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**COHERENT EXCLUSIVE EXPONENTIATION FOR PRECISION MONTE  
CARLO CALCULATIONS OF FERMION PAIR PRODUCTION /  
PRECISION PREDICTIONS FOR (UN)STABLE  $W^+W^-$  PAIRS**

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We present the new Coherent Exclusive Exponentiation (CEEX), in comparison to the older Exclusive Exponentiation (EEX) and the semi-analytical Inclusive Exponentiation (IEX), for the process  $e^+e^- \rightarrow f\bar{f}+n\gamma$ ,  $f = \mu, \tau, d, u, s, c, b$ , with validity for centre of mass energies from  $\tau$  lepton threshold to 1 TeV. We analyse  $2f$  numerical results at the  $Z$ -peak, 189 GeV and 500 GeV. We also present precision calculations of the signal processes  $e^+e^- \rightarrow 4f$  in which the double resonant  $W^+W^-$  intermediate state occurs using our YFSWW3-1.14 MC. Sample  $4f$  Monte Carlo data are explicitly illustrated in comparison to the literature at LEP2 energies. These comparisons show that a TU for the signal process cross section of 0.4% is valid for the LEP2 200 GeV energy. LC energy results are also shown.

## 1 Introduction

At the end of the LEP2 operation, the total cross section for the process  $e^-e^+ \rightarrow f\bar{f}+n\gamma$  will have to be calculated with the precision 0.2% - 1%, depending on the event selection. In addition, the awarding of the 1999 Nobel Prize to G. 't Hooft and M. Veltman emphasises the importance of the on-going precision studies of the Standard Model processes  $e^+e^- \rightarrow W^+W^- + n(\gamma) \rightarrow 4f + n(\gamma)$  at LEP2 energies, as well as the importance of the planned future higher energy studies of such processes in LC physics programs.

In what follows, we present precision predictions for both sets of processes, using our new coherent exponentiation (CEEX) <sup>1</sup> theory ( $\mathcal{K}\mathcal{K}$  MC) for the former set and our older and firmly established exclusive exponentia-

tion (EEX) <sup>2</sup> theory (YFSWW3-1.14 MC <sup>4</sup>) for the latter set. Both CEEX and EEX are based on the YFS exclusive exponentiation theory of Yennie, Frautschi and Suura <sup>3</sup>. A detailed description <sup>1,4,2</sup> of our two approaches to the precision exponentiation theory may be found in Refs. <sup>1,4,2</sup>. As we indicate below, we have compared our  $\mathcal{K}\mathcal{K}$  MC calculations with EEX, its semianalytical partner IEX, and ZFITTER 6.21 <sup>5</sup> and we have compared our YFSWW3-1.14 MC calculations with RacoonWW <sup>6</sup> and with the Beenakker *et al.* <sup>7</sup> semi-analytical approach.

The paper proceeds as follows. In Sec. 2 we discuss the implementation of CEEX in our  $\mathcal{K}\mathcal{K}$  MC in relation to EEX. In Sec. 3 we present some of its new results for  $2f + n(\gamma)$  processes at high energies. In Sec. 4 we present the EEX theory realization in our

YFSWW3-1.14 MC. In Sec. 5 we present some of its new results on  $WW + n(\gamma) \rightarrow 4f + m(\gamma)$  processes at high energies. Sec. 6 contains our summary remarks.

## 2 $\mathcal{K}\mathcal{K}$ MC

The main differences between CEEEX and EEX are best illustrated by focusing on the process of interest, which is

$$e^-(p_1, \lambda_1) + e^+(p_2, \lambda_2) \rightarrow f(q_1, \lambda'_1) + \bar{f}(q_2, \lambda'_2) + \gamma(k_1, \sigma_1) + \dots + \gamma(k_n, \sigma_n). \quad (1)$$

