GOALS AND METHODS OF FLAVOUR PHYSICS

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Abstract
These lectures describe the motivation for flavour physics, and the theoretical and experimental methods. I discuss the determination of the fundamental parameters from inclusive and exclusive decays of the flavoured mesons K, B, and the phenomena of CP violation. Theoretical tools for mastering the problems of QCD, such as heavy-quark effective theory and factorization are explained.

1. INTRODUCTION

1.1 Flavours in the Standard Model (SM)

The particles (or fields) of the standard model (SM) and most gauge theories [1] are divided into gauge fields associated with the gauge group and 'matter fields'. In the SM (see the lectures by Bardin), the gauge group is $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ which is spontaneously broken to $SU(3)_C \otimes U(1)_Q$ by the Higgs field. The gauge fields (gauge bosons), i.e. the photon, the W and Z bosons, and the gluons, are determined by the gauge group. All have spin one.

On the other hand, the number and properties of matter fields (quarks, leptons, neutrinos) are largely arbitrary. Their number, transformation properties, and other characteristics (mass, couplings, etc.) are essentially determined by experiment (see the particle data book [2] for the current numbers). Theoretical considerations, such as anomaly cancellations [3] only weakly limit their freedom. The Higgs field is introduced in a similarly arbitrary way as the matter fields, but because it triggers 'symmetry breaking' by which the W and Z bosons become massive, it is usually viewed as a 'different' kind of field.

In the SM, all matter fields, the quarks and leptons, are fermions. Because the gauge bosons have spin one, left- and right-handed fields can transform in different ways under the gauge group*. The absence of right-handed couplings to the W bosons implies that the right-handed fields are $SU(2)$ singlets. The left-handed ones are doublets, because there are only two different charges of quarks and leptons. This situation implies that the particles are massless if the gauge symmetry is unbroken.

There are in fact three complete 'generations' of particles. The generations all have the same interactions with the gauge particles; only their masses (and the couplings to the Higgs field) are different. In a notation† adapted to their $SU(2)$ properties, we have three generations

$$\begin{pmatrix} \nu_{eL} \\ e^-_L \end{pmatrix} \quad \begin{pmatrix} u_L \\ d'_L \end{pmatrix} \quad e_r \quad u_r \quad d_r \tag{1}$$

$$\begin{pmatrix} \nu_{\mu l} \\ \mu^-_l \end{pmatrix} \quad \begin{pmatrix} c_l \\ s'_l \end{pmatrix} \quad \mu_r \quad c_r \quad s_r \tag{2}$$

$$\begin{pmatrix} \nu_{\tau l} \\ \tau^-_l \end{pmatrix} \quad \begin{pmatrix} t_l \\ b'_l \end{pmatrix} \quad \tau_r \quad t_r \quad b_r \quad . \tag{3}$$

*The coupling of fermion fields ψ to gauge fields is proportional to $\psi^+\gamma_0\gamma^\mu\psi$. A fermion field ψ can be projected into left- and right-handed components via the projectors P_L, P_R defined by $1 \pm \gamma_5/2$. Since $P_L\gamma_0\gamma^\mu P_R = 0$, etc., the gauge bosons only 'connect' left-handed fields to left-handed fields, etc.

†We denote the fields by the name of the corresponding particle. Therefore, u means the field that in the usual free field description destroys an up-quark and generates an anti up-quark.

Fig. 1: Generic couplings of a vector and a scalar to two fermions.

The index L or R means that these particles have chirality -1 or 1. We call the particles left- and right-handed, respectively. The primes will be discussed below. The term **Flavour** denotes the different particles; therefore, we have u flavour, d flavour, etc.

We also often have the notation

$$Q_1 = \begin{pmatrix} u_L \\ d'_L \end{pmatrix} \qquad Q_2 = \begin{pmatrix} c_L \\ s'_L \end{pmatrix} \qquad Q_3 = \begin{pmatrix} t_L \\ b'_L \end{pmatrix} \tag{4}$$

and d_{1R} instead of d_R etc.

Problem

Discuss helicity and chirality (see, for example, the textbook by Itzykson and Zuber).

In principle, one can also add to each generation a right-handed neutrino. In the absence of neutrino masses, these are unnecessary. As we discuss here only the physics issues unrelated to the neutrino masses, I shall leave these out (see S. Bilenky's lectures for a thorough description of this field and its exciting developments).

The interaction vertex (see Fig.1) of fermion fields ψ_i, ψ_j with spin one (gauge) particles and spin zero particles (Higgs particle) is in general (i, j label the different fields)

$$ig_{ij}\gamma_\mu(a + b\gamma_5) \,, \tag{5}$$

$$ih_{ij}(a_y + b_y\gamma_5) \,. \tag{6}$$

The non-Abelian gauge invariance of the theory implies the important fact that the coupling constants g_{ij} can be chosen to be of the form $g\delta_{ij}$ without loss of generality[‡]. Thus, the coupling to the SM gauge bosons A_μ takes the diagonal form

$$\mathcal{L}_{\rm gauge} = \sum_i \bar{Q}_i \bar{\gamma}^\mu A_\mu Q_i + ... \tag{7}$$

We see that all generations couple in exactly the same way.

On the other hand, the h_{ij} are arbitrary (complex) matrices. In the SM, the only scalar is the Higgs field. The interactions of the matter fields[§] are conveniently written in the form of the interaction Lagrangian density[¶]

[‡]This choice is stable; for instance, it cannot be changed by higher orders in perturbation theory.

[§]I write here only the quark fields. The leptons can be treated in exactly the same way.

[¶]The $\tilde{\Phi}$ in the second term stands only for mathematical correctness; the relation to ϕ is $\tilde{\phi} = i\tau_2\phi^*$.

$$\mathcal{L}_{\rm int} = \bar{Q}_i Y^d_{ij} d_j \phi + \bar{Q}_i Y^u_{ij} u_j \tilde{\phi} + \text{h.c.} \tag{8}$$

The basis of the fields in which the couplings $\mathcal{L}_{\rm gauge}$ to gauge bosons take the simple diagonal form above, is called the interaction basis. It is, however, not physically useful. To see this, we consider further the coupling to the scalar field where we introduce spontaneous symmetry breaking. From Bardin's lectures we know that the scalar field is then replaced by a non-zero constant plus a space-time dependent field, the physical Higgs field, which in these lectures it will not be necessay to consider explicitely; thus

$$\phi \sim \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}_L , \tag{9}$$

where $v \approx 250$ GeV. This means that the interaction [Eq. (8)] is replaced by 'mass terms' for the quarks. Using the notation $M^u_{ij} = Y^u_{ij} v/\sqrt{2}$ and $M^d_{ij} = Y^d_{ij} v/\sqrt{2}$ we now have[‖]

$$\mathcal{L}_{\rm mass} = \bar{d}'_{Li} M^d_{ij} d_j + \bar{u}_{Li} M^u_{ij} u_j + \text{h.c.} \tag{10}$$

We see that these mass terms are not diagonal; it means that a free up-quark could become a charm quark, etc. This is of course wrong. We therefore need to diagonalize this expression. Without loss of generality M^u_{ij} can be chosen to be diagonal. The down-quark mass matrix M^d_{ij} is then diagonalized by a unitary transformation to the mass or physical basis of the fields, denoted again by d_{Li} but without the primes![**] Thus,

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V \begin{pmatrix} d \\ s \\ b \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} . \tag{11}$$

The unitary matrix V is the famous Cabibbo–Kobayash–Maskawa matrix [4]. It is obtained by diagonalization of M^d, i.e. by demanding that $V^\dagger M M^\dagger V$ be real and diagonal.

We now have to replace in Eq. (7) the fields d' by the unprimed physical ones, in order to obtain the weak interactions of the physical quarks. It is straightforward to see that because of unitarity the couplings to the electrically-neutral gauge bosons (the Z, the photon and the gluons) are diagonal in flavour. The $V^\dagger$ and V matrices associated with the d' in $\bar{Q}$ and Q in Eq. (7) cancel by virtue of the unitarity relation $V^\dagger V = 1$. Only the couplings to the W violate flavour, i.e. include transitions from one flavour to another. Therefore, whilst the neutral interactions remain as in Eq. (7), the charged ones pick up the matrix V:

$$\mathcal{L}_{\rm int} = \mathcal{L}_{\rm CC} + \mathcal{L}_{\rm NC} , \tag{12}$$

where

$$\mathcal{L}_{\rm CC} = \frac{g_2}{2\sqrt{2}} \sum_i (\bar{u}_{Li} \gamma^\mu V_{ij} d_{Lj}) W^+_\mu + \frac{g_2}{2\sqrt{2}} \sum_i (\bar{d}_{Lj} \gamma^\mu V^*_{ij} u_{Li}) W^-_\mu , \tag{13}$$

and

$$\mathcal{L}_{\rm NC} = e \sum_f q_f \bar{f} \gamma_\mu f A^\mu + \frac{g_2}{2 \cos\Theta_W} \sum_f \bar{f} \gamma_\mu (v_f - a_f \gamma_5) f Z^\mu . \tag{14}$$

The sum goes over all fermion fields f u_{iL}, u_{iR}, etc. The coefficients q_f are the charges of the fermion f and

$$v_f = T^f_3 - 2 q_f \sin^2\Theta_W \qquad a_f = T^f_3 . \tag{15}$$

The T^f_3 are zero for the right-handed fields and $1/2$ (or $-1/2$) for up-quarks and neutrinos (or down-quarks and charged leptons). The products of the fields multiplying the gauge fields are called currents; for instance, $J_\mu = e \sum_f q_f \bar{f} \gamma_\mu f$ is the electromagnetic current. Similarly, we have a charged current, coupled to W, and a neutral current coupled to Z. The couplings are shown in Fig. 2.

[‖] Terms quadratic in the fields are called mass terms. When they are diagonalized in flavour, they give the physical masses.

[**] In the SM, the right-handed fields d_R, like the u_i, are also the same in both of the bases.

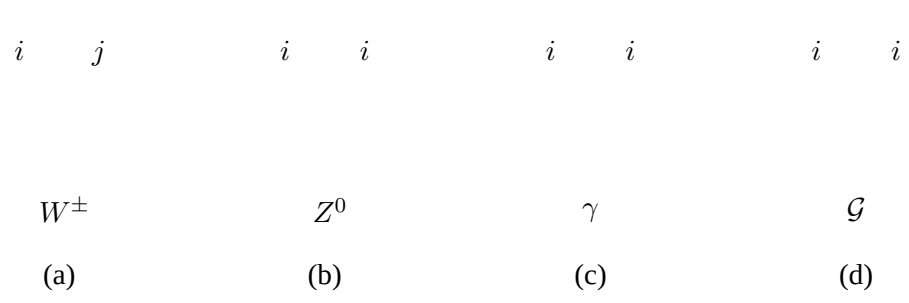


Fig. 2: The basic vertices representing the interactions of the quarks with the gauge bosons. The labels i and j represent the flavour quantum number ($i, j = u, d, c, s, t, b$).

Fig. 3: Approximation of the W-exchange interaction, by the four-fermion current–current vertex.

From these Lagrangians one obtains an effective charged current Hamiltonian, valid at energies below the mass of the W by exchanging a W between two currents. The W exchange yields a factor $1/M_W^2$ and thus the effective coupling for the resulting four-fermion interaction (Fig. 3) is

$$\frac{G_F}{\sqrt{2}} V_{ij} V_{kl}^* = \frac{g_2^2}{8 M_W^2} V_{ij} V_{kl}^* \,, \tag{16}$$

for a transition $d_{jL} \to u_{iL} \bar{u}_{kL} d_{lL}$. Here, G_F is the Fermi constant,

$$G_F = 1.10^{-5}\ \mathrm{GeV}^{-2} \tag{17}$$

which is determined with high precision from the μ decay.

The fact that neutral currents are diagonal in flavour (see Eq. (7) is trivial, because the three generations have exactly the same gauge interactions. Before the experimental discovery of charm in 1974 (the strange quark was discovered much earlier) this was not the case, and the SM was rejected because it seemed to have sizeable flavour-changing neutral currents (FCNC), contradicting experiments (smallness of the rate $K \to \mu\bar{\mu}$). For this reason, Glashow, Iliopoulos, and Maiani (GIM) introduced (theoretically) a charm quark in 1970 [5] to complete the generation of the s quark and to be able to make use of the unitarity of V. The discovery of charm in 1974 was thus a great success.

Much as the W, Z exchange also gives rise to an effective four-fermion interaction. Since it does not change flavour, we shall not consider it further. However, it contributes to fine parity violating effects in atoms and nuclei and played an important role in the development of the SM.

As already mentioned, only the charged currents contain flavour changes. Therefore, it is the charged weak interactions from which much can be learned about new flavours, and it is therefore essential to study them with high precision. Flavour physics is the study of all aspects related to the different flavours. Some of these issues are mentioned below.

The values of the flavour-related parameters

The flavour parameters in the SM include the fermion masses

$$m_u \qquad m_d \qquad m_s \qquad m_c \qquad m_b \qquad m_t \tag{18}$$

$$m_e \qquad m_\mu \qquad m_\tau \qquad m_{\nu_e} \qquad m_{\nu_e} \qquad m_{\nu_e} \tag{19}$$

and the CKM matrix elements

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} . \tag{20}$$

Most of the present flavour physics is concerned with determining these parameters with the highest precision possible. It turns out that quite an elaborate theoretical machinery is needed to isolate the values from the data. For instance, the up-quark is always inside the hadrons, so its mass cannot be determined without taking into account the binding energy of the hadron, but this is very difficult to do. For light quarks a method known as chiral perturbation theory (applicable to low-energy hadronic physics) can be employed [6]. The general rule is that only quark masses above roughly 1 GeV (proton mass) are directly related to the mass of the hadron in which they live. Various technologies yield the approximate values [2]

$$m_u \sim 4\ \mathrm{MeV} \quad m_d \sim 4\ \mathrm{MeV} \quad m_s \sim 120\ \mathrm{MeV} \tag{21}$$

$$m_c \sim 1.5\ \mathrm{GeV} \quad m_b \sim 4.5\ \mathrm{GeV} \quad m_t \sim 170\ \mathrm{GeV}\ . \tag{22}$$

Similar and different techniques are employed to find the values of the CKM matrix elements. Roughly,

$$V \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ -\lambda & 1 & \lambda^2 \\ \lambda^3 & -\lambda^2 & 1 \end{pmatrix} , \tag{23}$$

where $\lambda \sim 0.22$ is the sine of the Cabibbo angle. Both sets of values show a characteristic hierarchical pattern. What is the reason for this? Various ideas exist about how to model this structure, including the recent excitement about extra dimensions [7], but most appear to be mathematical games [8].

Why are there different flavours; how many are there?

In the SM, the flavours are added largely according to phenomenological need. There are several theoretical constraints, the invariance structure (dictated by the gauge symmetry), and the so-called anomaly cancellation [3]. They essentially amount to the requirement that new particles must come as a whole generation. Attempts exist to introduce so-called flavour groups which unify the flavours, or to link the number of flavours and their properties to geometrical structures in higher dimensional spaces. In all these theories, the different generations would not only have different masses, but also different interactions. By investigating very carefully all the properties of the various flavours, these fine details might one day be found.

CP violation

The properties of the fields under transformation play an essential role in modern physics theory. Of particular importance have been the Lorentz transformations (which lead to relativity) and the discrete transformations P, C, and T (or CP) which we consider here [9, 10]. The parity transformation P inverts the spatial coordinate, whilst the time reversal operation T inverts time. In a quantum field theory, these transformations also affect the fields. In general we write for a transformation $(x,t) \longrightarrow (x',t')$

$$\Phi(x,t) \longrightarrow (R_\Phi \Phi)(x',t') . \tag{24}$$

Here, R_Φ is a 'representation' of the transformation group. An example is the transformation of a fermion field Ψ under parity. We have $\Psi(x,t) \longrightarrow \gamma_0 \Psi(-x,t)$. A theory is said to be P invariant if this transformation (supplemented with a few others) does not change the Lagrangian used to describe it.

The charge conjugation operation C transforms particles into antiparticles. Its action on the fermions can be written as $\Psi \longrightarrow C\bar{\Psi}^T$ where C is a combination of Dirac matrices (in the usual notation, C$= i\gamma_2\gamma_0$ and $\bar{\Psi}^T$ is the transposed $\bar{\Psi} = \Psi^\dagger\gamma_0$.

Instead of T, one usually uses the combined transformation CP, because it is mathematically easier to handle (T is a so-called anti-unitary operator). It is well known that any field theory (but not necessarily strings) obeys the so-called CPT theorem [11], i.e. they are invariant under the combined transformation. Therefore, T and CP are equivalent. (The issue of CPT violation has received renewed interest in the context of string theories: see Ref. [11] for some new developments.)

Whilst the strong, electric and gravitational forces obey all these symmetries, parity violation has been an integral part of weak interactions since 1957 [12]. CP violation, discovered in 1964 [13], is not so well understood. It can be implemented in a straightforward way into the SM, as we see below, but many people feel that CP violation could actually shed light on new physics; it is therefore in the centre of modern research. The occurence of CP violation is crucial. As shown by Sakharov in 1967 [14], it is responsible for the fact that there is hardly any antimatter in the universe. Since antimatter would annihilate with matter, the stability of the universe requires CP violation. But it is known that SM CP violation cannot account for matter–antimatter asymmetry [15] (for a overview of astroparticle physics, see the lectures by J. Garcia-Bellido). Thus other sources of CP violation must be present, and flavour physics might be a way to investigate this.

Using the above rules for the transformations, it follows that under the CP transformation

$$W_\mu^+ \longrightarrow W_\mu^- \tag{25}$$

$$\bar{\Psi}_1 \gamma\mu \Psi_2 \longrightarrow \bar{\Psi}_2 \gamma\mu \Psi_1 \; . \tag{26}$$

Using this in the charged current interaction

$$\mathcal{L}_{\rm CC} = \frac{g_2}{2\sqrt{2}} \sum_i (\bar{u}_{Li} \gamma^\mu V_{ij} d_{Lj}) W_\mu^+ + \frac{g_2}{2\sqrt{2}} \sum_i (\bar{d}_{Lj} \gamma^\mu V_{ij}^* u_{Li}) W_\mu^- \; , \tag{27}$$

we find that the CP transformed expression is

$$\mathcal{L}_{\rm CC}^{CP} = \frac{g_2}{2\sqrt{2}} \sum_i (\bar{d}_{Lj} \gamma^\mu V_{ij} u_{Li}) W_\mu^- + \frac{g_2}{2\sqrt{2}} \sum_i (\bar{u}_{Li} \gamma^\mu V_{ij}^* d_{Lj}) W_\mu^+ \; . \tag{28}$$

The two expressions are different if the elements of V are complex: phases indeed indicate CP violation. But since phases in quantum mechanics are not measured, the argument is a bit more complicated because it is often possible to redefine fields such that certain phases are absorbed. In our case, V is a unitary 3×3 matrix. Since a unitary $N \times N$ matrix has N^2 parameters, $N(N-1)/2$ of which are real and $N(N+1)/2$ are phases, and since for N quark doublets Q we can redefine $2N-1$ phases for N quark doublets Q, we have altogether

$$N(N+1)/2 - (2N-1) = (N-1)(N-2)/2 \tag{29}$$

physical phases. Thus for one or two generations, there is no physical phase and no CP violation within the SM, and for three there is one phase [††]. This result of Kobayashi and Maskawa in 1973 [4] indicated that there must be three generations if CP violation is to be described by the SM. This shows that CP violation and flavour are strongly connected.

