BPS Solitons in M5-Brane Worldvolume Theory with Constant Three-Form Field

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Abstract

We study BPS solutions for a self-dual string and a neutral string in M5-brane worldvolume theory with constant three-form field. We further generalize such solitons to superpose with a calibrated surface. We also study a traveling wave on a calibrated surface in the constant three-form field background.

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1 Introduction

Recently, noncommutative theories have received renewed interest, after it is founded out that noncommutative theories can be naturally realized in M-theory compactified in the presence of constant three-form field background [1] and within string theories as the worldvolume theory of D-brane with nonzero constant NS B-field [2, 3]. In the case of D-brane with constant magnetic NS B-field, one can take a decoupling limit of the D-brane worldvolume theory to achieve a noncommutative field theory with space/space noncommutativity. However, in the case of electric NS B-field background, one cannot take a zero slope limit in such a way that the noncommutativity parameter is nonzero to obtain a field theory with space/time noncommutativity, since the electric NS B field cannot be scaled to infinity due to its critical value beyond which the open string parameters are not well-defined [4]. It is conjectured [4, 5] that nearby the critical electric B field with an appropriate scaling limit a novel noncritical open string theory, called noncommutative open string (NCOS) theory, which decouples from closed strings (and therefore also gravity) in the bulk, emerges. It is conjectured [6, 7] that the strong coupling limit of the NCOS theory is the so-called OM theory, which is the decoupled (from gravity) theory of light open M2-brane ending on M5-brane in the constant three-form field background.

It would be therefore interesting to study worldvolume solitons of the M5-brane with constant 3-form field. The solitons of the brane worldvolume theories are interpreted as the intersections of the interesting branes [8], with the intersection being the source for the charge carried by the worldvolume soliton [9, 10, 11]. For example, the M5-brane worldvolume soliton counterpart to the open M2-brane ending on M5-brane is the self-dual string [12] and the three-brane soliton [13] on the M5-brane worldvolume is interpreted as the three-brane intersection of intersecting two M5-branes. The solitons in the D-brane worldvolume theory, i.e. BI solitons or BIons [14, 15, 12], with constant NS B field were previously studied, for example, in Refs. [16, 17, 18, 19, 20]. In the case of the zero-brane soliton, corresponding to fundamental or D string ending on a D-brane, force on the endpoint due to nonzero constant NS B field is shown [16] to cause the string to be tilted. The non-locality of the suspended string due to such tilting is in accordance with the fact that the D-brane worldvolume is noncommutative, which leads to uncertainty in measurement.

In this paper, we study the BPS self-dual string and neural (or instanton) string solitons in the M5-brane worldvolume theory with constant three-form field, and their generalization on calibrated surfaces. We also show that traveling wave on the calibrated surface with constant three-form field preserves supersymmetry. In studying such worldvolume solitons, we follow the covariant equations of motion approach [21, 22, 23] of M5-brane theory.

