

# Massive sterile neutrinos as warm Dark Matter

A.D. Dolgov<sup>1,2</sup>, S.H. Hansen<sup>3</sup>

*INFN section of Ferrara*

*Via del Paradiso 12, 44100 Ferrara, Italy*

## Abstract

We show that massive sterile neutrinos mixed with the ordinary ones may be produced in the early universe in the right amount to be natural warm dark matter particles. Their mass should be in the range 1-40 keV and the corresponding mixing angles  $\sin^2 2\theta = 10^{-8}$  to  $10^{-11}$  for mixing with  $\nu_\mu$  or  $\nu_\tau$ , and mixing with  $\nu_e$  is marginally allowed with mass about 30 keV and  $\sin^2 \theta \approx 10^{-11}$ .

## 1 Introduction

There seems to be convincing experimental evidence for non-zero neutrino masses and mixing angles (for a review see e.g. [1]), and if all the present day data are correct, there must exist at least one sterile neutrino species. These neutrinos should be very light (sub eV range) and hence contribute negligibly to the cosmological energy density [2]

$$\Omega_\nu h^2 = \frac{m_\nu}{92\text{eV}} \quad , \quad (1)$$

if they were produced with the equilibrium number density in the early universe at a temperature below  $\sim 10$  MeV. If their number density was smaller than the equilibrium one, then the permitted value of the mass could be correspondingly higher.

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<sup>1</sup>Also: ITEP, Bol. Chermushkinskaya 25, Moscow 113259, Russia.

<sup>2</sup>e-mail: [dolgov@fe.infn.it](mailto:dolgov@fe.infn.it)

<sup>3</sup>e-mail: [sthansen@fe.infn.it](mailto:sthansen@fe.infn.it)

One could easily envisage more than one sterile neutrino species. The masses and mixing angles of these extra neutrinos are essentially free parameters. If we consider sterile neutrinos with masses  $10 - 200$  MeV, then big bang nucleosynthesis and energy loss arguments for SN 1987A allow one to exclude mixing angles in the range  $\sin^2 2\theta = 10^{-1} - 10^{-12}$  [3]. On the other hand, direct terrestrial experiments exclude supplementary mixing angles in the range  $\sin^2 2\theta = 0.001 - 1$  [4]. We should mention that there is still some non-excluded parameter space for  $m < 40$  MeV and  $\sin^2 2\theta = 0.01 - 1$ .

The hypothesis that sterile neutrinos could make a considerable contribution to cosmological dark matter has a rather long history. The idea that right-handed sterile neutrinos may form warm dark matter was briefly discussed in ref. [5] and was further pursued in the paper [6]. In more detail warm dark matter cosmology was considered in ref. [7]. There are some other warm dark matter candidates discussed in the papers [8, 9]. Sterile neutrinos coming from mirror world may also make WDM in our universe; they were discussed in the papers [10]. More models and references can be found in the recent works [11, 12]. A dark matter model with sterile neutrinos but with a non-thermal spectrum was considered in ref. [13]. Such neutrinos could be produced by the resonance oscillations in the early universe in the presence of a large lepton asymmetry. This model was further considered in ref. [14], where constraints originating from consideration of decays of  $\nu_s$ , especially of the radiative one, were presented.

In this paper we find the allowed values of mass and mixing angle of a sterile neutrino,  $\nu_s$ , so that the latter could be a dominant dark matter particle. We will consider the production of  $\nu_s$  in the early universe, calculate their energy spectrum, discuss their different decay modes, and derive bounds on mass/mixing from cosmology and astrophysics.

## 2 Production of $\nu_s$ in the early universe

Let us consider for simplicity a two-neutrino mixing scheme, where one of the active neutrinos,  $\nu_a = \nu_e, \nu_\mu$  or  $\nu_\tau$ , mixes with a heavy mainly sterile neutrino,  $\nu_s$ ,

$$\begin{aligned}\nu_a &= \cos \theta \nu_1 + \sin \theta \nu_2, \\ \nu_s &= -\sin \theta \nu_1 + \cos \theta \nu_2,\end{aligned}\tag{2}$$

where  $\nu_1$  and  $\nu_2$  are assumed to be the light and heavy mass eigenstates respectively, and  $\theta$  is the vacuum mixing angle. We will consider small mixing angles, and hence sometimes refer to the light neutrino mass eigenstate as the active neutrino and the heavy one as sterile neutrino.