The respective EEX total cross section

$$\sigma = \sum_{n=0}^{\infty} \int_{m_\gamma} d\Phi_{n+2} e^{Y(m_\gamma)} D_n(q_1, q_2, k_1, \dots, k_n) \quad (2)$$

corresponds to the attendant  $\mathcal{O}(\alpha^1)$  distributions  $D_n$  as given in Ref. <sup>2</sup> by formulas such as, for  $n = 0, 1$ ,  $D_0 = \bar{\beta}_0$  and  $D_1(k_1) = \bar{\beta}_0 \tilde{S}(k_1) + \bar{\beta}_1(k_1)$ , where the real soft factors  $\tilde{S}(k)$  are defined as usual <sup>2</sup>. The important point is that the IR-finite building blocks  $\bar{\beta}_n$ , for example,  $\bar{\beta}_0 = \sum_\lambda |\mathcal{M}_\lambda^{\text{Born}}|^2$ , in the multi-photon distributions are all in terms of  $\sum_{spin} |\dots|^2$ ! Here,  $\lambda =$  fermion helicities and  $\sigma =$  photon helicity. In contrast, in the analogous  $\mathcal{O}(\alpha^1)$  case of CEEEX

$$\sigma = \sum_{n=0}^{\infty} \int_{m_\gamma} d\Phi_{n+2} \sum_{\lambda, \sigma_1, \dots, \sigma_n} |e^{B(m_\gamma)} \mathcal{M}_{n, \sigma_1, \dots, \sigma_n}^\lambda(k_1, \dots, k_n)|^2 \quad (3)$$

the differential distributions for  $n = 0, 1$  photons are, for example,  $\mathcal{M}_0^\lambda = \hat{\beta}_0^\lambda$ ,  $\lambda =$  fermion helicities and  $\mathcal{M}_{1, \sigma_1}^\lambda(k_1) = \hat{\beta}_0^\lambda \mathbf{s}_{\sigma_1}(k_1) + \hat{\beta}_1^\lambda(k_1)$ , with the IR-finite building blocks  $\hat{\beta}_0^\lambda = (e^{-B} \mathcal{M}_\lambda^{\text{Born+Virt.}})|_{\mathcal{O}(\alpha^1)}$  and  $\hat{\beta}_{1, \sigma}^\lambda(k) = \mathcal{M}_{1, \sigma}^\lambda(k) - \hat{\beta}_0^\lambda \mathbf{s}_\sigma(k)$ . Explicitly, this time everything is in terms of  $\mathcal{M}$ -spin-amplitudes! This is the basic difference

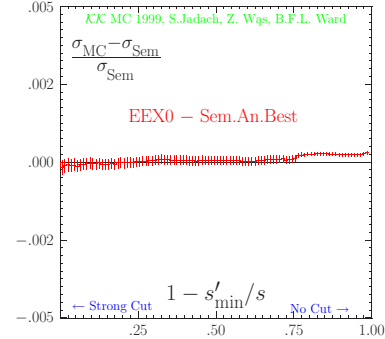


Figure 1. Results for 189 GeV in the  $\mu\bar{\mu}$  channel, for  $v < 0.999$ . We plot the difference between the  $\mathcal{K}\mathcal{K}$  MC result and semi-analytical (IEX) result divided by the latter.

between EEX/YFS AND CEEEX. Complete expressions for spin amplitudes with CEEEX exponentiation,  $n_\gamma$  arbitrary, are given in Phys. Lett. **B449**, 97 (1999) for the  $\mathcal{O}(\alpha^1)$  case and in CERN-TH/2000-087,UTHEP-99-09-01, for the  $\mathcal{O}(\alpha^2)$  case, all are based on GPS spinor conventions as given in CERN-TH-98-235, hep-ph/9905452.

## 3 Results: CEEEX

In Figs. 1, 2 and 3 we show the baseline technical precision test with the  $\bar{\beta}_0$  level matrix element and physical precision tests of  $\sigma_{tot}$ ,  $A_{FB}$ , and the IFI at LEP2 energies as effected in the LEP2 MC Workshop <sup>8</sup>. With these and related tests we achieve the technical precision tag of 0.02% at LEP2 energies, the physical tags of 0.2%(0.2 – 0.4%) for the  $\sigma_{tot}$ (the  $A_{FB}$ ), and firm control on the IFI <sup>1</sup>:

we see that the IFI  $\cong 1.5\%$  for energy cut 0.3, that a  $|\cos\theta| < 0.9$  cut reduces the IFI by 25%, and that the IFI is very small at the Z return, for example.

