# Excited State Contributions to the Heavy Baryon Fragmentation Functions in a Quark-Diquark Model*

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## Abstract

Spin dependent fragmentation functions for heavy flavor quarks to fragment into heavy baryons are calculated in a quark-diquark model. The production of intermediate spin 1/2 and 3/2 excited states is explicity included. The resulting $\Lambda_b$ production rate and polarization at LEP energies are in agreement with experiment. The $\Lambda_c$ and $\Xi_c$ functions are also obtained. The spin independent $f_1(z)$ is compared to data. The integrated values for production rates agree with the data.

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# 1 Introduction.

Fragmentation of quarks and gluons into hadrons is an important subject for which QCD, in both the soft and hard regions, should be applicable. Because of the entanglement of both of these regions, it is difficult to calculate fragmentation functions *in general*. However, when fragmentation involves heavy flavor quarks and hadrons the situation is clearer – factorization of the soft and hard parts can obtain. Heavy quark physics lends itself to perturbative QCD. The calculations are considerably simplified by the peculiar feature of heavy physics just as in atomic physics, where the presence of the heavy nucleus, with a mass much larger than the average momentum transfer inside of the system, effectively reduces the number of the degrees of freedom. A heavy quark or a heavy atomic nucleus can be considered effectively immobile. Of course, such a description is vitiated as soon as the momentum transfers become of the order of the heavy mass, but then an expansion in powers of the inverse mass works well [1].

Until recent years, heavy quark fragmentation functions have been outside the scope of experimental verification. Theoretical predictions for fragmentation into light flavor baryons have been based on a range of phenomenological models [2, 3] and Monte-Carlo simulations [4]. Subsequently, spin dependent fragmentation functions were put on a firm foundation employing light-cone field theory [5] and the theoretical implications of the heavy quark effective theory were incorporated [6]. As these developments occurred it was realized that the masses of the heavy flavor quarks allow for perturbative calculations in the case of the doubly heavy mesons [7, 8]. Later we extended these calculations to include the heavy quark fragmentation of baryons [9, 10]. (At the same time a Russian group [11, 12] made a similar extension to baryons. More recently the light cone expansion of the fragmentation functions was used by the Amsterdam group [13] with an algebraic model to generate predictions similar to ours.) The results of our phenomenological approach were interesting for several reasons. Working in the rather general parameter space we obtained the result that the direct production rate of the spin excited baryons is on the order of or higher than that of the ground state baryons of the same flavor [10]. We also found a functional dependence quite distinct from the simple Peterson parameterization [2].

That excited baryon result may turn out to be a sensible way to explain the tendency seen throughout the scarce experimental data to indicate the
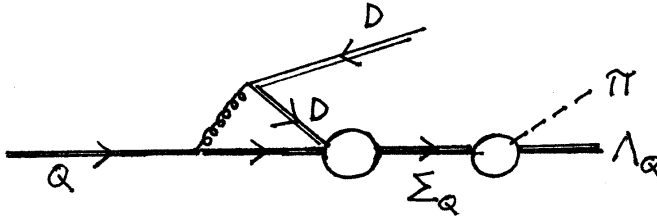
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Figure 1: Diagram representing the combined perturbative and soft QCD calculation of fragmentation through excited baryons.

high depolarization of the heavy flavor baryons. If most of the singly heavy baryons are produced in the ground state, then fragmentation can be seen as the heavy quark picking up a couple of light sea quarks. Since the momentum transfers associated with the quark fragmentation are not large enough to make the heavy quark flip its spin, the expected depolarization is minimal, i.e. the spin of the heavy baryon is expected to be nearly the same as that of the heavy quark. However, if we include spin excited baryons into consideration, the resulting polarization of the final baryon is less clear. The excited states will decay strongly into the ground state almost entirely via pion emission (with the exception of one radiative decay for the $\Xi_c(\frac{1}{2}')$) and mix with the directly fragmented sample.

It is our purpose herein to explore the fragmentation into excited heavy baryons which then decay and populate the observable ground state fragmentation. Since we are interested in the spin dependent fragmentation functions for ground state heavy baryons, the intermediate state resonance production will have a significant effect. In the next section we present the particulars of the model calculation, incorporating QCD, heavy quark approximations and the quark-diquark model of the baryons. The spin dependent functions are developed and discussed in the following section, including the relevant baryon wave functions in the quark-diquark model, along with the comparison with data, where available. The final section discusses the implications of these results in a broader context. An Appendix presents the many scalar products that arise in the final sum over states.

## 2 Calculation details.

The indirect fragmentation process is shown in the Fig. 1. The heavy quark emits a hard gluon that splits into the diquark pair. Diquarks can be either scalar or vector, with the vector diquark having the larger mass (as suggested by the nucleon-$\Delta$ mass difference). If the diquark is scalar, the heavy quark

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forms a ground state spin $\frac{1}{2}$ baryon with it. Since the scalar diquark is spinless and the ground state has no orbital angular momentum, the heavy baryon has nearly the same helicity as the heavy quark. Vector diquarks have spin 1 and the resulting helicity of the baryon is not the same as that of the single heavy quark, but naturally depends on the helicity of the diquark. That does not produce any depolarization of the heavy quark by itself - the heavy quark inside of the baryon does not change its helicity. However, depolarization may occur during the "relaxation" of the baryon into the ground state. The depolarizing features of the spin excited baryon decay into the ground state were discussed by Falk and Peskin [14]. Two parameters are crucial in determining the amount of depolarization relaxation. Let $\Delta M$ be the mass splitting between the $\frac{3}{2}$ and $\frac{1}{2}'$ excited states of the heavy baryon. So $\Delta M$ will be related to the spin-dependent part of the QCD interaction (which has a scale determined by $1/m_Q$) that, in turn, is responsible for flipping the quark spin. The excited baryon states lifetimes will be nearly the same, $\frac{1}{\Gamma}$. The longer this lifetime is, in comparison to the time required for heavy quark spin flipping, the more likely it is for the diquark and quark to mix together forming a randomized quantum spin state. The time it would take for the heavy quark to flip its helicity is proportional to $\frac{1}{\Delta M}$. The relative magnitude of these two parameters determines the nature of the decay. Three distinct regions in the parameter space can be identified:

1) $\Gamma >> \Delta M$. In this case the heavy quark would not have enough time to "mix" with the diquark or light degrees of freedom and the ground state baryon helicity would be the same as the initial helicity of the heavy quark.

2) $\Gamma << \Delta M$. The decay proceeds very slowly, so the resulting ground state baryon is completely depolarized..

3) $\Gamma \approx \Delta M$. This is the case of partial decoherence, the borderline between cases 1 and 2.

It is important to note that the decay width $\Gamma$ is independent of the heavy quark mass. Indeed, in a simplified model the decay may be due to the diquark and light degrees of freedom only. On the other hand the mass splitting $\Delta M \propto \frac{\Lambda_{QCD}^2}{m_Q}$ is inversely proportional to the mass of the heavy quark. This shows that in the limit of an infinitely heavy quark mass there is no depolarization. However, for any given finite mass of the heavy quark all of the above possibilities may occur.

There is very little experimental data on the absolute magnitude of the

decay widths for the heavy flavor excited baryons, but theoretical predictions based on the heavy quark effective theory and potential models (see Ref. [15] and the summary of Falk and Peskin [14]) indicate that the decay width is of the same order of magnitude as the mass splitting for $\Sigma_b$ and smaller than the mass splitting for $\Sigma_c$. That supports baryon relaxation as the source for the spin depolarization. We will return to this issue after presenting an outline of the calculation.