We assume that sterile neutrinos were initially absent in the primeval plasma and were produced through the mixing with active ones. The production rate is usually approximated as [15]

$$\Gamma/H = \frac{\sin^2 2\theta_M}{2} \left( \frac{T}{T_W} \right)^3,\tag{3}$$

where  $H$  is the Hubble expansion parameter,  $T$  is the plasma temperature, and  $T_W$  is the decoupling temperature of the active neutrinos, which is approximately taken about  $T_W = 3$  MeV. Instead of this approximate equation, below we will write down and solve the exact momentum dependent Boltzmann equation, taking into account the processes of production of  $\nu_s$  but neglecting inverse reactions. The latter are not important if the number density of  $\nu_s$  is small. In this more precise approach the question of the value of  $T_W$  never appears, it is solved automatically. Another advantage of our approach here is that it permits to calculate the energy spectrum of  $\nu_s$ , while the previous method permitted only to estimate the total number density. Before doing these calculations it may be instructive to make the standard simplified estimates and later compare them with the exact results found below.

The mixing angle  $\sin^2 2\theta_M$  is suppressed at large temperatures due to matter effects, and for  $\nu_\mu$  or  $\nu_\tau$  mixing it can be written as [16]

$$\sin 2\theta_M \approx \frac{\sin 2\theta}{1 + 0.8 \times 10^{-19} (T_\gamma/\text{MeV})^6 (\delta m^2/\text{MeV}^2)^{-1}}, \quad (4)$$

where the coefficient in front of the second term in the denominator was obtained by a rather arbitrary procedure of thermal averaging of the factor  $\langle E^2 \rangle \approx 12T^2$ , entering the ratio of the neutrino refraction index to the vacuum term  $\delta m^2/2E^2$ . We see from this expression that matter effects become essential and suppress the mixing for  $T_\gamma > 0.15 \text{ GeV} (m/\text{keV})^{1/3}$  (a similar argument was made in ref. [6] for the left-right neutrino mixing). For the  $(\nu_e - \nu_s)$ -mixing the factor in the denominator should be 2.7 instead of 0.8. We will assume here that  $\delta m^2 = m_2^2 - m_1^2$  is positive (specifically we assume  $m_1 \ll m_2$ ), and if instead the active neutrino is heavy the analysis somewhat changes (see section 4 and ref. [13]).

For the energy dependent calculations we need the expression for the matter effects in the denominator of eq. (4) prior to averaging over the thermal bath. The latter can be read off from the relevant equations of refs. [16, 15]

$$\sin 2\theta_M = \frac{\sin 2\theta}{1 + 3.73 \cdot 10^{-20} c_2 m(\text{MeV})^{-2} (y^2/x^6)}, \quad (5)$$

where the  $\nu_s$  mass,  $m$ , is measured in MeV and we introduced the new variables,  $x = 1\text{MeV} \times a$ ,  $y = Ea$ , and neglected a possible entropy release so that the temperature drops according to  $T = 1/a$ . The numerical coefficient  $c_2$  depends upon the neutrino flavour:  $c_2 = 0.61$  for  $\nu_e$  and  $c_2 = 0.17$  for  $\nu_\tau$  and  $\nu_\mu$ . However, for the temperatures close to or above the muon mass  $c_2$  becomes the same for  $\nu_e$  and  $\nu_\mu$ .

The Boltzmann equation describing the evolution of the sterile neutrino distribution function,  $f_s$ , in terms of these new variables takes the form:

$$xH\partial_x f_s = I_{\text{coll}}, \quad (6)$$

where the collision integral is given by

$$I_{\text{coll}} = \frac{1}{2E_s} \int \frac{d^3 p_2}{(2\pi)^3 2E_2} |A|^2 f_3 f_4 d\tau_{3,4} \quad (7)$$

where  $d\tau_{3,4}$  is the phase space element (together with the energy-momentum  $\delta$ -function) of the particles  $l_3$  and  $l_4$  in whose collision  $\nu_s$  is produced,

$$l_3 + l_4 \rightarrow \nu_s + l_2, \quad (8)$$

and  $f_{3,4}$  are their distribution functions. We assume that the latter are equal to their equilibrium values and then the conservation of energy gives  $f_3 f_4 = \exp(-y_1 - y_2)$  in the Boltzmann approximation. Integrating the probabilities of all the relevant processes over phase space (see the appendix) allows to find the collision integral and to solve the equation (7) analytically