## 4 YFSWW3-1.14 MC

Starting from the underlying process of interest, Eq.(1), its cross section, Eq.(2), and the attendant  $W^+W^-$  produc-

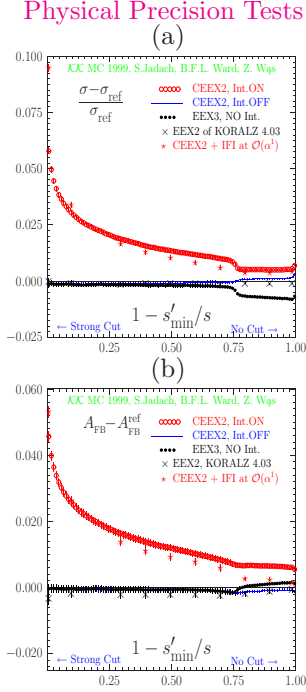


Figure 2. Absolute predictions for  $\sigma_{tot}$ ,  $A_{FB}$ :  $\mu\bar{\mu}$ , 189 GeV.

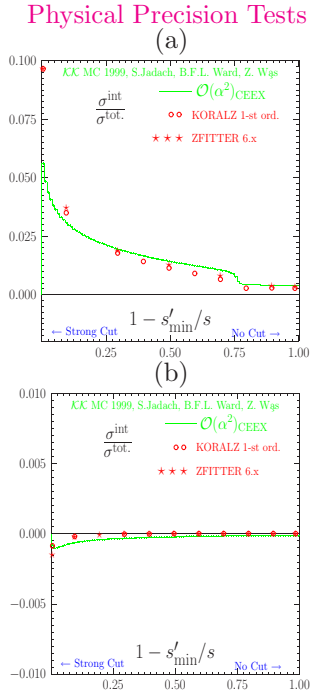


Figure 3.  $s'$ -cut dependence of  $\delta\sigma$ , No  $\theta$ -cut: (a), 189 GeV; (b),  $M_Z$ .

**tion and decay**,  $e^-(p_1) + e^+(p_2) \rightarrow W^-(q_1) + W^+(q_2)$ ,  $W^-(q_1) \rightarrow f_1(r_1) + \bar{f}_2(r_2)$ ,  $W^+(q_2) \rightarrow \bar{f}'_1(r'_1) + f'_2(r'_2)$ , we may isolate the **Leading Pole Approximation (LPA<sub>a,b</sub>)** as follows:

$$\begin{aligned}
 & \mathcal{M}_{4f}^{(n)}(p_1, p_2, r_1, r_2, r'_1, r'_2, k_1, \dots, k_n) \stackrel{LPA}{=} \\
 & \mathcal{M}_{LPA}^{(n)}(p_1, p_2, r_1, r_2, r'_1, r'_2, k_1, \dots, k_n) \\
 & = \sum_{\gamma \text{ Part'ns}} \mathcal{M}_{Prod}^{(n), \lambda_1 \lambda_2}(p_1, p_2, q_1, q_2, k_1, \dots, k_n) \\
 & \times \frac{1}{D(q_1)} \mathcal{M}_{Dec1, \lambda_1}^{(n)}(q_1, r_1, r_2, k_{a+1}, \dots, k_b) \\
 & \times \frac{1}{D(q_2)} \mathcal{M}_{Dec2, \lambda_2}^{(n)}(q_2, r'_1, r'_2, k_{b+1}, \dots, k_n),
 \end{aligned} \tag{4}$$

in an obvious notation <sup>4</sup> for the  $W^\pm$  propagator denominators  $D(q_i)$ , etc. Here, we can identify two different realizations, LPA<sub>a,b</sub>, of the leading pole residues in Eq. (4) by following the prescriptions of **Eden et al.** <sup>9</sup> and **Stuart** <sup>10</sup>: in  $\mathcal{M} = \sum_j \ell_j A_j(\{q_k q_l\})$ , the complete set of spinor covariants  $\{\ell_j\}$  may (b) or may not (a) be evaluated at the pole positions for the respective Lorentz scalar functions  $\{A_j(\{q_k q_l\})\}$ , as these latter already realize the analyticity properties of the S-matrix by themselves. **We do both.**

The standard YFS methods <sup>2</sup> (**EEX-Type**) give us the corresponding analog of Eq.(2). In realizing the exact  $\mathcal{O}(\alpha)$  corrections in the latter equation in the LPA, we have chosen, for our renormalization scheme, the  **$G_\mu$ -Scheme** of Fleischer et al. <sup>11</sup> in version 1.13 and the schemes **A** and **B** in version 1.14, where in **A** only the hard EW correction has  $\alpha_{G_\mu}$  whereas in **B** the entire  $\mathcal{O}(\alpha)$  correction has  $\alpha(0)$ . The analysis in Ref. <sup>12</sup> tells us that the schemes **A** and **B** are improvements over the  $G_\mu$  scheme in version 1.13, as we have verified in the context of the LEP2 MC Workshop comparisons with Denner et al. As a consequence, we have a

