

Testing A (Stringy) Model of Quantum Gravity

Nick E. Mavromatos

Department of Physics, Theoretical Physics, King's College London, Strand, London WC2R 2LS, United Kingdom, and
CERN, Theory Division, CH-1211 Geneva 23, Switzerland

Abstract. I discuss a specific model of space-time foam, inspired by the modern non-perturbative approach to string theory (D-branes). The model views our world as a three brane, intersecting with D-particles that represent stringy quantum gravity effects, which can be real or virtual. In this picture, matter is represented generically by (closed or open) strings on the D3 brane propagating in such a background. Scattering of the (matter) strings off the D-particles causes recoil of the latter, which in turn results in a distortion of the surrounding space-time fluid and the formation of (microscopic, i.e. Planckian size) horizons around the defects. As a mean-field result, the dispersion relation of the various particle excitations is modified, leading to non-trivial optical properties of the space time, for instance a non-trivial refractive index for the case of photons or other massless probes. Such models make falsifiable predictions, that may be tested experimentally in the foreseeable future. I describe a few such tests, ranging from observations of light from distant gamma-ray-bursters and ultra high energy cosmic rays, to tests using gravity-wave interferometric devices and terrestrial particle physics experiments involving, for instance, neutral kaons.

1 Introduction

The theory of Quantum Gravity still eludes us, despite considerable efforts of theorists for more than fifty years. This may be partly due to the fact that the theory by its very nature is associated with the structure of space and time, and as such it may be completely different from theories that describe the rest of the (known) fundamental interactions in Nature. In fact, many of the properties that characterize the known field theories, such as locality, renormalizability, unitarity etc, may not be features of a complete theory of quantum gravity. Even symmetries, such as Lorentz invariance, may not be exact at Planck length scales, $\ell_P \sim 10^{-35}$ m, which is the characteristic scale at which quantum gravity effects are expected to set in.

At present, there is no complete mathematical model for quantum gravity. However, there is a considerable number of attempts, which may be classified, roughly, into three major categories. The first is the *canonical approach*, where one tries to formulate the model in a *background independent* way, i.e. to give space time at Planckian scales a polymer-like structure, similar to ‘spin networks’, and from these (rather abstract) building blocks to construct the interactions [1] and the observable universe. The second approach is the one in which the quantum gravitational interactions are represented as a ‘stochastic medium’, which gives space time non-trivial optical properties (‘space-time foam’) [2]. This

approach, which is more phenomenological than the first one, is based on the expectation that any consistent field theory of quantum gravity should involve microscopic event horizons (foam), surrounding black holes or other singularities of space time. The latter are classical solutions of Einstein theory (or its extensions), and therefore such configurations should also represent quantum fluctuations, which should be part of the (still unknown) complete integration measure of the gravitational path integral. The third approach, and so far the most developed formally, is string theory and its modern non-perturbative extension (D-branes) [3,4]. The discovery of D branes has revolutionised the study of black-hole physics. Now one has quasi-realistic string models of black holes in different dimensions, which one can use to study profound issues concerning the reconciliation of general relativity and quantum mechanics. A key breakthrough was the demonstration that the entropy of a stringy black hole corresponds to the number of its distinct quantum states [5]. Thus D branes offer the prospect of accounting exactly for the flow of information in processes involving particles and black holes.

At first instance, the string approach to quantum gravity may seem to have overcome the loss of unitarity that is believed to characterise quantum gravitational interactions in the second ('space-time foam') approach. However it is not immediately apparent that an observer in string theory will not perceive loss of information in any given particle/D-brane interaction: the answer depends whether she/he is able to recover all the information transferred from the scattering particle to the recoiling black hole. It is important to address this issue at both the macroscopic and microscopic levels, where the answers may differ. In the case of a macroscopic black hole, it is difficult to see how *in practice* all the quantum information may be recovered without a complete set of observations of the emitted Hawking radiation [6]. However, even if this is possible *in principle*, the problem of the microscopic 'end-game' that terminates the Hawking evaporation process is unsolved, in our view. In this sense, the string theory approach to quantum gravity may lead to effective stochastic models. However, such stochastic models usually employ *non-equilibrium* physics, and as such cannot be described by critical string theory. Indeed, as we shall discuss below, it is our belief that proper quantum gravitational interactions in string theory involve at a certain stage departure from equilibrium, which, in terms of string-theory nomenclature, implies [7] the inclusion of non-critical (Liouville) strings [8,9]. The latter quantify the process of information loss in a mathematically consistent way, as we shall discuss in the next section.

It may be useful for what follows to recall one of the intuitive ways of formulating the information loss in the process of Hawking radiation from a macroscopic black hole, whose stringy analogue we discuss in this talk. Consider the quantum-mechanical creation of a pure-state particle pair $|A, B\rangle$ close to the (classical) black-hole horizon of such a macroscopic black hole. One can then envisage that particle B falls inside this horizon, whilst particle A escapes as Hawking radiation. The quantum state of the particle B is apparently unobservable, and hence information is apparently lost.

This argument is very naive, and one would like to formulate a more precise treatment of this process at the microscopic level, suitable for describing space-time foam [2]. The purpose of this talk is to review such a specific stringy treatment [10–12] of the interaction between closed-string particle ‘probes’ and D-brane black holes (defects). We have developed an approach capable of accommodating the recoil of a D-brane black hole struck by a closed-string ‘probe’, including also quantum effects associated with higher-genus contributions to the string path integral. We have shown explicitly [13,11] how the loss of information to the recoiling D brane (assuming that it is unobserved) leads to information loss, for both the scattered particle and also any spectator particle. This information loss can be related to a change in the background metric following the scattering event, which can be regarded as creating an Unruh-like ‘thermal’ state.

In a recent paper [14], which we shall review in the next section, we took this line of argument a step further, by demonstrating that closed-string particle/D-brane scattering leads in general to the formation of a microscopic event horizon, within which string particles may be trapped. The scattering event causes expansion of this horizon, which is eventually halted and reversed by Hawking radiation [6]. Thus we have a microscopic stringy realization of this process. A peculiarity of this approach is that the conformal invariance conditions select preferentially backgrounds with three spatial dimensions. This leads to a consistent formulation of the interaction of D3 branes with recoiling D particles, which are allowed to fluctuate independently only on the D3-brane hypersurface. Some physical consequences of the model and their (possible) experimental tests, which could be generic to other models of space-time foam, will be also reviewed in this talk.

2 A Stringy Model of Space-Time Foam

In this section we shall review the basic features of a theoretical model of space-time foam, proposed in [14], which is based on a modern version of non-critical

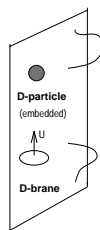


Fig. 1. The world as a D3 brane ‘punctured’ by D particles. The scattering on the D-particles of string states, either closed (gravitons) or open (matter fields) that live on the D3 brane, cause the D-particle to recoil, leading to stochastic effects in the propagation of the low-energy states, as well as to non-zero ‘vacuum’ energy on the D3 brane.

string theory. According to this model, our world is viewed as a fluctuating D(irichlet) 3-brane, embedded in a higher-dimensional (bulk) space time (see fig. 1). For formal reasons, we start with a Euclideanized $D4$ brane and follow the procedure of identifying the Liouville mode (arising from recoil) with the (Euclidean) time coordinate X^0 . As discussed in [14] this procedure results eventually in a Minkowskian signature. The D3 brane is punctured by D-particles, that represent defects in the fabric of the D3 space time, and thus can be viewed as genuine quantum-gravity singular effects (c.f. analogy with microscopic black holes). The foamy structure of space time is caused as a consequence of the distortion of the space-time fluid surrounding the D-particle, which occurs due to the recoil of the D-particle as a result of the scattering of matter strings off it. The recoil of the (massive, hyper-Planckian) defect causes the formation of microscopic horizons, which surround the defect, trapping part of the scattered matter in the interior.

In the language of (perturbative) world-sheet (σ -model) string theory, which suffices to describe the recoil effects to leading order, target-space quantum fluctuations are incorporated by appropriate summation over world-sheet topologies [12]. Such higher-genus effects lead to an oscillating behaviour of the horizon, characterized by initial expansion, stasis, and shrinking, which in physical terms may be interpreted as a result of a phenomenon analogous to Hawking radiation in conventional field theory [6]. The above situation involves a single-scattering event. However, conceptually one may think of a statistical ensemble of (virtual) defects, whose scattering with matter strings will create analogous phenomena involving a statistical distribution of dynamical horizons. This is the stringy picture of space time foam, whose consequences we shall explore in this talk.

2.1 Formulation of D-Brane Recoil

We now proceed to review briefly the mathematical formalism underlying the above model. As discussed in references [10–12], the recoil of a D-brane string soliton after interaction with a closed-string state is characterized by a σ model on the string world sheet Σ , that is deformed by a pair of logarithmic operators [15]:

$$C_\epsilon^I = \epsilon \Theta_\epsilon(X^I), \quad D_\epsilon^I = X^I \Theta_\epsilon(X^I), \quad I \in \{0, \dots, 3\} \quad (1)$$

defined on the boundary $\partial\Sigma$ of the string world sheet. Here $X^I, I \in \{0, \dots, p\}$ obey Neumann boundary conditions on Σ , and denote the D-brane coordinates, whilst $\epsilon \rightarrow 0^+$ is a regulating parameter and $\Theta_\epsilon(X^I)$ is a regularized Heaviside step function. The remaining $y^i, i \in \{p+1, \dots, 9\}$ in (1) denote the transverse bulk directions (c.f. fig. 1). For reasons of convergence of the world-sheet path integrals we take the space-time $\{X^I, y^i\}$ to have *Euclidean* signature.