$$f_s = 3.6 \cdot 10^8 \sin^2 \theta \left(1 + g_L^2 + g_R^2\right) c_2^{-1/2} m(\text{MeV}) \left(\frac{10.75}{g_*}\right)^{1/2} f_a, \quad (9)$$

where  $f_a$  is the distribution function of any of the active neutrinos, and  $g_*$  is the number of relativistic degrees of freedom at the time when the sterile neutrinos were produced. Subsequent to the production there will be a dilution of the active neutrinos relative to the sterile ones. This is described by another factor  $(g_*/10.75)$ .

The coefficient relating  $f_s$  to  $f_a$  in eq. (9) is independent on the energy of neutrinos, so the spectrum of  $\nu_s$  remains the same as that of active neutrinos. This is somewhat surprising because the reaction rate is proportional to the neutrino energy. However for smaller  $E$  the rate becomes efficient at higher temperature, as one can see from the expression (5) describing the suppression of neutrino mixing in matter. This effect compensates the factor  $y$  in the kinetic equation. However at very large  $T \sim M_{W,Z} \sim 100$  GeV the weak reaction rate drops down so that the spectrum would be somewhat distorted at very small  $y$ 's.

It is interesting to compare the accurate results presented above with the simplified calculations based on the solution of the following approximate kinetic equation:

$$Hx \partial_x f_s = \frac{1}{2} \sin^2 2\theta_M \Gamma_W f_a, \quad (10)$$

where the mixing angle and interaction rate can be taken from eqs. (3, 4). This equation is easily integrated, and we find that the result for the total number density of  $\nu_s$  agree within a factor of 2 with the more accurate result (9).

Up to now we have seen how the produced amount of sterile neutrinos depends on the mass and mixing angle, so let us instead ask: how many sterile neutrinos should be produced in order for them to be a dark matter candidate? Let us take  $\Omega_{\text{DM}} = 0.3$ , which means that we must demand  $\rho_s = 3 h^2 \text{keV}/\text{cm}^3$ . Using  $h = 0.65$  and  $n_\alpha^{\text{today}} = 100/\text{cm}^3$  one finds

$$n_s = 1.27 \times 10^{-5} n_\alpha \left( \frac{\text{MeV}}{m} \right) \left( \frac{\Omega_{\text{DM}}}{0.3} \right) \left( \frac{h}{0.65} \right)^2, \quad (11)$$

and comparing eqs. (9) and (11) obtains

$$\sin^2 \theta = 3.6 \times 10^{-14} \frac{c_2^{1/2}}{(1 + g_L^2 + g_R^2)} \left( \frac{10.75}{g_*} \right)^{1/2} \left( \frac{\text{MeV}}{m} \right)^2. \quad (12)$$

This equation thus describes a line in mass-mixing parameter space, where the sterile neutrino must lie, if it indeed is the dominant dark matter particle. Let us now see how decay processes and supernovae can further restrict this parameter space.

### 3 Decay

The mixing couples the heavier  $\nu_2$  to the Z-boson, and allows the decay channel

$$\nu_2 \rightarrow \nu_1 + \ell + \bar{\ell}, \quad (13)$$

where  $\nu_1$  is mostly an active flavour and  $\ell$  is any lepton with a mass smaller than half the mass of the heavy neutrino. This mixing angle can be translated into decay time

$$\tau = \frac{10^5 f(m)}{m(\text{MeV})^5 \sin^2 2\theta} \text{sec}, \quad (14)$$

where  $f(m)$  takes into account the open decay channels (for  $m < 1$  MeV only the neutrino channels are open, and  $f(m) = 0.86$ , while for  $m_s > 2m_e$  the  $e^+e^-$ -channel is also open and  $f = 1$ ). Now, for the sterile neutrino to be a dark matter candidate we must demand that it does not decay on cosmic time scales, which means  $\tau > 4 \times 10^{17}$  sec, and hence from eq. (14) we get

$$\sin^2 2\theta < 2.5 \times 10^{-13} \frac{f(m)}{m(\text{MeV})^5}. \quad (15)$$

