Hard Parton Rescatterings and Minijets in Nuclear Collisions at the LHC

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Abstract

The average number of minijets and the corresponding transverse energy in heavy ion collisions are evaluated by including explicitly semi-hard parton rescatterings in the dynamics of the interaction. At the LHC semi-hard rescatterings have a sizable effect on global characteristics of the typical inelastic event. An interesting feature is that the dependence on the cutoff which separates soft and hard parton interactions becomes less critical after taking rescatterings into account.

PACS: 11.80.La, 24.85.+p, 25.75.-q

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1 Introduction

Semi-hard physics is one of the most important issues in the interaction of heavy ions at the LHC. Given the total energy involved and the large number of participants, the component of the inelastic interaction which can be described within a perturbative approach is in fact rather substantial in heavy ion collisions at the LHC [1, 2]. The result is the production of a large number of minijets in the typical inelastic event. The description of the semi-hard component of the interaction, adopted by the majority of the papers on the subject, follows the approach of ref.[3]: the semi-hard component of the inelastic cross section, in a collision of two nuclei with atomic mass numbers A and B, is written as:

$$\begin{aligned} \sigma_H &= \int d^2\beta \left(1 - e^{-\sigma_J T_{AB}(\beta)}\right) \\ &= \sum_{n=1}^{\infty} \int d^2\beta \, \frac{(\sigma_J T_{AB}(\beta))^n}{n!} \, e^{-\sigma_J T_{AB}(\beta)} \; , \end{aligned} \tag{1.1}$$

where

$$\begin{aligned} \sigma_J &= \int_{xx's>4p_0^2} dx dx' \sum_{ff'} G_A^f(x) \sigma^{ff'}(xx') G_B^{f'}(x') \\ T_{AB}(\beta) &= \int d^2b \, \tau_A(b) \tau_B(b-\beta) \; . \end{aligned} \tag{1.2}$$

In Eq.(1.2) $\tau_A(b)$ and $\tau_B(b-\beta)$ are the thickness functions of the two interacting nuclei, normalized to one and depending on the transverse coordinates of the interacting partons b and $b-\beta$, where β is the impact parameter of the nuclear collision. σ_J is the single scattering cross section to produce jets, expressed as a convolution of the nuclear parton distributions $G_A^f(x)$, $G_B^{f'}(x')$ and of the partonic cross section $\sigma^{ff'}$. The indices f and f' label the different kinds of interacting partons and the momentum fractions x, x' are defined with respect to the single nucleon momentum. The partonic cross section is integrated on the momentum transfer with the cutoff p_0, which sets also the lower limits of the integrals on x and x'. The dependence on the scale factor is implicit in all quantities.

The semihard cross section, as expressed in Eq.(1.1), represents the contribution to the total inelastic cross section of all events with at least one semi-hard parton interaction, so that one may write $\sigma_{inel} = \sigma_H + \sigma_{soft}$, the two components being separated by means of p_0. Eq.(1.1) may be derived in an eikonal approach to nucleus-nucleus interactions, by writing the eikonal phase as the sum of a soft and of a hard component [1, 4]. The physical picture corresponding to Eq.(1.1) is that of a distribution of multiple independent parton collisions localized in different points in transverse space and with the average number depending on the nuclear impact parameter. The average number of parton interactions, at fixed β, is $\langle N(\beta)\rangle = \sigma_J T_{AB}(\beta)$ so that, if σ_J is at the lowest order in α_S, the average multiplicity of minijets in a given nuclear collision (that is at β fixed) is $2\langle N(\beta)\rangle$. The integrated inclusive cross section for producing minijets is therefore given by $2\int d^2\beta \langle N(\beta)\rangle$. To obtain the inclusive cross section one, in fact, needs to count the jets produced with their multiplicity, so that, in the case of multiple parton interactions, the inclusive cross section is obtained by taking the average of the distribution in the number of parton collisions. The result is that the integrated inclusive cross section is given by the single scattering expression, σ_J, multiplied by the average multiplicity of jets produced in a single partonic interaction, which shows that the description of the process, as given by Eq.(1.1), is consistent with the AGK cancellation [5] (all unitarity corrections cancel in the inclusive). The cancellation property obviously holds for all averages, that are therefore equal to the result obtained by means of the single scattering expression, so that the transverse energy produced by minijets is given by

$$\langle E_t(\beta)\rangle = 2\,T_{AB}(\beta)\int_{p_t\geq p_0} p_t\frac{d\sigma_J}{d^2p_t}\,d^2p_t\ . \tag{1.3}$$