In the case of D particles [10–12], the index I takes the value 0 only, in which case the operators (1) act as deformations of the conformal field theory on the world sheet. The operator $u_i \int_{\partial\Sigma} \partial_n X^i D_\epsilon$ describes the movement of the D brane

induced by the scattering, where u_i is its recoil velocity, and $y_i \int_{\partial\Sigma} \partial_n X^i C_\epsilon$ describes quantum fluctuations in the initial position y_i of the D particle. It has been shown rigorously [12] that the logarithmic conformal algebra ensures energy–momentum conservation during the recoil process: $u_i = (k_i^1 + k_i^2)/M_D$, where $k^1(k^2)$ is the momentum of the propagating closed string state before (after) the recoil, and $M_D = 1/(\ell_s g_s)$ is the mass of the D brane, where g_s is the string coupling, which is assumed here to be weak enough to ensure that the D brane is very massive, and ℓ_s is the string length.

In order to realize the logarithmic algebra between the operators C and D (1), one uses as a regulating parameter [10]

$$\epsilon^{-2} \sim \ln[L/a] \equiv A, \quad (2)$$

where L (a) is an infrared (ultraviolet) world–sheet cutoff. The recoil operators (1) are relevant, in the sense of the renormalization group for the world–sheet field theory, having small conformal dimensions $\Delta_\epsilon = -\epsilon^2/2$. Thus the σ -model perturbed by these operators is not conformal for $\epsilon \neq 0$, and the theory requires Liouville dressing [8,9,11]. The consistency of this approach is supported by the above-mentioned proof of momentum conservation during the scattering process [12].

As discussed in [11,13], the recoil deformations create a local distortion of the space-time surrounding the recoiling D brane, which may also be determined using the method of Liouville dressing. In [11,13] we concentrated on describing the resulting space-time in the case when a D-particle defect embedded in a D -dimensional space-time recoils after the scattering of a closed string. To leading order in the recoil velocity u_i of the D particle, the resulting space-time was found, for times $t \gg 0$ long after the scattering event at $t = 0$, to be equivalent to a Rindler wedge, with apparent ‘acceleration’ ϵu_i [13], where ϵ is defined above (2). For times $t < 0$, the space-time is flat Minkowski. There is hence a discontinuity at $t = 0$, which leads to particle production and decoherence for a low-energy spectator field theory observer who performs local scattering experiments long after the scattering, and far away from the location of the collision of the closed string with the D particle [13].

This situation is easily generalized to Dp branes [16]. The folding/recoil deformations of the Dp brane are relevant deformations, with anomalous dimension $-\epsilon^2/2$, which disturbs the conformal invariance of the world-sheet σ model, and restoration of conformal invariance again requires Liouville dressing [8,9,11], as discussed above. To determine the effect of such dressing on the space-time geometry, it is essential to write [11] the boundary recoil deformations as bulk world-sheet deformations

$$\int_{\partial\Sigma} \bar{g}_{Iz} x \Theta_\epsilon(x) \partial_n z = \int_\Sigma \partial_\alpha (\bar{g}_{Iz} x \Theta_\epsilon(x) \partial^\alpha z) \quad (3)$$

where the \bar{g}_{Iz} denote renormalized folding/recoil couplings [12]. Such couplings are marginal on a flat world sheet, and the operators (3) are marginal also on a curved world sheet, provided [9] one dresses the (bulk) integrand by multiplying

it by a factor $e^{\alpha_{Ii}\phi}$, where ϕ is the Liouville field and α_{Ii} is the gravitational conformal dimension. This is related to the flat-world-sheet anomalous dimension $-\epsilon^2/2$ of the recoil operator, viewed as a bulk world-sheet deformation by [9]: $\alpha_{Ii} = -\frac{Q_b}{2} + \sqrt{\frac{Q_b^2}{4} + \frac{\epsilon^2}{2}}$, where Q_b is the central-charge deficit of the bulk world-sheet theory. In the recoil problem at hand, as discussed in [13], $Q_b^2 \sim \epsilon^4/g_s^2 > 0$, for weak folding deformations g_{Ii} , and hence one is confronted with a *supercritical* Liouville theory. This implies a *Minkowskian-signature* Liouville-field kinetic term in the respective σ model [17], which prompts one to interpret the Liouville field as a time-like target field.

There are two approaches which one can follow. In the first of them [18], this time is considered as a *second* time coordinate [7], which is independent of the (Euclideanized) X^0 . The presence of this second ‘time’ does not affect physical observables, which are defined for appropriate slices with fixed Liouville coordinate, e.g., $\phi \rightarrow \infty$ or equivalently $\epsilon \rightarrow 0$. From the expression for Q we conclude that $\alpha_{Ii} \sim \epsilon$, to leading order in perturbation theory in ϵ , to which we restrict ourselves here. In the second approach [7], which we shall mainly follow here, the (Minkowskian) Liouville field ϕ is identified with the (initially Euclidean) coordinate X^0 , and hence one is no longer considering constant Liouville field slices. In this approach, however, one still identifies ϵ^{-2} with the target time, which in turn implies that the perturbative world-sheet approach is valid, provided one works with sufficiently large times t , i.e. small ϵ^2 .

The X^I -dependent field operators $\Theta_\epsilon(X^I)$ scale with ϵ as [11]: $\Theta_\epsilon(X^I) \sim e^{-\epsilon X^I} \Theta(X^I)$, where $\Theta(X^I)$ is a Heaviside step function without any field content, evaluated in the limit $\epsilon \rightarrow 0^+$. The bulk deformations, therefore, yield the following σ -model terms:

$$\frac{1}{4\pi\ell_s^2} \int_\Sigma \sum_{I=0}^3 (\epsilon^2 \bar{g}_{Ii}^C + \epsilon \bar{g}_{Ii} X^I) e^{\epsilon(\phi_{(0)} - X_{(0)}^I)} \Theta(X_{(0)}^I) \partial_\alpha \phi \partial^\alpha y_i \quad (4)$$

where the subscripts (0) denote world-sheet zero modes, and $\bar{g}_{0i}^C = y_i$.

Upon the interpretation of the Liouville zero mode $\phi_{(0)}$ as a (second) time-like coordinate, the deformations (4) yield metric deformations of the generalized space-time with two times. The metric components for fixed Liouville-time slices can be interpreted [11] as expressing the distortion of the space-time surrounding the recoiling D-brane soliton. For clarity, we now drop the subscripts (0) for the rest of this paper, and we work in a region of space-time such that $\epsilon(\phi - X^I)$ is finite in the limit $\epsilon \rightarrow 0^+$. The resulting space-time distortion is therefore described by the metric elements

$$\begin{aligned} G_{\phi\phi} &= -1, & G_{ij} &= \delta_{ij}, & G_{IJ} &= \delta_{IJ}, & G_{iI} &= 0, \\ G_{\phi i} &= (\epsilon^2 \bar{g}_{Ii}^C + \epsilon \bar{g}_{Ii} X^I) \Theta(X^I), & i &= 4, \dots, 9, & I &= 0, \dots, 3 \end{aligned} \quad (5)$$

where the index ϕ denotes Liouville ‘time’, not to be confused with the Euclideanized time which is one of the X^I . To leading order in $\epsilon \bar{g}_{Ii}$, we may ignore the $\epsilon^2 \bar{g}_{Ii}^C$ term. The presence of $\Theta(X^I)$ functions and the fact that we are working

in the region $y_i > 0$ indicate that the induced space-time is piecewise continuous (the important implications for non-thermal particle production and decoherence for a spectator low-energy field theory in such space-times were discussed in [13,11], where the D-particle recoil case was considered).

We next study in more detail some physical aspects of the metric (5), restricting ourselves, for simplicity, to the case of a single Dirichlet dimension z that plays the rôle of a bulk dimension in a set up where there are Neumann coordinates X^I , $I = 0, \dots, 3$ parametrizing a D4 (Euclidean) brane, interpreted as our four-dimensional space-time. Upon performing the time transformation $\phi \rightarrow \phi - \frac{1}{2}\epsilon\bar{g}_{Iz}X^I z$, the line element (5) becomes:

$$ds^2 = -d\phi^2 + \left(\delta_{IJ} - \frac{1}{4}\epsilon^2\bar{g}_{Iz}\bar{g}_{Jz} z^2 \right) dX^I dX^J + \left(1 + \frac{1}{4}\epsilon^2\bar{g}_{Iz}\bar{g}_{Jz} X^I X^J \right) dz^2 - \epsilon\bar{g}_{Iz} z dX^I d\phi, \quad (6)$$

where ϕ is the Liouville field which, we remind the reader, has Minkowskian signature in the case of supercritical strings that we are dealing with here. One may now make a general coordinate transformation on the brane X^I that diagonalizes the pertinent induced-metric elements in (6) (note that general coordinate invariance is assumed to be a good symmetry on the brane, away from the ‘boundary’ $X^I = 0$). The so-diagonalized metric becomes [18,14]

$$ds^2 = -d\phi^2 + (1 - \alpha^2 z^2) (dX^I)^2 + (1 + \alpha^2 (X^I)^2) dz^2 - \epsilon\bar{g}_{Iz} z dX^I d\phi, \quad \alpha = \frac{1}{2}\epsilon\bar{g}_{Iz} \sim g_s |\Delta P_z| / M_s \quad (7)$$

where the last expression is a reminder that one can express the parameter α (in the limit $\epsilon \rightarrow 0^+$) in terms of the (recoil) momentum transfer ΔP_z along the bulk direction.