These questions and similar ones constitute the entire issue of flavour physics. Unlike interactions between gauge bosons, the values of these parameters are not known with high precision. In particular, the phases of the CKM matrix elements are very poorly determined. Therefore, it is widely believed that flavour physics offers the possibility to investigate and discover new physics. In the SM, all flavour physics has its origin in the couplings to the Higgs fields. But there is no underlying theoretical framework. In the following I will discuss some of the current issues:

- measuring the CKM matrix elements precisely;

[††] Illustrate below in some detail the freedom of choosing the phase. It is possible to give a reparametrization invariant measure of CP violation [16].

- CP violation in K and B physics;
- rare decays, mainly kaons;
- hadronic decays of B mesons.

The task of precisely determining parameters and finding deviations from the SM implies much theoretical technology and hard work. Although I shall try to be general, some topics must be discussed in detail; flavour physics at this stage is largely nitty-gritty work for phenomenologists and theorists, and often not very exhilarating. But the outcome may well be. So please be patient.

2. THE CKM MATRIX

A general 3×3 unitary matrix V can be written as

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -s_{23}c_{12} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \tag{30}$$

where $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$ with $i, j = 1, 2, 3$ [2], δ is the phase necessary for CP violation. c_{ij} and s_{ij} can all be chosen to be positive and δ may vary in the range $0 \leq \delta \leq 2\pi$. However, the measurements of CP violation in K decays force δ to be in the range $0 < \delta < \pi$. Phase δ can be eliminated if any two quarks of the same charge have the same mass and if any of the s_{ij} vanishes [‡‡].

Problem

Set $s_{12} = 0$ and show by redefinition of the fields that δ can be eliminated. Start with $u_1 \longrightarrow e^{i\delta}u_1$.

The first element, V_{ud} is obtained by measuring the rates of superallowed β decays of nuclei, where the d quark inside a neutron goes over into a u quark which is part of the proton. The rate is proportional to $|G_F V_{ud}|^2$ but the bound state and electromagnetic effects render the proportionality difficult. Most of these have been calculated, with the result [2]

$$V_{ud} = 0.9736 \pm 0.001 \; . \tag{31}$$

Chiral perturbation theory applied to $K \to \pi e\nu$ yields

$$|V_{us}| = 0.2205 \pm 0.0018 \; , \tag{32}$$

and somewhat less reliable methods yield

$$|V_{cs}| = 1.010 \pm 0.16 \qquad , |V_{cd}| = 0.224 \pm 0.016 \; . \tag{33}$$

Of much recent interest are the matrix elements occuring in B physics, V_{cb} and V_{ub}. Both are determined from semileptonic B decays which are fed by the transitions $b \to ce\nu$ and $b \to ue\nu$. Both inclusive and exclusive decays are employed; theoretically, the heavy quark method is used. At present, we have [17, 18, 19]

$$|V_{cb}| = 0.0385 \pm 0.001 \text{ (exclusive)} \qquad |\mathrm{V_{cb}}| = 0.0401 \pm 0.001 \text{ (inclusive)} \tag{34}$$

and

$$|V_{ub}| = 0.0041 \pm 0.0007 \; . \tag{35}$$

[‡‡] Any CP violation is proportional to $(m_u^2 - m_c^2(m_u^2 - m_c^2)(m_u^2 - m_t^2)(m_c^2 - m_t^2)(m_d^2 - m_s^2)(m_d^2 - m_b^2)(m_s^2 - m_b^2)s_{12}s_{23}s_{12}\sin\delta$.

The errors are essentially statistical; the systematic errors, such as theoretical uncertainties, can be larger. Much work is currently being carried out to reduce these unertainties [10]. At present, the various groups measuring these quantities report slight disagreements.

Using the approximate values in Eq. (23) we can rewrite the unitary matrix V to high precision in the so-called Wolfenstein parametrization [20]

$$V = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\varrho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\varrho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)\,. \tag{36}$$

Here $\lambda = |V_{us}| = 0.22$; we have four parameters λ, A, ϱ, and η, as expected for a unitary 3×3.

To test the validity of the SM, we need to compare this specific matrix with a large number of experiments. A convenient way to do this are the so-called unitarity triangles. Unitarity of V implies that its rows and columns are orthogonal. As an equation, this means

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0 \tag{37}$$

and more such relations. Since the V_{if} are complex numbers, this relation can be represented as a triangle in the plane of complex numbers. The products of the matrix elements appearing here are measured: for instance the decay amplitude for $\bar{b} \to \bar{u}u\bar{d}$ is proportional to $V_{ud}V_{ub}^*$.

To an excellent accuracy $V_{cd}V_{cb}^*$ is real and equal to $A\lambda^3$. It is therefore customary to rescale the triangle by dividing all the terms by this number. Then the basis has length 1 and the coordinates of the plane are ϱ and η. The triangle is shown in Fig. 4, the sides are related to the products by

$$\frac{1}{A\lambda^3}V_{ud}V_{ub}^* = \bar{\varrho} + i\bar{\eta}\,, \qquad \frac{1}{A\lambda^3}V_{td}V_{tb}^* = 1 - (\bar{\varrho} + i\bar{\eta})\,. \tag{38}$$

Here, following Buras [21], the quantities

$$\bar{\varrho} = \varrho(1 - \frac{\lambda^2}{2})\;\; \bar{\eta} = \eta(1 - \frac{\lambda^2}{2}) \tag{39}$$

are introduced to take into account even higher powers of λ.

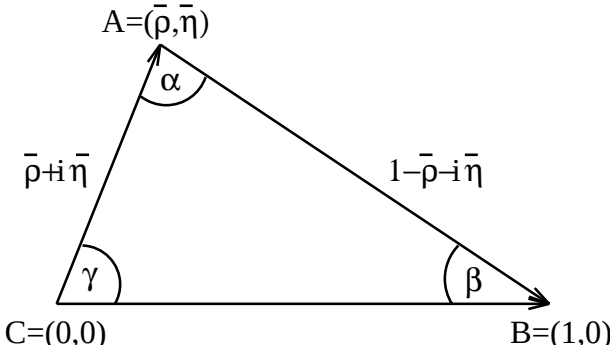


Fig. 4: Unitarity triangle in the complex $(\bar{\varrho}, \bar{\eta})$ plane.

I stress again that the triangle is just a graphical representation of the properties of the CKM matrix. It is convenient because it relates observables in a direct way. There are various other triangles that can be drawn, but this one is the most useful in connection with B physics. The major effort in phenomenological

flavour physics is to determine from experiment the elements of the CKM matrix with the largest possible accuracy. Certain observables give the length of the sides, but others can also determine the angles α, β, γ. Much theory is needed for this task. For instance, V_{td} involves the t quark, so it can only occur if the top quark is virtual, and thus the loops must be calculated. However, the biggest problem is that quarks are confined into hadrons. Thus in order to obtain reliable results, one must be able to deal with the strong interactions (QCD). Up to now there are huge difficulties in treating these in a satisfactory way, except for high-energy (above several GeV) processes, because of the asymptotic freedom of QCD.

3. A SCHEME FOR ORGANIZING QCD CORRECTIONS

3.1 Effective Hamiltonian, renormalization

As pointed out before, the major theoretical problem is to master QCD, the strong interaction effects. For this, the idea of the effective Hamiltonian has been devised [21, 22].

For processes at low energies, the W boson does not appear as a physical particle, but it affects the interactions of the lighter particles. As mentioned before, the exchanges of the W boson give rise to the weak interaction which we describe by an 'effective Hamiltonian' (the word 'effective' expresses the fact that it is not the fundamental interaction but the effective one at low energies). For a transition $b \to u\bar{u}d$ we have

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{ij} V_{kl}^* \left(\bar{d}\gamma^\mu (1-\gamma^5) u \right) \left(\bar{u}\gamma_\mu (1-\gamma^5) b \right) . \tag{40}$$

To this, we now must add QCD corrections coming from loops of gluons [Fig. 6 (a)]. We divide the gluon momentum q into two ranges, $q^2 \geq \mu$ and $q^2 \leq \mu$, where μ is the so-called renormalization scale whose value is chosen preferably in the energy region of the process considered. In the high q^2 regime, asymptotic freedom of QCD allows perturbative calculations. Below μ, non-perturbative techniques are needed because the gluon cannot be considered a simple free particle. The occurence of a scale μ can be understood from the simpler case of QED, quantum electrodynamics. We consider the scattering of an electron on a photon. The Hamilton operator is

$$\mathcal{H} = e(\bar{\Psi}\gamma^\mu A_\mu \Psi) , \tag{41}$$

where e is 'the electric charge' and Ψ, A^μ the electron and photon fields. We are interested in the matrix element $\langle \mathcal{H} \rangle$. The renormalization procedure leads to a 'renormalized' charge, $e_R(\mu)$, defined at an arbitrary energy scale μ, the renormalization point. To see this, we consider radiative corrections to the matrix element; as in Fig. (5) the momentum integral diverges and we introduce a cutoff Λ. To balance it, a so-called counterterm is added to the theory which can only depend on Λ and on μ (it cannot depend on the physical momentum p because we want to define a theory valid for all momenta. The loop integral over the internal momenta yields then for the total charge

$$e \longrightarrow e_R(\mu) + \text{correction} + \text{counterterm} \tag{42}$$

$$\sim e_R(\mu) + e_R^3 \log(\frac{\Lambda^2}{p^2}) - e_R^3 \log(\frac{\Lambda^2}{\mu^2}) \tag{43}$$

at momentum p^2. The dependence of e_R is necessary: because we do not want physical results which depend on an 'artificial' scale μ, the dependence of the counterterm must be cancelled by e_R.

At the renormalization point, as expected, this is simply e_R. The logarithmic behaviour is well known and follows from the form of the integral. Neglecting the electron mass, the integral is essentially

$$\int_k \frac{1}{k} \frac{1}{k} \frac{1}{k^2} \, d^4k . \tag{44}$$

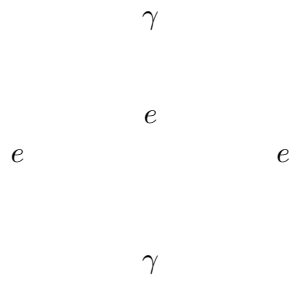


Fig. 5: Vertex correction in QED. The two electron lines bring a factor of $1/k$ each, the photon $1/k^2$.

There are four momenta in the denominator and a $d^4k \sim dkk^3$ in the numerator. This yields an integral over dk/k which is proportional to a logarithm. Thus we can write

$$\langle \mathcal{H} \rangle = e_R(\mu) \langle \bar{\Psi} \gamma^\mu A^\mu \Psi \rangle = e_R(\mu) \left(1 + \log(\frac{\mu^2}{p^2}) \right) . \tag{45}$$

Mathematically, we can view the last two terms as an integral over loop momenta from p^2 to μ^2. We see that the matrix element we wish to calculate depends on the product of a coupling, defined at μ, and the matrix element of the operator which contains the integration up to μ. If μ is chosen near p^2, the matrix element is simple; thus it is often said that the renormalization scale should lie near the physical one. However, in a complete calculation, the dependence on μ cancels between the coupling constant and the matrix element. Usually this happens only when perturbation theory is carried to an arbitrary order. Since this is not possible, there always remains a μ dependence in a calculation at some order.

The same procedure is now applied to the QCD gluon corrections to the effective Hamiltonian as in Fig. 6(a). Since we are interested in momenta far below the W mass, we take $M_W \gg \mu$. The loop

Fig. 6: Radiative corrections renormalizing the basic vertices. (a) is the correction to an effective four-fermion vertex, (b) takes into account W exchange.

momenta must be integrated from the low scale to the cutoff λ. But for momenta higher than $M - W$ the picture is invalid, and must be replaced by Fig. 6(b). Thus we split the total calculation in an integral from p^2 to μ using Fig. 6(a) and from μ to Λ using Fig. 6(b). This gives us the sum of

$$\int_{k \text{ small}} \frac{1}{k} \frac{1}{k} \frac{1}{k^2} \, d^4k \, , \tag{46}$$

and

$$\int_{k \text{ large}} \frac{1}{k} \frac{1}{k} \frac{1}{k^2} \frac{1}{k^2 - M_W^2} \, d^4k \, , \tag{47}$$

which yields

$$\log(\frac{\mu^2}{p^2}) + \log(\frac{M_W^2}{\mu^2}) \; . \tag{48}$$

We see again that the integration is split into a low-energy part and a high-energy part; the W mass acts as a cutoff on the otherwise divergent integral.

In analogy with the example from QED we then write for a matrix element of a typical four-fermion operator

$$A(I \to F) = \frac{G_F}{\sqrt{2}} \, V_{\rm CKM} C(\mu) \langle \mathcal{F}| \, O(\mu) \, |\mathcal{I}\rangle \; , \tag{49}$$

where the so-called Wilson coefficient contains $\log(M_W^2/\mu^2)$ and the matrix element $log(\mu^2/p^2)$. $V_{\rm CKM}$ is the relevant product of CKM matrix elements. Again, the matrix element contains the low-energy excitations, the Wilson coefficient the high-energy one.

In general, here basic W exchange gives rise to several effective operators, by applying the operator product expansion (OPE). Instead of Eq. (49) one then writes

$$A = \frac{G_F}{\sqrt{2}} \, V_{\rm CKM} \sum_i C_i(\mu) \langle \mathcal{F}| \, O_i(\mu) \, |\mathcal{I}\rangle \; . \tag{50}$$

The list of operators includes the usual four-fermion operators plus the operators that can be obtained from it through the various gluon exchanges. For instance, a gluon can alter the colours of the quarks. To see this, look at the operator in Eq. (40). When we also write the colour indices i, j explicitely, it reads

$$\mathcal{O}_2 = \left(\bar{d}_i \gamma^\mu (1-\gamma^5) u_i \right) \left(\bar{u_j} \gamma_\mu (1-\gamma^5) b_j \right) \; , \tag{51}$$

where the indices i and j are separately summed over. This means that the two factors are colour singlets. Exchanging a gluon between two quark lines as in Fig. 6 (a) also leads to the colour structure

$$\mathcal{O}_1 = \left(\bar{d}_i \gamma^\mu (1-\gamma^5) u_j \right) \left(\bar{u_j} \gamma_\mu (1-\gamma^5) b_i \right) \; , \tag{52}$$

which must be considered a new operator. QCD not only induces four other Fermi operators, but also others, and often many contribute to a given process, even if their particle structure does not quite look right [21].

The gluon loop brings with it a factor of the strong coupling constant α (0.2 at the Upsilon mass) apart from $\log(M_W^2/\mu^2)$. Since the scales M_W^2 and μ^2 are far apart (remember μ is to be chosen in the physics range, say m_B), the logarithm can be large. Thus it is useful to sum up the powers of $\alpha \log(M_W^2/\mu^2)$ in the Wilson coefficients. This important fact is explained below.

3.2 Calculating the μ dependence in the Wilson coefficients

A typical gluon loop as in Fig. 6(a) yields a factor $\alpha_s \log(\mu^2)$. This follows again from the integrand in the momentum integral which is of course the same as in Eq. (41).

Similarly, a two-loop diagram with two gluons yields two parts, one proportional to $[\alpha_s \log(\mu^2)]^2$, the other proportional to $\alpha_s \alpha_s \log(\mu^2)$. Since the effective argument in the logarithm is M_W^2/μ^2, the quantity $\alpha_s \log(\mu^2)$ is not so small and its powers should be summed up. The terms proportional to $[\alpha_s \log(\mu^2)]^N$ with $N = 1, 2, 3, ...$ make up the leading log approximation (LLA) and those proportional to $\alpha [\alpha_s log(\mu^2)]^N$ the next-to-leading log approximation (NLLA). Using the renormalization group techniques (see again Ref. [21] for an excellent review) one finds, for instance, at LLA the typical result

$$C(\mu) = \left[\frac{\alpha_s(\mu_0)}{\alpha_s(\mu)} \right]^{\frac{\gamma^{(0)}}{2\beta_0}} \, C(\mu) \; , \tag{53}$$

where $\gamma^{(0)}$ is the coefficient in front of the logarithm and β_0 describes the running of α_s. This shows how the Wilson coefficient at scale μ depends on its value at another scale μ_0 where it can be easily calculated. Because of asymptotic freedom, μ_0 is usually chosen quite high $\mu_0 \sim M_W$.

The complete calculation consists of three steps:

- Determine $C(\mu)$ (matching calculation). As mentioned, this is done at the scale $\mu_0 \sim M_W$ where the Wilson coefficients are matched to the full theory, i.e. with all particles, including the W. In the LLA the lowest-order expression can be used, in the NLL the order α_s corrections must be included.
- Determine the 'anomalous dimensions' $\gamma^{(0)}/2\beta_0$ in the LLA (leading gluon loop) or the NLLA (one loop more).
- Calculate the μ dependence in the matrix elements. This is the hardest step, since for non-perturbative calculations the μ dependence is essentially impossible to determine. One exception is inclusive decays where the usual perturbative methods apply.

The μ dependence should cancel between all contributions. Since we cannot usually include very high orders, a small dependence remains, which becomes smaller with increasing order of the calculation. As mentioned, the value of μ must be chosen near the physical scale. For instance in B physics it should be around m_B. To quantify the uncertainty coming from neglecting higher orders or being unable to determine the μ dependence, one lets μ vary in the interval $m_B/2 \leq \mu \leq 2m_B$.

An example where this procedure has been very successful is the inclusive decay $b \to s\gamma$ where the matrix element can be calculated. The LLA result for the branching ratio is $(2-3)10^{-4}$, whilst at the NLLA one obtains $(2.8-3.2)10^{-4}$ in the SM. This is in excellent agreement with the experiment [23].

In kaon physics, the natural scale would be $\mu \sim m_K$. However, at this low scale, QCD becomes highly nonperturbative. And even if effective methods are available, such as chiral perturbation theory, the scale-dependence may be different from that of perturbative QCD. For instance, while QCD has a logarithmic dependence, another method may lead to a quadratic behaviour. In this situation one must choose, *ad hoc*, an intermediate value, say 1 GeV. For a list of relevant references, see Ref. [24].

4. LOOP PROCESSES AND THE UNITARITY TRIANGLE

As mentioned, a major effort in particle physics is the determination of the unitarity triangle. Whilst some elements are determined by tree level processes, the elements V_{td}, V_{ts}, V_{tb}, occurring for instance in the side AB of the unitary triangle, can be obtained only from loops where the t quark is virtual. There are many useful observables, among them the parameter ϵ from kaon physics or $B\bar{B}$ mixing.

Since flavour loops are of general importance, we shall discuss them in some detail; historically they have played a crucial role (GIM)[5].

4.1 The Glashow, Iliopoulos, Maiani (GIM) mechanism

Consider the effective neutral current interaction vertex in Fig. 7

$$\bar{d}\gamma^\mu s Z_\mu \tag{54}$$

which gives rise to the highly suppressed decay $K \to \mu^-\mu^+$. In the SM, only the W bosons can change flavour, and so the contribution to the vertex comes from loops (Fig. 8) with u, c or t quarks) and W bosons.