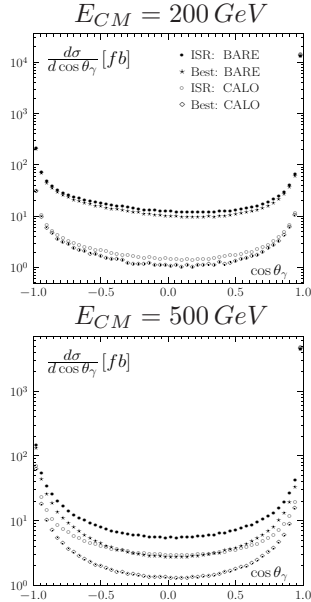


Figure 4. Distribution of  $\cos\theta_\gamma$  with respect to the  $e^+$  beam

$-0.3 \div -0.4\%$  shift of the **NORMALISATION** of version 1.14 relative to version 1.13. See G. Passarino<sup>13</sup> for more details and references.

## 5 Results: YFSWW3-1.14

In Fig. 4, we show the hardest photon angular distribution, both at 200 GeV and at 500 GeV. We see that the NL EW correction is relevant both for the BARE and CALO event selections as defined Ref.<sup>14</sup> away from the beam direction. Similar effects are discussed in Refs.<sup>4</sup>, where we find that the EW NL correction at LEP2 energies is large,  $\sim -2\%$ , and is in general a non-trivial function of the kinematical variables. The authors in Ref.<sup>6</sup> have reached the analogous conclusion.

Indeed, in Table 1 we show a comparison between the results from *RacoonWW* and YFSWW3-1.14, where we have chosen the case of with-

no cuts		$\sigma_{\text{tot}}$ [fb]	
final state	program	Born	best
$\nu_\mu \mu^+ \tau^- \bar{\nu}_\tau$	YFSWW3	219.770(23)	199.995(62)
	<i>RacoonWW</i>	219.836(40)	199.551(46)
	(Y-R)/Y	-0.03(2)%	0.22(4)%
$u\bar{d}\mu^-\bar{\nu}_\mu$	YFSWW3	659.64(07)	622.71(19)
	<i>RacoonWW</i>	659.51(12)	621.06(14)
	(Y-R)/Y	0.02(2)%	0.27(4)%
$u\bar{d}s\bar{c}$	YFSWW3	1978.18(21)	1937.40(61)
	<i>RacoonWW</i>	1978.53(36)	1932.20(44)
	(Y-R)/Y	-0.02(2)%	0.27(4)%

Table 1. Total cross sections, CC03 from *RacoonWW*, YFSWW3,  $\sqrt{s} = 200$  GeV without cuts. Statistical errors – last digits in ( ), etc.  $\Rightarrow 0.4\%$  TU.

out cuts, as carried out in the context of the LEP2 MC Workshop. From these results and others similar to them we arrive at the theoretical precision tag of 0.4% at 200 GeV for the WW signal cross section at LEP2. See G. Passarino<sup>13</sup> for more details and references.

## 6 Conclusions

Our conclusion for the CEEEX  $\mathcal{K}\mathcal{K}$  MC discussion is that the CEEEX is a clear upgrade path for the EEX in a spin amplitude level MC. We have shown that, for LEP2, the total TU is 0.2%(0.2-0.4%) for  $\sigma_{\text{tot}}(A_{FB})$ , for typical cuts – for the LC at 0.5 TeV, these are a factor of 2 worse, and for  $\gamma\gamma^*$  the TU is 0.3% for LEP2 (there is no firm result for LC). The IFI (ISR $\otimes$ FSR) is included and under firm control. Our conclusions for YFSWW3-1.14 are that the EW NL correction<sup>11</sup> in  $\mathcal{O}(\alpha)$ , which is also realized in *RacoonWW*, is important both for the normalisation and for the differential distributions. The TU at 200 GeV, based on comparisons with *RacoonWW*, is 0.4%.

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