2   Aspects of M5-Brane Worldvolume Theory

We discuss the relevant aspect of the M5-brane worldvolume theory [21, 22, 23, 24, 25, 26] for the purpose of fixing and defining notation. The convention for indices is as follows. The indices for the target space are underlined. The indices from the beginning [middle] of the alphabet refer to the tangent space [coordinate space] indices. The Latin [Greek] indices are for the bosonic [fermionic] coordinates. The primed indices are for directions normal to the M5-brane worldvolume. So, the coordinates of the target and the worldvolume superspaces are respectively $Z^{\underline{M}} = (X^{\underline{m}}, \Theta^{\underline{\mu}})$ and $z^M = (x^m, \theta^\mu)$, and, for example, the target space index $\underline{a}$ $[\underline{\alpha}]$ is decomposed as $\underline{a} = (a, a')$ with $a = 0, 1, ..., 5$ and $a' = 1', ..., 5'$ $[\underline{\alpha} = (\alpha, \alpha')$ with $\alpha = 1, ..., 16$ and $\alpha' = 1', ..., 16']$. The fermionic indices α and α', running from 1 to 16, are alternatively written respectively as αi and $\,^\alpha_i$ when appearing as subscript, and respectively as αi and $\,^i_\alpha$ when appearing as superscript, with α and i running from 1 to 4. For example, $\Theta^{\alpha'} \to \Theta^i_\alpha$, $\Theta^\alpha \to \Theta^{\alpha i}$ and $u_\alpha^{\;\beta'} \to u_{\alpha i\beta}^{\quad j}$. We denote the vielbeins of the target and the M5-brane worldvolume superspaces as $E_{\underline{M}}^{\;\underline{A}}$ and $E_M^{\;A}$, respectively. We denote the embedding matrix $\tilde{E}_A^{\;\underline{A}} = E_A^{\;M}\partial_M Z^{\underline{M}} E_{\underline{M}}^{\;\underline{A}}$ with a tilde to avoid confusion with vielbeins. The worldvolume theory of the M5-brane is described by the $(2,0)$ tensor multiplet containing five scalars $X^{a'}$, sixteen fermions $\Theta^{\alpha'}$ and a self-dual field strength h_{abc}. Here, $X^{m'}$ and $\Theta^{\alpha'}$ are identified as the transverse components of the target superspace coordinates $(X^{\underline{m}}, \Theta^{\underline{\mu}})$.

In this paper, we consider an M5-brane embedded in flat eleven-dimensional target space with the metric $\hat{g}_{\underline{mn}} = \eta_{\underline{mn}}$. Just as in the case of D-brane with the NS B-field, the induced metric $g_{mn} = \eta_{\underline{mn}}\partial_m X^{\underline{m}}\partial_n X^{\underline{n}}$ on the M5-brane worldvolume does not correspond to the metric on the M5-brane felt by the open M2-brane in the presence of the background three-form field strength $\mathcal{H}_{mnp}$. Here, the gauge invariant field strength $\mathcal{H}_{mnp}$ of the M5-brane worldvolume two-form potential b_{mn} given by

$$\mathcal{H}_{mnp} = \partial_{[m}b_{np]} + \partial_m X^{\underline{m}}\partial_n X^{\underline{n}}\partial_p X^{\underline{p}}C_{\underline{mnp}}, \tag{1}$$

where $C_{\underline{mnp}}$ is the three-form potential in the eleven-dimensional supergravity, is related to the self-dual field strength h_{abc} as

$$\mathcal{H}_{mnp} = E_m^{\;a}E_n^{\;b}E_p^{\;c}m_b^{\;d}m_c^{\;e}h_{ade} = e_m^{\;a}e_n^{\;b}e_p^{\;c}(m^{-1})_c^{\;e}h_{abe}. \tag{2}$$

The self-duality condition on h_{abc}, $h_{abc} = \frac{1}{6}\epsilon_{abcdef}h^{def}$, is translated to the following non-linear self-duality condition on $\mathcal{H}_{mnp}$:

$$\frac{\sqrt{-\det g}}{6}\epsilon_{mnpqrs}\mathcal{H}^{qrs} = \frac{1+K}{2}(G^{-1})_m^{\;r}\mathcal{H}_{npr}, \tag{3}$$

where $K \equiv \sqrt{1 + \frac{1}{24}\mathcal{H}_{mnp}g^{mq}g^{nr}g^{ps}\mathcal{H}_{qrs}}$. It is suggested in Ref. [27] that it is the metric $G_{mn} = E_m^{\;a}E_n^{\;b}\eta_{ab}$ associated with $E_m^{\;a}$ that is felt by an open M2-brane ending on the

M5-brane. The vielbein $e_m{}^a$ associated with g_{mn} is related to $E_m{}^a$ as $E_m{}^a = e_m{}^b(m^{-1})_b{}^a$, where $m_a{}^b \equiv \delta_a{}^b - 2h_{acd}h^{bcd}$. So, the open M2-brane metric G_{mn} is expressed in terms of g_{mn} and $\mathcal{H}_{mnp}$ as

$$G_{mn} = \frac{1+K}{2K}\left(g_{mn} + \frac{1}{4}\mathcal{H}_{mpq}g^{pr}g^{qs}\mathcal{H}_{nrs}\right). \tag{4}$$

As expected, the open M2-brane metric G_{mn} becomes g_{mn} when $\mathcal{H}_{mnp} = 0$ and reduces to the open string metric [3] on a D-brane upon double dimensional reduction.