From the form of the CKM matrix in Eq. (23) we see that the contributions of u and c are, respectively, proportional to

$$V_{us}V_{ud}^* F(m_u) \sim \lambda F(m_u/M_W) \tag{55}$$

$$V_{cs}V_{cd}^* F(m_c) \sim -\lambda F(m_c/M_W)\ . \tag{56}$$

Fig. 7: Effective $s\,d\,V^\mu$ coupling of a neutral vector to an s and a d quark.

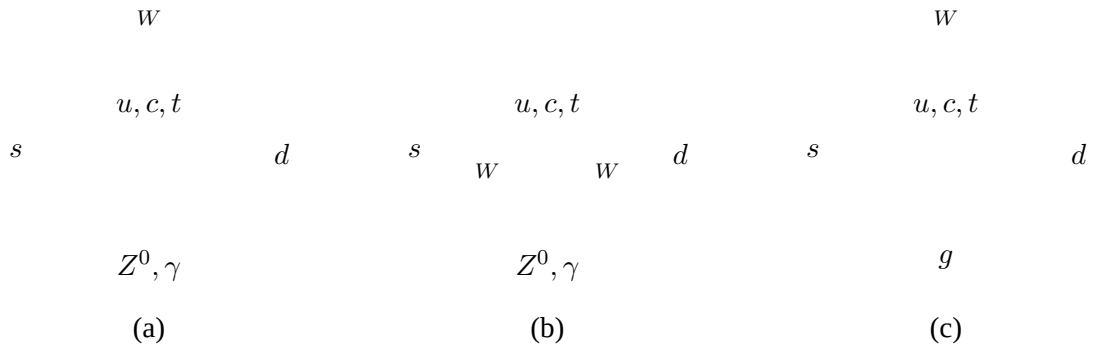


Fig. 8: Loop (so-called penguin) diagrams which contribute to the FCNC process $s \to d$. Diagrams (a) and (b) are electroweak penguins while diagram (c) is a gluonic penguin graph. Instead of external s and d quarks, one can also have b and s, etc. Then the diagrams contribute to FCNC transitions such as $b \to s$.

When we add the two (divergent) parts, we obtain the finite result

$$\lambda\Big(F(m_u/M_W) - F(m_c/M_W)\Big) \sim \lambda \frac{m_u^2 - m_c^2}{M_W^2} \; . \tag{57}$$

There is an important cancellation between the two parts; the result is finite and small, as desired. Without a c quark, the result would be useless. This was the original motivation for introducing the c quark before it was experimentally discovered [5].

This mechanism is universal: While the neutral gauge bosons do not change flavour in lowest order, loops of charged particles give rise to effective flavour-changing vertices. In theories beyond the SM, there can also be neutral loops, such as a squark–gluino loop if there are flavour changing gluino couplings in supersymmetric theories. In order to investigate the CKM matrix elements related to the t quark or investigate heavy virtual particles in general, we must consider loop diagrams. They allow processes not possible at the tree level (like $K \to \mu^- \mu^+$), often called rare decays, and are important for mixing and CP violation. Because they probe virtual particles, they can discern new physics. This so-called GIM mechanism, now so obvious, played a fundamental role in shaping the SM. As this GIM mechanism is very important, I would like to continue discussing it a bit more. In general, an s to d transition is proportional to

$$V_{us} V_{ud}^* F(m_u) + V_{cs} V_{cd}^* F(m_c) + V_{ts} V_{td}^* F(m_t) \; . \tag{58}$$

If all the quark masses were equal, this sum would be proportional to $V_{us} V_{ud}^* + V_{cs} V_{cd}^* + V_{ts} V_{td}^*$ which vanishes by the unitarity of V. This implies that the transition amplitude is proportional to the mass differences, which renders the effects finite and often small. Everything applies to any other neutral flavour transition, such as $b \to s$ or $c \to u$ with the obvious replacements. The dependence on the mass difference can be quadratic, $m_u^2 - m_c^2/M_W^2 \sim 10^{-3}$ as in Eq. 57. But in more complicated diagrams induced

through QCD corrections, it can also be logarithmic, i.e. proportional to $\log(m_u^2/m_c^2)$ which is considerably larger. This 'logarithmic GIM' can then lead to enhancements, which are important in $b \to s\gamma$ and particularly in the rare charm decay $c \to u\gamma$. In this case, similar loops contribute but the internal quarks are d, s. At lowest order, the quadratic GIM suppression factor $m_s^2 - m_d^2/M_W^2 \sim 10^{-6}$ is enormous and the resulting branching ratio is around 10^{-18}. The QCD corrections soften this to a logarithmic dependence $\alpha_s \log(m_s^2/m_d^2)$ and the resulting branching ratio becomes 10^{-8} [25].

4.2 K and B meson mixing

Another important flavour-changing effect is the 'mixing' between the K^0 and the $\bar{K}^0$ mesons and between B^0, $\bar{B}^0$. We introduce the subject here, and develop it as far as necessary to continue the discussion of the unitary triangle. More on this comes later. The formulas are written only for kaons, but the expressions for B mesons are analogous. Any good textbook on weak interactions gives these and more detailed expressions.

If CP is conserved, the states $|K^0\rangle$ and $|\bar{K}^0\rangle$, which themselves are not eigenstates of CP, combine to form eigenstates with definite CP properties:

$$|K_1\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\bar{K}^0\rangle\right) \qquad\qquad CP|K_1\rangle = |K_1\rangle \tag{59}$$

and

$$|K_2\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\bar{K}^0\rangle\right) \qquad\qquad CP|K_2\rangle = -|K_2\rangle\ . \tag{60}$$

But because of the small CP violation observed, the physical states (the mass eigenstates) are

$$|K_S\rangle = \frac{|K_1\rangle + \bar{\epsilon}\,|K_2\rangle}{(1+|\bar{\epsilon}|^2)^{\frac{1}{2}}} \quad \text{and} \quad |K_L\rangle = \frac{|K_2\rangle + \bar{\epsilon}\,|K_1\rangle}{(1+|\bar{\epsilon}|^2)^{\frac{1}{2}}} \tag{61}$$

(the parameter $\bar{\epsilon}$ depends on the phase convention chosen for the relation between $|K^0\rangle$ and $|\bar{K}^0\rangle$).

The 'mixing' between the states K^0 and $\bar{K}^0$ manifested in Eq. (59) can be seen as a transition K^0 to $\bar{K}^0$ and requires a $\Delta S = 2$ Hamiltonian. In the SM, it only arises through the well-known box diagram shown in Fig. 9. (Actually, this statement is not quite correct: There are so-called long-distance effects, commented on later, which also contribute. See, for example Ref. [26].) We use the Dirac notation with 'bras' and 'kets' to denote quantum mechanical states.

In the language of the effective Hamiltonians we have then

$$\begin{aligned}\mathcal{H}_{eff}^{\Delta S=2} \;&=\; \frac{G_F^2}{16\pi^2} M_W^2 \left[\lambda_c^2 \eta_1 S_0(x_c) \;+\; \lambda_t^2 \eta_2 S_0(x_t) + 2\lambda_c\lambda_t\eta_3 S_0(x_c,x_t)\right] \\ &\quad \times \left[\alpha_s^{(3)}(\mu)\right]^{-\frac{2}{9}} \left[1 + \frac{\alpha_s^{(3)}(\mu)}{4\pi} J_3\right] O^{\Delta S=2}(\mu) \;+\; \text{h.c.}\ .\end{aligned} \tag{62}$$

The renormalization scale μ must be chosen to lie between the kaon mass and 1 GeV and the usual notation is introduced: $x_i = m_i^2/M_W^2$ and $\lambda_i = V_{id}V_{is}^*$ (unitarity allows us to eliminate the dependence on λ_u and we set $x_u = 0$).

As we know (see previous and future discussions), the non-perturbative QCD effects for K^0–$\bar{K}^0$ mixing are all contained in the matrix elements of the single local composite operator

$$O^{\Delta S=2}(\mu) = \bar{s}\gamma^\mu(1-\gamma^5)d\ \bar{s}\gamma_\mu(1-\gamma^5)d\ . \tag{63}$$

The remaining terms in Eq. (62) can be calculated in perturbation theory. In particular, the functions S_i describe the loops, and the $\eta's$ the anomalous dimensions; J_3 plays a similar role. The values can be

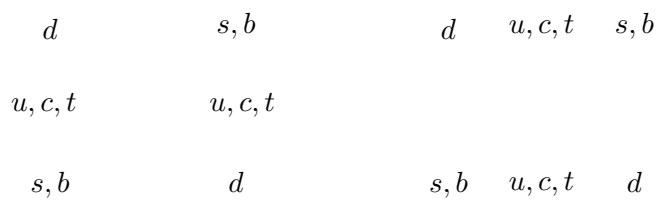


Fig. 9: Box diagrams which contribute to K^0–$\bar{K}^0$ and B^0–$\bar{B}^0$ mixing. The wavy lines are W bosons. A similar graph with obvious replacements also gives rise to D^0–$\bar{D}^0$ mixing, but it is small.

found in Ref. [21]. We note that for small x, $S_0(x)$ behaves as x. Also this diagram shows a (quadratic) GIM effect.

As mentioned repeatedly, the difficult part is evaluating the matrix element of the operator in Eq. (63), i.e.

$$\langle \bar{K}^0 \,|\, \bar{s}\gamma^\mu(1-\gamma^5)d\; \bar{s}\gamma_\mu(1-\gamma^5)d \,|K^0\rangle \equiv \frac{8}{3} m_K^2 f_K^2 B_K(\mu)\;. \tag{64}$$

The motivation for writing the matrix element in this form and introducing the parameter B_K comes from the vacuum saturation approximation. This is an often used concept and I digress a bit here. The matrix element of the operator above can be pictured as in Fig. 10 where the vertex represents the operator, and the $\bar{s}d$ pair on the left (and the $\bar{d}s$ on the right) are thought to merge into the K^0 (and $\bar{K}^0$) mesons. The calculation involves a most complicated maze of all possible gluon exchanges at all energies, quark–antiquark pairs, etc. Only one gluon is drawn.

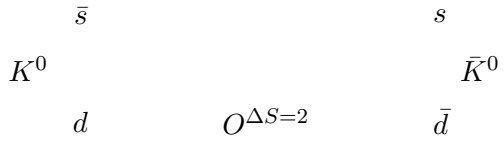


Fig. 10: One gluon contribution to the matrix element of the operator $O^{\Delta S=2}$.

Now, the quark pairs to the left and the right of the vertex are colour singlets, i.e. the quarks and the antiquarks left and right have 'opposite' colour. This means that the exchange of gluons from left to right is suppressed, since the monopole terms, depending on the charges only, cancel. There are four exchanges between left and right. It is like in electromagnetism: The forces between neutral objects are weaker. Thus, as a first approximation, we neglect these gluons and consider only those within the pairs. In this case, the matrix element 'factorizes': When we write

$$\langle \bar{K}^0 \,|\, \bar{s}\gamma^\mu(1-\gamma^5)d\; \bar{s}\gamma_\mu(1-\gamma^5)d \,|K^0\rangle = \sum_I \langle \bar{K}^0 \,|\, \bar{s}\gamma^\mu(1-\gamma^5)d \,|I\rangle\langle I|\bar{s}\gamma_\mu(1-\gamma^5)d \,|K^0\rangle \tag{65}$$

and neglect all gluons from left to right, only the intermediate state $|I\rangle = 0$ occurs. This state now has strongly interacting particles. We now use the well known expression

$$\langle \bar{K}^0 \,|\, \bar{s}\gamma^\mu(1-\gamma^5)d \,|0\rangle \equiv -i f_K p_{K\mu}\;, \tag{66}$$

where p_K is the kaon momentum and f_K is the kaon decay constant measured in the leptonic decay to be $f_K = 159.8 \pm 1.8$ MeV. Then the total matrix element is proportional to $(f_K p_{K\mu})(f_K p_K^\mu) = f_K^2 m_K^2$. The remaining factor $8/3$ is group-theoretical. Note that all gluons which are only left or only right

(as the one in the figure) are included. Thus in the factorization approximation we have $B_K = 1$. $B_K(\mu)$ depends on the renormalization scale, one introduces the renormalization group invariant B_K by:

$$\hat{B}_K = B_K(\mu) \left[\alpha_s^{(3)}(\mu)\right]^{-\frac{2}{9}} \left[1 + \frac{\alpha_s^{(3)}(\mu)}{4\pi} J_3\right] . \tag{67}$$

This **factorization approximation** is important and much used in studying the hadronic matrix elements. It often gives a good starting value, but it is impossible to assess its validity without a careful calculation.

In the present case, there are several calculations of B_K. A compilation of lattice results gives $\hat{B}_K = 0.90 \pm 0.06$ [27], the $1/N$ approximation $\hat{B}_K = 0.70 \pm 0.10$ [21]. From the relation (107) that relates the parameter ϵ to the imaginary part of the $\Delta S = 2$ matrix element in Eq. (64) one obtains

$$\epsilon = C\, \hat{B}_K\, \mathrm{Im}\, \lambda_t \left\{\mathrm{Re}\, \lambda_c \left[\eta_1 S_0(x_c) - \eta_3 S_0(x_c, x_t)\right] - \mathrm{Re}\, \lambda_t\, \eta_2 S_0(x_t)\right\}\, e^{i\frac{\pi}{4}} , \tag{68}$$

where

$$C = \frac{G_F^2 f_K^2 m_K M_W^2}{6\sqrt{2}\pi^2 (\Delta M_K)} = 3.78 \times 10^4 \tag{69}$$

and the mass difference $\Delta M_K = M_{K_L} - M_{K_S}$. The experimental value of ϵ is $2.2810^{-3} e^{i\pi/4}$ [2].

When rewritten in terms of the parameters $A, \bar{\rho}, \bar{\eta}$ of the unitary triangle, this yields

$$\bar{\eta} \left[(1 - \bar{\varrho}) A^2 \eta_2 S_0(x_t) + P_0(\varepsilon)\right] A^2 \hat{B}_K = 0.224 . \tag{70}$$

This equation traces out a hyperbolic curve in the $\bar{\rho}, \bar{\eta}$ plane. Because of the uncertainty in B_K it becomes a rather wide area; nevertheless it substantially constrains the allowed region for the parameters $\bar{\rho}$ and $\bar{\eta}$.

Mixing has also been observed in the B-system. The contributing graphs are analogous, one just replaces the d quark by a b quark. For B_s mixing, the quarks are b, s instead of s, d. There is no mixing between a B_d and a B_s. Why? Remember basic quantum mechanics.

Because of the values of the CKM matrix elements, the major contribution comes from the box with two internal top quarks. The effective $\Delta B = 2$ operator is clearly proportional to $(V_{td} V_{td})$. Using the formulas given below [Eq. (81)] to calculate the physical quantities from the matrix element, we get for a generic B_q meson

$$\Delta M_q = \frac{G_F^2}{6\pi^2} \eta_B m_{B_q} (\hat{B}_{B_q} F_{B_q}^2) M_{\mathrm{W}}^2 S_0(x_t) |V_{tq}|^2 , \tag{71}$$

where F_{B_q} is the B_q-meson decay constant.

Applied to B_d and B_s we get, using the central values of the various parameters,

$$\Delta M_d = 0.50/\mathrm{ps} \cdot \left[\frac{\sqrt{\hat{B}_{B_d}} F_{B_d}}{200\,\mathrm{MeV}}\right]^2 \left[\frac{\overline{m}_{\mathrm{t}}(m_{\mathrm{t}})}{170\,\mathrm{GeV}}\right]^{1.52} \left[\frac{|V_{td}|}{8.8 \cdot 10^{-3}}\right]^2 \left[\frac{\eta_B}{0.55}\right] \tag{72}$$

and

$$\Delta M_s = 15.1/\mathrm{ps} \cdot \left[\frac{\sqrt{\hat{B}_{B_s}} F_{B_s}}{240\,\mathrm{MeV}}\right]^2 \left[\frac{\overline{m}_{\mathrm{t}}(m_{\mathrm{t}})}{170\,\mathrm{GeV}}\right]^{1.52} \left[\frac{|V_{ts}|}{0.040}\right]^2 \left[\frac{\eta_B}{0.55}\right] . \tag{73}$$

Here, we have defined the renormalization group invariant parameters $\hat{B}_q$ in analogy to Eq. (67).

There are many calculations of F_{B_d} and $\hat{B}_d$. A typical values is

$$F_{B_d} \sim 220\,\mathrm{MeV} \tag{74}$$

with an error of around 20% [27]. However, the ratio

$$\frac{\sqrt{\hat{B}_{B_d}}F_{B_d}}{\sqrt{\hat{B}_{B_s}}F_{B_s}} \sim 1.14 \pm 0.08 \tag{75}$$

is known with better precision.

From the measured B^0 mass difference and Eq. (72) one obtains the value of $|V_{td}|$, i.e. circles in the unitary triangle around the corner B. Because $\hat{B}_{B_d}F_{B_d}$ is not so well known, they are relatively wide bands. To go further, one also uses Eq. (73). For the central values of the parameters m_t, $|V_{ts}|$ and η_B (note that unitarity fixes $|V_{ts}|$ rather well) we get

$$\Delta M_d = 0.50/\mathrm{ps} \cdot 1.44 \frac{\Delta M_s}{15.1/\mathrm{ps}} \left[\frac{\sqrt{\hat{B}_{B_d}}F_{B_d}}{\sqrt{\hat{B}_{B_s}}F_{B_s}}\right]^2 \left[\frac{\overline{m}_t(m_t)}{170\,\mathrm{GeV}}\right]^{1.52} \left[\frac{|V_{td}|}{8.8\cdot 10^{-3}}\right]^2 \left[\frac{\eta_B}{0.55}\right] . \tag{76}$$

Here, all values on the right (except, of course $|V_{td}|$ which we want to determine) are relatively well known.

At the moment, there exist only lower bounds on ΔM_s which translate into upper bounds on $|V_{td}|$ (see Fig. 11). As the lower bound increases and is replaced eventually by a measurement, $|V_{td}|$ decreases. The measurement of ΔM_s is difficult because it is relatively big and the B_s mesons must travel fast enough to make the oscillation length longer than the spatial resolution of the experiment. This requires high energies, even beyond LEP energies.

Together, all these results yield the present unitary triangle shown in Fig. 11. There are clearly no problems, but the allowed ranges are small.

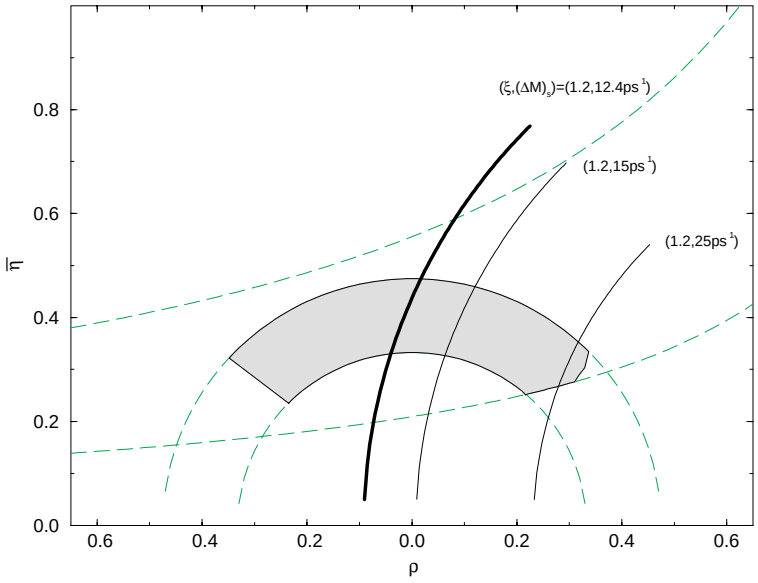


Fig. 11: Unitarity triangle. From Ref. [21].