We now proceed with the perturbative calculation of the fragmentation functions. For simplicity we will first consider the spin independent fragmentation function $f_1(z)$ (we use the notation of ref. [5] but without the carat in $\hat{f}_1$) . Once the formalism is established and the calculations are made, it will be easier to turn to the case of the spin dependent $h_1$ and $g_1$ in the next section. The partial width [8] for the inclusive decay process $Z^0 \to H + X$ can be written in general for any hadron $H$ as

$$d\Gamma(Z^0 \to H(E) + X) = \sum_i \int_0^1 dz \, d\hat{\Gamma}(Z^0 \to i(E/z) + X, \mu) \, D_{i \to H}(z, \mu), \quad (1)$$

where $H$ is the hadron of energy $E$ and longitudinal momentum fraction $z$ relative to the parton $i$, while $\mu$ is the arbitrary renormalization scale whose value will be chosen to avoid large logarithms. The function $D_{i \to H}$ is the fragmentation probability in a paricular channel. The parton $i$ is a heavy quark and the baryon $H = B$ could be $\Lambda_Q, \Sigma_Q$, or $\Xi_Q$ in our model. In order to extract the leading twist fragmentation function $f_1(z, \mu)$ out of the total fragmentation probability we take the high energy limit. More precisely the limit is of the proper light cone component $l_0 + l_3$. As long as we are only looking at leading twist functions, the high energy limit produces the same result. The expression, Eqn. 1 will be simplified if we restrict ourselves to the fragmentation channel shown in Fig. 1. Then, while $z$ is kept fixed, the limit of large mass of the $Z^0$ along with large energy of the heavy quark, $q_0$, and the baryon, $l_0$, yields

$$\lim_{l_0 \longrightarrow \infty} d\Gamma(Z^0 \to B(E) + X) = \lim_{q_0 \longrightarrow \infty} \int_0^1 dz \, d\hat{\Gamma}(Z^0 \to Q(E/z) + X, \mu) \, f_1(z, \mu).$$
$$(2)$$

and

$$\int_0^1 dz \, f_1(z, \mu) = \frac{\Gamma_1}{\Gamma_0}, \quad (3)$$

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where $\Gamma_1$ is the decay width of $Z^0$ into the ground state baryon and appropriate remnants - antiquark, spectator diquark and pion, while $\Gamma_0$ is the total decay width of the $Z^0$ into the heavy quark pair [9].

The decay width of the (infinitely massive) $Z^0$ into the inclusive heavy ground state baryon $B_Q$ is viewed as a direct decay into $Q + \bar{Q}$ wherein the quark is off-shell and subsequently fragments. The fragmentation is finally into the $B_Q$, an anti-diquark $\overline{D}$ and a single pion (or photon) from the intermediate resonant baryon decay and is represented by the integral:

$$\Gamma_1 = \frac{1}{2M_Z} \int [d\bar{q}][dl][dp'][d\overline{\pi}](2\pi)^4 \delta^4(Z - \overline{q} - l - p' - \overline{\pi}) |M_1|^2 \qquad (4)$$

where $\bar{q}$, $l$, $p'$ and $\pi$ are the 4-momenta of the $\bar{Q}$, $B_Q$,$\overline{D}$ and the pion (or photon), respectively, and the amplitude $M_1$ is summed and averaged over unobserved spins and colors. We use the notation $[dp] = d^3p/(16\pi^3 p_0)$ for the invariant phase space element. To isolate the fragmentation function, the production of the fragmenting quark ($d\hat{\Gamma}$ of Eq. 1) must be factored out. The fictitious decay width for the $Z^0 \rightarrow Q + \bar{Q}$, with the $Q$-quark on shell is

$$\Gamma_0 = \frac{1}{2M_Z} \int [d\bar{q}][dq] (2\pi)^4 \delta^4(Z - \bar{q} - q) \frac{1}{3} \sum |M_0|^2. \qquad (5)$$

with $q$ the heavy quark 4-momentum.

In order to factor the fictitious decay width out of Eq. 4 we have to transform the phase space variables. That can be achieved by introducing new, production independent variables $x_1 = \frac{p_0 + p_L}{q_0 + q_L}$ and $x_2 = \frac{l_0 + l_L}{p_0 + p_L}$ that can be loosely thought of as Feynman scaling variables for each subprocess, i.e. excited baryon production and decay. We will introduce a further simplification - the ratio of the narrow decay width to the mass of the excited baryons allows the narrow width approximation to be used. The square of the denominator for each excited state propagator that enters the squared amplitude can be factored out for each decay channel and is proportional to $\frac{1}{(p^2 - M^2) + M^2 \Gamma^2}$. Hence, in the limit of the small decay width to mass ratio, that factor can be approximated by $\frac{\pi}{M\Gamma} \delta(p^2 - M^2)$, effectively putting the excited baryon back on the mass shell. The cross terms that come from the cross products of the propagators of the $\frac{1}{2}'$ and $\frac{3}{2}$ spin channels of the decay disappear if the mass difference is much larger than the decay width.

The resulting phase space integral can be written as:

$$
\Gamma_1 = \frac{1}{2M_Z}\frac{1}{256\pi^4} \int [d\bar{q}][dq](2\pi)^4 \delta^4(Z - q - \bar{q})
$$
$$
\cdot \int ds_q \theta\left(s_q - \frac{M_\Sigma^2}{z} - \frac{m_d^2}{1-z}\right) \int d\phi d\varphi dx_1 dx_2 \, |A_1|^2 \tag{6}
$$

Here $|A_1|^2 \delta(p^2 - M^2) = |M_1|^2$. The two angles $\phi$ and $\varphi$ introduced here deserve some special attention and are defined carefuly in the Appendix. They are associated with the position of the transverse momentum vector in two frames of reference. The first is the frame determined by the three-momentum of the heavy quark and a fixed vector perpendicular to it, which is arbitrary unless it is the quark's transverse spin vector (which enters in $h_1$ only). The angle $\phi$ is the azimuthal angle between this plane and the transverse momentum vector (relative to the heavy quark direction) of the excited baryon. The second plane (or frame if we add the vector perpendicular to the first two) is constructed out of the three-momentum of the excited baryon and the spin vector perpendicular to that three-momentum but having no transverse component relative to the first frame. The second angle $\varphi$ is defined as the azimuthal angle between this latter plane and the transverse momentum (relative to the excited baryon direction) of the final baryon.

The spin averaged matrix element $|A_1|^2$ has no angular dependences, so we can safely integrate over the angles. That will not be true for the $g_1$ and especially $h_1$. The choice of $x_1$ and $x_2$ helps to keep the integral symmetric looking, but unlike the standard scaling variable $z = \frac{l_0 + l_L}{q_0 + q_L}$ these variables are not experimentally observable. Using $z = x_1 x_2$ we can finally rewrite the phase space integral to be:

$$
\Gamma_1 = \frac{1}{2M_Z}\frac{1}{16\pi^2} \int [d\bar{q}][dq](2\pi)^4 \delta^4(Z - q - \bar{q})
$$
$$
\cdot \int ds_q \theta(s_q - \frac{M_\Sigma^2}{z} - \frac{m_d^2}{1-z}) \int \frac{dx_2}{x_2} dz \, |A_1|^2 \tag{7}
$$

After factoring out the production decay width we are left with the somewhat simpler expression for $f_1$:

$$
f_1(z, \mu) = \frac{1}{16\pi^2} \lim_{q_0 \to \infty} \int_{s_{th}}^{\infty} ds \frac{dx_2}{x_2} \frac{|A_1|^2}{|M_0|^2} \tag{8}
$$

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The expression above is general for the four body final state. The model dependence is hidden inside of the $|A_1|^2$ with the delta function obtained in the narrow width approximation integrated out. The final ground state baryon can be produced via one of the two intermediate states. If the states are separated by a mass gap wider than the decay width they do no not interfere with each other. That is the scenario supported by the theoretical predictions and the experimental data we have at the moment. It follows that the two channels for the indirect baryon production can be considered independently. The amplitudes for both of them can be expressed as:

$$A = \overline{U}(l) \left\{ \frac{K_{1/2}(\not{p} + M_\Sigma)A_{1/2}}{\sqrt{\Gamma_{1/2}}} + \frac{K_{3/2}^\mu P_{\mu\nu}(\not{p} + M_\Sigma)A_{3/2}^\nu}{\sqrt{\Gamma_{3/2}}} \right\} \Pi \qquad (9)$$

where $\Pi$ is the quark production spinor, $K$ is the decay operator for $1/2'$ (into a pion or a photon) or $3/2$ baryon, the subscripted $A$ is the corresponding production operator for the excited baryon and $P_{\mu\nu}(\not{p} + M_\Sigma)$ is the spin sum of the $3/2$ baryon.

$$
\begin{aligned}
K_{1/2} &= g_1(\not{p} - \not{l})\gamma_5 \\
K_{3/2}^\nu &= -g_2(p - l)^\nu \\
P_{\mu\nu} &= -g_{\mu\nu} + \frac{1}{3}\gamma_\mu\gamma_\nu + \frac{1}{3M}(\gamma_\mu p_\nu - \gamma_\nu p_\mu) + \frac{2}{3M^2}p_\mu p_\nu \qquad (10)
\end{aligned}
$$

where $g_1$ and $g_2$ are coupling constants associated with the decays. Their exact value is irrelevant in this narrow width approximation for the resonance that decays into a single channel – both denominator and numerator of Eq.9 contain the second power of the decay coupling constant.