A last comment, which is important for our purposes here, concerns the case in which the metric (7) is *exact*, i.e., it holds to all orders in $\bar{g}_{Iz}z$. This is the case where there is no world-sheet tree-level momentum transfer. This naively corresponds to the case of static intersecting branes. However, the whole philosophy of recoil [10,12] implies that, even in that case, there are quantum fluctuations induced by the sum over genera of the world sheet. The latter implies the existence of a statistical distribution of logarithmic deformation couplings of Gaussian type about a mean-field value $\bar{g}_{Iz}^C = 0$. Physically, the couplings \bar{g}_{Iz} represent recoil velocities of the intersecting branes, hence these Gaussian fluctuations represent the effects of quantum fluctuations about the zero recoil-velocity case, which may be considered as quantum corrections to the static intersecting-brane case. We therefore consider a statistical average $\langle\langle \dots \rangle\rangle$ of the line element (6) where $\langle\langle \dots \rangle\rangle = \int_{-\infty}^{+\infty} d\bar{g}_{Iz} (\sqrt{\pi}\Gamma)^{-1} e^{-\bar{g}_{Iz}^2/\Gamma^2} (\dots)$, and the width Γ is found [12] after summation over world-sheet genera to be proportional to the string coupling g_s . In fact, it can be shown [12] that Γ scales as $\epsilon\bar{\Gamma}$, where $\bar{\Gamma}$ is independent of ϵ .

Assuming that $g_{Iz} = \mathcal{O}(|u_i|)$ where $u_i = g_s \Delta P_i / M_s$ is the recoil velocity [10,12], we see that the average line element ds^2 becomes:

$$\begin{aligned} \langle\langle ds^2 \rangle\rangle &= -d\phi^2 + (1 - \alpha^2 z^2) (dX^I)^2 + (1 + \alpha^2 (X^I)^2) dz^2, \\ \alpha &= \frac{1}{2\sqrt{2}} \epsilon^2 \bar{T} \end{aligned} \quad (8)$$

Thus the average over quantum fluctuations leads to a metric of the form (7), but with a parameter α determined by the width (uncertainty) of the pertinent quantum fluctuations [12].

An important feature of the line elements (7) and (8) is the existence of a *horizon* at $z = 1/\alpha$ for *Euclidean* Neumann coordinates X^I . Since the Liouville field ϕ has decoupled after the averaging procedure, one may consider slices of this field, defined by $\phi = \text{const}$, on which the physics of the observable world can be studied [18]. From a world-sheet renormalization-group view point this slicing procedure corresponds to selecting a specific point in the non-critical-string theory space. Usually, the infrared fixed point $\phi \rightarrow \infty$ is selected. In that case one considers (2) a slice for which $\epsilon^2 \rightarrow 0$. But any other choice could do, so α may be considered a small but arbitrary parameter of our effective theory. The presence of a horizon raises the issue of how one could analytically continue so as to pass to the space beyond the horizon. The simplest way, compatible, as we discussed in [18], with the low-energy Einstein's equations, is to take the absolute value of $1 - \alpha^2 z^2$ in the metric element (7) and/or (8).

We now pass onto the second approach [7], in which one identifies the Liouville mode ϕ with the time coordinate X^0 on the initial Dp brane. In this case, as we shall see, the situation becomes much more interesting, at least in certain regions of the bulk space time, where one can calculate reliably in a world-sheet perturbative approach. Indeed, far away from the horizon at $|z| = 1/\alpha$, i.e., for $\alpha^2 z^2 \ll 1$, the line element corresponding to the space-time (8) after the identification $\phi = X^0$ becomes:

$$ds^2 \simeq -\alpha^2 z^2 (dX^0)^2 + dz^2 + \sum_{i=1}^3 (dX^i)^2 \quad (9)$$

implying that X^0 plays now the rôle of a *Minkowskian*-signature temporal variable, despite its original Euclidean nature. This is a result of the identification $\phi = X^0$, and the fact that ϕ appeared with Minkowskian signature due to the supercriticality ($Q^2 > 0$) of the Liouville string under consideration.

Notice that the space time (9) is flat, and hence it satisfies Einstein's equations, formally. However, the space time (9) has a *conical* singularity when one compactifies the time variable X^0 on a circle of finite radius corresponding to an inverse 'temperature' β . Formally, this requires a Wick rotation $X^0 \rightarrow iX^0$ and then compactification, $iX^0 = \beta e^{i\theta}$, $\theta \in (0, 2\pi]$. The space-time then becomes a *conical* space-time of Rindler type

$$ds_{conical}^2 = \frac{1}{4\pi^2} \alpha^2 \beta^2 z^2 (d\theta)^2 + dz^2 + \sum_{i=1}^3 (dX^i)^2 \quad (10)$$

with deficit angle $\delta \equiv 2\pi - \alpha\beta$. We recall that there is a ‘thermalization theorem’ for this space-time [19], in the sense that the deficit disappears and the space-time becomes regular, when the temperature is fixed to be

$$T = \alpha/2\pi \tag{11}$$

The result (11) may be understood physically by the fact that α is essentially related to recoil. As discussed in [13], the problem of considering a suddenly fluctuating (or recoiling) brane at $X^0 = 0$, as in our case above, becomes equivalent to that of an observer in a (non-uniformly) accelerated frame. At times long after the collision the acceleration becomes uniform and equals α . This implies the appearance of a non-trivial vacuum [19], characterized by thermal properties of the form (11). At such a temperature the vacuum becomes just the Minkowski vacuum, whilst the Unruh vacuum [19] corresponds to $\beta \rightarrow \infty$. Here we have derived this result in a different way than in [13], but the essential physics is the same.

2.2 D-Particle Recoil, Vacuum Energy and the Dimensionality of the Brane World

As we have seen above, the recoil of a D-particle in the situation of fig. 1 induced a non-trivial distortion of the D3 hypersurface. The distortion is such as to induce non-trivial contributions to the vacuum energy on the D3 brane, as discussed in detail in [20,21,14].

To see this, we recall that the four-dimensional space-time, in which the defect is embedded, is to be viewed as a bulk space-time from the point of view of the world-sheet approach to the recoil of the D particle. Following the same approach as that leading to (9), involving the identification of the Liouville field with the target time, t , one observes again that there exists an (expanding) horizon, located at

$$r_h^2 \equiv x_1^2 + x_2^2 + x_3^2 = t^2/b'^2 \tag{12}$$

where $\{x_i\}, i = 1, \dots, 3$ constitute the bulk dimensions, obeying Dirichlet boundary conditions on the world sheet, and b' is related to the momentum uncertainty of the fluctuating D particle. The variance b' was computed [12] using a world-sheet formalism resummed over pinched annuli, which has been argued to be the leading-order effect for weak string coupling g_s :

$$(b')^2 = 4 \frac{g_s^2}{\ell_s^2} \left(1 - \frac{285}{18} g_s^2 \frac{E_{kin}}{M_D c^2} \right) + \mathcal{O}(g_s^6) \tag{13}$$

where E_{kin} is the kinetic energy scale of the fluctuating (heavy) D particle, $M_D = g_s/\ell_s$ is the D-particle mass scale, and ℓ_s is the string length. Note the dependence of the variance b' on the string coupling g_s , which arises because quantum corrections come from the summation over world-sheet topologies [7,12], and g_s is a string-loop counting parameter.

For the region of space-time *inside the horizon* one obtains the following metric on the D3 brane, as a result of recoil of the D particle embedded in it:

$$ds^{2(4)} \simeq \frac{b'^2 r^2}{t^2} (dt)^2 - \sum_{i=1}^3 (dx^i)^2, \quad r^2 = \sum_{i=1}^3 x_i^2 < t^2/b'^2 \quad (14)$$

Note that the scalar curvature corresponding to the metric (14) has the form $R = -4/r^2$, and as such has a singularity at the initial location $r = 0$ of the D-particle defect, as expected. This metric is a solution of Einstein's equations in a four-dimensional space-time $\{x_i, t\}$, with a non-trivial "vacuum" energy Λ , provided there exists a four-dimensional dilaton field of the form:

$$\varphi = \ln r + b' \ln t \quad (15)$$

which has non-trivial potential $V(\varphi)$:

$$V(\varphi_c) = \frac{1}{r^2}, \quad \Lambda = \frac{1}{r^2} \quad (16)$$

Above we have ignored the fluctuations of the D3 brane in the bulk directions. When these are taken into account there may be additional contributions to the vacuum and excitation energies on the D3 brane, which in fact are time-dependent, relaxing to zero asymptotically, as discussed in [7,22].