In this section we have described the standard measurements of the unitary triangle. Any further measurement can test the consistency of the triangle.

One idea is to look for alternative ways to determine V_{td}. Again we must look at loops. The graphs in Fig. 8 suggest looking for transitions of the type $b \to d$ or $s \to d$. The first ones can be seen in the

inclusive decays $b \to d\gamma$ or in exclusive decays such as $B \to \rho\gamma$. However, there also, there seem to be hadronic uncertainties, but they may be overcome [28].

Another option is to look for V_{ub} through so-called annihilation decays, as shown in Fig. 12. The

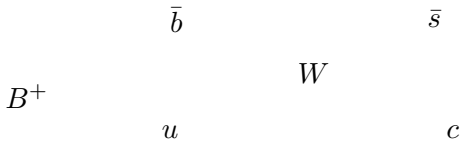


Fig. 12: Annihilation graph for the decay of a charged B meson into a $\bar{s}c$ state such as DK or $D_s^*\gamma$. The photon comes from radiation off any of the lines.

idea is clear from the graph; however, we must be certain that other decay mechanisms with other CKM factors are small. This has been proposed in Ref. [29] but so far the hadronic uncertainties are not clear.

Problem

Show that $B \to K\gamma$ is forbidden. Use gauge invariance (electromagnetic current conservation).

5. CP VIOLATION AND MIXING, RARE K-DECAYS

5.1 General formalism

We return to the meson mixing as begun above. We write here all formalism in terms of kaons, but the same applies with obvious changes also to the B system. From the states $|K^0\rangle$ and $|\bar{K}^0\rangle$ we construct the physical eigenstates by diagonalizing the free Hamiltonian for these particles. Instead of Eq. (61) we write in a slightly different notation

$$|K_S\rangle = \frac{p|K^0\rangle + q|\bar{K}^0\rangle}{\sqrt{|p|^2+|q|^2}} \qquad |K_L\rangle = \frac{p|K^0\rangle - q|\bar{K}^0\rangle}{\sqrt{|p|^2+|q|^2}} \; . \tag{77}$$

Comparison with Eq. (61) tells us that

$$\bar{\epsilon} = \frac{p-q}{p+q} \; . \tag{78}$$

We note that the relative phase of p and q are unimportant since the CP phases of the mesons are arbitrary. Thus only the magnitude $|q/p|$ is essential; if it is equal to one, there is no CP violation in the mass eigenstates.

Problem

Show this, by replacing the transformation in $K^0 \to \bar{K}^0$ used in Eq. (60) by the more general expression $K^0 \to e^{i\phi}\bar{K}^0$.

The Hamiltonian we consider here describes the time evolution of a state made up of K^0 and $\bar{K}^0$ mesons. In a quantum mechanical notation, we write

$$i\frac{d\psi(t)}{dt} = \hat{H}\psi(t) \qquad \psi(t) = \begin{pmatrix} |K^0(t)\rangle \\ |\bar{K}^0(t)\rangle \end{pmatrix} \; , \tag{79}$$

where

$$\hat{H} = \hat{M} - i\frac{\hat{\Gamma}}{2} = \begin{pmatrix} M_{11} - i\frac{\Gamma_{11}}{2} & M_{12} - i\frac{\Gamma_{12}}{2} \\ M_{21} - i\frac{\Gamma_{21}}{2} & M_{22} - i\frac{\Gamma_{22}}{2} \end{pmatrix} \tag{80}$$

with $\hat{M}$ and $\hat{\Gamma}$ being Hermitian matrices having positive (real) eigenvalues in analogy with the mass M and the width Γ of a single particle state. $\hat{M}$ is called the mass matrix and $\hat{\Gamma}$ the decay matrix. We note that because of the decays, $\hat{H}$ is not Hermitian and thus the diagonalization not necessarily unitary! Since $\hat{\Gamma}$ refers to decays, its elements $\hat{\Gamma}_{ij}$ are obtained from physical intermediate states, whereas the $\hat{M}_{ij}$ ones are obtained from virtual ones.

Problem

Think about these statements.

By standard algebra we diagonalize the system to find the physical quantites ($M = M_{11}$)

$$M_{L,S} = M \pm \mathrm{Re}Q \qquad \Gamma_{L,S} = \Gamma \mp 2\mathrm{Im}Q \tag{81}$$

$$Q = \sqrt{(M_{12} - i\frac{1}{2}\Gamma_{12})(M_{12}^* - i\frac{1}{2}\Gamma_{12}^*)}\,. \tag{82}$$

$$\Delta M = M_L - M_S = 2\mathrm{Re}Q \qquad \Delta\Gamma = \Gamma_L - \Gamma_S = -4\mathrm{Im}Q\,. \tag{83}$$

The quantities $\bar{\epsilon}$ and p, q are given by

$$\frac{1-\bar{\epsilon}}{1+\bar{\epsilon}} = \sqrt{\frac{M_{12}^* - i\frac{1}{2}\Gamma_{12}^*}{M_{12} - i\frac{1}{2}\Gamma_{12}}} = \frac{\Delta M - i\frac{1}{2}\Delta\Gamma}{2M_{12} - i\Gamma_{12}} \equiv r\exp(i\kappa) \tag{84}$$

and

$$\frac{q}{p} = \frac{2M_{12} - i\Gamma_{12}}{\Delta M - i\frac{1}{2}\Delta\Gamma}\,. \tag{85}$$

Depending on the actual values of the parameters, further simplifications are possible. In the kaon system, $\bar{\epsilon}$ is of order 10^{-3} and thus we can write

$$\Delta M_K = 2\mathrm{Re}M_{12}, \qquad \Delta\Gamma_K = 2\mathrm{Re}\Gamma_{12}\,, \tag{86}$$

where the subscript K indicates that these formulae apply only to the $K^0 - \bar{K}^0$ system.

The $K_\mathrm{L} - K_\mathrm{S}$ mass difference is experimentally measured to be [2]

$$\Delta M_K = M(K_\mathrm{L}) - M(K_\mathrm{S}) = (3.489 \pm 0.009)\cdot 10^{-15}\,\mathrm{GeV}\,. \tag{87}$$

In the SM roughly 70% of the measured ΔM_K is described by the real parts of the box diagrams with charm-quark and top-quark exchanges, whereby the contribution of the charm exchanges is by far dominant. This is related to the smallness of the real parts of the CKM top-quark couplings compared with the corresponding charm-quark couplings. A non-negligible contribution comes from the box diagrams with simultaneous charm and top exchanges. The remaining 20% of the measured ΔM_K is attributed to long-distance contributions which are difficult to estimate [26]. Further information with the relevant references can be found in Ref. [21]. The situation with $\Delta\Gamma_K$ is different. It is fully dominated by long-distance effects, due to the pion intermediate states. Experimentally one has the approximate relation $\Delta\Gamma_K \approx -2\Delta M_K$.

In the B system, the width difference is small, and Eq. (85) becomes

$$\frac{q}{p} \sim -\frac{M_{12}^*}{|M_{12}|}\,. \tag{88}$$

We see from this that in the B system (recall the last problem) there is essentially no CP violation in the eigenstates. If $|q/p| = 1$, the quantity $\bar{\epsilon}$ must be imaginary [as seen from Eq. (78)]. We also remember that the relative phase of $|q/p|$ is irrelevant, and thus we may choose $\bar{\epsilon} = 0$. In this case, there is no CP violation in mixing. Thus, CP violation in mixing requires $|q/p| \neq 1$. Since the physical situations in K and B are so different, it is easier to split the discussion at this point. Details about this can be found in Refs. [10], CP.

5.2 CP violation in the kaon system

In the kaon system, CP violation is seen in the decays

$$K \to \pi\pi \,. \tag{89}$$

This can be understood easily. Recall that without $\bar{\varepsilon}$ the eigenstates are K_1 and K_2 (no CP violation). The two-pion state always has CP parity $+1$, since Bose statistics forces them into a symmetric state and the angular momentum is zero. Since K_1 and K_2 have opposite CP parity, only one state can decay into two pions if CP is conserved. But experimentally, *both* states decay into two pions. While the decay of the state K_S with positive CP parity is expected, the state K_L also decays into two pions, in violation of the CP symmetry.

This can come about because of the admixture of K_1 (as indicated above) or from a CP violation in the decay amplitude of K_1. The situation is sketched below:

$$K_L \;\propto\; K_2 \;+\; \bar{\epsilon} K_1 \,. \tag{90}$$

$$\begin{array}{ccc} \text{Direct } (\varepsilon') & & \pi\pi \\ & & \text{Indirect } (\varepsilon_K) \\ \pi\pi & & \end{array}$$

The violation of CP invariance in the mixing is a $\Delta S = 2$ effect (strangeness change by two units) while the direct one is $\Delta S = 1$. Until recently, there was no evidence for direct CP violation.

To continue, we repeat some well-known algebra. Since CP violation means that both, the K_L and the K_S decay into two pions, one defines the CP violating ratios

$$\eta_{00} = \frac{A(K_{\rm L} \to \pi^0\pi^0)}{A(K_{\rm S} \to \pi^0\pi^0)} \,, \qquad \eta_{+-} = \frac{A(K_{\rm L} \to \pi^+\pi^-)}{A(K_{\rm S} \to \pi^+\pi^-)} \,. \tag{91}$$

To relate them to the calculable quantities, one decomposes the kaon decay amplitudes into isospin components as follows:

$$A(K^+ \to \pi^+\pi^0) = \sqrt{\frac{3}{2}} A_2 e^{i\delta_2} \tag{92}$$

$$A(K^0 \to \pi^+\pi^-) = \sqrt{\frac{2}{3}} A_0 e^{i\delta_0} + \sqrt{\frac{1}{3}} A_2 e^{i\delta_2} \,. \tag{93}$$

$$A(K^0 \to \pi^0\pi^0) = \sqrt{\frac{2}{3}} A_0 e^{i\delta_0} - 2\sqrt{\frac{1}{3}} A_2 e^{i\delta_2} \,. \tag{94}$$

Here the subscript $I = 0, 2$ denotes states with isospin $0, 2$ equivalent to $\Delta I = 1/2$ and $\Delta I = 3/2$ transitions, respectively, and $\delta_{0,2}$ are the corresponding strong phases. The weak CKM phases are contained in A_0 and A_2. The isospin amplitudes A_I are complex quantities which depend on phase conventions.

The corresponding decay amplitudes of the antiparticles take the same form, but with the A_0, A_2 replaced by their complex conjugate.

Now, we can insert Eqs. (92)–(94) into the definition of the $\eta's$. While for K_1 we get the real parts of the A_0, A_2, it will be the imaginary part for K_2. When we neglect small terms like $\bar{\epsilon}\,(A_2/A_0)$ or $\bar{\epsilon}^2$ we are lead to

$$\eta_{+-} \sim \frac{\bar{\epsilon} + i{\rm Im}A_0/{\rm Re}A_0 + i{\rm Im}A_2/{\rm Re}A_0 e^{i(\delta_2-\delta_0)}/\sqrt{2}}{1 + {\rm Re}(A_2/A_0) e^{i(\delta_2-\delta_0)}/\sqrt{2}} \tag{95}$$

and

$$\eta_{00} \sim \frac{\bar{\epsilon} + i{\rm Im}A_0/{\rm Re}A_0 - 2i{\rm Im}A_2/{\rm Re}A_0 e^{i(\delta_2-\delta_0)}/\sqrt{2}}{1 - 2{\rm Re}(A_2/A_0) e^{i(\delta_2-\delta_0)}/\sqrt{2}}. \tag{96}$$

These hold in any phase convention, where $|\bar{\epsilon}\mathrm{Im}A_0/\mathrm{Re}A_0|$ is small.

Now, defining

$$\varepsilon = \bar{\epsilon} + \frac{i\mathrm{Im}A_0}{\mathrm{Re}A_0} \tag{97}$$

$$\epsilon_2 = \frac{i\mathrm{Im}A_2}{\sqrt{2}\mathrm{Re}A_0} e^{i(\delta_2-\delta_0)} \tag{98}$$

$$\zeta = \frac{\sqrt{2}}{\mathrm{Re}(A_2/A_0)} e^{i(\delta_2-\delta_0)} \tag{99}$$

$$\varepsilon' = \epsilon_2 - \frac{i\mathrm{Im}A_0}{\mathrm{Re}A_0}\zeta, \tag{100}$$

we can write

$$\eta_{+-} = \frac{\varepsilon + \epsilon_2}{1-2\zeta} \sim \varepsilon + \varepsilon' \tag{101}$$

$$\eta_{00} = \frac{\varepsilon - 2\epsilon_2}{1-2\zeta} \sim \varepsilon - 2\varepsilon'. \tag{102}$$

The isospin decomposition also yields

$$\varepsilon = \frac{A(K_\mathrm{L} \to (\pi\pi)_{I=0})}{A(K_\mathrm{S} \to (\pi\pi)_{I=0})} \tag{103}$$

$$\sqrt{2}\epsilon_2 = \frac{A(K_\mathrm{L} \to (\pi\pi)_{I=2})}{A(K_\mathrm{S} \to (\pi\pi)_{I=0})} \tag{104}$$

$$\sqrt{2}\zeta = \frac{A(K_\mathrm{S} \to (\pi\pi)_{I=2})}{A(K_\mathrm{S} \to (\pi\pi)_{I=0})}\,. \tag{105}$$

From this, one also gets

$$\varepsilon' = \frac{1}{\sqrt{2}}\mathrm{Im}\left(\frac{A_2}{A_0}\right)\exp(i\Phi_{\varepsilon'})\,, \qquad \Phi_{\varepsilon'} = \frac{\pi}{2} + \delta_2 - \delta_0\,. \tag{106}$$

Using the theoretical expressions in Eq. (79) we arrive at

$$\varepsilon = \frac{\exp(i\pi/4)}{\sqrt{2}\Delta M_K}\,(\mathrm{Im}M_{12} + 2\xi\mathrm{Re}M_{12})\,, \qquad \xi = \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\,, \tag{107}$$

where the term involving $\mathrm{Im}M_{12}$ represents $\bar{\epsilon}$ defined in Eq. (84). The phase-convention dependence of the term involving ξ cancels the convention dependence of $\bar{\varepsilon}$ so that ε is free from this dependence. On the other hand, ε' measures the difference between the phases of A_2 and A_0 and is a physical quantity. The strong phases $\delta_{0,2}$ can be extracted from $\pi\pi$ scattering. Then $\Phi_{\varepsilon'} \approx 40^\circ$. (The situation is more complicated than this. Because of isospin violations, one cannot just add strong phase shifts δ; the electromagnetic part will make this a two-channel problem, see Ref. [30].)

Comment

When we choose a convention where $\mathrm{Im}A_0$, then $\varepsilon = \bar{\epsilon}$.

Since the discovery of CP violation in 1964 [13], manifested in the non-zero value of ε, the question remained whether also ε' was different from zero. Since ε is due to mixing, that is to a $\Delta S = 2$ transition, one could imagine a new interaction, the superweak force [31] that only violated CP invariance and had only a $\Delta S = 2$ part. Then ε' would vanish. Indeed, for over thirty years, only upper limits were measured and the superweak model could not be excluded. But it was understood that the SM would give a very small value of ε' if the top quark were heavy. This is indeed the case, and much effort (unfortunately not

enough) was spent calculating ε' in the SM. Various estimates gave $\varepsilon/\varepsilon' \sim 10^{-4} - 10^{-3}$, however, with a large error. It came then as a surprise that in 1999 the two experiments at CERN and at Fermilab obtained a definite non-zero value of [32] [33]

$$\varepsilon/\varepsilon' = (21.4 \pm 4.0) \times 10^{-4} \tag{108}$$

finally establishing direct CP violation. At the moment it is too early to draw also quantitative conclusions from this result, since the theoretical calculation is so involved. The problem is the calculation of the matrix elements as in Eq. (50). There are many operators contributing to ε/ε'. It is common belief that two of them, the penguin penguin operator

$$O_6 = (\bar{s}_\alpha d_\beta)_{V-A} \sum_{q\,=\,u,d,s} (\bar{q}_\beta q_\alpha)_{V+A} \tag{109}$$

and the electroweak penguin operator

$$O_8 = \frac{3}{2}\,(\bar{s}_\alpha d_\beta)_{V-A} \sum_{q\,=\,u,d,s} e_q (\bar{q}_\beta q_\alpha)_{V+A} \tag{110}$$

contribute most. Here, the subscript $V\pm$ simply means that both quarks are left handed or both right handed. In other words

$$(\bar{q}q)_{V\pm A}(\bar{q}q)_{V\pm A} = (\bar{q}\gamma^\mu(1\pm\gamma_5)q)(\bar{q}\gamma_\mu(1\pm\gamma_5)q)\,. \tag{111}$$

A complete list of operators and numerical methods can be found in Refs. [21, 34]. Since the isospin state of the two pions is important, one considers the isospin properties of the operators. Each operator contributes both to the isospin $1/2$ and $3/2$ changing amplitudes. O_6 has isospin $1/2$, however, isospin breaking (difference of up and down quark masses, electromagnetic corrections) induces also a $3/2$ contribution, thereby 'thinning' out the $1/2$ component. This is not so important for the $3/2$ part, dominated by O_8, which obtains a major contribution from the top quark.

The matrix elements of definite pion isospin I, $\langle(\pi\pi)_I|Q_i|K\rangle$, are usually given in units of their so-called vacuum insertion value or factorization value, $\langle(\pi\pi)_I|Q_i|K\rangle_0$. This is similar to the discussion after Eq. (64) and will be elaborated on further below. As before, a B will describe the proportionality. The vacuum insertion value can be expressed in terms of measured quantities, like decay constants, meson masses, and less well known parameters, such as the quark masses. In the present case, we have

$$\begin{aligned}
\langle(\pi\pi)_0|Q_6|K\rangle &= -4\sqrt{\frac{3}{2}}\left[\frac{m_{\rm K}^2}{m_{\rm s}(\mu)+m_{\rm d}(\mu)}\right]^2 \frac{F_\pi}{\kappa}\,B_6^{(1/2)}\,, &(113)\\
\langle(\pi\pi)_2|Q_8|K\rangle &= -\left[\frac{\kappa}{2\sqrt{2}}\langle\overline{Q_6}\rangle_0 + \frac{\sqrt{2}}{6}X\right] B_8^{(3/2)}\,, &(114)
\end{aligned}$$

where

$$\kappa = \frac{\Lambda_\chi^2}{m_{\rm K}^2 - m_\pi^2} = \frac{F_\pi}{F_{\rm K} - F_\pi}\,, \tag{115}$$

$$X = \sqrt{\frac{3}{2}}F_\pi\left(m_{\rm K}^2 - m_\pi^2\right)\,. \tag{116}$$

Using these quantities, we finally arrive at [21, 34]

$$\frac{\varepsilon'}{\varepsilon} \approx 13\,{\rm Im}\,\lambda_t\left[\frac{110\,{\rm MeV}}{m_{\rm s}(2\ {\rm GeV})}\right]^2\left[B_6^{(1/2)}(1-\Omega_{\eta+\eta'}) - 0.4\cdot B_8^{(3/2)}\left(\frac{m_{\rm t}}{165\,{\rm GeV}}\right)^{2.5}\right]\left(\frac{\Lambda_{\overline{\rm MS}}^{(4)}}{340\ {\rm MeV}}\right)\,. \tag{117}$$

The presence of the strange-quark mass is due to the parametrization of the matrix element in terms of the vacuum insertion value; the B's should be proportional to it. The quantity $\Omega_{\eta+\eta'}$ represents the 'dilution' due to isospin breaking as discussed above. While most authors take it to be ≈ 0.25, a recent paper [35] advocates a lower value of ≈ 0.16.