The semi-hard cross section σ_H is a smooth function of p_0 for small values of the cut-off: the limiting value of σ_H is in fact the geometrical limit $\pi(R_A+R_B)^2$, R_A and R_B being the two nuclear radii. On the contrary $\langle N(\beta)\rangle$ and $\langle E_t(\beta)\rangle$ are singular at small p_0 and their behavior may be roughly estimated on dimensional grounds to be $\langle N(\beta)\rangle \simeq 1/p_0^2$ and $\langle E_t(\beta)\rangle \simeq 1/p_0$. The singular behavior at low p_0 can be used to set the limits of validity of the picture. Indeed, for the picture of the interaction to be valid one should take a realtively large value of the cut-off p_0; in this way the whole semihard interaction takes place in a relatively dilute system and the overall number of interactions will be relatively small. To deal with a regime where the number of parton interactions and the density of the interacting partons are large, the main modification adopted by the majority of papers is to include shadowing corrections in the nuclear parton distributions [6, 7]. In this way one obtains a substantial reduction of the number of projectile and target partons at low x and the picture can be extended to sizably lower values of the cutoff p_0. Even so, when p_0 is further reduced, one reaches the condition of a highly dense interacting system where the whole picture ceases to be valid, which sets the lower limit for a sensible choice of the cutoff p_0 [8, 9].
The overall resulting features are therefore that p_0 is different when varying the atomic mass number of the interacting nuclei and their energy, and that the distribution in the number of hard collisions at a fixed value of the impact parameter β is a Poissonian in the whole semi-hard regime so that all average quantities are computed, as above, with the single scattering expression.

The clean physical interpretation of the approach, which incorporates the geometrical features of the nuclear process, unitarity, the factorization of the hard component of the interaction and the AGK cancellation rule, justifies the great success of the picture. Still there are a few delicate points which deserve further investigation and where the description of the interaction might be improved.
The issue of the dense system is accounted for by introducing shadowing in the nuclear structure functions: when p_0 is reduced, the transverse size that, because of the indetermination principle, can be assigned to each colliding parton grows in such a way that all the transverse area of the nucleus is filled with partons. When measuring the nuclear structure function with some resolution p_0 in deep inelastic scattering, this regime corresponds to the conditions where the interaction of the virtual photon with the nucleus is almost certain, then the virtual $q\bar{q}$ pair may interact several times with the target nucleus. However the measured structure function is proportional to the total virtual $q\bar{q}$-pair nucleus cross section and therefore to the probability of having at least one interaction, which is the reason why it saturates when the number of interactions grows. In nuclear collisions, in the regime where the probability of a projectile parton to interact

with momentum transfer larger than p_0 is close to one, there is consequently also a large probability for it to interact more than once with momentum transfer larger than p_0. The effect of these semi-hard re-interactions may however not be always taken properly into account by using nuclear parton distributions with shadowing corrections in Eq.(1.2). These corrections are in fact the result of the presence of re-interactions on a well defined physical quantity: the total virtual-photon nucleus cross section. When considering different physical observables, as the transverse energy produced in the nuclear collision, the effect of re-interactions may not be equivalent to a simple reduction of the incoming flux. Hence it could be worth trying to give a more detailed representation of the dynamics, by including explicitly semi-hard re-interactions in the collision process.

An attempt to introduce a more elaborate interaction dynamics which includes explicitly semi-hard parton rescatterings was done in ref. [10] and [11]. Both the average number of minijets and the average transverse energy are modified by semi-hard rescatterings and an interesting feature is that both quantities, develop a less singular dependence on the cutoff, in such a way that the choice of p_0 becomes less critical when semi-hard parton rescatterings are taken into account. The average number of minijets and the transverse energy produced in heavy ion collisions have been recently discussed in several papers, with the purpose to determine the initial conditions for the further evolution and termalization of the system (see e.g. [7, 12]). We think that it might be interesting to have an indication of the effects of rescatterings on these quantities at the LHC, and we'll discuss this topic in the present paper. In the next section, after including rescatterings in the picture of the interaction, we'll recall the expression of the average number of minijets and derive the corresponding average transverse energy. Then we'll give some quantitative indication on the effect and comment the qualitative features induced by the more structured interaction dynamics.

