2$ in the same limit. For $\hat{s}$ comparable with $M_S \sim 1$ TeV — which occurs at the Tevatron for a non-negligible fraction of the events — $x_s \sim \mathcal{O}(1)$, and then $\lambda(x_s)$ shows appreciable variation with x_s. For the RS model, where $m_0 \sim 100$ GeV is possible, we are never in the $x_s \to 0$ regime for $t\bar{t}$ production at the Tevatron and it is imperative to consider its variation. The low-energy approximation becomes even worse at the LHC, unless we choose the scales M_S and m_0 to be very high [1].

Calculation of $\lambda_{ADD}(x_s)$ is done by assuming the graviton KK spectrum to form a quasi-continuum and then integrating over the states, after having defined a suitable density-of-states function. This has been done explicitly in Refs. [12] and [13], and an elegant set of formulae are presented in Appendix A of Ref. [13]. Using their results, one can account for the full-energy dependence of $\lambda_{ADD}(x_s)$ and this turns out to be dependent on the number d of extra dimensions as well as the energy. It is, threfore, worthwhile to include the full energy dependence of $\lambda_{ADD}(x_s)$ and redo the analysis for the ADD case for both the Tevatron and the LHC.

For the RS model, given the masses and the widths of the individual graviton resonances, we have to sum over all the resonances to get the value of $\lambda_{RS}(x_s)$. We perform this sum numerically, using the fact that the higher zeros of the Bessel function become evenly-spaced. The summation is somewhat tricky because we have to sum over *several* resonances. The heavier resonances are rather wide and tend to overlap.

[1] This would defeat the spirit of theories with low-energy quantum gravity.

In our numerical procedure, for a given value of $x_s = \frac{\sqrt{s}}{m_0}$, we retain all resonances which contribute with a significance greater than one per mil, and treat the remaining KK modes as virtual particles (in which case the sum can be done analytically).

We can now compute the integrated $t\bar{t}$ production cross-section at the Tevatron and the LHC, using the equation

$$\sigma(AB \to t\bar{t}) = \sum_{a,b} \int dx_1 dx_2 \; d\hat{t} \; [f_{a/A}(x_1) \; f_{b/B}(x_2) + x_1 \leftrightarrow x_2] \; \frac{d\hat{\sigma}}{d\hat{t}}(ab \to t\bar{t}) \; , \qquad (14)$$

where A, B are the initial hadrons (either $p\bar{p}$ or pp), and $f_{a(b)/A}$ denotes the probability of finding a parton $a(b\ )$ in the hadron A (similarly $f_{a(b)/B}$). The sum in Eq. 14 runs over the contributing sub-processes.

The sub-process cross sections for the new physics that we have calculated are at leading order. For the SM contribution to $t\bar{t}$ production in hadronic collisions, significant progress has been made in computing higher-order corrections. Not only have the next-to-leading order corrections been calculated a long time ago[19], but the resummation of soft gluons and its effect on the total cross-section have been computed[20]. In principle, a reliable estimate of the cross-section for the case under consideration can also be made only when we have (at least) the corrections to these processes at next-to-leading order. For want of such a calculation, however, the best we can do is to use the leading order QCD cross-section and the resummed QCD cross-sections[20] to extract a 'K-factor'. We work within the approximation that the new physics will also be affected by QCD corrections in a similar fashion so that we can fold in our cross-sections for this case by the *same* K-factor. This is clearly an approximation but it turns out to affect the constraints that we extract from the data only marginally.

For our numerical studies, the above formulae have been incorporated into a simple parton-level Monte Carlo event generator, where we have used the CTEQ4M parametrisations[21] for the parton distributions. Since it is only the total $t\bar{t}$ cross-section that is being compared, we expect the results from a parton-level study to be of reasonably high quality. In order, now, to determine the constraints on the relevant model, we simply compare the new physics contributions with the maximum allowed deviation (at 95% confidence level) from the SM cross-section which is allowed by the experimental determination of the total $t\bar{t}$ cross-section in $p\bar{p}$ collisions at 1.8 TeV at the Tevatron. The latter has been estimated to be $(6.5^{+1.7}_{-1.4})$ pb from 109 pb^{-1} of data collected by the CDF Collaboration [22], and to be (5.9 ± 1.7) pb from 125 pb^{-1} of data collected by the D0 Collaboration [23]. These results are, of course, consistent at the 1σ level, but the slight difference in errors estimated by the two collaborations, in turn, slightly changes the bounds on the model parameters derived in the two cases.