The equations of motion of the M5-brane can be obtained by analyzing the torsion equation subject to the superembedding condition $\tilde{E}_\alpha{}^{\underline{a}} = 0$. In a flat target superspace background in the static gauge, where the fermionic field $\Theta^{\underline{\alpha}} = (\Theta^\alpha, \Theta^{\alpha'})$ is zero and $X^m = x^m$, the bosonic equations of motion are

$$
\begin{aligned}
G^{mn}\nabla_m\nabla_n X^{a'} &= 0, \\
G^{mn}\nabla_m\mathcal{H}_{npq} &= 0,
\end{aligned}
\tag{5}
$$

where the connection in the covariant derivative ∇_m is defined from g_{mn}, i.e. $\Gamma_{mn}{}^p = \partial_m\partial_n X^{a'}\partial_q X^{b'}g^{pq}\delta_{a'b'}$.

We are particularly interested in solutions that preserve part of supersymmetry. The condition that the static gauge condition $\Theta^\alpha = 0$ be preserved under the combined rigid supersymmetry transformation of a flat target superspace and super-reparametrization of the worldvolume induced on the target superspace leads to the following form of the supersymmetry transformation for the fermions:

$$\delta\Theta^{\alpha'} = -\epsilon^\alpha(\tilde{E}^{-1})_\alpha{}^\beta\tilde{E}_\beta{}^{\alpha'}, \tag{6}$$

where the non-linearly realized symmetry parameterized by $\epsilon^{\alpha'}$ is set to zero. Making use of the projection operators $(\tilde{E}^{-1})_{\underline{\alpha}}{}^\beta\tilde{E}_\beta{}^{\underline{\gamma}} = \frac{1}{2}(1+\Gamma)_{\underline{\alpha}}{}^{\underline{\gamma}}$ and $(\tilde{E}^{-1})_{\underline{\alpha}}{}^{\beta'}\tilde{E}_{\beta'}{}^{\underline{\gamma}} = \frac{1}{2}(1-\Gamma)_{\underline{\alpha}}{}^{\underline{\gamma}}$, one can put the variation (6) for $\Theta^{\alpha'}$ into the form:

$$\hat{\delta}\Theta^{\alpha'} \equiv \delta\Theta^{\gamma'}\left(\frac{1-\Gamma}{2}\right)_{\gamma'}{}^{\alpha'} = -\frac{1}{2}\epsilon^\gamma\Gamma_\gamma{}^{\alpha'}. \tag{7}$$

This has to be set to zero to make the condition $\Theta^{\alpha'} = 0$ to be invariant under the supersymmetry transformation. In terms of the bosonic fields of the M5-brane worldvolume theory, the supersymmetry variation (7) is expressed as

$$
\begin{aligned}
\hat{\delta}\Theta_\beta{}^j =\ & -\frac{1}{2}\epsilon^{\alpha i}\Big[\det(e^{-1})\partial_m X^{a'}(\gamma^m)_{\alpha\beta}(\gamma_{a'})_i{}^j \\
& -\frac{1}{3!}\det(e^{-1})\partial_{m_1}X^{a'_1}\partial_{m_2}X^{a'_2}\partial_{m_3}X^{a'_3}(\gamma^{m_1 m_2 m_3})_{\alpha\beta}(\gamma_{a'_1 a'_2 a'_3})_i{}^j \\
& +\frac{1}{5!}\det(e^{-1})\partial_{m_1}X^{a'_1}\cdots\partial_{m_5}X^{a'_5}(\gamma^{m_1\ldots m_5})_{\alpha\beta}(\gamma_{a'_1\ldots a'_5})_i{}^j
\end{aligned}
$$

$$-h^{m_1 m_2 m_3} \partial_{m_2} X^{a'_2} \partial_{m_3} X^{a'_3} (\gamma_{m_1})_{\alpha\beta} (\gamma_{a'_2 a'_3})_i{}^j$$
$$-\frac{1}{3} h^{m_1 m_2 m_3} (\gamma_{m_1 m_2 m_3})_{\alpha\beta} \delta_i{}^j \Big], \tag{8}$$

where $\gamma_m = \delta_m^a \gamma_a$.