2 Multiple Parton Scatterings, Average Number of Minijets and Average Transverse Energy

The introduction of semi-hard parton rescatterings can be obtained in a picture of the interaction where the soft part is factorized in a Poissonian multi-parton distribution and the hard part is expressed in terms of perturbative parton-parton collisions. The nuclear multi-parton distributions are unknown quantities and the reason of the choice of the Poisson distribution is that it corresponds to the case where the information on the initial state is minimal, since the whole distribution is expressed in terms of its average value only. One has moreover the possibility of introducing systematically further informations on the nuclear partonic structure in terms of correlations among partons [13]. The hard part is written in terms of two-body collisions by introducing the probability of having at least one interaction between the two configuration, in a way analogous to the expression of the inelastic nucleus-nucleus cross section [14]. The process is therefore represented as the sum of all possible interactions between all configurations with a definite number of partons of the two nuclei. In this way the semi-hard component of the cross section can be written as

$$
\begin{aligned}
\sigma_H \quad = \quad & \int d^2\beta \sum_{n=1}^{\infty} \frac{1}{n!} \Gamma_A(x_1,b_1)\dots\Gamma_A(x_n,b_n)\ e^{-\int dx d^2b\, \Gamma_A(x,b)} \,. \\
& \cdot \sum_{l=1}^{\infty} \frac{1}{l!} \Gamma_B(x'_1,b'_1-\beta)\dots\Gamma_B(x'_l,b'_l-\beta)\ e^{-\int dx' d^2b' \Gamma_B(x',b')} \,. \\
& \cdot \Big[1-\prod_{i=1}^{n}\prod_{j=1}^{l}(1-\hat\sigma_{ij})\Big]\, dx_1 d^2b_1\dots dx_n d^2b_n dx'_1 d^2b'_1 \dots dx'_l d^2b'_l \ , \qquad (2.4)
\end{aligned}
$$

where

$$
\Gamma_A(x,b)=\tau_A(b)G(x) \ .
$$

To keep the notation as simple as possible, the indices labeling the different kinds of partons have been suppressed and the dependence on the cutoff p_0 is implicit. The two Poissonian distributions, with average numbers $\Gamma_A(x,b)$ and $\Gamma_B(x',b'-\beta)$, represent the multi-parton distributions of the two interacting nuclei. The probability to have at least one partonic interaction, given a configuration with n and l partons in the two nuclei, is represented by the square parenthesis in Eq.(2.4) and is constructed by means of the probability $\hat\sigma_{ij}$ of interaction of a given pair of partons i and j. Since the distance over which the hard interactions are localized is much smaller than the typical nuclear radius, the probability of interaction can be approximated by $\hat\sigma(x_ix_j, b_i-b_j)=\sigma(x_ix_j)\delta^{(2)}(b_i-b_j)$. The most important features of Eq.(2.4) are that all possible interactions between the two partonic configurations are included, and that probability conservation is explicitly taken into account by the term in square parentheses. The Eikonal cross-section in Eq.(1.1) is obtained when one neglects all rescatterings in Eq.(2.4), in such a way that each parton is allowed to interact at most once [15]. If, on the contrary, one keeps rescatterings into account one cannot write a closed expression for σ_H. However, it is possible to obtain simple expressions from Eq.(2.4) for many relevant quantities.
If one works out the average number of parton collisions $\langle N(\beta)\rangle$ one obtains, as in the traditional approach, the single scattering expression $\langle N(\beta)\rangle=\sigma_J T_{AB}(\beta)$ [10]. So the overall average number of parton collisions satisfies the AGK cancellation and is not affected by any unitarity correction; however it is not a simple quantity to observe. A more directly observable quantity, or at least one which can be more directly related to observable quantities, is the average number of produced minijets. An important effect of including rescatterings is that the number of produced minijets is no more proportional to the number of collisions, because now each projectile parton is allowed to interact more than once with the target. As a consequence, while unitarity corrections do not change the average number of collisions they affect the average number of minijets produced in the nuclear collision. The correction term can be derived in a straightforward way from Eq.(2.4) [10], but one may use also a more direct argument, as will be discussed in the following.