We first present our improved results on the string scale M_S for the case of the ADD model. In Table 1, we have listed, for different numbers of large extra dimensions, the lower bounds on M_S arising out of the experimental numbers at Tun I of the Tevatron, as well as estimates for the discovery limits at RUN II and at the LHC.

d	CDF (GeV)	D0 (GeV)	Run II (GeV)	LHC (GeV)
2	1890 (1963)	1984(2088)	2678 (3228)	6947 (13411)
3	1650 (1625)	1722 (1699)	2272 (2252)	6234 (6061)
4	1516 (1367)	1575 (1428)	2049 (1888)	6068 (4956)
5	1428 (1235)	1478 (1291)	1904 (1705)	5883 (4454)
6	1366 (1149)	1411 (1202)	1800 (1586)	5779 (4127)

Table 1: *95% C.L. lower bounds on the string scale in the ADD model derived from the $t\bar{t}$ cross-section, for different numbers d of large extra dimensions. The first two columns show actual bounds derivable from existing data from Run-I of the Tevatron, while the last two columns show estimates of the discovery limits from Run-II of the Tevatron and from the LHC. Numbers in parantheses show those that would have been obtained by ignoring the energy dependence of $\lambda_{ADD}(x_s)$, i.e. by setting $\lambda(x_s)$ to its $x_s \ll 1$ limit irrespective of $\sqrt{\hat{s}}$.*

In order to make an estimate of the discovery limits on M_S obtainable from Run-II of the Tevatron, we assume that the $p\bar{p}$ collisions take place at a centre-of-mass energy of 2 TeV, and (conservatively) that an integrated luminosity of 2 fb^{-1} will be collected. Taking improvements in the b-tagging efficiency into account, this has been estimated [24] to lead to an uncertainty in the cross-section of 8-10%, which can be translated, using the SM cross-section, into an errorbar of about 0.7 pb. Since this is significantly smaller than the errors from Run-I, we predict significantly better results from Run-II of the Tevatron, assuming that the experimental central value will be very close to the SM prediction.

It may appear that the bounds shown in Table 1 are considerably better than the bounds on the effective string scale $M_S/\lambda^{1/4}$ earlier derived [18], which were in the range 600–700 GeV. There is no paradox, involved, however, since the value of λ is not actually $\mathcal{O}(1)$, but is more like 16π. Accordingly, the fourth root provides a factor somewhat larger than 2, which accounts for the numbers being in the present ballpark. It is also obvious that the low-energy approximation for $C(x_s)$ is not a very good one, except perhaps, for $d = 3$.

Going from the Tevatron to the LHC (pp collisions at $\sqrt{s} = 14$ TeV) affects the results quite significantly. This is because of the higher energy, and the fact that the gg channel is dominant at the LHC energy. We recall that the interference between the SM and the ADD model is present only in the gg channel. In performing the ADD analysis for the LHC, we encounter the technical problem that there is a significant part of the phase-space for which $x_s > 1$, which means that the low-energy effective theory approach becomes meaningless. In order to handle this problem we have used kinematic cuts on $\sqrt{\hat{s}}$ to make sure that x_s remains in the acceptable range. To derive bounds, we assume that no new physics effect will be seen, in which case it is likely that the experimental central value of the cross-section will coincide with the SM prediction, about 868 pb — which means that we can expect more than eight-and-a-half million $t\bar{t}$ events at the LHC, for a projected (integrated) luminosity[2] of 10 fb^{-1}. The statistical error is, therefore, negligibly small and the experimental error is expected to be dominated by the systematics. We assume a systematic error of 5 pb in the cross-section for the results shown in Table 1, which may be regarded as an educated guess. We have verified that reasonable changes in this number do not affect the discovery limits very significantly, since the cross-section has a generic M_S^{-4} dependance.