It is the purpose of this paper to study solitons in the M5-brane worldvolume theory in the constant three-form field $\mathcal{H}$ background. When $\mathcal{H}$ is constant, up to a Lorentz transformation the nonzero components of $\mathcal{H}$ satisfying the non-linear self-duality condition (3) are $\mathcal{H}_{012}$ and $\mathcal{H}_{345}$ [3, 27]. Equivalently, nonzero components of the worldvolume field h_{abc} are

$$h_{012} = -h_{345} = h = \text{constant}, \tag{9}$$

which corresponds to $\mathcal{H}_{012} = \frac{h}{1+4h^2} \equiv \frac{1}{4} \sin\theta$ and $\mathcal{H}_{345} = -\frac{h}{1-4h^2} = -\frac{1}{4} \tan\theta$, according to Eq. (2). In the infinite momentum frame (boosted along the x^5-direction), the nonzero components of h_{abc} are

$$h_{012} = -h_{034} = -h_{512} = h_{534} = \text{constant}. \tag{10}$$

The derivation of M5-brane solitons in the constant $\mathcal{H}$ background is along the same line as the case with zero $\mathcal{H}$ background [12, 13, 26], except that one has to impose the boundary condition (9) or (10) at the infinity of the worldvolume.

3 Self-Dual String

The self-dual string in the M5-brane worldvolume theory is interpreted as the boundary of a M2-brane ending on a M5-brane:

$$\begin{array}{ccccccc} M5: & 1 & 2 & 3 & 4 & 5 & \\ M2: & 1 & & & & & 6 \end{array} \tag{11}$$

All fields of the self-dual string soliton are independent of the worldvolume coordinates (x^0, x^1) of the string soliton. We denote the four M5-brane worldvolume indices for the directions transverse to the string soliton with tilde, i.e. $\tilde{a}, \tilde{m} = 2, ..., 5$. We let only one of the scalar fields, which we choose $X^{1'} \equiv \phi$, to be active. The bosonic worldvolume field Ansatz for the string soliton is

$$X^{1'} = \phi, \qquad h_{01\tilde{a}} = v_{\tilde{a}}, \qquad h_{\tilde{a}\tilde{b}\tilde{c}} = \epsilon_{\tilde{a}\tilde{b}\tilde{c}\tilde{d}} v^{\tilde{d}}, \tag{12}$$

with the remaining components of h_{abc} vanishing, along with the boundary condition $h_{01\tilde{a}} = h\delta_{\tilde{a}}^2$ at infinity.

4

Substituting the field Ansatz (12) into Eq. (8), one obtains the following supersymmetry variation of fermions Θ:

$$\hat{\delta}\Theta = -\frac{1}{2}\epsilon\left[\det(e^{-1})\partial_m\phi\gamma^m\gamma_{1'} - 2\gamma^{01}\left\{v^{\tilde{m}}\gamma_{\tilde{m}} + \det(e^{-1})v_{\tilde{m}}\gamma^{\tilde{m}}\right\}\right]. \tag{13}$$

If one requires spinors to satisfy the following constraint

$$\epsilon = \epsilon\gamma^{01}\gamma_{1'}, \tag{14}$$

then from the Killing spinor equation $\hat{\delta}\Theta = 0$ one obtains the following Bogomol'nyi condition on the fields:

$$v_{\tilde{a}} = \frac{1}{2}\frac{\delta_{\tilde{a}}^{\tilde{m}}\partial_{\tilde{m}}\phi}{1 + \sqrt{1 + (\partial_{\tilde{m}}\phi)^2}}, \tag{15}$$

where $(\partial_{\tilde{m}}\phi)^2 \equiv \delta^{\tilde{m}\tilde{n}}\partial_{\tilde{m}}\phi\partial_{\tilde{n}}\phi$.