The single scattering cross section to produce jets at large p_t is written, in the perturbative QCD-parton model, as:

$$
\sigma_J=\int d^2\beta\, d^2b\, dx\, dx'\, \Gamma_A(x,b)\,\sigma(xx')\,\Gamma_B(x',b-\beta)=\int d^2\beta\,\langle N(\beta)\rangle \ . \qquad (2.5)
$$

This expression needs to be unitarized also when $\sigma(xx')$ is obtained by integrating on the transverse momenta with a rather large value of the cutoff p_0, since σ_J is proportional to the large factor $A \times B$. The cross section in Eq.(1.1) is in fact the result of the s-channel unitarization of σ_J. In Eq.(2.5) $\langle N(\beta)\rangle$ is dimensionless and may be understood as the average number of parton interactions at a given impact parameter β. One obtains the s-channel unitarized cross section σ_H after replacing the average number $\langle N(\beta)\rangle$ with the interaction probability, which, if the distribution in the number of interactions is a Poissonian, is just $1-\exp(-\langle N(\beta)\rangle)$. Hence the unitarized cross section σ_H represents the contribution to the total cross section of all events with at least one couple of partons interacting with a transverse momentum exchange above p_0, as it is clear from the second line of Eq.(1.1). On the other hand σ_J, that includes also the multiplicity of the interactions, represents rather the integrated inclusive cross section (apart from the factor representing the average multiplicity of jets produced in a single collision).
When the cutoff is moved towards low values and rescatterings need to be taken into account, the average number of jets produced is no more proportional to the average number of collisions. In this case one may proceed by applying to $\langle N(\beta)\rangle$ the argument previously used to unitarize σ_J. By looking at Eq.(2.5) we can identify

$$\langle n_B(x, b-\beta)\rangle = \int dx'\, \Gamma_B(x', b-\beta)\, \sigma(xx')$$

as the average number of collisions of each interacting parton. Then we can write the average number of produced minijets at fixed impact parameter as

$$2\langle N(\beta)\rangle \;\;=\;\; \int d^2b dx \Gamma_A(x,b)\, \langle n_B(x, b-\beta)\rangle + \int d^2b dx' \Gamma_B(x', b-\beta)\, \langle n_A(x', b)\rangle \;,$$

that is as the average number of incoming partons from the A nucleus times their average number of collisions against partons from the B nucleus, plus the analogous term with A and B interchanged. Then, if one replaces in Eq.(2.6) the average number of scatterings of each parton with its interaction probability, one obtains the average number of "wounded partons" $W_A(x,b)$ and $W_B(x', b-\beta)$ of the two nuclei. These ones are the partons of the two nuclei that had at least one hard interaction. The expression of the average number of wounded partons of nucleus A (with transverse coordinate b and momentum fraction x, in an event with nuclear impact parameter β) is therefore

$$W_A(b,x) = \Gamma_A(x,b)\Big[1 - \; e^{-\langle n_B(x, b-\beta)\rangle}\Big] \;. \tag{2.6}$$

Every wounded parton obtained in this way will produce a minijet in the final state and the transverse energy produced by semi-hard interactions is the transverse energy carried by the wounded partons. As a consequence both the average number of minijets and their average transverse energy are quantities affected by the presence of rescatterings and the corresponding correction term is more and more important when the average number of rescatterings $\langle n_B\rangle$ become larger and larger, namely at low values of the cutoff p_0 and (or) for large atomic mass numbers.
The overall number of produced minijets, i.e. the sum of the wounded partons of nucleus A with those of nucleus B, obviously coincides with the usual result $2\langle N(\beta)\rangle$ when the average number of rescatterings is small. When the number of rescatterings is large the

two quantities are however different and, while the average number of collisions $\langle N(\beta)\rangle$ may be divergent in the saturation limit, the average number of wounded partons is on the contrary well defined. In fact one obtains that the square parenthesis in Eq.(2.6) has 1 as a limiting value and in this limit the average number of wounded partons is just the sum of the average number of partons of the two interacting nuclei.