We can, however, get even a stronger bound by considering the radiative decay

$$\nu_s \rightarrow \nu_a + \gamma, \quad (16)$$

where  $\nu_a$  is any of the active neutrinos. This decay will contribute with a distinct line into the diffuse photon background near  $m/2$ . The branching ratio for the reaction (16) was found [17] to be:  $BR \approx 1/128$ . The flux of electromagnetic radiation from the decay was calculated in the papers [18, 19] (see also refs. [20, 14]). In the case of a large life-time, larger than the universe age, and of the matter dominated flat universe the intensity of the radiation in the frequency interval  $d\omega$  is equal to:

$$dI = (BR) \frac{n_s^{(0)}}{H\tau_s} \frac{\omega^{1/2} d\omega}{(m_s/2)^{3/2}} \quad (17)$$

where  $n_s^{(0)}$  is the present day number density of  $\nu_s$  and  $H$  is the Hubble constant. We neglected here corrections related to the a possible dominance of the lambda-term in the latest history of the universe.

Assuming the following rather conservative upper limit for the flux (see e.g. ref. [20]):

$$\frac{d\mathcal{F}}{d\Omega} < 0.1 \left( \frac{1\text{MeV}}{E} \right) \text{cm}^{-2} \text{sr}^{-1} \text{sec}^{-1} \quad (18)$$

and taking the accepted now values  $\Omega_s = 0.3$  and  $h = 0.65$  we find:  $\tau > 4 \times 10^{22}$ , which leads to the bound

$$\sin^2 2\theta < 2.5 \times 10^{-18} \frac{f(m)}{m(\text{MeV})^5}. \quad (19)$$

A mass independent lower bound can be found by considering the energy loss argument for SN 1987A, for  $m_s < 3T_{SN} \approx 100$  MeV. Sterile neutrinos produced due to mixing with the active ones inside the supernova would carry away too much energy, hence shortening the explosion. The excluded mixing angles have been calculated several times for SN 1987A, and the results are about  $\sin^2 2\theta < 3 \times 10^{-8}$  for  $\nu_\mu$  or  $\nu_\tau$  mixing [3] and  $\sin^2 2\theta < 10^{-10}$  for the  $\nu_e$  mixing [21].

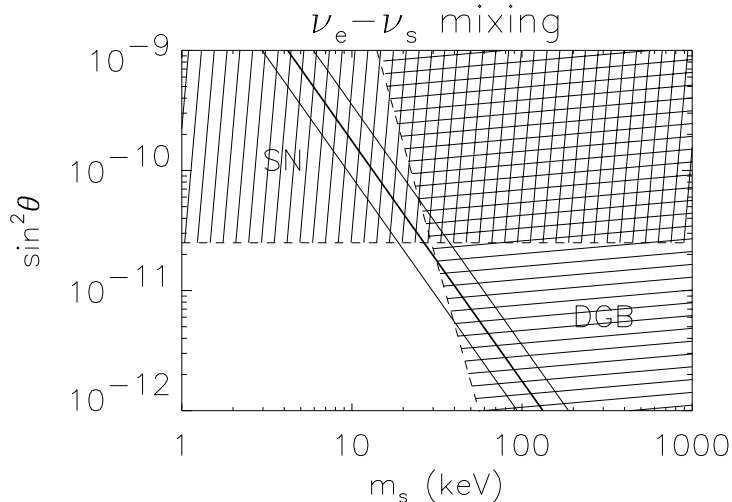


Figure 1: Bounds from  $\nu_e - \nu_s$  mixing. The middle full line describes the mass-mixing relationship if sterile neutrinos are the dark matter. The two other full lines allow a factor 2 uncertainty in the amount of dark matter,  $\Omega_{DM} = 0.15 - 0.6$ . The hatched region for big masses is excluded by the Diffuse Gamma Background. The hatched region for big mixing angles is excluded by the duration of SN 1987A..