We see that for positive B's there is a substantial cancellation between the two matrix elements; this, together with the large uncertainty in the values of the B's themselves and of m_s, results in a very large uncertainty of the prediction. I do not want to enter into the technical discussions surrounding these values. At the moment, a strange-quark mass of 90–120 MeV seems reasonable [27]. For the matrix elements, various methods have been used. There are results from lattice QCD simulations, large N methods or the chiral quark model. For a review see Ref. [34]; see also Refs. [36, 37] and recent work which includes final state interactions [38]. The present typical ranges are 1.1–1.6 for $B_6^{(1/2)}$ and 0.5–1 for $B_8^{(3/2)}$. These result in a value of around 10^{-3} for ε'/ε, somewhat below the experimental value. Recently, it was also pointed out [38] that final-state rescatterings (among the two pions) which are not taken into account properly by all the above methods would increase the value of $\epsilon'\epsilon$, because $B_6^{(1/2)}$ would be enlarged. This last issue is also under debate in the lattice community.

The situation is very interesting, and much effort will be made to pin down the theoretical number. At the moment it is too early to claim a disagreement with the SM.

5.3 The decays $K \to \pi\nu\bar{\nu}$

Also the decays $K \to \pi\nu\bar{\nu}$ may give important information on the unitarity triangle and new physics. As stressed especially by Buras, the QCD-related uncertainties largely cancel, and so clean predictions are possible. Therefore, I discuss these decays in some detail, although experiments seem far away. The decays $K^+ \to \pi^+\nu\bar{\nu}$ and $K^0 \to \pi^0\nu\bar{\nu}$ occur in the SM through penguin diagrams with the Z (see Fig. 8) where the Z couples to $\nu\bar{\nu}$ and boxes of two W (see Fig. 9). This makes them particularly sensitive to the top quark and thus to the not-so-well known CKM matrix elements. Because in this case the QCD corrections are well under control, these decays are an important future input for scrutinizing the SM. The importance of this decay has been particularly emphasized by Buras and collaborators [21] and we follow his treatment. The drawbacks are the relatively low branching fractions of around 10^{-11} and the difficulties in detecting the neutral decay. One decay $K \to \pi\nu\bar{\nu}$ has been seen at Brookhaven, resulting in a branching ratio of $(4.6 + 9.7 - 3.5)10^{-10}$ [39].

The Z penguin and the two-W box give rise to an effective Hamiltonian of the form (see Fig. 8 where the Z couples to a a neutrino pair or the box diagram of the left graph of Fig. 9 where one quark line is replaced by external neutrinos and internal leptons)

$$\mathcal{H}_{\text{eff}} = \frac{G_{\text{F}}}{\sqrt{2}} \frac{\alpha}{2\pi \sin^2 \Theta_{\text{W}}} \sum_{l=e,\mu,\tau} \left(V_{cs}^* V_{cd} X_{\text{NL}}^l + V_{ts}^* V_{td} X(x_t) \right) (\bar{s}d)_{V-A} (\bar{\nu}_l \nu_l)_{V-A} \,. \tag{118}$$

The X are loop functions, X_{NL}^l is due to charm and $X(x_t)$ to top. The matrix element of the operator can be related by the isospin argument

$$\langle \pi^+ | (\bar{s}d)_{V-A} | K^+ \rangle = \sqrt{2} \langle \pi^0 | (\bar{s}u)_{V-A} | K^+ \rangle \tag{119}$$

to that of the operator

$$(\bar{s}u)_{V-A} (\bar{\nu}_e e)_{V-A} \tag{120}$$

which governs the charged-current decay $K \to \pi e\bar{\nu}$. Thus this uncertainty disappears and one gets

$$\text{Br}(K^+ \to \pi^+\nu\bar{\nu}) = \kappa_+ \left[\left(\frac{\text{Im}\lambda_t}{\lambda^5} X(x_t) \right)^2 + \left(\frac{\text{Re}\lambda_c}{\lambda} P_0(X) + \frac{\text{Re}\lambda_t}{\lambda^5} X(x_t) \right)^2 \right] , \tag{121}$$

where

$$\kappa_+ = r_{K^+} \frac{3\alpha^2 Br(K^+ \to \pi^0 e^+ \nu)}{2\pi^2 \sin^4 \Theta_{\rm W}} \lambda^8 = 4.11 \times 10^{-11} \,. \tag{122}$$

Here, $P_0(X) = 0.42 \pm 0.06$ is a particular combination of the $X^l_{\rm NL}$. Putting in all the numbers, one arrives at

$$\mathrm{Br}(K^+ \to \pi^+ \nu\bar{\nu}) = 4.11 \times 10^{-11} A^4 X^2(x_t) \frac{1}{\sigma} \left[(\sigma\bar{\eta})^2 + (\varrho_0 - \bar{\varrho})^2 \right] , \tag{123}$$

where

$$\sigma = \left(\frac{1}{1 - \frac{\lambda^2}{2}} \right)^2 \tag{124}$$

$$\varrho_0 = 1 + \frac{P_0(X)}{A^2 X(x_t)} \,. \tag{125}$$

This cuts out an ellipse in the $(\bar{\varrho}, \bar{\eta})$ plane.

The neutral decay $K^0 \to \pi^0 \nu\bar{\nu}$ is even more interesting. In the one-Z exchange approximation it violates CP invariance. Therefore it is sensitive to $Im\lambda_t$ and thus only the top quark loop contributes.

Problem

Show this. Use the CP properties of the currents involved and of the Z.

The effective Hamiltonian is in analogy to above

$$\mathcal{H}_{\rm eff} = \frac{G_{\rm F}}{\sqrt{2}} \frac{\alpha}{2\pi \sin^2 \Theta_{\rm W}} V^*_{ts} V_{td} X(x_t) (\bar{s}d)_{V-A} (\bar{\nu}\nu)_{V-A} + \mathrm{h.c.} \,, \tag{126}$$

where the function $X(x_t)$, present already in $K^+ \to \pi^+ \nu\bar{\nu}$, includes NLO corrections and is given in Ref. [21].

Strong-interaction corrections can be taken into account in the same way as before, by using

$$\langle \pi^0 | (\bar{d}s)_{V-A} | \bar{K}^0 \rangle = \langle \pi^0 | (\bar{s}u)_{V-A} | K^+ \rangle \tag{127}$$

and expressing the square of this matrix element through the decay rate of $K^+ \to \pi^0 e^+ \nu$. This then leads to

$$\frac{\mathrm{Br}(K_L \to \pi^0 \nu\bar{\nu})}{\mathrm{Br}(K^+ \to \pi^0 e^+ \nu)} = 3 \frac{\tau(K_L)}{\tau(K^+)} \frac{\alpha^2}{|V_{us}|^2 2\pi^2 \sin^4 \Theta_{\rm W}} \left[\mathrm{Im}\lambda_t \cdot X(x_t) \right]^2 \tag{128}$$

where $\lambda_t = V^*_{ts} V_{td}$. Note that the neutrino flavours were summed over here. Using all the numerical values and expressing the CKM matrix elements through the parameters $\bar{\rho}$ and $\bar{\eta}$, one ends up with

$$\mathrm{Br}(K_{\rm L} \to \pi^0 \nu\bar{\nu}) = 3.0 \times 10^{-11} \left[\frac{\eta}{0.39} \right]^2 \left[\frac{\overline{m}_{\rm t}(m_{\rm t})}{170\ \mathrm{GeV}} \right]^{2.3} \left[\frac{|\ V_{cb}\ |}{0.040} \right]^4 . \tag{129}$$

We see that this decay yields a very precise determination of the height of the unitary triangle. There are essentially no theoretical uncertainties, but the experiments for which there are plans at Fermilab and Brookhaven are very difficult. It should be noted that, because of the theoretical accuracy, these decays are ideal to search for new interactions. For instance, as shown in Ref. [40], supersymmetry may increase the rates of the decays $K \to \pi\nu\bar{\nu}$. Thus, a branching ratio well above the SM prediction is a clear sign of new physics.

Also rare decays of B mesons are of great interest. The most discussed decay is $b \to s\gamma$ which was briefly discussed above. A discussion of rare B decays and their possible implications for new physics can be found in Ref. [10]. I will not discuss this interesting field further, mainly because I have worked on it myself for a long time.

6. CP VIOLATION IN THE B SYSTEM

6.1 General formalism and mixing

CP violation in B meson physics is in the centre of interest in flavour physics. Indeed, two new B factories, BaBar at SLAC and Belle at KEK with the goal of finding CP violation have come into operation.

In these experiments, a $B^0\bar{B}^0$ pair is produced at time zero; then each state develops in time (and corresponding distance) and we want to know its composition at time t. Quantum mechanics determines their development in time; see also Ref. [9] (in particular the article by Bigi et. al.) and [10].

As in the kaon system [compare Eq. (77)], mixing between the two different B mesons $B^0 = \bar{b}d$ and $\bar{B}^0 = \bar{d}b$ yields two eigenstates B_1 and B_2

$$|B_1 = \frac{p|B^0\rangle + q|\bar{B}^0\rangle}{\sqrt{|p|^2+|q|^2}} \qquad |B_2 = \frac{pB^0\rangle - q\bar{B}^0\rangle}{\sqrt{|p|^2+|q|^2}} \,. \tag{130}$$

Diagonalizing the $\Delta B = 2$ Hamiltonian and using the Schrödinger equation (79), one obtains for the time development of an initial $B^0 =$ state

$$|B^0_{\rm phys}(t)\rangle = g_+(t)\,|B^0\rangle + \left(\frac{q}{p}\right) g_-(t)|\bar{B}^0\rangle \,, \tag{131}$$

where the functions $g_+(t)$ and $g_-(t)$ give the amplitude (square root of the probability) that the state $|B^0_{\rm phys}(t)\rangle$ is a B^0 or a $\bar{B}^0$, respectively. They follow from solving the Schrödinger equation (79) and are given by

$$g_+(t) \;=\; \exp\left[-\frac{\Gamma t}{2}\right] \exp[-iMt]\,\cos\left(\frac{\Delta M\, t}{2}\right) \,, \tag{132}$$

$$g_-(t) \;=\; \exp\left[-\frac{\Gamma t}{2}\right] \exp[-iMt]\, i \sin\left(\frac{\Delta M\, t}{2}\right) \,. \tag{133}$$

We have neglected the tiny width difference between the eigenstates; M is the average mass of B_1 and B_2 (recall, in the kaon system the widths are very different). For an initial $\bar{B}^0$ p and q (and $\bar{B}^0$ and B^0) must be exchanged. We also need the amplitudes for the decay of a B^0 into a final state f:

$$A_f \equiv \langle f|\mathcal{H}|B^0\rangle \;\text{ and }\; \bar{A}_f \equiv \langle f|\mathcal{H}|\bar{B}^0\rangle \,. \tag{134}$$

Similarly, we define the amplitudes $A_{\bar{f}}$ and $\bar{A}_{\bar{f}}$ for the decays into the CP conjugated state $\bar{f}$.

Finally, we write

$$\lambda_f \equiv \frac{q}{p}\,\frac{\bar{A}_f}{A_f} \qquad \text{and} \qquad \lambda_{\bar{f}} \equiv \frac{p}{q}\,\frac{A_{\bar{f}}}{\bar{A}_{\bar{f}}} \tag{135}$$

which are independent of the phase convention chosen for the CP transformation. Note that these quantities are useful only if both B^0 and $\bar{B}^0$ can decay into the same final state f. Of course, there are final states (so-called flavour-specific states) where this is impossible.

For the amplitude of the decay of the initial B^0 and $\bar{B}^0$ state into f or $\bar{f}$ at time t one gets

$$\langle f\,|\,B^0_{\rm phys(t)}\rangle = A_f\,[g_+(t) \,+\, \lambda_f\, g_-(t)] \;\text{ and }\; \langle \bar{f}\,|\bar{B}^0_{\rm phys(t)}\rangle = \bar{A}_{\bar{f}}\,[g_+(t) \,+\, \lambda_{\bar{f}}\, g_-(t)] \,. \tag{136}$$

Squaring this, we get the probabilities that the initial B^0 and $\bar{B}^0$ states end up as f and $\bar{f}$ at time t:

$$\begin{aligned}
\Gamma(B^0_{\rm phys}(t) \to f) \;&=\; |A_f|^2\, e^{-\Gamma t}\left[\frac{1+|\lambda_f|^2}{2} \,+\, \frac{1-|\lambda_f|^2}{2}\cos(\Delta M\, t) - \mathrm{Im}\,\lambda_f\, \sin(\Delta M\, t)\right] \\
\Gamma(\bar{B}^0_{\rm phys}(t) \to \bar{f}) \;&=\; |\bar{A}_{\bar{f}}|^2\, e^{-\Gamma t}\left[\frac{1+|\lambda_{\bar{f}}|^2}{2} \,+\, \frac{1-|\lambda_{\bar{f}}|^2}{2}\cos(\Delta M\, t) + \mathrm{Im}\,\lambda_{\bar{f}}\, \sin(\Delta M\, t)\right] .
\end{aligned} \tag{137}$$

We note again that these expressions should only be used if the amplitudes A_f and $\bar{A}_{\bar{f}}$ are non-zero. Then they also apply for flavour-specific states where $\bar{A}_f$ and $A_{\bar{f}}$ vanish; in this case the λ's vanish. The formulas also hold for charged B decays if we set $\sin(\Delta M\, t\, =\, 0)$. The square brackets are then simply 1.

If, however, A_f and $\bar{A}_{\bar{f}}$ vanish, but not the $\bar{A}_f$ and $A_{\bar{f}}$, instead of going through a mathematical limiting procedure, we go back to Eq. (131) and use simply

$$\langle f \,|\, B^0_{\mathrm{phys}(t)}\rangle = \bar{A}_f \frac{q}{p}\, g_-(t) \quad \text{and} \quad \langle \bar{f}\,|\bar{B}^0_{\mathrm{phys}(t)}\rangle = A_{\bar{f}} \frac{p}{q}\; g_-(t)\;. \tag{138}$$

For CP studies, we are interested in the difference of decay probabilities at time t of particles and antiparticles. We define therefore the **CP-violating asymmetries**

$$\mathcal{A}_f(t) \equiv \frac{\Gamma(B^0_{\mathrm{phys}}(t) \to f) - \Gamma(\bar{B}^0_{\mathrm{phys}}(t) \to \bar{f})}{\Gamma(B^0_{\mathrm{phys}}(t) \to f) + \Gamma(\bar{B}^0_{\mathrm{phys}}(t) \to \bar{f})}\;. \tag{139}$$

A non-zero value of this asymmetry is a signal for CP violation.

We see that there are many effects that can induce CP violation, because the rates and therefore the asymmetry is a complicated expression. The first we consider are the amplitudes A_f and $\bar{A}_{\bar{f}}$. CP violation occurs when their absolute values are different. Since this violation resides in the decay amplitudes and not in the mixing (which resides in the square brackets in Eq. (138) one calls it **direct**. In particular for charged decays, it is the only source of CP violation since there the square bracket is 1. This asymmetry is time-independent and takes the form

$$\mathcal{A}_f = \frac{|A_f|^2 - |\bar{A}_f|^2}{|A_f|^2 + |\bar{A}_f|^2}\;. \tag{140}$$

There are usually several decay mechanisms contributing to the total decay amplitude A_f. For instance, a particular decay can go via a tree-level vertex or by a loop where new particles are formed as an intermediate state which then decays into f. Such diagrams pick up a so-called rescattering phase, δ. This phase has nothing to do with CP violation and is the same for particles and antiparticles. Thus the amplitude takes the general form

$$A_f = \sum_i A_i\, e^{i\delta_i}\, e^{i\phi_i} \quad \text{and} \quad \bar{A}_{\bar{f}} = \sum_i A_i\, e^{i\delta_i}\, e^{-i\phi_i}\;, \tag{141}$$

where the ϕ_i are the **weak** phases and the δ_i the rescattering (or **strong**) phases described above. This form holds if the CP phases of f and B are equal, otherwise a correcting phase must be included in $\bar{A}_{\bar{f}}$ (as noted above, under the CP transformation a particle goes into its antiparticle times the so-called CP phase).

When we calculate the squares of the amplitudes in Eq. (141), we see that they are different (required for a non-zero $\mathcal{A}_f$) only if there are at least two-decay mechanisms, with different weak *and* strong phases. In fact, for two-decay chains we get

$$\mathcal{A}_f \sim \sin(\phi_2 - \phi_1)\sin(\delta_2 - \delta_1)\;. \tag{142}$$

Problem

Show Eq. (142).

Measuring then the asymmetries gives information on the weak phases ϕ which come from CKM matrix elements or new interactions. But this requires a theoretical calculation of the strong phases δ_i which is very difficult, and no accurate predictions can be made. We shall come back to this later [see Eq. (215)].

We return to the asymmetry Eq. (139) for neutral B mesons. We consider first the flavour-specific decays where the final state is semileptonic, $f = l^- \nu X$. Since only the $\bar{B}^0$ can decay into this, we have the situation described in Eq. (138). The decay is mediated by the standard tree-level weak decay; there is only one weak phase and thus $\bar{A}_f = A^*_{\bar{f}}$. The rates [see Eq. (138)] are proportional to $|\frac{q}{p}|^2$ and $|\frac{p}{q}|^2$ and the asymmetry becomes

$$\mathcal{A}_f(t) = \frac{1 - |\frac{q}{p}|^4}{1 + |\frac{q}{p}|^4} \, . \tag{143}$$

This asymmetry completely depends on the mixing mechanism; it is called **indirect or mixing-induced CP violation**. As discussed above [Eq. (88)], $|q/p| \sim 1$; an estimate gives around 10^{-2}. We may safely take $|q/p| = 1$ in the following. It is difficult to measure such a small number; the hope is that exotic new physics could enhance the asymmetry.