In summary, the effect of rescatterings on the average number of the produced minijets is to reduce the number obtained by means of the single scattering expression, not differently, qualitatively, from the result of including shadowing corrections in the nuclear parton distribution. On the other hand the overall distribution in the number of minijets produced is modified. In the traditional approach there is a strong correlation in the distribution of the number of minijets, since each minijet has a recoiling companion; when the average number of rescatterings increases this correlation gets weaker and weaker, so that in the high density limit no correlation is left and the distribution tends to a Poissonian [10]. A further important difference is that, since W_A and W_B are well defined in this limit, after including rescatterings the average number of minijets becomes much less dependent on the choice of the cutoff at low p_0.

The average number of minijets is not the only quantity modified by this more elaborate interaction dynamics, which in fact has a non-trivial effect also on the transverse energy carried by the minijets [11]. The wounded partons of nucleus A (at a given x and b and in a nuclear interaction with impact parameter β) are obtained by multiplying the average number of partons of A (with given x and b) by the corresponding interaction probability, which is a Poisson probability distribution in the number of scatterings, with average $\langle n_B(x,b-\beta)\rangle$. Therefore the differential distribution in the transverse momentum p_t of the wounded A-partons is

$$\begin{aligned}
\frac{dW_A(x,b)}{d^2p_t} \quad = \quad & \Gamma_A(x,b)\sum_{\nu=1}^{\infty}\frac{1}{\nu!}\int \Gamma_B(x_1',b_1)\ldots\Gamma_B(x_\nu',b_\nu)\,\cdot \\
& \cdot\, e^{-\int dx'\Gamma_B(x',b-\beta)\int d^2k\frac{d\sigma}{d^2k}}\,\cdot \\
& \cdot\,\frac{d\sigma}{d^2k_1}\cdots\frac{d\sigma}{d^2k_\nu}\,\delta(k_1+\ldots+k_\nu-p_t)\,d^2k_1\ldots d^2k_\nu\,dx_1\ldots dx_\nu\,. \qquad (2.7)
\end{aligned}$$

To obtain the corresponding average transverse energy $\langle p_t(\beta,b,x)\rangle_A$ one has to integrate Eq.(2.7) with an additional factor p_t. A convenient way to proceed is to introduce the Fourier transform of the parton-parton scattering cross section

$$\tilde{\sigma}(u)=\int d^2ke^{i\mathbf{k}\cdot\mathbf{u}}\frac{d\sigma}{d^2k}\quad ; \quad \tilde{\sigma}(0)=\sigma \qquad (2.8)$$

and to express the δ-function in Eq.(2.7) in an integral form. Since $\tilde{\sigma}(u)$ depends only on the modulus of $\mathbf{u}$ one obtains

$$\begin{aligned}
\langle p_t(\beta,b,x)\rangle_A \quad = \quad & \Gamma_A(x,b)\int_0^\infty dp_t\,p_t^2\int_0^\infty du\,uJ_0(p_tu)\,\cdot \qquad (2.9) \\
& \cdot\sum_{\nu=1}^{\infty}\frac{1}{\nu!}\left[\int dx'\Gamma_B(x',b-\beta)\tilde{\sigma}(u)\right]^\nu e^{-\int dx'\Gamma_B(x',b-\beta)\tilde{\sigma}(0)}\,.
\end{aligned}$$

After multiplying the integrand by the exponential factor $\exp(-\lambda p_t)$ one can exchange the two integrals and first do the integral on p_t. The result is

$$\int_0^\infty e^{-\lambda p_t} J_0(p_t u) p_t^2 dp_t = \frac{2\lambda^2 - u^2}{(\lambda^2+u^2)^{5/2}} = \frac{1}{u}\frac{d}{du}\left[\frac{\lambda^2 u^2 + u^4}{(\lambda^2+u^2)^{5/2}}\right] , \tag{2.10}$$

which allows one to perform the integral on u by parts. After differentiating the series in ν one obtains the factor

$$\tilde{\sigma}'(u) = -2\pi \int_0^\infty k^2 J_1(ku) \frac{d\sigma}{d^2k}\, dk , \tag{2.11}$$

which is proportional to u because of the argument of the J_1 Bessel function. The integrand in u is therefore regular for $u=0$ also in the limit $\lambda \to 0$. Then one takes this limit and sums the series in ν. The final expression has a closed analytical form:

$$\langle p_t(\beta, b, x)\rangle = -\,\Gamma_A(x,b)\int_0^\infty du\, dx' \frac{\tilde{\sigma}'(u)}{u}\Gamma_B(x', b-\beta)\, e^{\int dx'\Gamma_B(x', b-\beta)[\tilde{\sigma}(u) - \tilde{\sigma}(0)]} \tag{2.12}$$