From (16),(15) we observe that, in general, there is an explicit r or t dependence in the dilaton potential V and the vacuum energy Λ , which cannot be all absorbed in the field φ . This implies *violation of Lorentz invariance* in the bulk, as a result of the recoil of the D3 brane. The result is in agreement with the thermalization theorem (11), discussed in subsection 2.1.

It is interesting to remark [14] that the metric equations are satisfied for the simple case of a free scalar (dilaton) field φ of the form (15), *only for $d = 3$* spatial Neumann coordinates, independent of the value of b' . It seems therefore that the restoration of conformal invariance in the case of recoiling D particles embedded in a Dp brane, or equivalently the satisfaction of the corresponding equations of motion in the Liouville-dressed problem, constrains the number of longitudinal dimensions on the Dp brane to three. In other words, *only a D3 brane can intersect with recoiling (fluctuating) D particles* in a way consistent with the restoration of conformal invariance in the manner explored here.

2.3 Energy Conditions and Horizons in Recoil-Induced Space-Times

It is interesting to look at the energy conditions of such space times, which would determine whether ordinary matter can exist within the horizon region displayed above. There are various forms of energy conditions [23], which may be expressed as follows:

$$\text{Strong} \quad : \quad \left(T_{\mu\nu} - \frac{1}{D-2} g_{\mu\nu} T^\alpha_\alpha \right) \xi^\mu \xi^\nu \geq 0,$$

$$\begin{aligned}
\text{Dominant} & : T_{\mu\nu}\xi^\mu\eta^\nu \geq 0, \\
\text{Weak} & : T_{\mu\nu}\xi^\mu\xi^\nu \geq 0, \\
\text{Weaker} & : T_{\mu\nu}\zeta^\mu\zeta^\nu \geq 0.
\end{aligned} \tag{17}$$

where $g_{\mu\nu}$ is the metric and $T_{\mu\nu}$ the stress-energy tensor in a D -dimensional space-time, including vacuum-energy contributions, ξ^μ and η^μ are arbitrary future-directed time-like or null vectors, and ζ^μ is an arbitrary null vector. The conditions (17) have been listed in decreasing strength, in the sense that each condition is implied by all its preceding ones.

It can be easily seen from Einstein's equations for the metric (14) that inside the horizon $b'^2 r^2 \leq t^2$ the conditions are satisfied, which implies that stable matter can exist *inside* such regions of the recoil space-time. On the other hand, *outside the horizon* the recoil-induced metric assumes the form:

$$ds^{2(4)} \simeq \left(2 - \frac{b'^2 r^2}{t^2}\right) (dt)^2 - \sum_{i=1}^3 (dx^i)^2, \quad r^2 > t^2/b'^2 \tag{18}$$

The induced scalar curvature is easily found to be:

$$R = -4b'^2 (-3t^2 + b'^2 r^2) / (-2t^2 + b'^2 r^2)^2.$$

Notice that there is a *curvature* singularity at $2t^2 = b'^2 r^2$, which is precisely the point where there is a signature change in the metric (18).

Notice also that, in order to ensure a Minkowskian signature in the space-time (18), one should impose the restriction $2 > \frac{b'^2 r^2}{t^2} > 1$. Outside this region, the metric becomes *Euclidean*, which matches our (formal) construction of having initially a (static) Euclidean D4 brane embedded in a higher-dimensional (bulk) space time. Notice that in such a region one can formally pass onto a Minkowskian four-dimensional space time upon a Wick rotation of the (Euclidean) time coordinate X^0 . In this (Wick-rotated) framework, then, the space time inside the bubbles retains its Minkowskian signature due to the specific form of the metric (14).

The above metric (18) does not satisfy simple Einstein's equations, but this was to be expected, since the formation of such space-times is not necessarily a classical phenomenon. In ref. [14] we linked this fact with the failure of the energy conditions in this exterior geometry. These considerations suggest that matter can be trapped *inside* such horizon regions around a fluctuating D-particle defect. This sort of trapping is interesting for our space-time-foam picture, as it implies that such *microscopic D-brane horizons* act in a similar way as the intuitive description of a *macroscopic black-hole horizon* discussed in the Introduction, as illustrated in Fig. 2.

To reinforce the interpretation that matter is trapped in the interior of a region described by the metric (14), we now show that a matter probe inside the horizon 'bubble' experiences an energy-dependent velocity of light. First rewrite

the metric in a Friedmann-Robertson-Walker (FRW) form:

$$ds^2 = e^{2\ln r} \left(b'^2 dt_{FRW}^2 - \frac{1}{r^2} \sum_{i=1}^3 (dx_i)^2 \right) \quad (19)$$

where we were careful when performing coordinate redefinitions *not* to absorb in them the factor b' , which, depends (13) on the energy scale of the matter probe. We are interested in matter at various energies propagating *simultaneously* in such a space-time, and performing a coordinate transformation could not absorb an energy-dependent factor such as b' . When we consider the encounter of a matter probe, such as a photon, with a fluctuating D-particle defect, the kinetic-energy scale E_{kin} may be identified with the energy scale E of the matter probe. We recall that energy conservation has been proven rigorously in the world-sheet approach to D-brane recoil [12], and survives the resummation over higher genera.

We observe from (19) that the overall scale factor may be absorbed into a redefinition of the spatial part of the dilaton (15), implying that stable matter experiences an energy-dependent ‘light velocity’

$$c_{int}(E) = b'c = 2cg_s \left(1 - \frac{285g_s^2 E}{18M_D c^2} \right)^{1/2} \quad (20)$$

in the space-time (14), where $M_D = M_s/g_s$ is the D-particle mass scale. The energy-independent factor $2g_s$ may in fact be absorbed into the normalization of the FRW time coordinate t_{FRW} , thereby making a smooth connection with the velocity of light *in vacuo* in the limiting case of $E/M_D c^2 \rightarrow 0$. *It is important to note that, because of the specific form (13) of the variance b' , the resulting effective velocity (20) in the interior of the bubble is subluminal [24].* On the other hand, we see from (18) that matter propagates at the normal *in vacuo* light velocity c in the exterior part of the geometry.

If one considers pulses containing many photons of different energies [25,26], then the various photons will experience, as a result of the dynamical formation

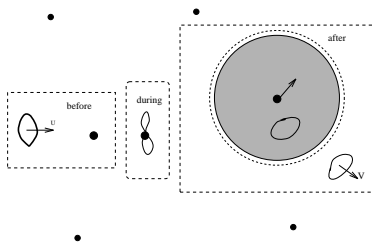


Fig. 2. A schematic representation of scattering in D-foam background. The dashed boxes represent events just before, during and after the scattering of a closed-string probe on one particular D brane defect. The scattering results in the formation of an shaded bubble, expanding as indicated by the dotted line, inside which matter can be trapped and there is an energy-dependent refractive index.

of horizons, changes in their mean effective velocities corresponding on average to a *refractive index* $\Delta c(E)$, where the effective light velocity:

$$c(E) = c \left(1 - \xi \frac{g_s E}{M_s c^2} \right). \quad (21)$$

Here ξ is a quantity that depends on the actual details of the scenario for quantum space-time foam, in particular on the density of the D-brane defects in space. In a dilute-gas approximation, ξ might plausibly be assumed to be of order one, as can be seen as follows. Consider a path L of a photon, which encounters \mathcal{N} fluctuating D-particle defects. Each defect creates a bubble which is expected to be close to the Planckian size ℓ_s , for any reasonable model of space-time foam. Inside each bubble, the photon propagates with velocity (20), whereas outside it propagates with the velocity of light in vacuo c . The total time of flight for this probe will therefore be given by:

$$t_{total} = \frac{L - \mathcal{N}\ell_s}{c} + \mathcal{N} \frac{\ell_s}{c} \left(1 - g_s^2 \frac{285}{18} \frac{E}{M_D c^2} \right)^{-1/2} \quad (22)$$

In a ‘dilute gas approximation’ for the description of space time foam, it is natural to assume that a photon encounters, on average, $\mathcal{O}(1)$ D particle defect in each Planckian length ℓ_s , so that $\mathcal{N} \sim \xi L/\ell_s$, where $\xi \leq 1$. From (22), then, one obtains a delay in the arrival time of a photon of order

$$\Delta t \sim \xi g_s^2 \frac{285}{36} \frac{L E}{M_D c^3} + \dots, \quad (23)$$

corresponding to the effective velocity (21). In conventional string theory, $g_s^2/2\pi \sim 1/20$, and the overall numerical factor in (23) is of order 4.4ξ . However, g_s should rather be considered an arbitrary parameter of the model, which may then be constrained by phenomenological observations [26] through limits on (23).

2.4 Breathing Horizons in Liouville String Theory and the emergence of Space-Time Foam

The tendency of the horizon (12) to expand is a classical feature. Upon quantization, which corresponds in our picture to a proper resummation over world-sheet topologies, one expects a phenomenon similar to Hawking radiation. Such a phenomenon would decelerate and stop the expansion, leading eventually to the shrinking of the horizon. This would be a dynamical picture of space-time foam, which unfortunately at present is not fully available, given that at microscopic distances the world-sheet perturbative analysis breaks down. However, we believe that this picture is quite plausible, and we can support these considerations formally by recalling that time t is the Liouville field in our formalism.