6.2 Mixing-decay asymmetries

The most popular situation is when f is not flavour-specific and also a CP eigenstate, e.g. $f = \bar{f}$ (Ref. [9] and in particular the article by Bigi et. al.). Then we find that $\lambda_{\bar{f}} = 1/\lambda_f$ and we obtain from Eq. (138)

$$\Gamma(B^0_{\rm phys}(t) \to f) = |A_f|^2 \, e^{-\Gamma t} \left[\frac{1 + |\lambda_f|^2}{2} \; - \; \frac{1 - |\lambda_f|^2}{2} \cos(\Delta M \, t) - {\rm Im}\, \lambda_f \, \sin(\Delta M \, t) \right] |\frac{p}{q}|^2 . \tag{144}$$

The CP-violating asymmetry takes the form

$$\mathcal{A}_f(t) = \frac{(1 - |\lambda|^2) \cos(\Delta M \, t) - 2{\rm Im}\, \lambda \, \sin(\Delta M \, t)}{1 + |\lambda|^2} \, . \tag{145}$$

We need to estimate the λ's. From Eq. (88) we have

$$\frac{q}{p} = -\frac{M^*_{12}}{|M_{12}|} = -\frac{(V_{td} V^*_{tb})^2}{|V^*_{td} V_{tb}|^2} = \frac{V_{td} V^*_{tb}}{V^*_{td} V_{tb}} = e^{-2i\beta} \tag{146}$$

where β is the corresponding angle in the unitary triangle. Of course, in a more general model than the standard one, we would replace β by some angle $\phi^d_{\rm mix}$. (For the B_s meson, V_{td} must be replaced by V_{ts} in the above formulas. We see from the values of the CKM matrix elements that then $\frac{q}{p} \sim 1$; however, in a general model, we could write $e^{i\phi^s_{\rm mix}}$.)

The choice of the final state f is important. From Eq. (141) we see that the ratio $\frac{\bar{A}_f}{A_f}$ occurring in λ is difficult to express in terms of weak phases because there are many contributions with different strong phases. An exception is when there is only one term; then the strong phase cancels and we are left with $\frac{\bar{A}_f}{A_f} = e^{-2i\phi_{dec}}$ where ϕ_{dec} is a weak angle, basically that of the CKM matrix elements. In this case, $|\lambda| = 1$ and the asymmetry simplifies further to

$$\mathcal{A}_f(t) = -{\rm Im}\, \lambda \, \sin(\Delta M \, t) \tag{147}$$

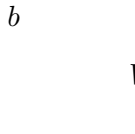
where

$${\rm Im}\, \lambda = \sin(2\phi^d_{mix} + 2\phi_{dec}) \, . \tag{148}$$

In this ideal case, the asymmetry is just given by CKM matrix elements and the measurement yields them without theoretical noise. The CP violation that follows from this type of asymmetry is called **mixing-decay CP violation**. It is the most discussed one, and one particular final state, $J/\Psi \, K_S$, where $Im\lambda = \sin(2\beta)$, has become the favourite and will be discussed further below.

The experimental aspects are discussed by P. Harrison. There are many amusing points here, such as the need for an asymmetric B factory to measure the asymmetry Eq. (147).

We return to the typical final states that are presently under discussion. As mentioned above, the first is $J/\Psi\, K_S$. The major decay mechanism is the tree-level decay $b \to sc\bar{c}$,



$$\text{for which} \quad \frac{\bar{A}}{A} = \frac{V_{cb}V_{cs}^*}{V_{cb}^*V_{cs}} \; . \tag{149}$$

One chooses the K_S (instead of any kaon) because one wants a CP eigenstate and K_S is better experimentally. The above decay mechanism yields a K^0 from a B^0 and a $\bar{K}^0$ from a $\bar{B}^0$. Since the final state is a K_S (and not a K^0 or a $\bar{K}^0$) the amplitudes A_f and $\bar{A}_f$ obtain a factor p and q, respectively: according to Eq. (77) $K_s = \frac{pK^0+q\bar{K}^0}{\sqrt{|p|^2+|q|^2}}$. (Note that these are the p and q for the kaon system.) Thus in the ratio $\frac{\bar{A}}{A}$, a correction factor $(q/p)_K$ must be included. Since $(q/p)_K = \frac{V_{cs}V_{cd}^*}{V_{cs}^*V_{cd}}$, we arrive at

$$\lambda(B \to J/\Psi K_S) = \frac{V_{td}V_{tb}^*}{V_{td}^*V_{tb}} \, \frac{V_{cs}V_{cd}^*}{V_{cs}^*V_{cd}} \, \frac{V_{cb}V_{cs}^*}{V_{cb}^*V_{cs}} \; = -\sin(2\beta) \; . \tag{150}$$

The first factor in Eq. (150) is $(q/p)_{B_d}$ in Eq. (146), the second the analogous one for the final-state kaon, and the third one is the 'naive' $\bar{A}_f/A_f$, as in Eq. (149). As expected, λ is independent of the choice of phases in the CKM matrix; this is insured by the $(q/p)_K$ factor.

Problem

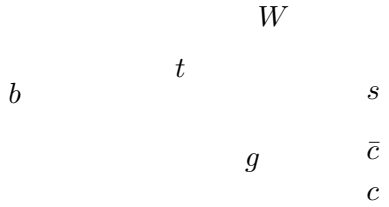
Show that the phase ambiguity remains if the factor $(q/p)_K$ is not included.

Since

$$\beta = \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right) \; , \tag{151}$$

the decay $B \to J/\Psi K_S$ measures the angle β in the unitary triangle.

In principle, this decay also receives contributions from penguin diagrams as pictured below



which are proportional to $V_{ib}V_{is}^*$. Using unitarity of the CKM matrix, we have

$$V_{ub}V_{us}^* + V_{cb}V_{cs}^* + V_{tb}V_{ts}^* = 0 \; . \tag{152}$$

Since the first term is small, $V_{cb}V_{cs}^* \sim -V_{tb}V_{ts}^*$ and both the c and the t penguins are proportional to $V_{cb}V_{cs}^*$, like the tree-level amplitude. Thus, even when we include these penguins, the factor $\frac{\bar{A}}{A}$ remains the same. The error is only around 1%.

Next we want to find the angle

$$\alpha = \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{td}V_{tb}^*}\right) \tag{153}$$

of the unitary triangle. For this we need a decay of the form $\bar{b} \to u\bar{u}\bar{d}$ whose tree-level amplitude is proportional to $V_{ud}V_{ub}^*$. The simplest such decays are $B_d \to \pi\pi$. From the tree-level decay one would obtain

$$\bar{A}_f/A_f = \frac{V_{ub}V_{ud}^*}{V_{ub}V_{ud}^*} \tag{154}$$

and thus Im $\lambda = \sin(2\alpha)$. However, penguin diagrams contribute substantially since the factor $V_{ub}V_{ud}^*$ which governs the decay is relatively small. Using unitarity (152), one can always re-express the top contribution in terms of the other two; thus we write the amplitude in the form

$$\bar{A}_f = V_{ub}V_{ud}^* A_T + V_{cb}V_{cd} A_P \ , \tag{155}$$

where the amplitudes A_T include the tree and the penguin, and A_P the penguin diagrams. One estimates that A_P/A_T is around 20–30%, giving a large error on α. Theoretically, one can disentangle A_T and A_P by an isospin analysis of all three decay $B_d \to \pi^+\pi^-$ and $B^+ \to \pi^+\pi^0$ [41] because A_P changes isospin by $1/2$, and A_T by $3/2$ and $1/2$. This requires all three decays $B_d \to \pi^0\pi^0$. However, the first of these is considered practically unmeasurable. More realistic ways have been discussed in the literature, especially via the decay $B \to \rho\pi$, see Ref. [10].

Similarly, the angle γ can be determined from $B_s \to \rho^0 K_S$. The above arguments now yield

$$\lambda(B_s \to \rho K_S) = \frac{V_{ts}V_{tb}^*}{V_{ts}^*V_{tb}} \frac{V_{ub}V_{ud}^*}{V_{ub}^*V_{ud}} \frac{V_{cd}V_{cs}^*}{V_{cd}^*V_{cs}} = -\sin(2\gamma) \ , \tag{156}$$

since

$$\gamma = \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right) . \tag{157}$$

Again, it is thought that penguin contributions spoil this clean picture and at the moment this method is not considered useful for determining γ.

Obviously there are many decays that yield, in principle, the three angles via the formula for λ; the angle β, for instance, can also be obtained from the decay $B \to D\pi$, (look at the quark diagram). However, many decays have unwanted extra contributions, are harder to detect, etc. Therefore one usually limits oneself to a few.

The consistency of the SM requires that the sum of the measured three angles be 180°. But this is also true for models where all CP violation is in the CKM matrix. Then the triangle still closes, but its corners are moved. An example can be found in Ref. [42], where supersymmetry adds new forces, but only the CKM couplings include CP violation.

Also, when there are new CP violating forces, the triangle might still close. For instance if there were a new interaction that only contributed to the p and q in the B_d system. Then the angles β and α would be shifted, but the sum $\alpha + \beta$ would remain the same, and the triangle would just be rotated.

A possible way of finding new CP violating interactions would be to measure the 'same' angle in several decays, i.e. to measure λ in decays which should have the same value in the SM, but not in general. See for example, Ref. [43].

6.3 Determining the angle γ from amplitude measurements

A very different approach to the angle γ comes from measuring a well chosen set of amplitude measurements. The basic observation is the following. The various contributions to one decay amplitude are proportional to products of the form $V_{ij}V_{lb}^*$. If there are relations among at most three of these contributions, the relations can be written as triangles in the complex plane (these are not unitary, but amplitude triangles). The angles between two sides (amplitudes) of these triangles are given by the difference of the weak and strong angles of the two amplitude contributions [see Eq. (141)]. If there are enough such triangles, and the magnitudes of the amplitudes can be measured, one may reconstruct both the weak and the strong angles. In other words, knowing the sides of a triangle also gives its angles!

To make this more concrete, we consider the decays $B \to DK$ [44, 10], but the ideas are, in some sense, at the base of more recent ideas, such as proposed by Mannel and Fleischer [45, 46], and later by Neubert and Rosner [47]. We shall discuss them later.

$\sqrt{2}\,A(B^- \to D^0_+ K^-)$

$A(B^+ \to \bar{D}^0 K^+)$

$A(B^- \to \bar{D}^0 K^-)$

$\sqrt{2}\,A(B^+ \to D^0_+ K^+)$

2γ

29 $A(B^+ \to \bar{D}^0 K^+) = A(B^- \to D^0 K^-)$

Fig. 13: Six amplitudes from which the angle γ can (in principle at least) be determined.

The decay $B^+ \to \bar{D}^0 K^+$ receives only a tree-level contribution proportional to $V_{us}V^*_{cb}$. This is real, and the amplitude has only a strong phase δ. The charge conjugated decay $B^- \to D^0 K^-$ has the same strong phase, and because $V_{us}V^*_{cb} \sim V^*_{us}V_{cb}$ in the usual convention, the two amplitudes are the same. On the other hand, the decay $B^+ \to D^0 K^+$ is proportional to $V_{cs}V^*_{ub}$ (check it!). The amplitude now has a weak phase γ (from V^*_{ub}) and a strong phase δ'. Its conjugated amplitude has the phase $-\gamma+\delta'$ [always compare with Eq. (141)]. On the other hand, we have $D^0_+ = \frac{1}{\sqrt{2}}(D^0 + \bar{D}^0)$ which implies the amplitude relation

$$A(B^+ \to \bar{D}^0 K^+) + A(B^+ \to D^0_+ K^+) = \sqrt{2}\,A(B^+ \to D^0_+ K^+) \tag{158}$$

and its complex conjugate. The situation is shown in Fig. 13. The angle δ can be chosen equal to zero, only the difference $\delta' - \delta$ matters. We see that the angle γ appears through this geometric construction!

The measurement of all six amplitudes is difficult. More suitable decays have been proposed [10]; and maybe there are even better ones. Find them!

7. LEPTONIC AND RARE DECAYS

Before going onto hadronic decays, I shall briefly mention the purely leptonic and so-called rare decays. The simplest leptonic decays are the 'tree-level' decays $B^+ \to l^+\nu_l$ where $l = e, \mu, \tau$ and where the B meson annihilates with a factor $V_{ub}f_B$. The matrix element factorizes as in Eq. (201) and the hadronic factor is

$$\langle\, 0 \,|\, (\bar{b}\gamma_\mu(1-\gamma_5)u)_A \,|\, B^+ \,\rangle \;= i f_B p_\mu \;. \tag{159}$$

Because of the famous helicity suppression factor $(m_l/m_B)^2$ in the formula for the rate, the largest branching ratio of around 10^{-5} is for the τ, whilst for the μ the branching ratio is around 10^{-7}. These are difficult decays to measure.

These leptonic decays give important information; they are the only direct measurements on the decay constant f_B, presently known to only 20% or so, can be fixed more accurately, this decay could give V_{ub} with substantial precision. Also of interest are the radiative decays, such as $B^+ \to l^+\nu_l\gamma$. It was shown that it is not supressed for the μ and the electron as compared to the leading decay $B^+ \to l^+\nu_l$ [48]. Using heavy quark arguments (next section) also these decays may yield f_B or other important quantities.

A second class of leptonic decays are the rare decays induced by loops, such as $B \to \mu^+\mu^-$. Like all rare decays, they test the SM at the loop level and are sensitive to new physics. I cannot cover this interesting topic more, but see, for instance the BaBar book [10] for results and references.

Also semihadronic rare decays are much discussed. We already mentioned the decay $b \to s\gamma$, but many others are being investigated with great care; an example is $b \to s e^- e^+$. Much on them can be found in the BaBar book [10].

8. HQET: A SHORT EXPOSITION

8.1 The ideas

Since the Heavy Quark Effective Theory is so intuitive and is modelled after well-known concepts of atomic physics, it is worthwhile and instructive to briefly expose it here. There are several extensive reviews [49], and shorter descriptions can be found in Refs. [50] or [10].

The theory of strong interactions, QCD, enjoys the property of asymptotic freedom according to which the interactions become weaker at higher momentum exchange. Roughly, a scale $\Lambda_{\rm QCD} \sim$ 0.5–1 GeV separates regions of strong and weak couplings. Hadron radii are typically of the order $1/\Lambda_{\rm QCD}$. Heavy quarks are those whose mass is way above $\Lambda_{\rm QCD}$ and which include c, b, t. Accordingly, their Compton wavelength $\lambda_q \sim 1/m_q$ is much smaller than the hadronic radius. This leads to several simplifications, also known from atomic physics where the tiny nucleon is much smaller than the atom. The mass of the nucleon is essentially irrelevant, only its charge (and the electron mass) determine all levels, transition rates, etc). Furthermore, the interaction of the electronic hull with the spin of the nucleus (hyperfine interactions) is supressed by $1/M_{\rm nucleus}$. This is of course well known and follows from simple physical arguments: If the nucleus has a very large mass, it does not move in the atom's centre of mass. Therefore its charge does not generate the B field necessary to interact with the spin of the electron. Only when it moves, and the motion is clearly of the order $1/M_{\rm nucleus}$, does the spin of the nucleus generate a B field as a relativistic effect.

Translated into hadrons, this means that the flavour and spin of the heavy quark become irrelevant as the mass of the heavy quark is large (this concerns c and b quarks). There are relations between hadrons containing a c and those containing a b quark, i.e. between D and B and also between hadrons of different spin, for example between B and B^* or between D and D^*. This **heavy quark symmetry** yields certain approximate relations, such as the one between the decay constants f_B and f_D of B and D mesons, etc.

In HQET, one follows the notion of big and small components introduced in the study of the Dirac equation. A Dirac spinor Ψ which describes a relativistic fermion decomposes into large and small components as follows:

$$\Psi = \begin{pmatrix} \Psi_u \sim 1 \\ \Psi_d \sim 1 \\ \Psi'_u \sim E/m \\ \Psi'_d \sim E/m \end{pmatrix}. \tag{160}$$

Here, E is the energy and m the mass of the particle; E/m is small (binding energy over mass). The two first (large) parts describe the spin-up and spin-down components of the particle, the other two (small) components, the two spin states of the antiparticle.

In treating atoms with the Dirac equation, one systematically expands in powers of $1/m_e$, using the so-called Foldy Wouthuysen Transformation (see for example, Ref. [51]). Basically the idea is the following. The Dirac equation connects large and small components. One can express the large ones in terms of the small ones and obtain in this way a more complicated equation for the small components. In the first step, this gives for instance the so-called Pauli equation, which gives rise to a magnetic dipole coupling between the spin and the B field felt by the moving electron.

In our case of quark field theory, one expresses in a similar way the small 'fields' through the large ones. This leads to an 'effective' field theory in terms of the large components, HQET. I would like to carry out a few steps in some detail; maybe this will also help to understand some of the beautiful physics in the Dirac equation.

As already mentioned, the heavy quark carries most of the hadron's momentum. Thus we express its momentum in the form

$$p_\mu = m_Q v_\mu + k_\mu \ , \tag{161}$$

where $|k_\mu| \ll m_Q$ and v_μ is the so-called four velocity of the hadron containing the heavy quark ($v^2 = 1$). For instance, in the rest frame of the hadron, $v = (1, 0, 0, 0)$, and k_μ describes the internal motion of the heavy quark. We now consider the Lagrangian density and the corresponding Dirac equation for the field $Q(x)$ of the heavy quark. Recall that if the Lagrangian density has the form

$$\mathcal{L} = \bar{Q}(\not{D} - m_q)Q \tag{162}$$

then the Euler–Lagrange procedure yields the Dirac equation

$$(i\,\not{D} - m)Q = 0 \ . \tag{163}$$

Here, $\not{D} = D_\mu \gamma^\mu$ and the covariant derivative D_μ is given by $D_\mu = \partial_\mu - igA_\mu$. The γ are the usual Dirac matrices, A is the gauge field, and g the gauge coupling.

To obtain some insight, consider the simple case where the quark is free and at rest. Then the Dirac equation reduces to

$$(\gamma_0 \partial_t - m_q)Q = 0 \tag{164}$$

whose simple solution is

$$Q \sim e^{-im_q t} \ . \tag{165}$$

We see that $e^{-im_q t}$ is a 'trivial' phase and does not contain physics of interest to us here. It is therefore useful to redefine the fields as much as possible by 'taking out' this phase. We define $Q = e^{-im_q t} Q_v$. Recall that the Dirac matrix γ_0 has the form

$$\gamma_0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} . \tag{166}$$

When we apply the Dirac equation to Q, a factor of m_q which comes from differentiating the exponential cancels the explicit mass term for the large component of Q_v. For this field there is therefore no mass term. This expresses the physical fact, discussed above, that the mass is irrelevant.

We can formalize this discussion [52, 49]. For general v, we must replace the time t by the scalar product $v \cdot x$, where x is the four coordinate. Large and small components are formed via the projection operators

$$P_+ = \frac{1+ \not{v}}{2}, \quad P_- = \frac{1- \not{v}}{2} \tag{167}$$

Note that for $v = (1, 0, 0, 0)$ the $P_\pm$ reduce to the well known form $P_\pm = \frac{1 \pm \gamma_0}{2}$, given by

$$P_+ = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad P_- = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} . \tag{168}$$

We can now write

$$h_v = e^{im_q vx} P_+ Q(x), \ H_v = e^{im_q vx} P_- Q(x) \tag{169}$$

or

$$Q = e^{-im_q vx}(h_v + H_v) \ , \tag{170}$$

where $\not{v} h_v = h_v$ and $\not{v} H_v = -H_v$. h_v correspond to the large, and H_v to the small component. The price to pay for the disappearance of the mass is that the fields become velocity dependent.