Plotting the equation describing the production, eq. (12), together with the bounds from SN 1987A and radiative decay, makes it obvious that  $\nu_e - \nu_s$  mixing (see fig. 1) as the producer of dark matter is only marginally allowed with  $m \approx 30$  keV and  $\sin^2 2\theta \approx 10^{-11}$ . The two thinner full lines in the figure allow for a factor 2 uncertainty in the amount of dark matter,  $\Omega_{DM} = 0.15 - 0.60$ . In fig. 2 we see that  $\nu_\tau - \nu_s$



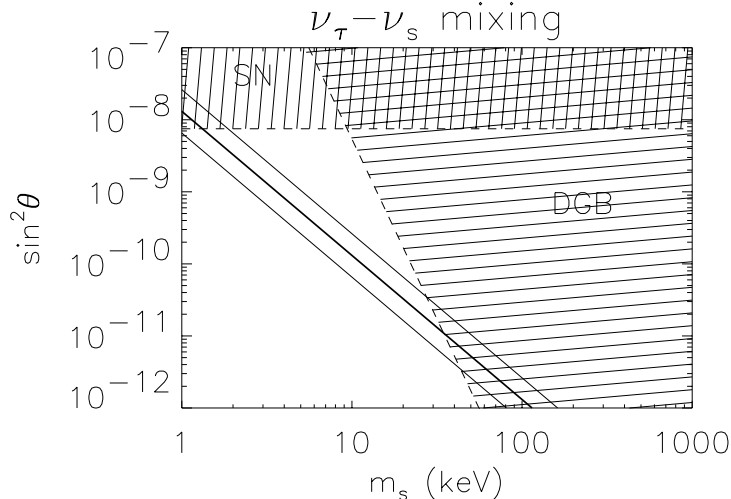


Figure 2: Same as fig. 1, for  $\nu_\tau - \nu_s$  mixing..

(or  $\nu_\mu - \nu_s$ ) mixing is a far more promising possibility with an allowed mass range  $m = 1 - 40$  keV and corresponding mixing angles  $\sin^2\theta = 10^{-8} - 10^{-11}$ .

Several comments are in order here. First we must check that the sterile neutrinos are indeed relativistic when produced. This is the case, because the temperature  $T_{\max}$  is about 1.3 GeV for  $m = 1\text{MeV}$ , and about 0.13 GeV for  $m = 1\text{keV}$ . Further, the dilution factor is somewhere between 1 and 4 depending upon whether the production happens before or after the QCD transition, and can thus enlarge the allowed region slightly compared to the figures, where we for simplicity used  $g_* = 10.75$ . Looking at eq. (9) it seems that the sterile neutrinos follow an equilibrium distribution function. This is not quite the case, because the small momentum neutrinos are produced first, and hence their relative importance is increased by the subsequent entropy release (which dilutes the active neutrinos). A different non-thermal effect can appear for  $\nu_\mu - \nu_s$  mixing, since the factor  $c_2$  is 0.17 when the  $\mu$ 's are absent (for  $T \ll m_\mu$ ), whereas

it grows to  $c_2 = 0.61$  when the muons are fully present in the plasma. This means that bigger momenta will be produced with a factor 1.9 more efficiently [22].

## 4 Discussion and conclusion

There are several other warm dark matter candidates as mentioned in the introduction, and one particularly interesting and related possibility is that the sterile neutrino is lighter than the active,  $\delta m^2 < 0$ , which allows a resonant transition, giving a non-thermal spectrum [13]. In that resonant transition model the adiabaticity condition demands the mixing angle in the region  $\sin^2 2\theta > 10^{-9} - 10^{-7}$ , which for  $\nu_\mu$  or  $\nu_\tau$  mixing is near the excluded region from SN 1987A, and certainly will be covered with the next nearby supernova. For  $\nu_e$  mixing this possibility is already excluded by SN 1987A.

Let us return to the model considered in this paper, which undoubtedly is the simplest, oldest and, as we have seen, a very natural for warm dark matter. The value of the  $\nu_s$  mass can be found from the detailed analysis of large scale structure formation. In the case of mixed dark matter scenario when both cold and warm dark matter are cosmologically significant the  $\nu_s$  parameter space is less constrained and in particular permits masses below 1 keV which may be interesting for galaxy formation problem [11]. With a future nearby supernova we will definitely reach deeper into the relatively small allowed parameter space. Furthermore, better observations of the diffuse  $\gamma$  background around  $E = 1 - 20$  keV should be able to cut away more of the parameter space, or potentially make an indirect observation of dark matter.