The dynamics of the Liouville field exhibits a ‘bounce’ behaviour, when considered from a world-sheet view point [27,7], as illustrated in Fig. 3. This is a general feature of non-critical strings, whenever the Liouville field is viewed as

a local renormalization-group scale of the world sheet. The flow of the Liouville scale is in both directions between fixed points of the world-sheet renormalization group: Infrared fixed point \rightarrow Ultraviolet fixed point \rightarrow Infrared fixed point.

This formalism is similar to the Closed-Time-Path (CTP) formalism used in non-equilibrium quantum field theories [28]. The absence of factorization is linked to the evolution from a pure state $|A, B\rangle$ to a mixed density matrix, ρ , which cannot be described by a conventional S matrix.

In our approach, the logarithmic algebra of the recoil operators forces the regularizing parameter ϵ (2) to be identified with the logarithm of the world-sheet area scale $A = |L/a|^2$, and hence with the target time. In the bounce picture outlined above, there will be a ‘breathing mode’ in the recoil-induced space-time, *characterized by two directions of time*, corresponding to the processes of expansion, stasis and shrinking of the horizon in the recoil-induced space-time (14), all within a few Planckian times. This is the Liouville-string description of Hawking radiation.

3 Physical Consequences of the Model

3.1 Modified Dispersion relations and Analogies with superfluids

The result (21) implies a *modified dispersion relation* for matter propagation in the above-model of space time foam, which violates *linearly* Lorentz invariance on the brane D3 (we remind the reader that violation of Lorentz symmetry (LIV) is a generic feature of the recoil formalism we discuss here, and already occurs in the bulk, as discussed in section 2.2):

$$E^2 - p^2 = m^2 + p^2 f\left(\frac{p}{M_{QG}}\right) \quad (24)$$

where M_{QG} denotes an effective scale at which quantum-gravitational interactions set in. We note that such modifications of dispersion relations appear as a generic feature of the non-critical string theory approach to quantum gravity, where the time is identified as the Liouville mode [29].

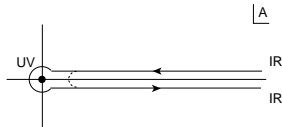


Fig. 3. Contour of integration appearing in the analytically-continued (regularized) version of world-sheet Liouville string correlators. The quantity A denotes the (complex) world-sheet area. This is known in the literature as the Saalschutz contour [28], and has been used in conventional quantum field theory to relate dimensional regularization to the Bogoliubov-Parasiuk-Hepp-Zimmermann renormalization method. Upon the identification of the Liouville field with target time, this curve resembles closed-time paths in non-equilibrium field theories.

In the specific case discussed here (20) the function $f(\frac{p}{M_{QG}})$ is linear in the small quantity p/M_{QG} , where $M_{QG} \sim \frac{36}{285g_s^2} M_D$. In general, other models of quantum gravity foam may yield more suppressed modifications, where the function $f(p/M_{QG})$ starts from quadratic (or higher-order) terms in p/M_{QG} [30]. Clearly such modified dispersion relations are direct consequences of the induced violations of Lorentz symmetry. Some experimental tests will be discussed later on.

For the moment we note that [31] such modifications of dispersion relation is a typical effect occurring in open systems in condensed matter. In fact the non-diagonal form of the original induced metric (5) is common in condensed-matter situations where one encounters the motion of fermions in superfluids in the Landau-Khalatnikov two-fluid framework [32]. We now turn briefly to a discussion of this analogy.

Indeed, it was observed in [32] that relativistic fermionic quasiparticle excitations appear near the nodes of such systems, with a spin-triplet pairing potential $V_{\mathbf{p},\mathbf{p}'} \propto \mathbf{p} \cdot \mathbf{p}'$ and an energy gap function $\Delta(\mathbf{p}) \sim cp_x$ in the polar phase, where p_x denotes the momentum component along, say, the x direction, and c denotes the effective ‘speed of light’ in the problem. This is, in general, a function of the superflow velocity w : $c(w)$, that is determined self-consistently by solving the Schwinger-Dyson-type equations that minimize the effective action.

This system was considered in the context of ^3He in a container with stationary rigid walls and a superflow velocity w taken, for simplicity, also along the x direction. The Doppler-shifted energy of the fermions in the pair-correlated state with potential $V_{\mathbf{p},\mathbf{p}'}$ is given by

$$E(p_x, \epsilon_p) = \sqrt{\epsilon_p^2 + c^2 p_x^2} + wp_x, \quad (25)$$

where $\epsilon_p = (p^2 - p_F^2)/2m$, is the energy of the fermion in the absence of the pair correlation, p_F is the Fermi momentum and m is the mass of a superfluid (e.g. Helium) atom [32]. The term wp_x appearing in the quasiparticle energy spectrum (25), as a result of the motion of the superfluid, yields an effective off-diagonal (1+1)-dimensional metric $G_{\mu\nu}$ with (contravariant) components

$$G^{00} = -1, \quad G^{01} = w, \quad G^{11} = c^2 - w^2 \quad (26)$$

The off-diagonal elements of the induced metric (26) are analogous to those of our metric (5) upon the interpretation of the Liouville field as target time [7,31]. In this analogy, the role of the recoil velocity \mathbf{u} in our quantum-gravitational case is played by the superflow velocity field w .

An important feature of the superfluid case is the appearance of an horizon that characterizes the metric (26). This arises when the superflow velocity $w = c$, in which case the metric element G^{11} in (26) crosses zero, leading to a signature change for superluminal flow $w > c$. In fact, as shown in [32] by an analysis of the gap equation, the superluminal flow branch is not stable, because it corresponds to a saddle point rather than a minimum of the effective action. This suggests that the intactness of the analogy with our problem, in which on the one hand

we do have the formation of horizons (12), and on the other the special dynamics that governs the recoil problem [12] keeps the photon velocity subluminal (20), may be maintained.

3.2 Other Stochastic Effects: light-cone fluctuations

The above-mentioned modified dispersion relation (24) and the induced refractive index (21) may be viewed as a sort of *mean field effects* in the full quantum theory. As discussed in [24], the summation over world-sheet topologies of the stringy σ -model we are examining here, leads to additional stochastic fluctuations of the widths of the pulses of massless particle probes (photons) propagating in the space-time-foam background. Such fluctuations are associated with non-zero elements of the following (target-space) correlation functions between two gravitons:

$$\langle h_{\mu\nu} h_{\alpha\beta} \rangle_{QG} \neq 0 \quad (27)$$

where the correlators are taken with respect to the full theory of quantum-gravity foam. In our σ -model D-brane framework, to leading order such fluctuations incorporate resummation of a sub-class of world-sheet topologies (annuli) [12], but in more complete situations other topologies must be included, which makes the full expression unknown. Nevertheless, from the leading order calculations we have the result that such stochastic fluctuations will lead to a stochastic spread on the arrival times of photons (massless probes) with the *same energy*, in contrast to the the (mean-field) refractive effect (21) which relates photons in different energy channels. Moreover, the stochastic effect will be suppressed by extra powers of the string coupling g_s as compared to (23). This phenomenon is similar with the one predicted in the context of conventional quantum-field theory of gravity involving graviton coherent states [33] and is probed independently of any possible modification of dispersion relations.

3.3 Charged Particles and Transition Radiation?

As we have stressed above, the basic feature of our model [14] is the formation of ‘bubbles’ (see fig. 2) with non-trivial refractive index (21), thereby giving the notion of a ‘medium’. So far we have examined the propagation of massless neutral probes in such a medium. When charged particles are considered one is prompted to draw an analogy [26] with the situation encountered in electrodynamics of interfaces between two media with different dielectric constants (and refractive indices). In such a situation the phenomenon of *transition radiation* (TR) takes place [34]: when a charged particle crosses the interface separating two media with different refractive indices and dielectric constants radiation is emitted in the forward direction, which is appreciable for highly energetic particles. The physical reason for TR is the fact that the moving electromagnetic fields of the charged particle induce a time-dependent polarization in the medium which emits radiation. The radiated fields from different points in space combine *coherently* in the neighbourhood of the path and for a certain depth (formation

depth) in the medium, giving rise to TR with a characteristic angular distribution and intensity that depends on the Lorentz factor (and hence the energy) of the charged particle. For the case of relevance to us here, in which the particle crosses an interface separating the vacuum from a medium with refractive index close to unity, the bulk of the TR spectrum comprises of highly energetic photons.

In the case of the space-time foam model discussed in [14] and reviewed here, one should expect similar effects if he/she views the D-brane defects as “real” (some sort of “material reference system” [20,35]), the recoil of which gives rise to the ‘physical’ bubble picture of fig. 2. Of course this is only an analogy, and the actual calculation differs from the electromagnetic case, especially due to the microscopic (Planckian) size of the bubbles, which approaches the uncertainty limits. However, we believe that qualitatively similar phenomena take place, but we expect them to be suppressed, due to trapping of a significant part of the emitted radiation inside the microscopic fluctuating horizons. It goes without saying that the characteristics of the associated TR, if any, depend crucially on the *details* of the space-time foam picture. It may even be averaged out. Nevertheless, the possibility of detection of energetic photons accompanying charged particles due to space time foam quantum-gravity effects is an interesting issue deserving a separate study, which we shall turn to in a forthcoming publication.