Note that $\Xi_c$ has to be treated differently. The spin excited states are widely separated, so the lowest lying spin $\frac{1}{2}'\Xi_c(2574)$ state can no longer decay via $\pi$. That only leaves the possibility of photon decay with a branching ratio of nearly 100%. For the radiative decay we will consider the simple electric dipole transition amplitude, $g_3\overline{U}(l) \not{\epsilon}(\gamma)U(p)$, where the photon's polarization vector $\epsilon(\gamma)$ enters. This leads to a $K_{1/2} = g_3 \not{\epsilon}$ replacing the value in Eq. 10.

The ground state baryon is composed of a heavy quark and a scalar diquark. For direct production of the ground state there is one coupling constant for the scalar diquark to couple to the gluon field via a color octet

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vector current – a color charge strength, along with a possible form factor $F_s$.

$$J_\mu^{A(S)} = g_s F_s(k^2)(p + p')_\mu S^{\alpha\dagger}\lambda_{\alpha\beta}^A S^\beta, \tag{11}$$

where $p$ and $p'$ are the scalar diquark 4-momenta and $k = p' - p$. Then the amplitude for direct production becomes

$$A_{S\,1/2} = -\frac{\psi(0)}{\sqrt{2m_d}} F_S(k^2)\bar{U}_B g_s[k_\lambda - 2m_d v_\lambda]P^\lambda, \tag{12}$$

where

$$P^\lambda = \triangle^{\lambda\nu} g_s \gamma_\nu \frac{m_Q(1 + \mathbf{v}) + \mathbf{k}}{(s - m_Q^2)}\Gamma. \tag{13}$$

The amplitudes for the production of the $\frac{1}{2}'$ and $\frac{3}{2}$ states involve the gluon coupling to the vector diquark. For the vector diquark, the color octet current (which couples to the gluon field vector) is more complicated [16]. There are three constants - color charge, anomalous chromomagnetic dipole moment $\kappa$, and chromoelectric quadrupole moment $\lambda$, along with the corresponding form factors, $F_E$, $F_M$, and $F_Q$,

$$\begin{aligned}
J_\mu^{A(V)} &= g_s(\lambda^A)_{\beta\alpha} \Big\{ F_E(k^2)[\epsilon^\alpha(p) \cdot \epsilon^{\beta\dagger}(p')](p + p')_\mu \\
&\quad + (1 + \kappa)F_M(k^2)[\epsilon_\mu^\alpha(p)p \cdot \epsilon^{\beta\dagger}(p') + \epsilon_\mu^{\beta\dagger}(p')p' \cdot \epsilon^\alpha(p)] \\
&\quad + \frac{\lambda}{m_D^2}F_Q(k^2)[\epsilon_\rho^\alpha(p)\epsilon_\nu^{\beta\dagger}(p') + \frac{1}{2}g_{\rho\nu}\epsilon^\alpha(p) \cdot \epsilon^{\beta\dagger}(p')]k^\rho k^\nu(p + p')_\mu \Big\},
\end{aligned} \tag{14}$$

where $A$ is the color octet index, $\alpha$, $\beta$, ..., are color anti-triplet indices, the $\epsilon$'s are polarization 4-vectors for the diquarks. The chromoelectric part of the matrix element contributing to the spin $\frac{1}{2}'$ baryon is

$$A_{E\,1/2} = -\frac{\psi(0)}{\sqrt{3m_d}} F_E(k^2)\gamma_5\gamma^\mu \frac{1 + \mathbf{v}}{2} g_s \bar{\epsilon}_\mu^*[k_\lambda - 2m_d v_\lambda]P^\lambda. \tag{15}$$

The chromomagnetic contribution to the spin $\frac{1}{2}'$ baryon is taken to be small based on earlier extimates from baryon spectroscopy [9, 10, 16]. The quadrupole coupling is assumed inconsequential due to the more rapid fall-off with momentum transfer from dimensional counting rules. For the spin $\frac{3}{2}$ baryon the corresponding chromoelectric amplitude is

$$A_{E\,3/2}^\nu = -\frac{\psi(0)}{\sqrt{2m_d}} F_E(k^2)g_s\bar{\epsilon}^{*\nu}[k_\lambda - 2m_d v_\lambda]P^\lambda, \tag{16}$$

After some simplification we can write the above equations including the decays:

$$\overline{U}(l)K_{1/2}(\not{p} + M_\Sigma)A_{1/2} = -\frac{\psi(0)(M_\Sigma + M_\Lambda)g_{1/2}}{\sqrt{6}m_d}F_E(k^2)\frac{2g_s^2}{M_\Sigma(s - m_Q^2)}$$
$$\overline{U}(l)(\not{p} - M_\Sigma)(M_\Sigma \not{\epsilon}^* + (\epsilon^*p))$$
$$[2M_\Sigma^2(1 - r) - 2\frac{(np)}{(nk)}(kp) + M_\Sigma \not{k}] \qquad (17)$$

for $\pi$ decay of the $1/2'$;

$$\overline{U}(l)K_{1/2}(\not{p} + M_\Sigma)A_{1/2} = -\frac{\psi(0)(M_\Sigma + M_\Lambda)g_{1/2}}{\sqrt{6}m_d}F_E(k^2)\frac{2g_s^2}{M_\Sigma(s - m_Q^2)}$$
$$\overline{U}(l)\gamma_5 \not{\epsilon}(\pi^2)(\not{p} - M_\Sigma)(M_\Sigma \not{\epsilon}^* + (\epsilon^*p))$$
$$[2M_\Sigma^2(1 - r) - 2\frac{(np)}{(nk)}(kp) + M_\Sigma \not{k}] \qquad (18)$$

for the photon $(\epsilon(\pi^2))$ decay of the $1/2'$;

$$\overline{U}(l)K_{3/2}^\mu P_{\mu\nu}(\not{p} + M_\Sigma)A_{3/2}^\nu = \frac{\psi(0)M_\Sigma g_{3/2}}{\sqrt{2}m_d}F_E(k^2)\frac{2g_s^2}{M_\Sigma(s - m_Q^2)}$$
$$\overline{U}(l)(p^\mu - l^\mu)P_{\mu\nu}(\not{p} + M_\Sigma)\epsilon^{*\mu}$$
$$[2M_\Sigma^2(1 - r) - 2\frac{(np)}{(nk)}(kp) + M_\Sigma \not{k}] \,(19)$$

for the $\pi$ decay of the $3/2$ state, where $r = m_d/M_\Sigma$ is a measure of the departure from the heavy quark limit. Note that the $\Xi_c(\frac{1}{2}')$ radiative decay is somewhat different from $K_{1/2}$ above, but easily accommodated in the sum over intermediate states.

The fragmentation functions must be independent of the heavy quark production mechanism. For simplicity then, we actually calculate the decay of a heavy scalar breaking into the heavy quark antiquark pair with the quark further fragmenting into the baryon. The remaining calculations are straightforward, but quite complex. We will sketch them in the next section. The results for the different heavy quark flavors are presented following that.

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# 3 Spin dependent fragmentation functions.

The leading twist fragmentation functions $g_1$ and $h_1$ carry valuable information about the helicity and transversity transfer in the system. The meaning of these functions first introduced by Jaffe and Ji [5] is straightforward. The function $g_1$ is the difference between probabilities for having the final baryon with helicity aligned with and opposite to the original helicity of the heavy quark; $h_1$ similarly represents the difference in the no flip and flip rates, but in the sense of the transverse direction of spin (more carefully, the transversity [17]).