Making use of Eq. (2) along with the field Ansatz (12), one obtains the following nonvanishing components of $\mathcal{H}$:

$$\mathcal{H}_{01\tilde{m}} = \frac{1}{1+4v^2}e_{\tilde{m}}^{\tilde{a}}v_{\tilde{a}},$$

$$\mathcal{H}_{\tilde{m}\tilde{n}\tilde{p}} = \frac{\sqrt{1+(\partial_{\tilde{m}}\phi)^2}}{1-4v^2}\epsilon_{\tilde{m}\tilde{n}\tilde{p}\tilde{q}}e_{\tilde{a}}^{\tilde{q}}v^{\tilde{a}}, \tag{16}$$

where $v^2 \equiv \delta^{\tilde{a}\tilde{b}}v_{\tilde{a}}v_{\tilde{b}}$. The vielbein e_m^a associated with the induced metric g_{mn} on the M5-brane worldvolume is given by $(e_m^a) = \mathrm{diag}(1,1,e_{\tilde{m}}^{\tilde{a}})$ with $e_{\tilde{m}}^{\tilde{a}} = \delta_{\tilde{m}}^{\tilde{a}} + c\partial_{\tilde{m}}\partial^{\tilde{a}}\phi$, where $c \equiv (-1 + \sqrt{1 + \delta^{\tilde{m}\tilde{n}}\partial_{\tilde{m}}\phi\partial_{\tilde{n}}\phi})/\delta^{\tilde{m}\tilde{n}}\partial_{\tilde{m}}\phi\partial_{\tilde{n}}\phi$. These nonzero components of $\mathcal{H}$ simplify to the following forms after the Bogomol'nyi condition (15) is substituted:

$$\mathcal{H}_{01\tilde{m}} = \frac{1}{4}\partial_{\tilde{m}}\phi,$$

$$\mathcal{H}_{\tilde{m}\tilde{n}\tilde{p}} = \frac{1}{4}\epsilon_{\tilde{m}\tilde{n}\tilde{p}\tilde{q}}\delta^{\tilde{q}\tilde{r}}\partial_{\tilde{r}}\phi, \tag{17}$$

where $\epsilon_{\tilde{m}\tilde{n}\tilde{p}\tilde{q}} = e_{\tilde{m}}^{\tilde{a}}e_{\tilde{n}}^{\tilde{b}}e_{\tilde{p}}^{\tilde{c}}e_{\tilde{q}}^{\tilde{d}}\epsilon_{\tilde{a}\tilde{b}\tilde{c}\tilde{d}}$.

To find the expression for the scalar ϕ, one has to solve the equation (5) for the scalar $X^{5'} = \phi$. It is shown [28] that generally the equation of motion $G^{mn}\nabla_m\nabla_n X^{a'} = 0$ for the scalar $X^{a'}$ implies $G^{mn}\partial_m\partial_n X^{a'} = 0$. Note, the worldvolume fields for self-dual string solution are independent of the worldvolume coordinates x^0 and x^1. And it can be shown by applying Eq. (15) that $G^{\tilde{m}\tilde{n}} \propto \delta^{\tilde{m}\tilde{n}}$. So, the scalar ϕ satisfies the flat Laplace's equation:

$$\delta^{\tilde{m}\tilde{n}}\partial_{\tilde{m}}\partial_{\tilde{n}}\phi = 0. \tag{18}$$

From Eq. (17), one can see that the solution to Eq. (18), satisfying the boundary condition $\mathcal{H}_{012} = \frac{h}{1+4h^2} = \frac{1}{4}\sin\theta$ at infinity, that describes array of strings with charge Q_K located at $x^{\tilde{m}} = y_K^{\tilde{m}}$ is given by

$$\phi = \phi_0 + \sum_K \frac{2Q_K}{|x - y_K|^2} + x^2 \sin\theta. \tag{19}$$

Due to the self-duality condition on $\mathcal{H}$, the string at $x^{\tilde{m}} = y_K^{\tilde{m}}$ carries the same electric and magnetic charges $Q_E = Q_M = Q_K$. From the second equation of Eq. (16) or (17), one can see that the boundary condition $\mathcal{H}_{345} = -\frac{h}{1-4h^2} = -\frac{1}{4}\tan\theta$ at infinity is automatically satisfied by Eq. (19). From the expression (19) for the scalar ϕ, one can see that the M2-brane is tilted towards the x^2-direction due to the force felt by the self-dual string at the boundary of the suspended M2-brane in the background of constant $\mathcal{H}$ field. This force due to the constant $\mathcal{H}$ field is canceled by the tension of the M2-brane.