3.4 Time-dependent “vacuum” energy

An important feature of our (non-critical) stringy model is the appearance of a non-trivial “vacuum” energy, which actually is time dependent, relaxing to zero asymptotically. This is a generic feature of D-brane space-time models [20] and may have important physical consequences. The recoil of the D-particles in our picture, as a result of the scattering of strings on them, *excites* the ground state of the system, with the inevitable consequence of the appearance of a non-zero (time-dependent) excitation energy, which plays the rôle of a time-dependent “vacuum energy”. Although the characterization “vacuum” is really misleading in the sense that in the recoil picture one is dealing with an excited state of the D-brane system, however for our purposes we shall continue to use it. This is on account of two facts: (i) in the framework of our model, an observer living on the recoiling D3 brane of fig. 1 cannot tell the difference between living in an excited state and in the ground state of the system. (ii) in our scenario for the space-time foam the recoil may represent *virtual quantum fluctuations*, which can be attributed to properties of the “space-time-foam” non-equilibrium state.

It is important to notice that in the situation depicted in fig. 1 one encounters two kinds of contributions to the “vacuum” energy. The first occurs as a result of recoil effects from the D-particles embedded in the D3 brane. As discussed in [20], such effects lead to *positive* excitation (or “vacuum”) energies exhibiting a $1/t^2$ time dependence as $t \rightarrow \infty$. In addition to this, one encounters contributions to the “vacuum” energy coming from the bulk space time, as a result of the recoil (quantum) fluctuations of the D3 brane along the bulk directions. Such

contributions may lead to *negative* (anti-de-Sitter type) contributions to the vacuum energy on the brane [22], which in certain models also scale as $1/t^2$.

Both types of contributions to the vacuum energy are responsible for a supersymmetry *obstruction*, in the sense that the excited state of the system of recoiling D-branes (non-critical strings) is not supersymmetric, despite the fact that the true ground state of the system (no recoil, critical strings) is. The possibility of opposite sign contributions may imply cancellations in certain model yielding strongly suppressed or even zero vacuum energy. However, it should be remarked that a small positive contribution to the vacuum energy, scaling as $1/t^2$, is still compatible with (if not desirable on account of) recent astrophysical data, coming, for instance, from high-redshift supernovae [36].

4 Experimental Tests

4.1 Astrophysical Tests of Modified Dispersion Relations and the associated Lorentz Symmetry Violation

It is interesting to remark that the modified dispersion relations (24) may have falsifiable consequences in the foreseeable future, especially if one looks at distant astrophysical probes such as light from Gamma-Ray-Bursts (GRB) [25,37] and ultra-high-energy cosmic rays [38]. Such tests fall into the general category of experimental probes of Lorentz symmetry [39,30,40]. Indeed, in our framework Lorentz Invariance is only a symmetry of the *low energy* effective theory, which is reminiscent of the situation encountered in condensed-matter systems with relativistic excitations near certain points (nodes) of their fermi surfaces (e.g. *d*-wave superconductors). Near these points, the effective low-energy field theory appears relativistic, which is not the case of the full theory.

In a similar manner, for energies much lower than the Planck energy, M_P , which is the characteristic scale of Quantum Gravity, Lorentz symmetry seems a good symmetry of the effective theory, whilst there may be violations of it for energies near M_P where the concept of space and time may change drastically. In the model considered above, we have seen that this is indeed the case, and such manifestations of Lorentz Invariance Violations (LIV) are the modified dispersion relations (24) and the induced refractive index effects (21). We should notice, however, that the LIV in our model are different from the ones suggested by Coleman and Glashow [39]. In that model the violations of Lorentz symmetry result in the propagation velocity of massless particle species (e.g. neutrinos) being species dependent c_i , whilst in our case (21) the violation depends only on the energy content of the particle, being otherwise universal, and in this sense the induced LIV are compatible with the equivalence principle.

Gamma-Ray-Burster Observations We presented in [26] a detailed analysis of the astrophysical data for a sample of Gamma Ray Bursters (GRB) whose redshifts z are known (see fig. 4 for the data of a typical burst: GRB 970508). We looked (without success) for a correlation with the redshift, calculating a

regression measure for the effect (23) and its stochastic counterpart associated with (27). Specifically, we looked for linear dependences of the ‘observed’ Δt and the spread $\Delta\sigma/E$ (with E denoting energies) on $\tilde{z} \equiv 2 \cdot [1 - (1/(1+z))^{1/2}] \simeq z - (3/4)z^2 + \dots$, which expresses the cosmic-expansion-corrected redshift. We determined limits on the respective quantum gravity scales M_{QG} and M_{stoch} by constraining the possible magnitudes of the slopes in linear-regression analyses of the differences between the arrival times and widths of pulses in different energy ranges from five GRBs with measured redshifts, as functions of \tilde{z} . Using the current value for the Hubble expansion parameter, $H_0 = 100 \cdot h_0$ km/s/Mpc, where $0.6 < h_0 < 0.8$, we obtained the following limits [26]

$$M_{QG} \geq 10^{15} \text{ GeV}, \quad M_{\text{stoch}} \geq 2 \times 10^{15} \text{ GeV} \quad (28)$$

on the possible quantum-gravity effects.

This is one kind of tests that could yield useful limits on space-time foam models in the future. However, I should remark that such analyses have to be performed with care. The regression index should yield reliable information only in case one has a statistically-significant population of data, with known redshifts, something which at present is not feasible (it is worthy of pointing out, however, recent claims [41] according to which a systematic study of the available luminosity and spectral lag data of GRB’s can lead to an estimate of the respective redshifts. If true, such studies can provide us with statistically significant samples of GRB data relevant to our quantum-gravity precision tests). Moreover, detailed knowledge on the emission mechanisms [42] at the source are essential in order to disentangle effects that may be due to conventional physics,

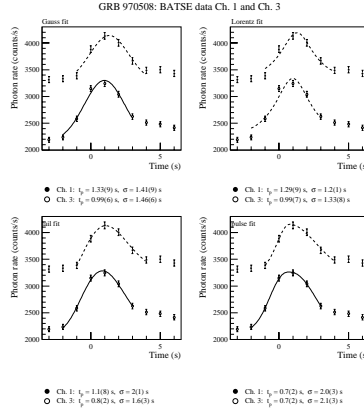


Fig. 4. Time distribution of the number of photons observed by BATSE in Channels 1 and 3 for GRB 970508, compared with the following fitting functions [26]: (a) Gaussian, (b) Lorentzian, (c) ‘tail’ function, and (d) ‘pulse’ function. We list below each panel the positions t_p and widths σ_p (with statistical errors) found for each peak in each fit. The various fitting functions provide systematic errors in precision tests of quantum-gravity effects [26].

i.e. physics which is unrelated to foam effects. For instance, it is known [43] that non-trivial vacua in effective (non-linear) theories of quantum electrodynamics, associated with thermalized fermions or photons (e.g. cosmic microwave background radiation), lead to non-trivial refractive indices. However, the energy dependence on the probe energy in such cases is different from our effect (21), in the sense that it either leads to an energy-independent light velocity, which simply changes value (jumps) as the Universe expands, or it leads to an energy and temperature dependent refractive index, which however decreases with increasing probe energy, and hence leads to the opposite effect than (23). Such conventional physics effects, therefore, have to be disentangled from the pure space-time foam effects in all the relevant analyses. It is probably worthy of mentioning that a systematic study of the observed GRB indicates [44] that the pulses of light become narrower, and the arrival times shorter, as one goes from the low- to the high- energy channels. This is opposite to the quantum-gravity effect (23), and is probably related to conventional physics phenomena at the source.

Ultra-High-Energy Cosmic Rays Recently, ultra-high-energy cosmic rays (UHECR) [38], with energies higher than 10^{20} eV, have been invoked [45–48] as very sensitive probes of Lorentz invariance violations, and in particular of the modified dispersion relations (24), with sensitivities that, depending on the model, reach and/or exceed by far Planckian energy scales. In particular, it has been argued [46] that possible modifications of the dispersion relation, due to quantum gravity effects, could provide an explanation of the observed violation of the GZK cut-off [49], and also be responsible for a significant increase in the transparency of the Universe [45], in such a way so as the sources of UHECR could be extragalactic, lying much further (at cosmological distances), in contrast with the common belief, based on Lorentz-invariant models, that the origin of UHECR should be within at most 50 Mpc radius from us [38]. Such modifications in the dispersion relation have also been invoked as a possible explanation of certain discrepancies between the observed γ spectrum of Markarian 501 and expectations based on a new estimate of the infrared background [47].