In the strict limit of the heavy quark effective theory one expects to find $f_1(z) = g_1(z) = h_1(z) = P\delta(1-z)$, with $P$ being an overall production rate for the corresponding baryon. This is not quite so for any given finite mass of the heavy quark. The only restrictions come from the probabilistic interpretation of the fragmentation function (for example, $f_1(z) \geq g_1(z)$) and the analog of the proposed structure function inequality [18] $f_1(z) + g_1(z) \geq |2h_1(z)|$. Both of these restrictions are satisfied in our model.

The theoretical prediction made in ref. [14] indicates that the polarization of the $\Lambda_Q$ is heavily dependent on the production rates for the excited baryons. We only consider the lowest spin excited baryons $\Sigma_Q(\frac{1}{2}')$ and $\Sigma_Q^*(\frac{3}{2})$ in the present paper. Other excitations can be included in two ways: considering excited diquarks, either radially or orbitally, or considering quark-diquark baryon configurations excited radially and/or orbitally. The former case creates a lot of theoretical uncertainties, since the diquark is less bound and can hardly be considered as a parton. In the latter case, the wave functions at the origin are expected to be smaller, so that the production rate of such states is smaller. In general, such states may provide for corrections to our calculations, but the main features of the fragmentation should remain unaltered.

In our model the spin dependent fragmentation functions can be obtained directly, using the modified Eq.8:

$$g_1(Q,z) = \frac{1}{256\pi^4} \lim_{q_0\to\infty} \int_{s_{th}}^{\infty} ds \frac{dx_2}{x_2} d\phi d\varphi \frac{|A_{1+}|^2 - |A_{1-}|^2}{|M_0|^2} \qquad (20)$$

$$h_1(Q,z) = \frac{1}{256\pi^4} \lim_{q_0\to\infty} \int_{s_{th}}^{\infty} ds \frac{dx_2}{x_2} d\phi d\varphi \frac{|A_{1y+}|^2 - |A_{1y-}|^2}{|M_0|^2} \qquad (21)$$

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with new indices specifying the spin alignment ($|y+>\sim |+> +i|->$). Angular integration is especially complicated for $h_1$, because matrix elements involve spin projections that make them no longer azimuthally symmetric. Naturally, the transverse spin vector would set a preferred azimuthal direction that manifests itself in the scalar products of the transverse spin vectors with other vectors that have transverse components. These scalar products are listed in the Appendix.

The amplitude $A$ is defined in Eq. 9 and in general can be represented as:

$$A = \overline{U}O\Pi \tag{22}$$

where $O$ is the expression in curly brackets in Eq. 9. The square of the matrix element can then be written as:

$$|A_{1\alpha}|^2 = Tr(\Pi\overline{\Pi}\gamma_0 O^\dagger \gamma_0(\not{p} + M_\Lambda)\frac{1 + \gamma_5 \not{S}_\alpha}{2}O) \tag{23}$$

with $\alpha$ being the spin projection index and $S_\alpha$ being the spin four-vector corresponding to that projection. The two cases for this problem are the longitudinal or helicity and transverse spin vectors. The longitudinal spin vectors are defined unambiguously by defining the four-vector in the rest frame of the particle with time component equal to zero and space component equal to the unit vector pointed in the direction of the Lorentz boost that would take the particle back into the lab frame. The transverse spin vector (transversity [17]) has to be perpendicular to the direction of the Lorentz boost and has zero energy component. It will not be affected by the Lorentz boost, so it will still be perpendicular to the three-momentum of the particle even in the lab frame. The tip of the spin vector could be anywhere on the unit circle that has its origin at the base of the spin vector and is perpendicular to the three-momentum of the particle. The ambiguity is resolved by choosing the positive direction of the transverse spin of the final baryon to "align" (or anti-align) with the arbitrarily chosen positive direction of the initial quark's transversity. By taking the three-momentum of the quark to be along the z-axis and transverse spin direction to be along the x-axis the transverse momentum will be defined to be in the xz plane with it's x component positive. Such a definition of the transverse spin vector also naturally introduces the new coordinate system with z along the three -momentum of the baryon and the x-axis parallel to the transverse spin. In exactly the

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same fashion we can introduce the transverse spin vector of the intermediate baryon. We set up yet another coordinate frame. The angles $\phi$ and $\varphi$ are azimuthal angles of the intermediate baryon in the frame of the heavy quark and the final baryon in the frame of intermediate baryon (see the Appendix).

As was mentioned previously, we can proceed in explicitly calculating all matrix elements without losing any generality by assuming that the production mechanism is a simple decay of the scalar particle. This way $\Pi = \upsilon_{-\alpha}$, where $-\alpha$ indicates that the spin of the antiquark is opposite of the expected spin of the quark. After simplifying the squares of matrix elements and leaving only the leading terms in the high momentum limit, we end up with the scalar products of all the involved four-momenta and spin vectors that are given in the Appendix. We next look at the angular composition of the resulting integrands.

In general, the square matrix elements involve scalar products of the available four-vectors:

$\overline{q}, q, p, l, n, s^q, s^{\overline{q}}, s^l$.

The spin vectors can be either transverse or longitudinal and $n$ is the four-vector orthogonal to the quark momentum that enters in the axial gauge. The phase space integration variables are

$x_1, x_2, \phi, \varphi, s_q$.

We will use the notation $p_{qt}$ throughout this paper, where the first index stands for the three-momentum that originated the frame and the second index, if there, corresponds to the component of the vector. So $p_{qt}$ is the transverse component of the excited baryon three-momentum viewed in the first frame. Note that

$$p_{qt}^2 = s_q x_1 (1 - x_1) - M_\Sigma^2 (1 - x_1) - m_d^2 x_1, \qquad (24)$$

$$l_{pt}^2 = M_\Sigma^2 x_2 (1 - x_2) - M_\Lambda^2 (1 - x_2) - m_d^2 x_2. \qquad (25)$$

In the Appendix we consider each scalar product that can arise in the integrands of Eq. 20 and 21.

For the $f_1$ there are no spin vectors since we spin average everything. This way the only scalar product that will introduce angular dependence is $(ql)$ (as shown in the Appendix), the product of the four-momenta of the initial quark and baryon. The angular dependence is proportional to $\cos(\varphi + \phi)$. This term disappears after the single integral over one of the angles – introducing

intermediate baryon degrees of freedom breaks the azimuthal symmetry of the system, but the integration restores the symmetry.

The function $g_1$ has the same type of angular dependence. The only scalar products that produce azimuthal angles are $(ql)$, $(s_q l)$ and $(s_l q)$. All of them are proportional to $\cos(\varphi + \phi)$ and the angular dependence is removed by integrating over one of the angles, which is not surprising given that the azimuthal symmetry should be preserved again.

In the case of $h_1$ the situation becomes more complex. The scalar products of the type $(s_q p)$ and $(s_l p)$ are proportional to $\cos(\phi)$ and $\cos(\varphi)$ and we still have the scalar product $(ql)$ that is proportional to $\cos(\varphi + \phi)$. This type of angular dependence is removed only after both angles are integrated. The system is never azimuthally symmetric – in choosing the arbitrary transverse spin direction we break the rotational invariance.

In order to obtain the final fragmentation function we only have to integrate over the angles and change all the variables into $x_1$ and $x_2$, which can be done in the high momentum limit. This integration is performed numerically. There are several parameters that need to be specified. The masses of the diquarks are taken as (uu, ud) 0.6 $\mathrm{GeV}/c^2$ and (us, ds) 0.9 $\mathrm{GeV}/c^2$. The quark masses are 4.9 $\mathrm{GeV}/c^2$ for the b-quark and 1.6 $\mathrm{GeV}/c^2$ for the c-quark. The $\Lambda_b$ mass is 5.6 $\mathrm{GeV}/c^2$; the $\Sigma_b$ mass is 5.8 $\mathrm{GeV}/c^2$ (the mass of the $\Sigma_b*$ can be taken the same, because difference is in the hundredths); the $\Lambda_c$ is 2.3 $\mathrm{GeV}/c^2$; the mass of $\Sigma_c$ is 2.5 $\mathrm{GeV}/c^2$. The wave functions for the formation of the baryons are obtained from the power law potential of Eichten and Quigg [19],

$$V(r) = -8.064 \text{ GeV} + 6.898 \text{ GeV } (r \times 1 \text{ GeV})^{0.1}. \tag{26}$$

The resulting square moduli of the wavefunctions at the origin, $|\psi(0)|^2$, are 0.46 $\mathrm{GeV}^3$, 0.35 $\mathrm{GeV}^3$ and 0.51 $\mathrm{GeV}^3$ for $\Lambda_b, \Lambda_c$ and $\Xi_c$, respectively.