By compactifying the above self-dual string solution along the x^1-direction, one obtains the following 0-brane soliton (BIon) on the D4-brane worldvolume in the constant B field background studies in Refs. [16, 17]:

$$\mathcal{F}_{0\tilde{m}} = \frac{1}{4}\partial_{\tilde{m}}\phi, \qquad \phi = \phi_0 + \sum_K \frac{2Q_K}{|x - y_K|^2} + x^2 \sin\theta, \tag{20}$$

where $\mathcal{F}_{\tilde{m}\tilde{n}} \equiv \mathcal{H}_{\tilde{m}\tilde{n}1}$. Dimensional reduction along, say, the x^5-direction leads to the following string soliton on the D4-brane worldvolume:

$$\mathcal{F}_{\tilde{m}\tilde{n}} = \frac{1}{4}\epsilon_{\tilde{m}\tilde{n}\tilde{p}}\delta^{\tilde{p}\tilde{q}}\partial_{\tilde{q}}\phi, \qquad \phi = \phi_0 + \sum_K \frac{2Q_K}{|x - y_K|^2} + x^2 \sin\theta, \tag{21}$$

where $\mathcal{F}_{\tilde{m}\tilde{n}} = \mathcal{H}_{\tilde{m}\tilde{n}5}$.

It is straightforward to show that a self-dual string on a calibrated surface in the constant h_{abc} field background also preserves supersymmetry. The amount of supersymmetry preserved depends on the type of calibrated surface. We denote X^I as the scalars associated with the calibrated surface and, as above, $X^{1'} = \phi$ is the scalar of the self-dual string soliton. The supersymmetry variation of the fermion is given by

$$
\begin{aligned}
\hat{\delta}\Theta &= -\frac{1}{2}\epsilon \Big[\Big\{ \det(e^{-1})\partial_m X^I \gamma^m \gamma_I \\
&\quad -\frac{1}{3!}\det(e^{-1})\partial_{m_1} X^{I_1}\partial_{m_2} X^{I_2}\partial_{m_3} X^{I_3} \gamma^{m_1 m_2 m_3}\gamma_{I_1 I_2 I_3} \\
&\quad +\frac{1}{5!}\det(e^{-1})\partial_{m_1} X^{I_1}\cdots\partial_{m_5} X^{I_5} \gamma^{m_1...m_5}\gamma_{I_1...I_5} \Big\} \\
&\quad +\det(e^{-1})\partial_m \phi \gamma^m \gamma_{1'} - 2\gamma^{01}\left(v^{\tilde{m}}\gamma_{\tilde{m}} + \det(e^{-1})v_{\tilde{m}}\gamma^{\tilde{m}}\right) \Big]. \tag{22}
\end{aligned}
$$

The self-dual string soliton on the calibrated surface preserves supersymmetry, if it satisfies the Killing spinor equation $\hat{\delta}\Theta = 0$ with nonzero ϵ. We rather consider two

6

equations obtained by setting two separate terms, i.e. terms in the curly bracket of Eq. (22) and the remaining terms, equal to zero. The first equation (associated with the terms in the curly bracket) determines the amount of supersymmetry preserved by and the geometry of the calibrated surface. The second equation is for the self-dual string soliton on the calibrated surface. The amount of preserved supersymmetry can be determined by considering the supersymmetry projectors of the associated intersecting M5-brane configuration and of the added M2-brane. One can also add an M2-brane to an intersecting M5-brane configuration without breaking additional supersymmetry, if the supersymmetry projectors associated with M5-branes yield the supersymmetry projector for the added M2-brane. The scalar ϕ no longer satisfies the flat Laplace's equation, but satisfies an equation determined by the curved metric of the calibrated surface.