The next step is to eliminate the small component H_v sytematically. To illustrate the procedure, we take $v = (1, 0, 0, 0)$. The complete Dirac equation for Q reads

$$0 = (p_0\gamma_0 - \vec{p}\vec{\gamma} + gA_0\gamma_0 - g\vec{A}\vec{\gamma} - m_q)e^{-imt} \begin{pmatrix} h_v \\ H_v \end{pmatrix} . \tag{171}$$

Here, the p_μ are the partial derivatives $i\partial_\mu$. The gauge fields describe the 'colour' background in the hadron; they are of the order of the binding energy and thus small compared to the mass m_q. With

$$\vec{\gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix} \tag{172}$$

Eq. (171) splits into two equations

$$0 = (p_0 - m - A_0)e^{-im_q t}h_v + (\vec{p}\vec{\sigma} - g\vec{A}\vec{\sigma})e^{-im_q t}H_v \; . \tag{173}$$

$$0 = (p_0 + m - A_0)e^{-im_q t}H_v - (\vec{p}\vec{\sigma} - g\vec{A}\vec{\sigma})e^{-im_q t}h_v \; . \tag{174}$$

Because of the exponential, $p_0 \sim m_q$ and is large. Therefore, we can use the second equation to eliminate H_v in terms of h_v

$$H_v \sim \frac{\vec{p}\vec{\sigma} - g\vec{A}\vec{\sigma}}{2m_q} h_v \; . \tag{175}$$

This is inserted into the first one which becomes now

$$0 = (p_0 - m - A_0)e^{-im_q t}h_v + (\vec{p}\vec{\sigma} - g\vec{A}\vec{\sigma})e^{-im_q t}\frac{\vec{p}\vec{\sigma} - g\vec{A}\vec{\sigma}}{2m_q} h_v \; . \tag{176}$$

Using (see problem below)

$$(\vec{p} - g\vec{A})\vec{\sigma})((\vec{p} - g\vec{A})\vec{\sigma}) = (\vec{p} - g\vec{A})^2 - g\vec{B}\vec{\sigma} \tag{177}$$

we arrive at the 'Pauli' equation

$$0 = (p_0 - m - A_0)e^{-im_q t}h_v + \frac{1}{2m_q}((\vec{p} - g\vec{A})^2 - g\vec{B}\vec{\sigma})e^{-im_q t}h_v \; . \tag{178}$$

Comparing this equation with Eqs. (162,163), we can view this as an equation of motion for the field h_v and translate it 'back' into an effective Lagrangian density for h_v. This will give a leading 'kinematical' term, valid if $m_q \to \infty$, and two corrections proportional to $1/m_q$, namely

$$\delta\mathcal{L} \sim \frac{1}{2m_q}(\vec{p} - g\vec{A})^2 - g\vec{B}\vec{\sigma} \; . \tag{179}$$

A systematic treatment will give a series of such correction terms, with all powers of $1/m_q$. It can be formalized and adapted to the present notation. The effective Lagrangian for the field h_v up to order $1/m_q$ then reads

$$\mathcal{L} = \bar{h}_v iD \cdot v h_v + \frac{1}{2m_q}\bar{h}_v(iD_\perp)^2 h_v + \frac{g}{4m_q}\bar{h}_v\sigma_{\mu\nu}G^{\mu\nu}h_v \; . \tag{180}$$

We recognize the terms in Eq. (179), the square of the momentum, and the magnetic interaction.

Problem

Derive Eq. (177) using the matrix identity

$$\vec{a}\vec{\sigma}\vec{b}\vec{\sigma} = \vec{a}\vec{b} + (\vec{a}\times\vec{b})\vec{\sigma}\ . \tag{181}$$

Make use of the well-known expression $\vec{B} = rot\vec{A}$.

In order to see a little 'how it works' we calculate the mass of the pseudoscalar and vector mesons whose difference is the relative spin state of the constituents. We need to calculate the matrix element of the Hamiltonian between two heavy mesons. The additional interaction in Eqs. (179) and (180) gives the corrections

$$\lambda_1(B) = \frac{1}{2m_{H_b}}\langle H_b|\bar{h}_v(iD_\perp)^2 h_v|H_b\rangle \tag{182}$$

and

$$\lambda_2 = \frac{g}{4m_q}\langle H_b|\bar{h}_v\sigma_{\mu\nu}G^{\mu\nu}h_v|H_b\rangle\ . \tag{183}$$

The first of these is independent of the spins. However, the second is not. From the more physical form in Eq. (179), $g\vec{B}\vec{\sigma}$, we find that the correction is of the form $s_1\cdot s_2$ where the s_i are the spins of the heavy quark and the light quark, respectively. The first factor is already present in the interaction, whilst the second comes in because the B field must be proportional to s_2 (the relative angular momentum is zero for these states). Now, the total spin is $S = s_1 + s_2$. Squaring this, we find $s_1\cdot s_2 = S(S+1) - 2(3/4)$. This gives a factor $1/2$ for a vector and $-3/2$ for the pseudoscalar. We can then write for the masses:

$$m_{\rm vector} = M_0 + (\lambda_2)/(2m_q)\ \ ; m_{\rm scalar} = M_0 - 3(\lambda_2)/(2m_q)\ . \tag{184}$$

The difference between the masses of B and B^* is then $2(\lambda_2)/(m_b)$. From the experimental values and the approximate relation $m_B \sim m_b$ one obtains $\lambda_2 \sim 0.12\ {\rm GeV}^2$.

In this way one can fix the parameters of the effective Lagrangian from a few experimental numbers and then use it for the calculation of the various quantities of interest. The two major areas are inclusive decay from which one determines the quark mass, and semileptonic decay, which are important (as seen before) for determining the CKM matrix elements. We consider here only the latter.

8.2 Applications

The first application which comes to mind is the semileptonic decay $B\to De\nu$. The matrix element of the four-fermion Hamiltonian $(\bar{c}\gamma^\mu b)(e\gamma^\mu\nu)$ is proportional to

$$\langle D(p')|\bar{c}\gamma^\mu b|B(p)\rangle = f^+(q^2)(p+p')^\mu - f^-(q^2)(p-p')^\mu\ , \tag{185}$$

where $q = p - p'$ and f^+ and f^- are the form factors. This form is dictated by Lorentz covariance. As before, we define the four-velocities v and v' by $p = m_B\,v$ and $p' = m_D\,v'$ and take the fields b, c to be the large components, b_v, etc. Then Eq. (185) can be rewritten as

$$\begin{aligned}\langle D(v')|\,\bar{c}\gamma^\mu b\,|B(v)\rangle &= \frac{1}{2}\left[(m_B+m_D)f^+(q^2) - (m_B-m_D)f^-(q^2)\right](v+v')^\mu \\ &+ \frac{1}{2}\left[(m_B-m_D)f^+(q^2) - (m_B+m_D)f^-(q^2)\right](v-v')^\mu\ .\end{aligned} \tag{186}$$

If $v = v'$, the heavy quark symmetry tells us that in the transition from b to c nothing else happens, the surrounding quarks 'do not notice' the change since their states are independent of the heavy quark. Another way of saying this is that the Dirac equation for h_v is mass independent, and thus the current $\bar{c}\gamma^\mu$ is conserved. This implies that $(v-v')\mu\bar{c}\gamma^\mu = 0$. When multiplying the right-hand side of Eq. (186) by

Fig. 14: An impression of the complicated QCD interactions in an hadronic decay.

$v - v'$, the first term vanishes, since $(v - v')(v + v') = 0$. Thus the second term must be zero, and we arrive at

$$f^- = \frac{m_B - m_D}{m_B + m_D} f^+ \, . \tag{187}$$

We see that both functions are proportional to one single function; one usually sets

$$f^\pm = \frac{m_B \pm m_D}{2\sqrt{m_B m_D}} \, \xi(\omega) \, , \tag{188}$$

where $\xi(\omega)$ is the so-called Isgur–Wise function and the argument is $\omega = v{\cdot}v'$. For $vv' = 1$ (corresponding to $v = v'$) this function is the one reflecting the fact that 'nothing' happens. Using the relations $p = mv$ between the real momenta and the four velocities for the b and c quarks, the value $vv' = 1$ corresponds to the maximum momentum transfer $q^2 = (m_B - m_D)^2$.

Following the above exposition [such as Eq. (180)] one can calculate corrections to this. One finds that typically the Isgur–Wise function becomes, at $w = 1$,

$$\xi(\omega) = 0.91 \pm 0.03 \;\; (\mathrm{HQET}), \;\; \xi(\omega) = 0.935 \pm 0.03 \;\; (lattice) \, . \tag{189}$$

The semileptonic branching fraction calculated from this appears, at the moment, to be slightly higher than the experimental values measured at LEP and at the low-energy machines [18, 19].

9. HADRONIC DECAYS OF B-MESONS AND RELATED ISSUES

9.1 General comments

The decay of a hadron containing a b quark is illustrated in Fig. 14. The b quark decays by emitting the W and an up-type quark. The W may just decay into a pair of quarks, or combine with the other quarks in the hadron. Many gluons are exchanged between initial and final quarks.

The non-leptonic decays are difficult to treat. After the decay there are a number of light quarks and gluons around, with various momenta and energies. Because of the strong confining forces they must form hadrons. This can happen in many ways and there is now a reliable way to calculate it theoretically. Therefore one asks either limited questions or makes assumptions based on physical pictures. Some of these will be described in the following Section.

9.2 Inclusive decays

The mass of the b quark is rather large compared to the QCD binding energy ('bag') which holds together the hadrons. Thus, to a first approximation, we may say that in a B hadron at rest the b quark just 'sits' there, and the light quarks orbit around it. In this picture, the b quark decays without being influenced by the other constituents, they are just 'spectators'. After the decay, there are several light quarks. Since they are bound to form hadrons, we may consider the quark process (before hadronization) as a model for *inclusive* decays. Therefore inclusive decays are easier to treat than exclusive ones. Because of the obvious connections, inclusive decays can be treated within the Heavy Quark Effective Theory [53].

This rough picture implies that the lifetimes of all B-hadrons be the same. A little inspection of the present data for the lifetimes τ of the B hadrons gives the following picture:

$$\tau(B^-/B^0) = 1.07 \pm 0.02 \quad (190)$$

$$\tau(B_s/B^0) = 0.94 \pm 0.04 \quad (191)$$

$$\tau(\Lambda_b/B^0) = 0.79 \pm 0.05\ . \quad (192)$$

While the first two ratios are near one, as expected from the above discussion, the last value cannot be understood from the simple picture. (Note that for the charmed hadrons, the ratios are indeed near one.) Unless the experimental value is unreliable (a possibility the present author does not exclude) corrections to the above picture must be evaluated.

The first correction is to include the internal motion of the b quark in a hadron. This leads to corrections in powers of λ/m_b. Clearly this internal motion depends on the hadron in which it takes place and therefore differences arise between heavy mesons and baryons. The Heavy Quark Effective Theory', HQET, introduced above is ideally suited to tackle this question and many results exist [10].

The other effects come from direct spectator interactions. Clearly, such effects also depend on the hadron in which the b quark decays, and thus give rise to different lifetimes. Recent investigations [54] and [55] find small corrections unable to account for the experimental numbers, but again, such results are to be taken as an indication rather than as complete results.

9.3 Exclusive hadronic decays of B-mesons

As often mentioned, these decays are very difficult to calculate. Present theoretical work is essentially limited to decays of a B meson into two mesons. These decays are important for understanding QCD, but mostly for extensive studies of CP violation (as mentioned, semileptonic decays are not too promising in this respect). The best investigated ones are decays into charmed hadrons, such as $B \to D\pi$, but as we shall see later, the most interesting decays are the charmless ones, especially those into two π's or a π and a K. The experimental situation has greatly improved in 1999. Table 1 contains only the $\pi\pi$ and πK modes. There is an excellent review by M. Artuso ([17] which contains extensive results from which we note here only the relatively large branching ratios of $B^+ \to \eta' K^+$ and $B^0 \to \eta' K^0$.

These results already teach us some simple lessons. Consider the tree-level decays $\bar{b} \to \bar{u}u\bar{s}$ and the penguin graph process $\bar{b} \to \bar{s}q\bar{q}$ in Fig. 15.

The tree-level process always leads to a pair of $u\bar{u}$, whilst the q in the penguin can be u or d. Consider then the decay $B^{0+} \to \pi^0 K^0$. The final state contains a $d\bar{d}$ pair and thus it must come from a penguin graph. Similarly, the $\pi^-\pi^+$ final state is expected to originate from the tree-level process. The CKM factor of the penguin $|V_{cb}^* V_{cs}| \sim \lambda^2$ and that of the tree level graph is $|V_{ub}^* V_{ud}| \sim \lambda^3$. The table then tells us that

$$|\frac{\lambda^2 P}{\lambda^3}|^2 \sim \frac{15 \times 10^{-6}}{5 \times 10^{-6}}\ , \quad (193)$$

from which we conclude that the ratio of the penguin amplitude P to the tree level amplitude T is around 0.4. This implies rather large penguin effects and is also supported by the big branching ratios into η'. As a consequence, the measurement of α in pion decays is not as easy as once hoped.

Table 1: Experimental results. Branching fractions ($\mathcal{B}$) and 90% C.L. upper limits are given in units of 10^{-6}, from Ref. [17].

Mode	$\mathcal{B}$ (10^{-6})	Signif. (σ)
$\pi^+\pi^-$	$4.7^{+1.8}_{-1.5} \pm 0.6$	4.2
$\pi^+\pi^0$	< 12	3.2
$K^+\pi^-$	$18.8^{+2.8}_{-2.6} \pm 1.3$	11.7
$K^+\pi^0$	$12.1^{+3.0+2.1}_{-2.8-1.4}$	6.1
$K^0\pi^+$	$18.2^{+4.6}_{-4.0} \pm 1.6$	7.6
$K^0\pi^0$	$14.8^{+5.9+2.4}_{-5.1-3.3}$	4.7
K^+K^-	< 2.0	–
$K^+\overline{K}^0$	< 5.1	1.1

Fig. 15: Schematic tree and penguin amplitudes for a hadronic B decay.

9.4 General description of hadronic decays

We take as an example the effective Hamiltonian such as in Eq. (50) for a $b \to c\bar{u}d$ transition. Other flavour transitions just require appropriate changes of the flavours and CKM matrix elements.

$$\begin{aligned} H_{\rm eff} &= \frac{G_F}{\sqrt{2}} V_{cb}V_{ud}^* \left[c_1(\mu)\, O_1 + c_2(\mu)\, O_2\right] \\ &\quad + \text{penguin operators}\,. \end{aligned} \tag{194}$$

The local four-quark operators, renormalized at a scale μ, are written as products of colour-singlet currents,

$$\begin{aligned} O_1 &= (\bar{d}u)_{V-A}\,(\bar{c}b)_{V-A}\,, \\ O_2 &= (\bar{c}u)_{V-A}\,(\bar{d}b)_{V-A} \end{aligned} \tag{195}$$

where

$$(\bar{d}u)_{V-A} = (\bar{d}\gamma_\mu \frac{1-\gamma_5}{2} u) \tag{196}$$

is the left-handed current typical for the weak interactions. Naively, we may picture these operators as two currents, connected by a colourless exchange 'particle' of large mass. The corresponding Wilson coefficients $c_i(\mu)$ are at the scale relevant to B decays [21]

$$c_1 = 1.13 \;\text{ and }\; c_2 = -0.3\,. \tag{197}$$

To gain some intuitive ideas about the process, one expresses the effective Hamiltonian in a different way, using the Fierz identity, a mathematical formula. Instead of (194) we can write

$$\begin{aligned} H_{\rm eff} &= \frac{G_F}{\sqrt{2}} V_{cb}V_{ud}^* \left[(c_1(\mu)\, O_1 + \frac{c_2(\mu)}{3}\, O_1 + 2c_2(\mu)\, O_8\right] \\ &\quad + \text{penguin operators}\,. \end{aligned} \tag{198}$$

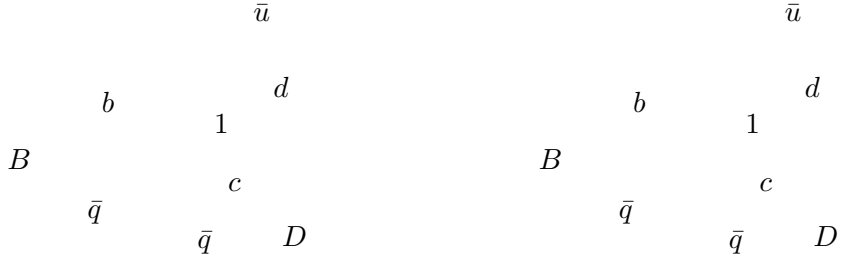


Fig. 16: (a) Schematic picture of the contribution of the operator O_1 to the decay $\bar{B} \to D\pi$ in the factorization approximation. The '1' at the dashed line indicates that the exchanged 'quantum' is a colour singlet. (b) A suppressed gluon correction.

or

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cb}V_{ud}^* \left[(c_2(\mu)\, O_2 + \frac{c_1(\mu)}{3}\, O_2 + 2c_1(\mu)\, O_8 \right] + \text{penguin operators} . \tag{199}$$

where the new operator O_8 is given by

$$O_8 = (\bar{d}\, \frac{\lambda_a}{2} \frac{1-\gamma_5}{2} u)\, (\bar{c}\, \frac{\lambda_a}{2} \frac{1-\gamma_5}{2} b) . \tag{200}$$

The λ_a are the Gell-Mann matrices. In contrast to the above operators, O_8 is the product of two colour octets. These two forms are referred to as I or II, respectively. We will understand their usefulness shortly.

We need the matrix elements of H_{eff}. From Fig. 16 the following simplified picture emerges. Since the B is quite heavy, the momentum of the emitted quark pair $\bar{u}d$ is large. Thus one expects the corrections from gluons 'across' the vertex to be small and takes them to be absent. For the operator O_1, this implies that the matrix element 'factorizes' and can be written as the product

$$F_I = \langle \pi^- |\, (\bar{d}u)_A \,|\, 0 \,\rangle\, \langle D^+ |\, (\bar{c}b)_V \,|\bar{B}^0\rangle , \tag{201}$$

since no quarks or gluons are exchanged between the two currents. The two factors account for all QCD effects on either side of the exchange, i.e. gluons between the $\bar{u}d$ pair and between all constituents of the B and D mesons are included. Recall the factorization idea in Eq. (65). Gluon corrections across the exchange can be parametrized by a contribution $F_1\varepsilon_1(\mu)$. The letter ε indicates that it is supposed to be a small quantity. The form (201) is called **factorization**. It is the basic approximation of most treatments of quark processes. Its validity is a theme of long theoretical debates; recent work has even heated up the discussion [56].

For the operator O_8 the exchange 'quantum' [in analogy to Fig. (16)] is a colour octet and cannot generate a physical (colourless) $\bar{u}d$ state. Therefore, at least one extra gluon is required, as in Fig. 16(b). We thus assume that this contribution is small and parametrize it by $F_1\varepsilon_8(\mu)$.

Note the scale-dependence of the $\varepsilon_i(\mu)$. And, of course, they are process dependent.