The integrations produce spin-dependent fragmentation functions that are defined at the scale $\mu_0 = m_Q + m_{diquark}$. To evolve them to higher values of the defining scale (or the typical $Q^2$) we utilize the appropriate spin-dependent Altarelli-Parisi integro-differential equations as determined by Artru and Mekhfi [20].

In Fig. 2 the unevolved fragmentation functions $f_1, g_1, h_1$ are plotted for $\Lambda_c$ as functions of $z$. The function $f_1(z)$ evolved to 5.5 GeV (half of the CESR energy) is shown in Fig. 3, along with the CLEO data [21] on $\Lambda_c$
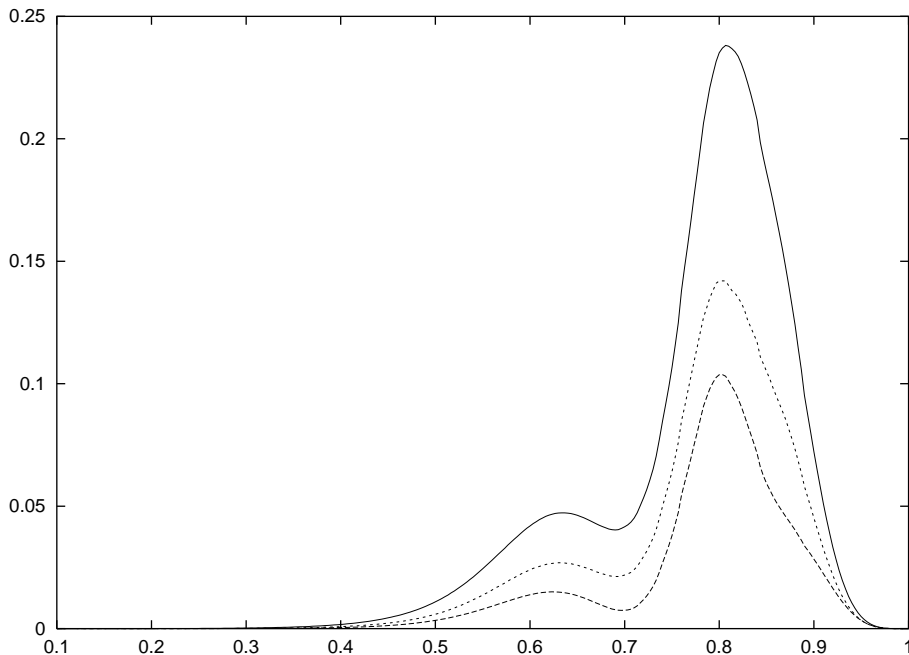
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Figure 2: Fragmentation functions for $\Lambda_c$ at $\mu_0$. At the peak $f_1$ is largest, followed by $h_1$ and $g_1$.

production from one of the decay channels, $pK^-\pi^+$. The data are given for bins of $x_+$, which is related to our $z$, but somewhat different at finite c-quark energy. The normalization of the data (which is arbitrary) was adjusted to correspond to the normalization of the predicted curve. There is considerable scatter about our curve for $f_1(z)$. A smooth curve through the data suggests a lower $z$ for the peak position. The data are not sufficiently spaced in $x_+$ to check whether or not there is evidence for the shoulder in our predicted curve. Caution is advised in interpreting the CLEO $\Lambda_c$ production data as a leading twist fragmentation function, given that the c-quark is being produced at half the $\Upsilon$ mass. This is far from a large energy compared to the fragmenting $\Lambda_c$, i.e. $\frac{1}{2}M_{\Lambda_c}/M_\Upsilon \sim 0.4$ which is quite sizeable.

The 45 GeV spin dependent functions are shown in Fig. 4 for completeness. Corresponding results for the b-quark are shown for $\Lambda_b$ in Fig. 5 and 6. The $\Xi_c$ fragmentation functions are presented in Figs. 7,8 and 9, for the unevolved, 5.5 GeV/c (for comparing with CESR data) and 45 GeV/c (for comparing with LEP data).

It is clear from these figures that the unevolved functions are all sharply peaked at high $z$ and get spread out and smoothed out with evolution. The double peak structure noted for the direct production of the ground state heavy baryons in our previous work [9, 10] has been moderated by the con-
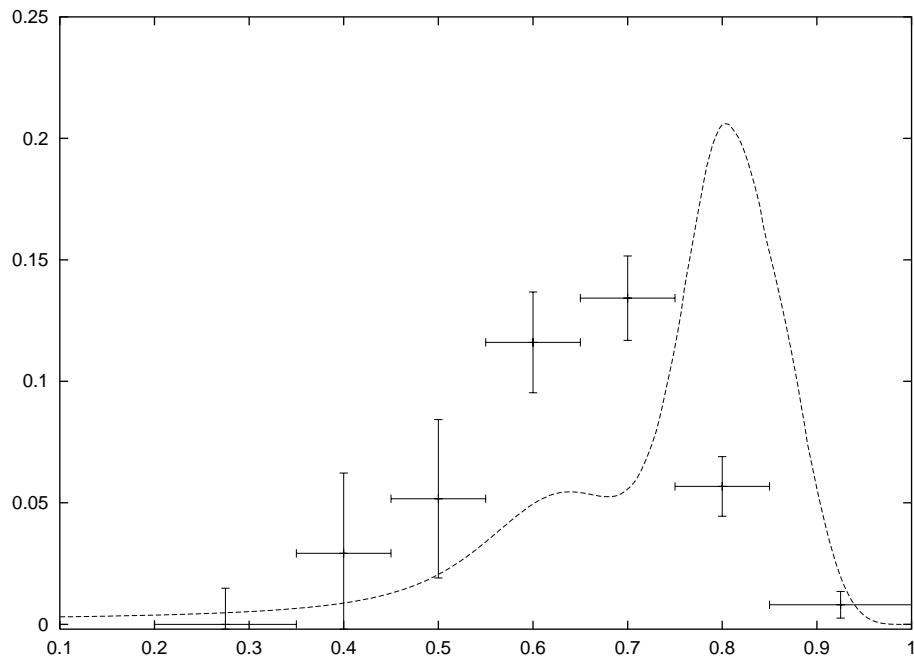
15

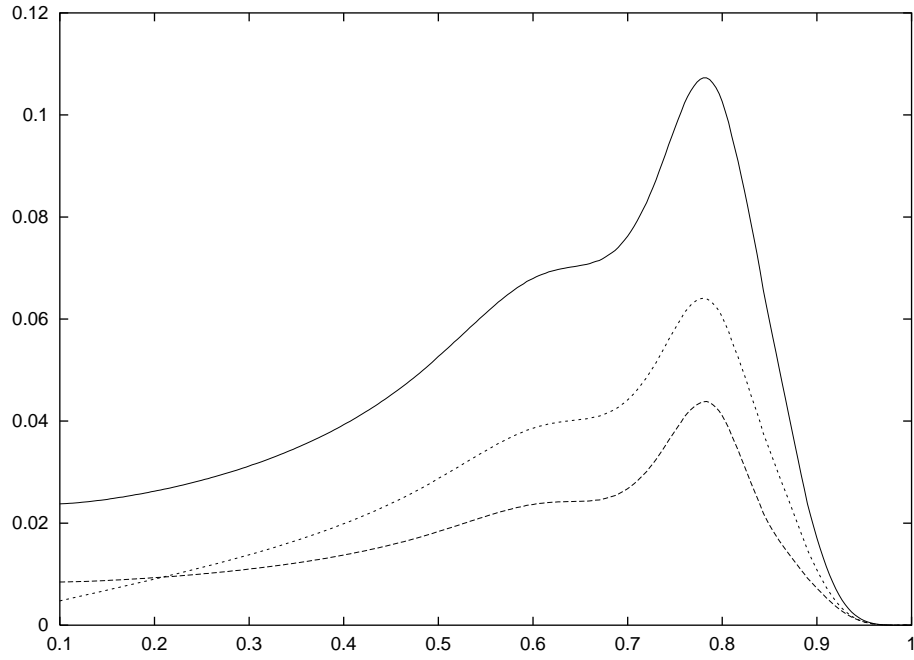Figure 3: Spin independent function $f_1(z)$ evolved to $\mu= 5.5$ GeV. The data are from CLEO [21].