4 Neutral String

We consider the worldvolume soliton counterpart to the following target space configuration where M-wave travels along an longitudinal direction of an M5-brane:

$$
\begin{array}{llllll}
M5: & 1 & 2 & 3 & 4 & 5 \\
MW: & & & & & 5
\end{array}
\tag{23}
$$

When the x^5-direction is compactified, this configuration becomes a D0-brane in a D4-brane, where the D0-brane is interpreted as an instanton of the D4-brane worldvolume theory [29, 30]. The worldvolume soliton counterpart to (23) has no active scalar and has non-vanishing $\mathcal{H}_{0\tilde{m}\tilde{n}}$ and $\mathcal{H}_{5\tilde{m}\tilde{n}}$, where $\tilde{m}, \tilde{n} = 1, 2, 3, 4$. The nonzero component $\mathcal{H}_{5\tilde{m}\tilde{n}} = F_{\tilde{m}\tilde{n}}$ is (anti-) self-dual as a two-form in four-dimensional Euclidean space and gives rise to a string-like soliton in the x^5-direction. But such string-like soliton does not carry charge of the $\mathcal{H}$ field. So, such soliton is called an instanton or a neutral string. The corresponding bosonic worldvolume field Ansatz is

$$
h_{0\tilde{a}\tilde{b}} = \pm h_{5\tilde{a}\tilde{b}} \equiv F_{\tilde{a}\tilde{b}},
\tag{24}
$$

where $\tilde{a}, \tilde{b} = 1, ..., 4$ and all of the scalar fields $X^{a'}$ are set to zero (hence, the induced metric is flat, $g_{mn} = \eta_{mn}$). To consider an instanton string solution on the noncommutative M5-brane worldvolume, one has to impose the boundary condition that h_{mnp} is nonzero constant at infinity. In order for the boundary condition to be compatible with the field Ansatz (24), one has to go to the infinite momentum frame (through infinite boost along the x^5-direction), in which the nonzero components of h_{mnp} at infinity are given by Eq. (10). This boundary condition can be imposed on the field Ansatz (24) only for the negative sign choice in Eq. (24), i.e. only when $F_{\tilde{m}\tilde{n}}$ is anti-self-dual [2].

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[2]The infinite boost along the negative x^5-direction would lead to the boundary condition on h_{abc} with opposite signs, and therefore select the self-dual $F_{\tilde{m}\tilde{n}}$ (i.e. the positive sign in Eq. (24)).

Then, the supersymmetry variation (8) of fermions takes the following form:

$$\hat{\delta}\Theta = -\frac{1}{2}\epsilon F^{\tilde{m}\tilde{n}}(\gamma_{0\tilde{m}\tilde{n}} - \gamma_{5\tilde{m}\tilde{n}}). \tag{25}$$

The resulting Killing spinor equation $\hat{\delta}\Theta = 0$ can be satisfied, if we impose the following constraint on spinors:

$$\epsilon\gamma_0\gamma_5 = \epsilon. \tag{26}$$

We show that a neutral string on a calibrated surface in the constant h_{abc} field background also preserves supersymmetry. For this purpose, it is convenient to introduce the light-cone coordinates:

$$u = \frac{1}{\sqrt{2}}(x^0 - x^5), \qquad v = \frac{1}{\sqrt{2}}(x^0 + x^5). \tag{27}$$