Although, most likely, conventional explanations could account for such violations of GZK cutoff and other UHECR related effects [50], however the fact that such phenomena exhibit sensitivities to Planckian (and even sub-Planckian) scales is by itself remarkable and one cannot exclude the possibility of even having experimental signatures of quantum-gravitational effects in the near future. For instance, it has been argued [48] that certain models of deformed Lorentz symmetry cannot lead to threshold effects that account for a violation of GZK cut-off, in contrast to our model, and hence UHECR data can be used to disentangle various models of space-time foam. In addition, since UHECR involve charged particles, the possibility of foam-induced transition radiation discussed in the previous section, should be taken into account as a means of excluding models.

4.2 Terrestrial Experiments on space-time foam

We shall conclude this section by mentioning briefly that space-time quantum-gravity models can also be tested in terrestrial particle-physics experiments, involving neutral kaons [51–53] and other sensitive probes of quantum mechanics, as well as gravity-wave interferometric experiments [54].

In the latter case, the effects are similar to the ones associated with the GRB experiment, and stem from the fact that space-time foam effects induce stochastic modifications of the dispersion relations for photons, and as such appear as noise in the gravity-wave interferometers. It is remarkable that the next-generation facilities of this kind could be sensitive (in principle) to Planck-size effects [54].

In the neutral-kaon or other meson experiments, the effects of quantum gravity are more subtle and are associated with the modification of the quantum mechanical evolution of the density matrix of (neutral kaon) matter propagating in a space-time foam background. A parametrization of possible deviations from the Schrödinger evolution has been given [51], assuming energy and probability conservation, in terms of quantities α, β, γ that must obey the conditions $\alpha, \gamma > 0$, $\alpha\gamma > \beta^2$ stemming from the positivity of the density matrix ρ . These parameters induce quantum decoherence and violate CPT [52]. Experimental data on neutral kaon decays so far agree perfectly with conventional quantum mechanics, imposing only the following upper limits [55]:

$$\alpha < 4.0 \times 10^{-17} \text{GeV}, \quad \beta < 2.3 \times 10^{-19} \text{GeV}, \quad \gamma < 3.7 \times 10^{-21} \text{GeV} \quad (29)$$

We cannot help being impressed that these bounds are in the ballpark of m_K^2/M_P , which is the maximum magnitude that we could expect any such effect to have.

It has been pointed out in ref. [56] that within the context of linear maps the requirement of complete positivity of ρ results in further restrictions, in particular only two of the coefficients, α, γ are non vanishing. Together with energy conservation, this requirement leads to the so-called Lindblad parametrization for the non-quantum mechanical evolution [57]. Recently, however, it has been argued [58] that such a special form of linear evolution leads to effects that are significantly more suppressed than (29), by many orders of magnitude, which casts doubt on claims [56] that complete positivity of linear maps can be tested experimentally in the foreseeable future.

However, in our Liouville-string case, which is the basis of the present model of quantum gravity foam, the deviations from the conventional quantum mechanical evolution in the respective density matrix are *non linear* [7,59]. In such a case, the parameters α, β, γ can only be viewed as appropriate averages, which themselves depend on the wave-functional of the system. This is a non-trivial feature of our formalism of identifying time with the Liouville mode [7]. In the particular case of recoiling D-particles the non-linear temporal evolution of the system has been computed explicitly [60]. Such non-linear evolution of the quantum-gravitational ‘environment’ also results, in general, in a non-linear evolution of low-energy matter propagating in this background. Moreover, in this specific example energy is not conserved during the scattering of strings with D-particle

defects, as a result of recoil of the latter. This is also in agreement with the explicit breaking of translational invariance by the presence of D-branes, and is captured formally by the very special properties of Liouville dynamics.

As discussed in [59], such features lead to important deviations from the (specific) Lindblad evolutionary form. which, in turn implies the possibility of significant enhancement of the effects: the latter can be as large as (29), thereby offering the possibility of experimental tests in the next-generation of neutral-kaon (or other meson) experiments [52,53], as well as future neutrino facilities [61].

5 Instead of Conclusions

We have discussed in this article a microscopic mechanism for the dynamical formation of horizons by the collisions of closed-string particle ‘probes’ with recoiling D-particle defects embedded in a p -dimensional space time, which may in turn be viewed as a Dp brane domain wall in a higher-dimensional target space. As we have argued before, the correct incorporation of recoil effects, which are unavoidable in any quantum theory of gravity that reproduces the conceptual framework of general relativity in the classical limit, necessitates a Liouville string approach in the context of a (perturbative) world-sheet framework.

The most important result of our approach is the demonstration of the dynamical formation of breathing horizons, which follows directly from the restoration of conformal invariance by means of Liouville dressing. The non-trivial optical properties induced by the propagation of light in such a fluctuating space-time may be subject to experimental verification in the foreseeable future, and are already constrained by existing data [25,26]. The fact that the refractive index in the bubbles of space-time foam is subluminal implies the absence of birefringence in light propagation, which is, however, possible in other approaches to space-time foam [40].

It is important to stress that the sensitivity to Planckian effects may not be so remote as one naively thinks. There are both terrestrial and astrophysical experimental tests, which are currently under way or about to be launched, that may not be far from excluding (or even verifying!) space-time foamy models of quantum gravity. It goes without saying that such a sensitivity is highly model dependent. For instance the sensitivity of the GRB test to the effect (23) depends crucially on the specific model of foam described above, in which the effect is linearly suppressed by the quantum gravity scale. If, for some reason, the effect is quadratically suppressed [30], such a sensitivity is lost [26], however such quadratic models may be experimentally testable by means of UHECR experiments. Within the framework of our model, the linear effect is undoubtedly a feature of the single scattering event of a string with a D-particle in the foam. In the case of many D-particles it is possible that the effect is further suppressed. This depend on the (yet unknown) details of the statistical dynamics of the foam.

Certainly much more work, both theoretical and ‘phenomenological’ is necessary before even tentative conclusions are reached on such important matters as an understanding and the possibility of experimental signatures of quantum

gravity. But as explained above, there are optimistic signs that this task may not be impossible. Allow me, therefore, to close this lecture by recalling a statement from Sherlock Holmes (by Sir A. Conan-Doyle), which was reminded to me by my collaborator A. Campbell-Smith: *If you eliminate the impossible, whatever remains, no matter how improbable, must be the truth.*

Acknowledgements

This work is dedicated to the memory of T. Ypsilantis. The work of N.E.M. is supported in part by PPARC (UK). The author wishes to thank Prof. H. Klapdor-Kleingrothaus (Chairman) and Dr. B. Majorovits for inviting him to the DARK2000 third international conference on Dark Matter in astro- and particle physics (Heidelberg, Germany, July 10-15 2000). He also thanks D. Fargion and V. Mitsou for discussions on cosmic rays and transition radiation respectively, and H. Hofer for his interest and support.

References

1. C. Rovelli and L. Smolin, Phys. Rev. Lett. **61**, 1155 (1988); Nucl. Phys. **B331**, 80 (1990); A. Ashtekar, C. Rovelli and L. Smolin, Phys. Rev. Lett. **69**, 237 (1992); A. Ashtekar, *Lectures on non-perturbative canonical gravity* (World Scientific, Singapore 1991); A. Ashtekar and J. Lewandowski, Class. Quant. Grav. **14**, A55 (1997); for a recent review see: C. Rovelli, Living Reviews Vol. 1; <http://www.livingreviews.org/Articles/>.
2. J.A. Wheeler, Annals of Physics (NY) **2**, 605 (1957); S.W. Hawking, Nuclear Physics **B144**, 349 (1978); S.W. Hawking, D.N. Page and C.N. Pope, Physics Letters **B86**, 175 (1979); J. Ellis, J. Hagelin, D. Nanopoulos and M. Srednicki, Nuclear Physics **B241**, 381 (1984); T. Banks, L. Susskind and M.E. Peskin, Nuclear Physics **B244**, 125 (1984); L. J. Garay, Physical Review **D58**, 124015 (1998).
3. M.B. Green, J.H. Schwarz and E. Witten, *String Theory* Vol. I and II (Cambridge Univ. Press 1987);
4. J. Polchinski, Physical Review Letters **75**, 4724 (1995); J. Polchinski, *TASI Lectures on D-branes*, hep-th/9611050.
5. A. Sen, Invited talk at the *29th International Conference on High-Energy Physics (ICHEP 98)*, Vancouver, Canada, 23-29 July 1998, hep-ph/9810356.
6. S. W. Hawking, Commun. Math. Phys. **43** (1975) 199.
7. For reviews, see: J. Ellis, N. E. Mavromatos and D. V. Nanopoulos, J. Chaos, Solitons and Fractals **10**, 345 (1999), hep-th/9805120; J. Ellis, N. E. Mavromatos and D. V. Nanopoulos, invited contribution to the 2nd International Conference on *Physics Beyond the Standard Model: Beyond the Desert 99: Accelerator, Nonaccelerator and Space Approaches*, Ringberg Castle, Tegernsee, Germany, 6-12 Jun 1999, gr-qc/9909085; J. Ellis, N. E. Mavromatos and D. V. Nanopoulos, gr-qc/0005100, presented at 4th International Symposium on *Sources and Detection of Dark Matter in the Universe (DM 2000)*, Marina del Rey, California, 23-25 Feb 2000.
8. F. David, Modern Physics Letters **A3**, 1651 (1988).
9. J. Distler and H. Kawai, Nuclear Physics **B321**, 509 (1989).