Figure 4: Fragmentation functions for $\Lambda_c$ evolved to $\mu= 45$ GeV. At the peak $f_1$ is largest, followed by $h_1$ and $g_1$.
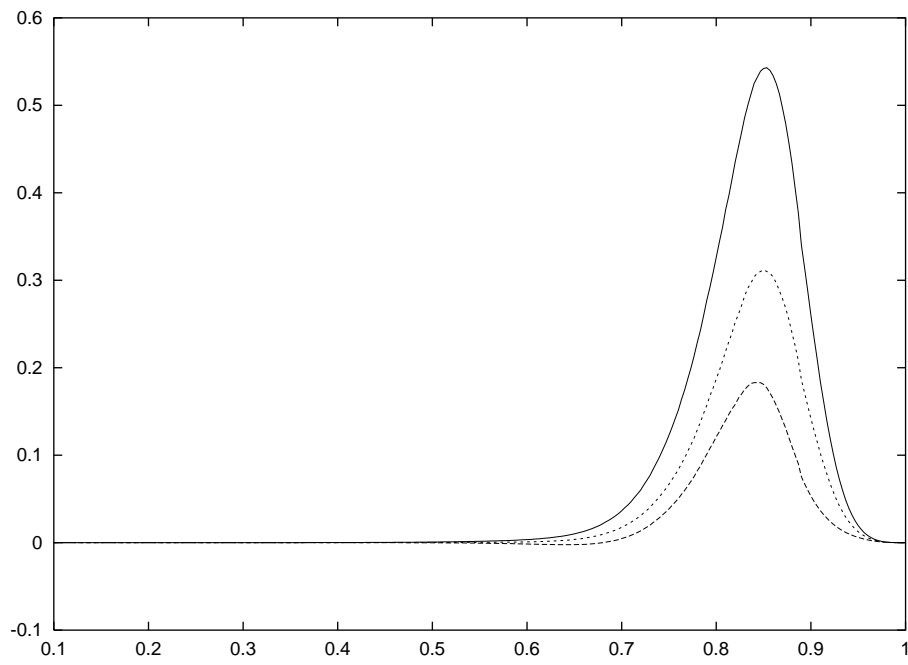
16

Figure 5: Fragmentation functions for $\Lambda_b$ at $\mu_0$. At the peak $f_1$ is largest, followed by $h_1$ and $g_1$.
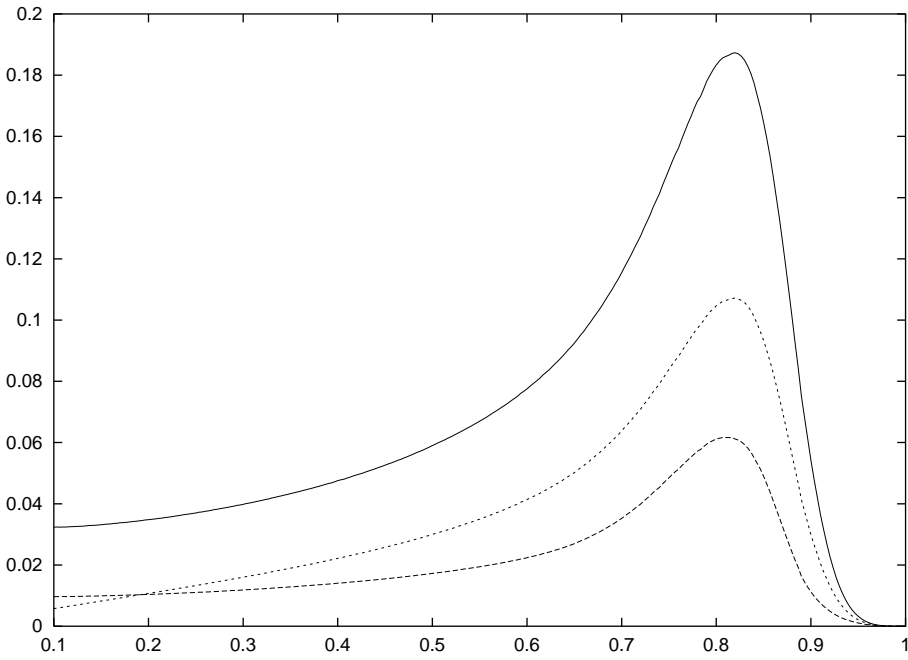


Figure 6: Fragmentation functions for $\Lambda_b$ evolved to 45 GeV. At the peak $f_1$ is largest, followed by $h_1$ and $g_1$.
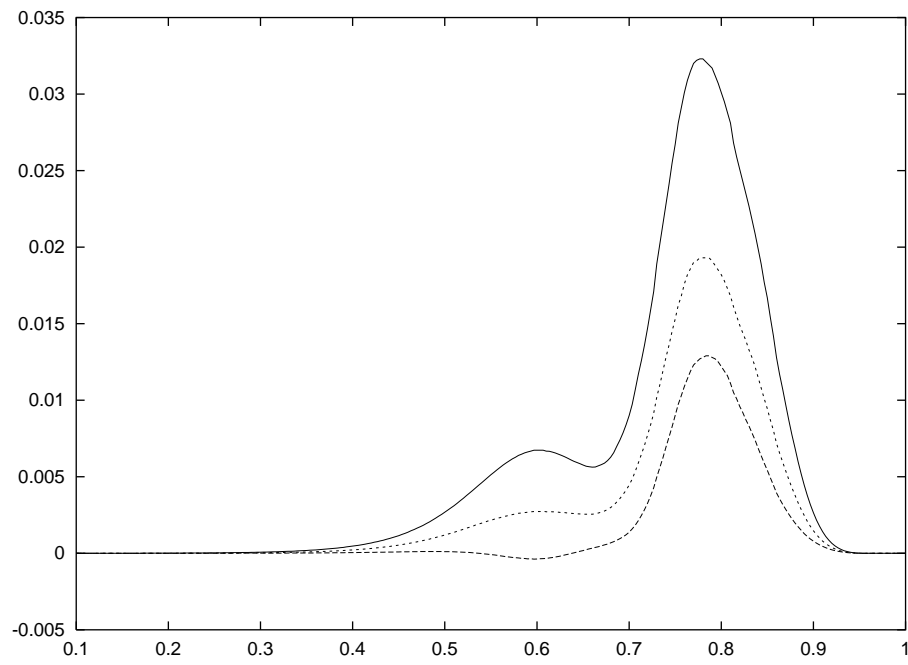
17