It is interesting to note that the low-energy approximation for M_S becomes wildly inaccurate for the case of LHC for the case $d = 2$. The actual lower bound on M_S, when the energy dependence is taken into account, is much more modest, and shows smaller variation with the number of extra dimensions, (about 20%, as we go from $d = 2$ to $d = 6$) than do the Tevatron bounds (about 35%, as we go from $d = 2$ to $d = 6$).

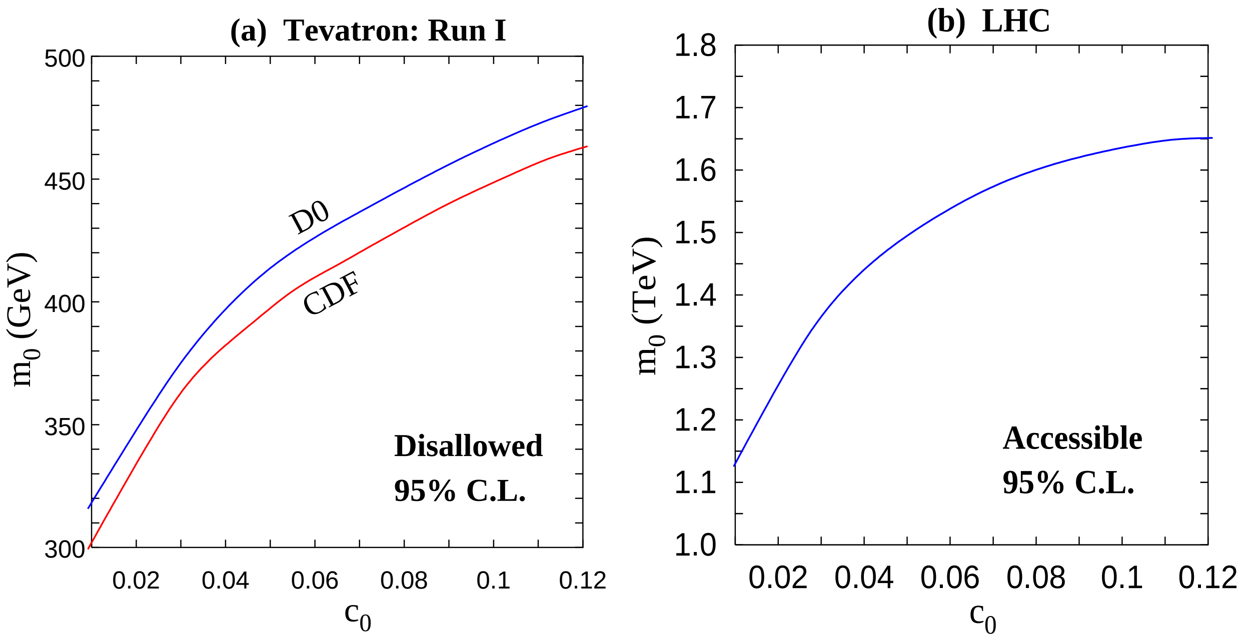

Figure 1. *Illustrating (a) 95% C.L. constraints on the $m_0 - c_0$ plane of the Randall-Sundrum model, using $t\bar{t}$ production data from Run-I of the Tevatron, and (b) discovery limits at 95% C.L. on the $m_0 - c_0$ plane at the LHC.*

We now turn to the more interesting case of the RS model, where the contribution from the $t\bar{t}$ cross-section have not been studied before. In this case, as we have seen, the parameter space is two-dimensional, with both m_0 and c_0 being free parameters within certain theoretical constraints. We present the results of our computations in Fig. 1. In Fig 1(a), we show the excluded region at 95% C.L. in the (m_0, c_0) plane

[2]This is actually a conservative estimate: estimates of around 100 fb^{-1} have also been made.