10. I. I. Kogan, N. E. Mavromatos and J. F. Wheeler, *Physics Letters* **B387**, 483 (1996).
11. J. Ellis, N. E. Mavromatos and D. V. Nanopoulos, *International Journal of Modern Physics* **A13**, 1059 (1998).
12. N. E. Mavromatos and R. J. Szabo, *Physical Review* **D59**, 104018 (1999).
13. J. Ellis, P. Kanti, N. E. Mavromatos, D. V. Nanopoulos and E. Winstanley, *Modern Physics Letters* **A13**, 303 (1998).
14. J. Ellis, N. E. Mavromatos and D. V. Nanopoulos, gr-qc/0006004, *Phys. Rev. D* in press.
15. V. Gurarie, *Nuclear Physics* **B410**, 535 (1993); J.S. Caux, I.I. Kogan and A.M. Tselik, *Nuclear Physics* **B466**, 444 (1996); M.A.I. Flohr, *International Journal of Modern Physics* **A11**, 4147 (1996); M.R. Gaberdiel and H.G. Kausch, *Nuclear Physics* **B489**, 293 (1996); I.I. Kogan and N.E. Mavromatos, *Physics Letters* **B375**, 111 (1996); M.R. Rahimi-Tabar, A. Aghamohammadi and M. Khorrani, *Nuclear Physics* **B497**, 555 (1997); I.I. Kogan, A. Lewis and O.A. Soloviev, *International Journal of Modern Physics* **A13**, 1345 (1998).
16. J. Ellis, N.E. Mavromatos and E. Winstanley, *Phys. Lett.* **B476**, 165 (2000).
17. I. Antoniadis, C. Bachas, J. Ellis and D.V. Nanopoulos, *Nucl. Phys.* **B328**, 117 (1989).
18. G. K. Leontaris and N. E. Mavromatos, *Phys. Rev.* **D61**, 124004 (2000).
19. W.G. Unruh, *Physical Review* **D14**, 870 (1976).
20. J. Ellis, N. E. Mavromatos and D. V. Nanopoulos, *Gen. Rel. Grav.* **32**, 943 (2000).
21. J. Ellis, N.E. Mavromatos and D.V. Nanopoulos, *International Journal of Modern Physics* **A13**, 5093 (1998).
22. A. Campbell-Smith and N. E. Mavromatos, *Phys. Lett.* **B488**, 199 (2000)
23. R.M. Wald, *General Relativity*, University of Chicago Press, Chicago (1984).
24. J. Ellis, N.E. Mavromatos and D.V. Nanopoulos, *Gen. Rel. Grav.* **32**, 127 (2000); *Physical Review* **D61**, 027503 (2000).
25. G. Amelino-Camelia, J. Ellis, N. E. Mavromatos, D. V. Nanopoulos and S. Sarkar, *Nature* **393**, 763 (1998), astro-ph/9712103;
26. J. Ellis, K. Farakos, N.E. Mavromatos, V.A. Mitsou and D.V. Nanopoulos, *Astro-phs. J.* **535**, 139 (2000) [astro-ph/9907340].
27. I. Kogan, *Phys. Lett.* **B265** (1991) 269.
28. J. Schwinger, *J. Math. Phys.* **2** (1961) 407; H. Umezawa, *Advanced Field Theory : Micro, Macro and Thermal Concepts* (American Inst. of Physics, N.Y., 1993). For recent discussions, see: E. Calzetta and B.L. Hu, *Phys. Rev.* **D37** (1988) 2878; E. Calzetta, S. Habib and B.L. Hu, *Phys. Rev.* **D37** (1988) 2901.
29. G. Amelino-Camelia, J. Ellis, N. E. Mavromatos and D. V. Nanopoulos, *Int. J. Mod. Phys.* **A12**, 607 (1997).
30. L. Gonzalez-Mestres, hep-ph/99050430 and physics/0003080 and references therein.
31. J. Ellis, N. E. Mavromatos, D. V. Nanopoulos and G. Volkov, gr-qc/9911055, *Gen. Rel. Grav.* in press.
32. N. B. Kopnin and G. E. Volovik, *Pisma Zh. Eksp. Teor. Fiz.* **67**, 124 (1998) [cond-mat/9712187]; G. E. Volovik, gr-qc/0005091 and references therein.
33. H. Yu and L. H. Ford, *Phys. Rev.* **D60**, 084023 (1999) and gr-qc/0004063, and references therein.
34. see for instance: J.D. Jackson, *Classical Electrodynamics* (John Wiley & Sons, New York 1975).
35. C. Rovelli, *Class. Quant. Grav.* **8**, 297 (1991); *ibid.* 317.

36. S. Perlmutter *et al.*, astro-ph/9712212; A.G. Riess *et al.*, astro-ph/9805201; P. Garnavich *et al.*, astro-ph/9806396.
37. S. D. Biller *et al.*, Phys. Rev. Lett. **83**, 2108 (1999); P. Kaaret, Astronomy and Astrophysics **345**, L32 (1999).
38. for recent reviews see: A.V. Olinto, astro-ph/0003013; G. Sigl, astro-ph/0008364.
39. For a sample of representative references see: V. A. Kostelecky and S. Samuel, Phys. Rev. **D39**, 683 (1989); S. Coleman and S. L. Glashow, Phys. Lett. **B405**, 249 (1997); S. L. Glashow, A. Halprin, P. I. Krastev, C. N. Leung and J. Pantaleone, Phys. Rev. **D56**, 2433 (1997); V. A. Kostelecky and C. D. Lane, Phys. Rev. **D60**, 116010 (1999); O. Bertolami and C. S. Carvalho, Phys. Rev. **D61**, 103002 (2000).
40. R. Gambini and J. Pullin, Phys. Rev. **D59**, 124021 (1999); J. Alfaro, H.A. Morales-Tecotl and L.F. Urrutia, Phys. Rev. Lett. **84**, 2318 (2000).
41. E. E. Fenimore and E. Ramirez-Ruiz, astro-ph/0004176; D. E. Reichart *et al.*, astro-ph/0004302.
42. T. Piran, Physics Reports **314**, 575 (1999).
43. J. I. Latorre, P. Pascual and R. Tarrach, Nucl. Phys. **B437**, 60 (1995); W. Dittrich and H. Gies, Phys. Rev. **D58**, 025004 (1998); K. Scharnhorst, Annalen Phys. **7**, 700 (1998) [hep-th/9810221], and references therein; V. A. De Lorenci, R. Klippert, M. Novello and J. M. Salim, Phys. Lett. **B482**, 134 (2000); M. Novello, V. A. De Lorenci, J. M. Salim and R. Klippert, Phys. Rev. **D61**, 045001 (2000); M. H. Thoma, hep-ph/0005282.
44. see for instance: J.P. Norris, G.F. Marani and J. T. Bonnell, Astrophys. J. **534**, 248 (2000).
45. T. Kiffune, astro-ph/9904164.
46. R. Aloisio, P. Blasi, P. L. Ghia and A. F. Grillo, Phys. Rev. **D62**, 053010 (2000).
47. R.J. Protheroe and H. Meyer, astro-ph/0005349.
48. G. Amelino-Camelia and T. Piran, hep-ph/0006 and astro-ph/0008107.
49. K. Greisen, Phys. Rev. Lett. **66**, 748 (1966); G.T. Zatsepin and V.A. Kuzmin, Pis'ma Zh. Eksp. Teor. Fiz. **4**, 114 (1966) (JETP Lett. **4**, 78 (1966)).
50. D. Fargion and B. Mele, astro-ph/9906451; see also D. Fargion, these proceedings.
51. J. Ellis, J. Hagelin, D. Nanopoulos and M. Srednicki, Nucl. Phys. B241 (1984), 381.
52. J. Ellis, N.E. Mavromatos and D.V. Nanopoulos, Phys. Lett. B293 (1992), 142; Int. J. Mod. Phys. A11 (1996), 1489; J. Ellis, J. L. Lopez, N.E. Mavromatos and D.V. Nanopoulos, Phys. Rev. D53 (1996), 3846.
53. P. Huet and M. Peskin, Nucl. Phys. B434 (1995), 3.
54. G. Amelino-Camelia, Nature **398**, 216 (1999) Phys. Rev. **D62**, 024015 (2000).
55. R. Adler, *et al.* (CPLEAR Collaboration), J. Ellis, J. Lopez, N.E. Mavromatos, and D.V. Nanopoulos, Physics Letters **B364**, 239 (1995).
56. F. Benatti and R. Floreanini, Nucl. Phys. **B488** (1997) 335. *ibid.* **B511**, 550 (1998); Phys. Lett. **B468**, 287 (1999).
57. G. Lindblad, Comm. Math. Phys. 48 (1976), 119; For a review see V. Gorini *et al.*, Rep. Math. Phys. 13 (1978), 149.
58. S. Adler, hep-ph/0005220.
59. J. Ellis, N.E. Mavromatos and D.V. Nanopoulos, gr-qc/0007044.
60. N. E. Mavromatos and R. J. Szabo, hep-th/9909129, Int. J. Mod. Phys. **A** in press.
61. E. Lisi, A. Marrone and D. Montanino, hep-ph/0002053.