Figure 7: Fragmentation functions for $\Xi_c$ unevolved. At the peak $f_1$ is largest, followed by $h_1$ and $g_1$.
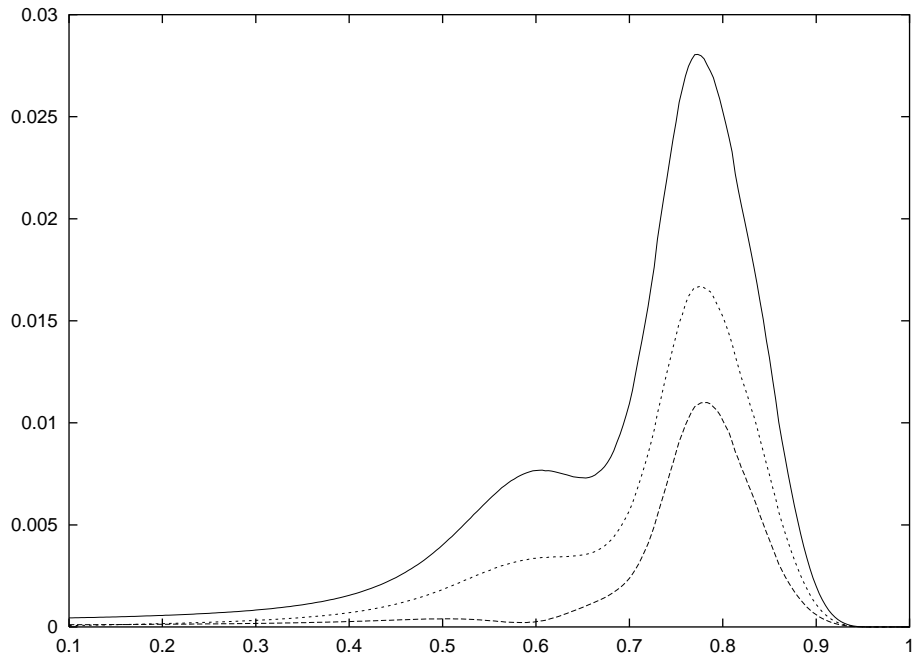


Figure 8: Fragmentation functions for $\Xi_c$ evolved to 5 GeV/c. At the peak $f_1$ is largest, followed by $h_1$ and $g_1$.
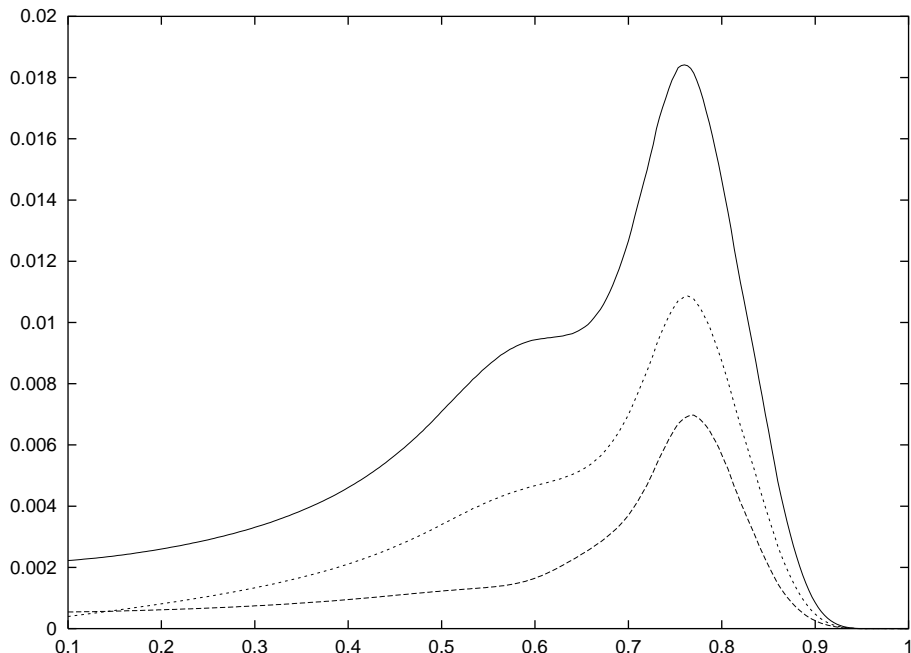
18

Figure 9: Fragmentation functions for $\Xi_c$ evolved to 45 GeV/c. At the peak $f_1$ is largest, followed by $h_1$ and $g_1$.

tributions from the excited states. The bound $f_1 + g_1 \geq 2|h_1|$ is satisfied [18] and nearly saturated, as expected in the heavy quark limit.

The overall rate for a heavy baryon state to be a fragment of the corresponding heavy quark is obtained by integrating $f_1(z)$ over $z$. The results, which include the excited intermediate states are tabulated here along with experimental data.

*Total Fragmentation Probabilities*

| Particle | Experiment | Prediction |
|---|---|---|
| $\Lambda_c$ | $5.6 \pm 2.6\%[OPAL~[22]]$ | 3.9% |
| $\Xi_c$ | | 0.59% |
| $\Lambda_b$ | $7.6 \pm 4.2\%[ALEPH~[23]]$ | 6.7% |

These results are consistent with experiment within errors. The total fragmentation probability for $\Xi_c$ is not given, although the fractional rates for different states are known [24] and were stated in our previous work [10].

The predicted overall net polarization transfer from quark to baryon is the integral of $g_1(z)/f_1(z)$. For these we obtain $0.288, 0.239$ and $0.302$ for $\Lambda_c, \Xi_c$ and $\Lambda_b$, respectively. The averaged value of the polarization measurements for $\Lambda_b$ produced at LEP [25] are $-0.45 \pm 0.18$ compared to the

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Standard Model expectation of $-0.94$ for the b-quark polarization. So the value of net polarization transfer for $b \to \Lambda_b$ is $0.48 \pm 0.19$, consistent with our prediction of 0.30. Note that our prediction is based on integrating over all $z$, whereas the experiment necessarily emphasizes a narrower range of $z$ centered on the peak production region and there is some fluctuation of the value of $g_1(z)/f_1(z)$ in that region.

# 4    Discussion of the results.

It is known that heavy quark fragmentation into *excited* charmonium and bottomonium states contributes significantly to the high energy hadronic production of $J/\psi$, $\psi'$ and $\Upsilon$ states [26]. The excited intermediate states do not make up completely for the overall theoretical underestimate of the cross sections for heavy quarkonium production (color octet [27] schemes or alternatives are needed), but they are quite significant. The situation in the production of heavy baryons is less clear at this time. However, the observation of the depolarization of the heavy quark provides an opportunity to pin down the contribution of excited states and to compare the theoretical model predictions based on QCD with the experimental data. Such an effect for the $\Lambda_b$ was first estimated by Falk and Peskin [14]. The heavy c and b baryons fall into their second or third category. The decay of the excited $\Sigma_b$, $\Sigma_c$, $\Xi_b$ or $\Xi_c$ baryons is primarily due to pion emission (with the exception of the radiative decay of $\Xi_c(2574)$ as we discussed above). The experimental data on the decay widths are either unavailable or come with large margins of error. They can be theoretically estimated for the heavy baryons [28]. The results clearly indicate that the decay of the excited heavy baryons should exhibit at least partial depolarization.

Such depolarization is more important in the light of the results obtained in the direct fragmentation model [9, 10]. Since the number of the excited states that can contribute is large, they may very well pollute the final sample of the ground state baryons and result in an overall depolarization. Based on general, model independent reasoning Falk and Peskin [14] estimated the amount of depolarization:

$$\frac{\Lambda_b(-\frac{1}{2})}{\Lambda_b(+\frac{1}{2})} = \frac{2(2-w_1)A}{9 + A(5 + 2w_1)} \tag{27}$$

with all heavy quarks initially taken to be in $+\frac{1}{2}$ state. The parameter $A$ is the probability of producing the vector diquark (versus the scalar); $w_1$ is the probability that the vector is in the helicity +1 or - 1 state. Note that when direct production of $\Lambda_b$ is much bigger than the direct production of the excited states, as naively may be assumed to be the case, the depolarization is minimal. On the other hand, the signature of our direct quark-diquark fragmentation model is the high ratio of the excited state production, which leads to a sizeable depolarization.

The original investigation of excited state contributions by Falk and Peskin [14] was made without any dynamical details. What we have done here is to go a step further, with an eye toward future high statistics experiments, and to look at the dynamics of the indirect fragmentation. We used a perturbative calculation of excited baryon fragmentation and decay. The idea was simply to incorporate pion decay into the direct quark-diquark model. The basic premise of the model still involves a heavy fast quark originating from any high energy source, such as the $Z^0$ decay. Following that the quark shakes off the gluon that breaks into a diquark antidiquark pair. If the diquark has velocity similar to that of the heavy quark they fragment into the baryon. The diquark could be either scalar, and then the baryon produced is in the ground state (i.e. $\Lambda_b$), or vector, so that the baryon is in one of the spin excited states.