obtained by comparing the excess predicted by the RS model with the experimental errors from the CDF and D0 experiments, respectively. It may be seen that the present data already constrain a significant region of the parameter space and for the values of c_0 between 0.01 and 0.1, suggested by naturalness considerations discussed earlier, m_0 values smaller than about 300 GeV (rising to about 500 GeV for larger values of c_0) are definitely excluded. This means that the first KK graviton resonance must lie above 1 TeV at the least. As in the case of the ADD model, the constraints are expected to get much better at Tevatron Run II, unless, indeed, a discovery is made. To underline the importance of undrstanding the error estimates at the Tevatron better, we have not presented discovery limits for the Tevatron, since the RS model case is more sensitive to small changes in the error estimate than the ADD model.

At the LHC, our analysis of the RS model lies on a firmer theoretical foundation, since the low-energy theory is valid for the entire energy range accessible at this machine. However, our numerical analysis must take care of more graviton resonances, which can become accessible because of the larger energy. Once this is done, we assume the same errors of 5 pb as before, and obtain discovery limits in the (m_0,c_0)-plane as in the case for the Tevatron. These are exhibited in Fig. 1(b). It is obvious that the accessible values of m_0 for $0.01 < c_0 < 0.1$ lie in the range between 1.1 TeV and 1.7 TeV, which means that the first graviton resonance can be probed up to around 4 TeV. These numbers change somewhat as the systematic error is varied from 5 pb, but so long as the latter is not changed drastically, the discovery limits remain in the range 1–2 TeV. Improving the systematic error to 1 pb or less, would, however, drive the discovery limits well above 2 TeV, and this may be a goal to set for experimental analyses, when the time is ripe to make serious estimates.

The fact that the LHC can probe KK graviton resonances of the RS model in the range of a few TeV is a rather exciting prospect. This is because the graviton resonances cannot be made indefinitely heavy in this model. For, if m_0 becomes very large, it would require either $\mathcal{K}$ to be large, or $\mathcal{K}R_c$ to be small (see Eq. 6). In either case, the curvature of the fifth dimension becomes very large, and this leads to problems in fine-tuning the cosmological constants to get a flat metric on the 3-branes. Moreover, a small warp factor $e^{-\pi\mathcal{K}R_c}$ would tend to increase the electrowak scale, leading to unitarity problems in the Higgs sector; this can be ameliorated by reducing the scale on the Planck brane which gives rise to the electroweak symmetry-breaking, but only at the cost of introducing a small hierarchy. In the simplest version of the RS model, therefore, we expect the graviton spectrum to start at a few TeV, and this, as we have shown, is precisely the range which can be probed at the LHC in $t\bar{t}$ collisions. The situation is different in the ADD model, since the string scale M_S can easily be pushed above the reach of the LHC without encountering any theoretical difficulties.

To summarize, we have analysed the effects of the interactions of the spin-2 Kaluza-Klein modes with SM fields in $t\bar{t}$ production at the Tevatron and LHC, both in the case of large extra dimensions, as suggested by ADD, as well as for a small warped extra dimension, as suggested by RS. Using the full dependance of the effective coupling of gravitons to SM particles on the parton-level centre-of-mass energy, we have re-derived 95% C.L. bounds from Run I data at the Tevatron on the string scale M_S in the ADD model. We find that these vary from about 0000 GeV to 0000 GeV, depending on the number of large extra dimensions. For the RS model, the respective bound on the mass scale m_0 of graviton resonances varies from around 300–500 GeV, depending on the coupling parameter c_0. These numbers can improve at Run-II of the Tevatron, but it is only at the LHC where we can expect the discovery limits to go up by about three times the present lower bounds. In particular, we expect that M_S values in the range 0.0 TeV to 0.0 TeV for the ADD model and m_0 in the range 1.1 TeV to 1.7 TeV for the RS model can be probed in $t\bar{t}$ production at the LHC. This shows that $t\bar{t}$ hadroproduction is a powerful tool to probe extra dimensions, and we may expect very interesting results to come out of such measurements in the near future.