The average transverse energy in an event with a given impact parameter β is the result of integrating Eq.(2.12) on b and x and of summing the two contributions of the wounded partons of the nuclei A and B. Notice that the expression in Eq.(2.12) is much less dependent on the choice of the cutoff p_0 than the usual average energy evaluated with the single scattering expression of the perturbative QCD-parton model: the Rutherford singularity of the parton-parton cross section is in fact smoothed in Eq.(2.11) by the Bessel function $J_1(ku)$, in such a way that the dependence on the cutoff p_0 is only logarithmic. The same logarithmic dependence on the cutoff is present in the argument of the exponential

$$\tilde{\sigma}(u) - \tilde{\sigma}(0) = 2\pi \int \Big[J_0(ku) - 1\Big] \frac{d\sigma}{d^2k}\, k\, dk \tag{2.13}$$

as a consequence of the behavior of $[J_0(ku) - 1]$ for $k \to 0$.

In summary, when the semi-hard cross section is expressed by Eq.(2.4) one may obtain, without further approximations, a closed analytic form both for the average number of minijets and for the corresponding average transverse energy and in both cases the singular dependence for small values of the cutoff is smoothed by rescatterings.

3 Concluding Discussion

Many papers have been recently devoted to the production of minijets in heavy ion collisions [2, 7, 12, 16, 9]. A rather general feature is that, because of the singular behavior of the elementary parton interaction at low momentum transfer, many relevant quantities depend rather strongly (typically like an inverse power) on the value of the cutoff which distinguishes soft and hard parton interactions. The feature is unpleasant since although one might find physical arguments to determine a meaningful value of the cutoff [8, 9, 18], it is rather difficult to fix it in a very precise way. We have therefore tried, in the present paper, to face this issue by studying the effect of a more elaborate interaction dynamics on the average number of minijets produced in a nuclear collision and on the corresponding average transverse energy. While in the traditional picture of the semi-hard processes each parton is allowed to interact with large momentum transfer only once, we have included semi-hard parton rescatterings in the dynamics of the interaction. Semi-hard rescatterings, that are negligible when the threshold between hard and soft processes is high, become more and more important when the threshold is lowered. Naively one would expect that the inclusion of rescatterings in the picture of the interaction might worsen the divergent behavior at low transverse momenta; on the contrary a more careful analysis, that takes probability conservation consistently into account, shows that the result is just the opposite. Following [10] and [11] we have in fact represented the semi-hard nuclear cross section with Eq.(2.4), where all possible multi-parton collisions, including rescatterings, are taken into account and the conservation of probability is explicitly implemented. The average number of minijets and the corresponding transverse energy, at fixed x, b and impact parameter β, are then expressed in a closed analytic form by (2.6) and (2.12), whose behavior with the cutoff is much less singular in comparison with the analogous averages obtained without taking rescatterings into account. The reason of this smoother behavior is that rescatterings introduce (through Γ_A and Γ_B) a new dimensional quantity in (2.6) and (2.12), the nuclear radius, which gives the dimensionality to the two average quantities at small p_0. When rescatterings are neglected the dimensionality at small p_0 is provided by the cutoff itself, and the result is that the two quantities behave as an inverse power of the cutoff for $p_0 \to 0$.