Now note that in Falk and Peskin the final net polarization of $\Lambda_Q$ (including $\frac{1}{2}$ and $\frac{3}{2}$ excited state contributions), obtained from Eq. 27 is :

$$P = \frac{1 + (1 + 4w_1)A/9}{1 + A} \tag{28}$$

That corresponds to the integral of $\frac{f_1}{g_1}$ over all values of $z$. Obviously that integrated value is not enough to pinpoint both of the parameters ($A$ and $w$). However, the net polarization of $\Lambda_Q$ produced purely via decays of excited states (direct fragmentation excluded) is:

$$P = \frac{4w_1 + 1}{9}. \tag{29}$$

Using this expression to obtain $w_1$ we find:

For $\Lambda_b$: $w_1 = 0.41$. Total polarization $P = 0.295$
For $\Lambda_c$: $w1 = 0.39$. Total polarization $P = 0.285$

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In our previous paper we had $w_1 = 0.46$ for $\Lambda_b$ using the direct fragmentation function of $\Lambda_b$ and $\Sigma_b$. The results here are very close to this value although we used different techniques to obtain it. Also, these results assume that $w$ and $A$ are independent of $z$, which generally is not correct (production rates may depend on the energies of the gluon and that can be translated into $z$ dependence).

The parameter $A$ has not changed much from the last paper.

For $\Lambda_b$: $A = 6$.

For $\Lambda_c$: $A = 6.3$.

The various fragmentation functions that have been obtained in our model have yet to be tested experimentally. The important features of those functions that should appear in the data include their overall normalizations, the resulting longitudianl polarizations, the ratios of the three spin dependent functions and the characteristic double hump dependence on $z$. The functional dependence on $z$ distinguishes this model from the qualitative parameterization of the Peterson model. The departure from a single sharp peak structure is a measure of the departure from the heavy quark limit. So careful measurements of the $z$ dependence of any of the functions, $f_1(z), g_1(z)$ or $h_1(z)$ will be very revealing.

# Appendix

In this appendix we list all relevant scalar products of two 4-vectors beginning with all combinations not involving spin. In each case we show the limiting value as the fragmenting quark's energy $q_0$ and 3-momentum $|\vec{q}|$ become large.

a) $(q\bar{q})$

$$(q\bar{q}) = q_0\bar{q}_0 + |\vec{q}|^2 = \sqrt{m_q^2 + |\vec{q}|^2}\sqrt{s_q + |\vec{q}|^2} + |\vec{q}|^2 \to 2\,|\vec{q}|^2$$

b) $(p\bar{q})$

$$(p\bar{q}) = p_0 q_0 + p_l\,|\vec{q}| \to 2p_l\,|\vec{q}| = 2x_1\,|\vec{q}|^2$$

c) $(l\bar{q})$

$$(l\bar{q}) \to 2z\,|\vec{q}|^2$$

d) $(n\bar{q})$

$$(n\bar{q}) = q_0 - |\vec{q}| = \sqrt{m_q^2 + |\vec{q}|^2} - |\vec{q}| \rightarrow \frac{m_q^2}{2|\vec{q}|}$$

e) $(qp)$

Note that the intermediate quark (momentum $q - k$) is taken as on-shell and the mass of the hadron (momentum $p$) $M_\Sigma$ is approximately $m_{diquark} + m_{quark}$. Hence $q = k + \frac{m_q}{M_\Sigma}p$ and $p' = k - \frac{m_d}{M_\Sigma}$. The quantity $s_q = q^2$. Hence:

$$(qp) = m_q M_\Sigma + (kp)$$

$$(qq) = m_q^2 + 2\frac{m_q}{M_\Sigma}(kp) + k^2 = s_q$$

$$k^2 = 2\frac{m_d}{M_\Sigma}(kp)$$

So finally

$$(qp) = m_q M_\Sigma + \frac{(s - m_q^2)}{2}.$$

f) $(nq)$

$$(nq) = q_0 + |\vec{q}| \rightarrow 2|\vec{q}|$$

g) $(lp)$

$$p = l + \overline{\pi}$$
$$(p - l)^2 = m_\pi^2$$
$$M_\Sigma^2 + M_\Lambda^2 - 2(pl) = m_\pi^2$$
$$(lp) = \frac{M_\Sigma^2 + M_\Lambda^2 - m_\pi^2}{2}$$

h) $(ql)$

Evaluating this expression requires careful definitions of coordinate systems. Let $\hat{z} = \hat{q}$ and $\hat{x} = \vec{S}_T(q)$ define the X-Z plane for the incoming quark (with transverse spin vector also called $s^{qt}$). Let $(\theta, \phi)$ be the polar and azimuthal angles for $\vec{p}$. Define a second frame of reference in which $\hat{z}' = \hat{p}$ is the polar axis and $\hat{x}' = [\hat{x}cos(\theta) - \hat{z}sin(\theta)cos(\phi)]/[1 - sin^2(\theta)sin^2(\phi)]^{\frac{1}{2}}$ so

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that $\hat{x}'$ is defined by the excited baryon's transverse spin direction. We can express $l$ in the primed coordinates as $l = (l_0, l_{pt}\cos(\varphi), l_{pt}\sin(\varphi), l_{pl})$. The azimuth of $\vec{q}$ in the primed coordinates is approximately $\pi - \phi$. It is more precisely $= \pi - \phi + O(\frac{1}{|p|})$ in the relevant large momentum limit. Using that relation we get:

$$(\vec{q} \cdot \vec{l}) \rightarrow |\vec{q}|\frac{p_{ql}}{|\vec{p}|}l_{pl} + \frac{|\vec{q}|}{|\vec{p}|}p_{qt}l_{pt}\cos(\phi' - \varphi)$$

Then after several steps the appropriate limiting value is given by

$$(ql) \rightarrow -\frac{x_2}{x_1}M_\Sigma^2 + (qp)x_2 + \frac{(lp)}{x_1} - \frac{p_{qt}l_{pt}}{x_1}\cos(\phi + \varphi)$$

i) $(np)$

$$(np) = p_0 + p_{ql} \rightarrow 2x_1|\vec{q}|$$

j) $(nl)$

$$(nl) = l_0 + l_{ql} \rightarrow 2z|\vec{q}|$$

These complete all scalar products not involving the spin vectors. Now we consider transverse spin vectors also. We only have two distinct ones: $s^{qt}$ and $s^{lt}$.

k) $(s^{qt}p)$

$$(s^{qt}p) = -p_{qt}\cos(\phi)$$

l) $(s^{qt}l)$

This also requires going into the primed frame of $p$. In that frame:

$$(s^{qt}p) = \left(0, (1 + O(\frac{1}{|\vec{p}|})), O(\frac{1}{|\vec{p}|}), (\frac{p_{qt}\cos(\phi)}{\vec{p}} + O(\frac{1}{|\vec{p}|^2}))\right)$$

$$(s^{qt}l) \rightarrow -l_{pt}\cos(\varphi) - \frac{l_{pl}}{|\vec{p}|}p_{qt}\cos(\phi) + O(\frac{1}{|\vec{p}|})$$

$$(s^{qt}l) \rightarrow -l_{pt}\cos(\varphi) - \frac{p_{qt}\cos(\phi)}{x_2}$$

m) $(s^{qt}n)$

$$(s^{qt}n) \rightarrow \frac{p_{qt}\cos(\phi)}{|\vec{p}|}$$

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n) $(s^{lt}\overline{q})$

In the primed frame:

$$(s^{lt}p) = \left(0, (1 + O(\frac{1}{|\overrightarrow{p}|})), O(\frac{1}{|\overrightarrow{p}|}), (-\frac{p_{qt}\cos(\phi)}{\overrightarrow{p}} + O(\frac{1}{|\overrightarrow{p}|^2}))\right)$$

$$(s^{lt}\overline{q}) \rightarrow -\frac{q_{pl}}{|\overrightarrow{p}|}p_{qt}\cos(\phi)$$

$$(s^{lt}\overline{q}) \rightarrow -\frac{1}{x_1}p_{qt}\cos(\phi)$$

o) $(s^{lt}q)$

$$(s^{lt}\overline{q}) \rightarrow \frac{1}{x_1}p_{qt}\cos(\phi)$$

p) $(s^{lt}p)$

$$(s^{lt}p) \rightarrow p_{qt}\cos(\phi)$$

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