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References

[1] P. Horava and E. Witten, *Nucl. Phys.* **B460** (1996) 506; J. Polchinski, *Lectures given at the SLAC Summer Institute: Gravity – From the Hubble Length to the Planck Length (SSI 98), Stanford, 3-14 Aug 1998*, hep-th/9812104; J.D. Lykken *Phys. Rev.* **D54** (1996) 3697; E. Witten, *Nucl. Phys.* **B471** (1996) 135.

[2] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, *Phys. Lett.* **B249** (1998) 263; I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. Dvali, *Phys. Lett.* **B436** (1998) 257.

[3] J. C. Long, H. W. Chan and J. C. Price, *Nucl. Phys.* **B539** (1999) 23.

[4] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, *Phys. Rev.* **D59** (1999) 086004.

[5] For a review of the phenomenology of the ADD model, see K. Sridhar, *Int. J. Mod. Phys* **A15**, (2000) 2397 (hep-ph/0004053); S. Raychaudhuri, Talk given at the Sixth Workshop on High-Energy Phenomenology, Chennai (India), January 2000. For details of the phenomenology see Ref. [25].

[6] S. Cullen and M. Perelstein, *Phys. Rev. Lett.* **83** (1999) 268.

[7] L. Randall and R. Sundrum, *Phys. Rev. Lett.* **83** (1999) 3370.

[8] W.D. Goldberger and M.B. Wise, Phys. Rev. Lett. 83 (1999) 4922; *Phys.Lett.* **B475**(2000) 275.

[9] R. Altendorfer, J. Bagger and D. Nemeschansky, CITUSC-00-015, hep-th/0003117.

[10] U. Mahanta and A. Datta, *Phys.Lett.* **B483** (2000) 196; U. Mahanta, Mehta Research Institute Preprint (2000) hep-ph/0008042; K. Cheung (2000) hep-ph/0009232.

[11] Chang S. Chan, Percy L. Paul and H. Verlinde, *Nucl. Phys.* **B581** (2000) 156.

[12] G. F. Giudice, R. Rattazzi and J. D. Wells, *Nucl. Phys.* **B544** (1999) 3;

[13] T. Han, J. D. Lykken and R-J. Zhang, *Phys. Rev.* **D59** (1999) 105006. (For the corrected formulae refer to hep-ph/9811350 v.4).

[14] H. Davoudiasl, J.L. Hewett and T.G. Rizzo, *Phys. Rev. Lett* **84** (2000) 2080; SLAC Preprint SLAC-PUB-8436 (2000) hep-ph/0006041.

[15] P. Das, S. Raychaudhuri and S. Sarkar, *JHEP* **0007:050** (2000).

[16] D. Ghosh and S. Raychaudhuri, TIFR Preprint TIFR-TH-00-38, hep-ph/0007534.

[17] J. L. Hewett, SLAC Preprint SLAC-PUB-8001, hep-ph/9811356 (1998).

[18] Prakash Mathews, Sreerup Raychaudhuri and K. Sridhar, *Phys. Lett.* **B 450** (1999) 343.

[19] P. Nason, S. Dawson and R.K. Ellis, *Nucl. Phys.* **B303**, 607 (1988); *Nucl. Phys.* **B327**, 49 (1989).

[20] S. Catani et al., *Phys. Lett.* B 378 (1996) 329; *Nucl. Phys.* **B 478**, (1996) 273.

[21] H.L. Lai et al., *Phys. Rev.* **D51**, 4763 (1995).

[22] S. Leone, in the Proceedings XXXVth Rencontres de Moriond: Electroweak and Unified Theories, Les Arcs, France, (2000) FERMILAB-CONF-00/115-E.

[23] B. Abbott et al., *Phys. Rev. Lett.* **80**, 666 (1998). For the latest cross-scetion values refer to the talk of S.Leone quoted above.

[24] P.C. Bhat , H. Prosper and Scott S. Snyder, *Int.J.Mod.Phys.* **A13** (1998) 5113.

[25] Prakash Mathews, Sreerup Raychaudhuri and K. Sridhar, *Phys. Lett.* **B 450** (1999) 343; *Phys. Lett.* **B 455** (1999) 115; *JHEP* **0007:008** (2000).