## A Solving the Boltzmann equation

All the relevant processes were presented in table 2 of ref. [3]. There are two kinds of matrix elements, namely  $(p_1 p_2)(p_3 p_4)$  and  $(p_1 p_4)(p_3 p_2)$ , and one finds from the

integral over phase space that

$$\int d\tau_{34}(p_1 p_2)(p_3 p_4) = 3 \int d\tau_{34}(p_1 p_4)(p_3 p_2) = \frac{(p_1 p_2)}{8\pi}. \quad (20)$$

Now one can count all the relevant processes, integrate over momenta and find

$$Hx\partial_x f_s = \frac{5 \times 2^4}{3\pi^3} \sin^2 \theta \left(1 + g_L^2 + g_R^2\right) G_F^2 E_1 T^4 f_a, \quad (21)$$

where  $Hx = 4.5 \times 10^{-22} \left(\frac{g_*}{10.75}\right) x^{-1}$  MeV and  $G_F = 1.1664 \times 10^{-5}$  GeV. With the variable  $\xi = y/x^3$  and  $\beta$  defined in eq. (5) the suppression of mixing angle is

$$\sin 2\theta_M = \frac{\sin 2\theta}{1 + \beta^2 \xi^2}, \quad (22)$$

and with the integral

$$\int_0^\infty d\xi \left(\frac{1}{1 + \beta^2 \xi^2}\right)^2 = \frac{4}{\pi} \frac{1}{\beta}, \quad (23)$$

we find the result in eq. (9).

## References

- [1] J. Ellis, hep-ph/0008334.
- [2] S.S. Gerstein, Ya.B. Zeldovich, ZhETF Pis'ma Red. **4** (1966) 174;  
R. Cowsik and J. McClelland, Phys. Rev. Lett. **29** (1972) 669.
- [3] A. D. Dolgov, S. H. Hansen, G. Raffelt and D. V. Semikoz, Nucl. Phys. **B580**  
(2000) 331;  
A. D. Dolgov, S. H. Hansen, G. Raffelt and D. V. Semikoz, To appear in Nucl.  
Phys., hep-ph/0008138.
- [4] NOMAD Collaboration, paper in preparation.
- [5] K.A. Olive and M.S. Turner, Phys. Rev. **D25** (1982) 213.

- [6] S. Dodelson and L. M. Widrow, Phys. Rev. Lett. **72** (1994) 17.
- [7] S. Colombi, S. Dodelson, and L.M. Widrow, Astrophys. J. 458 (1996) 1.
- [8] R. A. Malaney, G. D. Starkman and L. Widrow, Phys. Rev. **D52** (1995) 5480.
- [9] G. Dvali and Y. Nir, JHEP **9810** (1998) 014.
- [10] Z.G. Berezhiani and R.N. Mohapatra Phys. Rev. **D52** (1995) 6607;  
Z.G. Berezhiani, A.D. Dolgov, and R.N. Mohapatra, Phys.Lett. **B375** (1996) 26;  
Z.G. Berezhiani, Acta Phys.Polon. **B27** (1996) 1503.
- [11] J. Sommer-Larsen and A.D. Dolgov, astro-ph/9912166.
- [12] S. Hannestad, astro-ph/0008451.
- [13] X. Shi and G. M. Fuller, Phys. Rev. Lett. **82** (1999) 2832.
- [14] M. Drees and D. Wright, hep-ph/0006274.
- [15] R. Barbieri and A.D. Dolgov, Phys. Lett. **B237** (1990) 440;  
K. Kainulainen, Phys. Lett. **B244** (1990) 191;  
K. Enqvist, K. Kainulainen, and M.Thomson, Nucl. Phys. **B373** (1992) 498;  
A.D. Dolgov, hep-ph/0006103.
- [16] D. Nötzold and G. Raffelt, Nucl. Phys. **B307** (1988) 924.
- [17] V. Barger, R. J. Phillips and S. Sarkar, Phys. Lett. **B352** (1995) 365.
- [18] F.W. Stecker, Phys. Rev. Lett. **45** (1980) 1460.
- [19] R. Kimble, S. Boyer, and P. Jacobsen, Phys. Rev. Lett. **46** (1981) 80.
- [20] E. W. Kolb and M. S. Turner, “The Early Universe,” *Redwood City, USA: Addison-Wesley (1990).*

- [21] K. Kainulainen, J. Maalampi, and J.T. Peltoniemi, Nucl. Phys. **358** (1991) 435;  
G. Raffelt and G. Sigl, Astropart. Phys. **1** (1993) 165.
- [22] Lacking a better synonym than *lukewarm* dark matter, we decide to use the standard name *sterile neutrinos* (see [11, 13] for great names like *warmons* or *coolDM*).