Collecting all the intermediate results, we can write for the matrix element

$$A = \frac{G_F}{\sqrt{2}} V_{cb}V_{ud}^* F_I \left((c_1 + \frac{c_2}{3})(1+\epsilon_1) + 2c_2\epsilon_8 \right) + \text{penguin contributions} . \tag{202}$$

Factorization means that the ϵ_i are zero. A perturbative calculation of them would require graphs as shown in Fig. (16). In recent work [56] it has been shown that the sum of all possible one-gluon exchanges is infrared finite in the leading order in q/m_b. This is taken as evidence that factorization holds

at very large energy and can have only corrections proportional to $1/m_B$. But, of course, these can still be numerically large.

The point of factorization is that F_I is the product of known quantites or those that can be measured in semileptonic decays. In the above case,

$$\langle \pi^- | \, (\bar{d}u)_A \, | \, 0 \, \rangle \; = i f_\pi p \; , \tag{203}$$

where f_π is the decay constant of the pion. The matrix element $\langle D^+ | \, (\bar{c}b)_V \, | \bar{B}^0 \rangle$ occurs in the semileptonic decay $\bar{B}^0 \; \to \; D^+ \ell^- \nu$ (see Eq. (185). One can either determine it from the semileptonic decay and/or theoretically using HQET arguments [10] or other methods. Then, F_I is fixed. This scheme can be used for all type I processes, i.e. for all decays dominated by the operator O_1.

Next consider the transition $\bar{B}^0 \to D^0$. We see from Fig. 16 that it cannot go by O_1 (without other exchanges) since the c quark must combine with a $\bar{u}$ rather than with the $\bar{d}$. Therefore we need the second form of the effective Hamiltonian in Eq. (199). Following the same steps as above, the matrix element for the decay is now

$$\begin{aligned} A_{II} \;\; &= \;\; \frac{G_F}{\sqrt{2}} \, V_{cb} V_{ud}^* \, F_{II} \left((c_2 + \frac{c_1}{3})(1 + \tilde{\epsilon}_1) + 2 c_1 \tilde{\epsilon}_8 \right) \\ &\quad + \text{penguin contributions} \; , \end{aligned} \tag{204}$$

where the $\tilde{\epsilon}$ are supposed to be small again.

For this case factorization is not a good starting point because the first coefficient,$(c_2 + \frac{c_1}{3})$, is numerically much smaller than the second, $2c_1$, and therefore the second term cannot be neglected. Instead, following Bauer, Stech and Wirbel [57, 58] one introduces two effective phenomenological coefficients, a_1 and a_2, in the following way: one writes for each matrix element

$$\begin{aligned} A_I \;\; &= \;\; \frac{G_F}{\sqrt{2}} \, V_{cb} V_{ud}^* \, F_I a_1 \\ A_{II} \;\; &= \;\; \frac{G_F}{\sqrt{2}} \, V_{cb} V_{ud}^* \, F_{II} a_2 \; , \end{aligned} \tag{205}$$

where the $F_{I,II}$ are products of form factors and decay constants for the decay under consideration. However, the $a_{1,2}$ are taken to be **process independent**. We may call this model 'universality'.

In practice, one determines the $a_{1,2}$ from decays where the $F_{I,II}$ are well known; one finds

$$a_1 = 1 \;\; \text{and} \;\; \; a_2 = 0.3 \; . \tag{206}$$

One can then predict many other decay rates. Extensive tables show that this simplified picture is rather good [58].

From a more fundamental viewpoint, the success of this picture is not so easy to understand. Whilst for class I decays it may not be so unexpected, the agreement with experiments in class II is quite surprising. Several attempts have been made to calculate $a_{1,2}$ from basic QCD. It turns out that perturbative calculations give values which are too small. A sum rule calculation gives larger values, but it seems impossible to reproduce the sign of a_2 [59, 10]. As mentioned, it was argued recently that factorization is an exact result in the limit $m_b \to \infty$ [56].

Factorization can be used and tested in different ways. One obvious test is to compare the hadronic decay $B \; \to \; Dh$ (h is any hadron, such as a π, etc.) to the semileptonic one $B \; \to \; Dl\nu$ for a leptonic invariant momentum $q^2 = m_h^2$. In the ratio of the widths, the form factor $B \to D$ drops out and one gets

$$R_l = \frac{\Gamma(B \to Dh)}{\Gamma(B \to Dl\nu)_{q^2 = m_h^2}} = 6\pi^2 f_h^2 |a_1|^2 \; , \tag{207}$$

where some corrections have been dropped. f_h is the decay constant of the hadron h, defined in a similar way to that of the pion in Eq. (203).

Similarly, one can compare two hadronic rates and obtain essentially

$$R_h = \frac{\Gamma(B \to Dh_1)}{\Gamma(B \to Dh_2)} = (\frac{f_{h_1}}{f_{h_2}})^2 \; . \tag{208}$$

These equations can be exploited in several ways: As tests of factorization, or as the means to obtain unknown branching ratios or unknown parameters, such as decay constants. An example is D mesons, whose decay constant is still only poorly known.

Another test of factorization [60] follows from the fact that certain decays are absent in this limit, since the current matrix element

$$\langle \pi^- | \, (\bar{d}u)_A \, | \, 0 \, \rangle \tag{209}$$

vanishes for hadrons with a certain spin-parity assignment. An example is the state a_2, thus a measurement of the rate of

$$\bar{B}^0 \to D^+ a_2^- \tag{210}$$

would be clear and quantitative evidence for a departure from factorization.

9.5 Final-state interactions

Final-state interactions are very important and many discussions evolve around them. They occur because of the continuing strong and weak interactions among the decay products and quantum mechanical evolution. They generate phases for the amplitudes such as e^{-iEt}, but they also induce rescatterings into other channels, such as the rescattering of a state $\pi^0\pi^0$ into $\pi^+\pi^-$, etc. These phases and rescatterings are important because the phases influence the rates substantially through interference.

We can formulate final-state interactions through the scattering matrix (S matrix) of strong interactions. The necessity of final-state interactions can be understood from the unitarity of the full (weak and strong) $\mathcal{S}$-matrix, $\mathcal{S}^\dagger\mathcal{S} = 1$. As a consequence, the $\mathcal{T}$-matrix, defined by $\mathcal{S} = 1 + i\mathcal{T}$, obeys the equation (optical theorem)

$$\langle \bar{F}|\mathcal{T}|\bar{B}\rangle^* = \sum_I \langle F|\mathcal{S}^\dagger|I\rangle\langle I|\mathcal{T}|B\rangle \; . \tag{211}$$

Here, the states $|I\rangle$ represent all possible final states (including $|F\rangle$ itself) which can be reached from the state $|B\rangle$ by the weak transition matrix $\mathcal{T}$. The right-hand side of Eq. (211) can then be viewed as a (weak) decay of $|B\rangle$ into $|I\rangle$ followed by a strong rescattering of $|I\rangle$ into $|F\rangle$. Thus, $\langle F|\mathcal{S}^\dagger|I\rangle$ may be identified as a CP-conserving FSI rescattering of particles.

If $|I\rangle$ is an eigenstate of $\mathcal{S}$ with a phase $e^{2i\delta_I}$, then

$$\langle \bar{I}|\mathcal{T}|\bar{B}\rangle^* = e^{-2i\delta_I}\langle I|\mathcal{T}|B\rangle \; , \tag{212}$$

which implies equal rates for the charge conjugated decays and hence no CP asymmetry. Therefore, at least two different states with equal quantum numbers must exist which can be connected by strong rescattering (for example, states with different numbers of particles or states with different particle charges but the same total charge and isospin).

For the one-channel case, Eq. (211) is known as the Omnes problem and its solution can be found if the final states phases are known experimentally. Methods for more channels exist; a satisfactory solution for decays of the form $K \to 3\pi$ or $\eta \to 3\pi$ exists [61]. There is not even a partial treatment along these lines available for B decays.

The level of theoretical understanding of final-state scatterings of B meson decays is similar to that of factorization. On the phenomenological side one may argue as follows [62]: all known hadronic cross-sections can be parametrized by the form [63]

$$\sigma(s) = X \left(\frac{s}{s_0}\right)^{0.08} + Y \left(\frac{s}{s_0}\right)^{-0.56}, \tag{213}$$

where $s_0 \sim 1$ GeV^2 is a typical hadronic scale. This implies (optical theorem) that the imaginary part of the forward elastic scattering amplitude increases asymptotically as $s^{1.08}$. This result can be extended to larger momentum transfer, assuming an exponential fall-off [64]. When applied to B meson decays, we set $s = m_b$.

One can refine the argument further, since the phenomenology of high-energy scattering is well accounted for by Regge theory [64]. Scattering amplitudes are described by the exchanges of Regge trajectories (families of particles of different spins). The leading trajectory for high-energy scattering is the pomeron. Non-leading trajectories presumably vanish as $1/m_B$ in the infinite heavy-quark mass limit.

In Ref. [62] it was estimated that the pomeron contribution in $B \to \pi\pi$ is

$$\mathcal{I}m\, \mathcal{M}_{B\to\pi\pi}|_{\rm pomeron} \simeq \epsilon i \mathcal{M}_{B\to\pi\pi} \tag{214}$$

with $\epsilon \sim -0.21$. From this numerical result and from the nature of its derivation, one can conclude that additional individual soft final-state interactions are not vanishingly small. However, of chief significance is the naive expected weak dependence of ϵ on m_B — the $(m_B^2)^{0.08}$ factor in the numerator is attenuated by a $\ln(m_B^2/s_0)$ dependence. Support for large phases also comes from a recent phenomenological analysis [65] where large phases were found in decays of heavy mesons.

In the recent theoretical analysis of Ref. [56] it was argued that at the one-loop level (one gluon correction) the final-state phases vanish as m_b goes to zero, the only effects being the perturbative one-loop diagrams. This implies, in particular, that the older, perturbative calculations [66, 67] which yielded relatively small values for CP asymmetry in two-body decays are valid.

This result is in obvious contradiction to the phenomenological procedure outlined above. Given the incompleteness of both treatments, there several possible explanations:

- m_b is finite, and thus a large numerical value of the coefficient of the $1/m_b$ term can be large (this actually happens for f_B).
- Two loops or higher loops may spoil the clean result of Ref. [56].
- In the phenomenological description only the pomeron is included. Important cancellations might occur when more complete intermediate states are considered. This can be observed in other situations.

Obviously, the situation is rather interesting. The methods outlined in Ref. [65] might be useful to determine experimentally the size of the final-state phases in many instances.

The calculation of the final-state interaction is rather difficult, because of the intricacies of QCD. They share the same (or even worse) difficulties as factorization, discussed above. For instance, it is not clear whether they should be done using quark and gluons or physical hadrons. One often invokes the notion of duality where quarks and gluons are equivalent to hadrons if appropriate sums are employed.

We start with the perturbative approach, where we use quarks and gluons [66]. To show how it works, we consider the simple diagram in Fig. 17 where we take the photon momentum square q^2 to be arbitrary. The rescattering phase comes about when we consider the imaginary part of this diagram (apart from the complex values of the CKM matrix element). Using the straightforward Feynman rules of the so-called Cutkosky rules, one finds that the imaginary part is caused by the internal quarks q_i being on their mass shell. We can view this as follows: the b decays first into the state $sq\bar{q}$, and then the electromagnetic

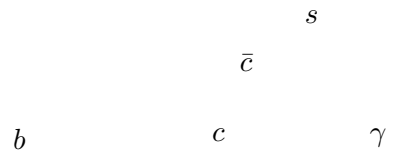
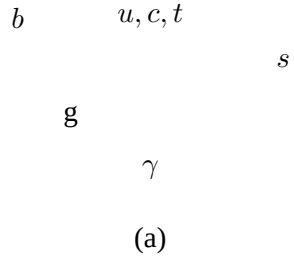


Fig. 17: The transition $b \to s\gamma$ drawn to adapt to Eq. (211). The dashed lines correspond to the states in that equation; the $c\bar{c}s$ state is the intermediate state I.

rescattering changes this state into the desired final state $s\gamma$. We can see from Eq. (211) that the existence of two states that can rescatter into each other leads to phases.

In the above case, the imaginary part depends on q^2 and the mass of the intermediate quark. It is zero for $q^2 \leq 4m_q^2$ because then the quarks cannot be on the mass shell and do not form a physical state required by Eq. (211). Thus, if $4m_u^2 \leq q^2 \leq 4m_c^2$, there is an imaginary part for the u quark, but none for c, let alone t. But even if the photon is on the mass shell, e.g. $q^2 = 0$, radiative corrections such as in Fig. 18 allow the phase, since the effective q^2 of the gluon (whose momentum can also be taken as the relevant one) assumes any value [68].



(a)

Fig. 18: One-gluon correction to the transition $b \to s\gamma$. Because the gluon has arbitrary momentum squared, there is an imaginary part, as discussed in the text.

We can now close the discussion started at Eq. (142). The total amplitude is the sum of the graphs with all possible intermediate quark antiquark intermediate states and restates. Each antiquark state can be viewed as a separate contribution to the decay as in Eq. (141). They all have different CKM factors, $V_{tb}V_{ts}^*, V_{cb}V_{cs}^*V_{ub}V_{us}^*$, and different strong phases: the masses are different and the phases depend on the variable q^2/m^2, as discussed above. The strong phase is near 1, since the imaginary and the real part of the diagrams are not that different. The weak phase can be estimated as follows: While $V_{tb}V_{ts}^*, V_{cb}V_{cs}^*$ are real, $V_{ub}V_{us}^*$ is not, and we have

$$\frac{V_{ub}V_{us}^*}{V_{cb}V_{cs}^*} \sim 10^{-2} \, . \tag{215}$$

Since the asymmetry is proportional to the weak phase (see Eqs. (141, 142), we expect it to be of the order 10^{-2}, and thus small. This leaves room for new physics to enhance the asymmetry. See, for example, Ref. [69].

This method can also be used to estimate asymmetries in exclusive hadronic decays [67]. For instance, the penguin graph in Fig. 15 gives an asymmetry of a few per cent for the πK decays. Because such asymmetries are very interesting, it is important to investigate closer the reliability of the calculation.

The perturbative method outlined above is often criticized. In particular, taking only the quark intermediate state might be false, since there are always strong confinement effects for physical states (recall, that state I must be physical to yield an imaginary part). Instead one may argue that one should use physical intermediate states. For instance, in the case of $B \to K\pi$, one could envisage the chain $B \to D\bar{D}_s \to K\pi$ with the $D\bar{D}_s$ intermediate state. However, a complete treatment must also include other states, such as $D^*\bar{D}_s^*$. It might be that their sum can indeed be represented by the quark state. There is no agreement to date. As in the case of factorization, it was shown recently [56] that the infrared divergences also cancel in this process, again implying the validity of the perturbative approach.

9.6 Determination of γ from $B \to K\pi$

A particularly interesting illustration of the effects of final-state interactions is the new methods begun by Fleischer and Mannel [45] to limit and find the angle γ of the unitarity triangle from the decays $B \to \pi K$. The method is also a consequence of the idea described previously, namely that this angle comes as a weak relative phase between amplitudes that are constrained by an equation.

The idea is to consider all possible $B \to \pi K$ decays. The tree-level amplitude T originates in the transition $b \to su\bar{u}$. This only contributes to the decays $B^- \to \pi^0 K^-$ and $B_d \to \pi^+ K^-$ because no d quarks are produced. On the other hand, penguin graphs contribute an amplitude P to all four decays $B^- \to \pi^0 K-$, $B_d \to \pi^+ K^-$, $B^- \to \pi^- \bar{K}^0$, and $B^- \to \pi 0 \bar{K}^0$. In addition, we consider the so-called annihilation amplitude A which only affects $B^- \to \pi^- \bar{K}^0$ and $B^- \to \pi^0 K-$. The amplitudes T and A contain a factor V_{ub} and thus a factor $e^{i\gamma}$. With these comments, we can write

$$A(B^+ \to \pi^+ K^0) = P\,, \quad A(B_d^0 \to \pi^- K^+) = -\left[P + Te^{i\gamma}\right]\,, \tag{216}$$

where the amplitudes T and P contain strong phases.

The A amplitude is usually considered to be small, since the annihilation of the two quarks is rather unlikely. Neglecting it, the decay $B^- \to \pi^- \bar{K}^0$ does not have an amplitude proportional to $e^{i\gamma}$. We define, following Fleischer and Mannel, the ratio

$$\begin{aligned} R &\equiv \frac{\mathrm{BR}(B_d^0 \to \pi^- K^+) + \mathrm{BR}(\overline{B_d^0} \to \pi^+ K^-)}{\mathrm{BR}(B^+ \to \pi^+ K^0) + \mathrm{BR}(B^- \to \pi^- \overline{K^0})} \\ &= \frac{|Te^{-i\gamma} + P|^2 + |Te^{i\gamma} + P|^2}{|P|^2 + |P|^2} = 1 + 2r\cos(\gamma) + r^2 \end{aligned} \tag{217}$$

where $r = |\frac{T}{P}|$. Using $\cos^2 + \sin^2 = 1$ we can rewrite this as

$$R = \sin^2(\gamma) + \Big(r + cos(\gamma)\Big)^2 \geq \sin^2(\gamma)\,. \tag{218}$$

Of course, this inequality only helps if R is below 1. When this idea was proposed, this was the case. However, more recent results indicate $R \geq 1$.

So far we have neglected final-state interactions. When we include these, we must also consider tree-level diagrams where the $u\bar{u}$ quark pair **rescatters** into a $d\bar{d}$ pair. Let the amplitude for this be $T'e^{i\gamma}$. It contains the same quark flavours as the annihilation graph. Then, the ratio R becomes

$$R = \frac{|Te^{-i\gamma} + P|^2 + |Te^{i\gamma} + P|^2}{|T'e^{-i\gamma} + P|^2 + |T'e^{i\gamma} + P|^2}\,. \tag{219}$$

In this case, no easy result is obtained, because there are two unknown ratios, $r = |\frac{T}{P}|$ and $r' = |\frac{T'}{P}|$.

A clever way out was proposed by Neubert and Rosner [47]. Instead of R, they propose the ratio

$$R_* = \frac{\mathrm{Br}(B^+ \to \pi^+ K^0) + \mathrm{Br}(B^- \to \pi^- \bar{K}^0)}{2[\mathrm{Br}(B^+ \to \pi^0 K^+) + \mathrm{Br}(B^- \to \pi^0 K^-)]} \equiv (1 - \Delta_*)^2 \,. \tag{220}$$

The point here is that the amplitudes are of the form $A + B$ and $A - B$, respectively, and therefore there is again only one unknown ratio. The treatment is quite well advanced and, as a result, rather reliable predictions on γ are possible. An up-to-date review can be found in Ref. [46].

Concluding remarks

In these lectures I have tried to show why flavour physics is important for a further insight into the fundamental structure of matter, and how various theoretical and experimental methods must combine in order to find fundamental parameters and forces. I have discussed, in some detail, flavour loops and their role in mixing and CP violation, both in kaon and B-meson physics. On a more technical level, some tools used to master QCD have been presented: operator expansion and renormalization, Heavy Quark Symmetry and Effective Theory, factorization approximations, and final-state effects. From lattice techniques only the results were borrowed. A summary of these methods can be found in a previous school in Ch. Sachrajda's lectures [70].

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