In terms of the light-cone coordinates, the boundary condition (10) on the h_{abc} field is

$$h_{u12} = -h_{u34} = \text{constant}, \tag{28}$$

and the bosonic field Ansatz (24) with negative sign becomes

$$h_{u\tilde{a}\tilde{b}} := \bar{F}_{\tilde{a}\tilde{b}}. \tag{29}$$

From Eq. (2), one can see that the only nonzero component of the corresponding $\mathcal{H}$ field is $\mathcal{H}_{u\tilde{m}\tilde{n}} = \bar{F}_{\tilde{m}\tilde{n}}$. The supersymmetry variation of fermions is

$$\tilde{\delta}\Theta = -\frac{1}{2}\epsilon\left[\bar{F}^{\tilde{m}\tilde{n}}\gamma_{v\tilde{m}\tilde{n}} + \bar{F}^{\tilde{m}\tilde{n}}\partial_{\tilde{m}}X^{I_1}\partial_{\tilde{n}}X^{I_2}\gamma_v\gamma_{I_1 I_2}\right], \tag{30}$$

where X^I are scalars associated with the calibrated surface and ϵ is the spinor for supersymmetry preserved by the calibrated surface, determined by setting the terms in the curly bracket of Eq. (22) equal to zero. By imposing the spinor constraint $\epsilon\gamma^u = 0$, one can set this supersymmetry variation to zero, making use of the relation $\gamma_v = \gamma^u$. The possibility of having such supersymmetric configuration depends on whether the supersymmetry projectors of the intersecting M5-brane configuration associated with the calibrated surface yields the projector $\epsilon\gamma^u = 0$ (or $\epsilon\gamma^0\gamma^5 = \epsilon$). Had one used the boundary condition on h_{abc} corresponding to the infinitely boosted frame along the negative x^5-direction, one would have had a self-dual instanton string on a calibrated surface with $\mathcal{H}_{v\tilde{m}\tilde{n}} := \bar{G}_{\tilde{m}\tilde{n}}$ and with the associated spinor constraint $\epsilon\gamma^v = 0$.

5 Traveling Wave on Calibrated Surface

In this section we show that supersymmetric traveling wave on a calibrated surface exists even in the nonzero constant h_{abc} field background. Traveling wave on the M5-brane worldvolume is regarded as fluctuations in the shape of the calibrated surface.

We consider the wave traveling in the x^5-direction. For the purpose of studying traveling wave, it is convenient to work with the light-cone coordinates (27) and constant background h_{abc} field of the form (28) in the infinite momentum frame. We expect that either purely left-moving or right-moving wave moving at the speed of light, i.e. the case when scalar fields $X^{a'}$ do not depend on either u or v, preserves supersymmetry. We consider the case when $X^{a'}$ are independent of v, i.e. $\partial_v X^{a'} = 0$. Then, the supersymmetry transformation (8) reduces to the following form:

$$
\begin{aligned}
\hat{\delta}\Theta &= -\frac{1}{2}\epsilon\Big[\det(e^{-1})\partial_u X^{a'}\gamma^u\gamma_{a'} \\
&\quad -\frac{1}{3!}\det(e^{-1})\partial_u X^{a'_1}\partial_{m_2}X^{a'_2}\partial_{m_3}X^{a'_3}\gamma^{um_2m_3}\gamma_{a'_1a'_2a'_3} \\
&\quad +\frac{1}{5!}\det(e^{-1})\partial_u X^{a'_1}\partial_{m_2}X^{a'_2}\cdots\partial_{m_5}X^{a'_5}\gamma^{um_2\ldots m_5}\gamma_{a'_1\ldots a'_5} \\
&\quad -h^{vm_2m_3}\partial_{m_2}X^{a'_2}\partial_{m_3}X^{a'_3}\gamma_v\gamma_{a'_2a'_3} \\
&\quad -\frac{1}{3}h^{vm_2m_3}\gamma_{vm_2m_3}\Big].
\end{aligned}
\tag{31}
$$

This supersymmetry variation can be set to zero by imposing the spinor constraint $\epsilon\gamma^u = 0$. So, traveling wave in the constant h_{abc} field background (28) with arbitrary dependence of $X^{a'}$ on u preserves supersymmetry. The analysis of the case with no dependence on u, i.e. $\partial_u X^{a'} = 0$, is along the same line. The corresponding supersymmetry projector is $\epsilon\gamma^v = 0$ (or $\epsilon\gamma^0\gamma^5 = -\epsilon$).

Note Added

While this work was being completed, there appeared the paper [31] which has overlapping results with the section 3 of our paper but with slightly different derivation of the self-dual string soliton from ours.

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