The effect of semi-hard parton rescatterings on the average number of minijets and on the average transverse energy produced in a central Pb-Pb collision at LHC energies is summarized in Fig.1. We plotted the average number of minijets and their transverse energy in the case of a very central rapidity window, $|y| < .9$, corresponding to the ALICE detector, and in a larger rapidity interval, $|y| < 4$, that will be covered by the CMS detector. Figure 1a) shows, in the two cases, the dependence of the average number of produced minijets on the choice of the cutoff p_0. The dashed curves are the results obtained by the single scattering expression, Eq.(2.5), while the continuous curves are the result of the inclusion of semi-hard parton rescatterings, Eq.(2.6) plus the analogous term for the B-partons. These expressions have been computed with the GRV98LO distribution functions [19] with no shadowing corrections included, and by representing the elementary partonic interaction at the lowest order in QCD; to account for higher order corrections the result of the elementary interaction has been multiplied by a factor $k = 2.5$. Both the k-factor and the scale $Q = p_0/2$ where chosen in order to reproduce the value of the $p\bar{p}$ mini-jet cross-section at $\sqrt{s} = 900$ GeV [20]. The sensitivity to the value of the k-factor is shown in Fig. 1b), where the curves have the same meaning as in Fig. 1a), and the cutoff has been fixed to the value $p_0 = 2$ GeV. Analogous curves for the average transverse energy carried by the produced minijets are shown in Fig. 1c) and 1d). The average transverse energy without rescatterings has been computed by using Eq.(1.3) (dashed curves) and with rescatterings (continuous curves) by using Eq.(2.12), after integrating on b and on x (inside the corresponding rapidity windows) and adding the analogous contribution of the B-partons.

In conclusion, the main features are that semi-hard rescatterings have a sizable effect on the average number of minijets and on the transverse energy produced in heavy ion collisions at the LHC, so that semi-hard rescatterings affect also global characteristics of the typical inelastic event. The induced correction increases with the value of the k-

factor, which represents higher orders in the elementary parton collision, and with the size of the rapidity window, since rescatterings are more frequent for partons with a larger momentum fraction. By looking at the dependence on the cutoff, both the average number of minijets and the corresponding average transverse energy are more regular at low p_0, showing a tendency to saturate below $p_0 \simeq 3$ GeV and making in this way the choice of the cutoff less critical.

Acknowledgment

This work was partially supported by the Italian Ministry of University and of Scientific and Technological Researches (MURST) by the Grant COFIN99.

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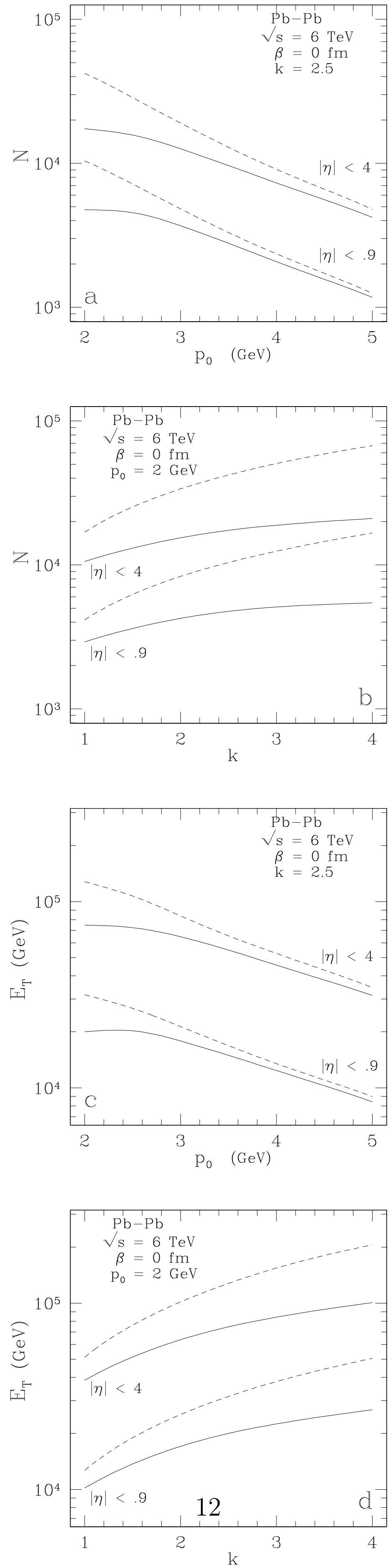

Figure 1: Average number N of minijets, *a*) *b*), and average transverse energy E_T, *c*) *d*), in a Pb-Pb central collision in two different rapidity windows, $|y| < 4$ and $|y| < .9$, as a function of the cutoff, *a*) and *c*), and of the k-factor, *b*) and *d*). The dashed curves are computed without including rescatterings, the