The Infrared behaviour of the gluon propagator in SU(2) and SU(3) without lattice Gribov copies*

C. Alexandrou[a], Ph. de Forcrand[b,c], E. Follana[a]

[a] *Department of Physics, University of Cyprus, CY-1678 Nicosia, Cyprus*
[b] *Inst. für Theoretische Physik, ETH Hönggerberg, CH-8093 Zürich, Switzerland*
[c] *CERN, Theory Division, CH-1211 Geneva 23, Switzerland*

We present lattice results for the gluon propagator for SU(2) and SU(3) in the Laplacian gauge which avoids lattice Gribov copies. In SU(3) we compare with the most recent lattice calculation in Landau gauge and with various approximate solutions of the Dyson Schwinger equations (DSE).

Introduction

We first summarize the results obtained within the Landau gauge[1]: By solving approximately the DSE, Mandelstam found an infrared enhanced gluon propagator of the form $D(q^2) \overset{q \to 0}{\sim} \frac{1}{q^4}$. Avoiding gauge copies, Gribov obtained $D(q^2) \sim \frac{q^2}{q^4+m^4}$. Using the "pinch technique", Cornwall [2] obtained a solution which fulfills the Ward identities, allows a dynamical mass generation, and also predicts a finite value for $D(0) \equiv D(q^2 = 0)$ consistent with our data.

Early results for the gluon propagator obtained directly from Lattice QCD on small lattices [4] were interpreted in terms of a massive scalar propagator. Results on larger lattices were accounted for by assuming a positive anomalous dimension [5]: $D(q^2) \sim \frac{1}{q^{2(1+\alpha)}+m^2}$. A recent, detailed study of the gluon propagator uses very large lattices [6]. Since we want to compare our results with these, we follow closely their analysis and refer to Refs. [6,7] for details. In the Laplacian gauge, the longitudinal part of the gluon propagator does not vanish; the transverse scalar function $D(q^2)$ can be extracted from $\mathcal{D}_{\mu\nu}^{ab}(q)$ as $D(q^2) = \frac{1}{3}\left\{\sum_\mu \frac{1}{8}\sum_a \mathcal{D}_{\mu\mu}^{aa}(q)\right\} - \frac{1}{3}\frac{F(q^2)}{q^2}$, where $F(q^2)$ is determined by projecting the longitudinal part of $\mathcal{D}_{\mu\nu}^{aa}(q)$ using the symmetric tensor $q^\mu q^\nu$.

Gauge Fixing Procedure

Previous lattice studies all fixed to Landau gauge by using a local iterative maximization algorithm, which converges to any one of many local maxima (lattice Gribov copy), but fails to determine the global one. To overcome this problem, we use a different gauge condition, the Laplacian gauge [3], which

---

*Talk given by C. Alexandrou

is Lorentz-symmetric and gives a smooth gauge field like the Landau gauge, but which specifies the gauge unambiguously. We consider the maximization of $Q = \mathrm{Re}\sum_{x,\mu} \mathrm{Tr}\left[g(x)U_\mu(x)g^\dagger(x+\hat\mu) - g(x)g(x)^\dagger\right]$. If one relaxes the requirement that $g \in SU(N)$, maximizing Q is equivalent to minimizing the quadratic form $\sum_{xy} f_x^* \Delta_{xy} f_y$, with $\Delta(U)$ the covariant Laplacian. Using the $(N-1)$ lowest-lying eigenvectors of $\Delta(U)$, one can construct $\Omega(x) \in SU(N)$ [7].

Results

In Fig.1 we show the transverse gluon propagator for SU(2) Yang-Mills theory in two different volumes; $m_0 \equiv \sqrt{D(0)^{-1}}$ for the 16^4 lattice. Changing the volume has little effect, in particular on $D(0)$. We observe similarly small volume effects in SU(3). This is strikingly different from Landau gauge, where Zwanziger has argued that $D(0)$ should vanish in the infinite lattice volume limit [8]. This prediction is indeed consistent with recent lattice results in SU(2) at finite temperature [9]. In contrast, in the Laplacian gauge, we find that $D(0)$ is finite and independent of the volume V for V larger than about $1/2\mathrm{fm}^4 \sim D(0)^2$. We find $D(0) = 58(2)$ in lattice units at $\beta = 6.0$, i.e. $D(0)^{-1/2} = 248(5)$ MeV (using $a^{-1} = 1.885$ GeV), corresponding to a length scale of about 0.8 fm.

In Fig.2 we compare results for the gluon propagator in SU(3) quenched QCD in Laplacian and Landau gauges. ($m_0 \equiv \sqrt{D(0)^{-1}}$ in the Laplacian gauge). Scaling is checked on the $16^3 \times 32$ lattice for $\beta = 5.8$ and 6.0. Making a cylindrical cut in the momenta [6] to minimize lattice artifacts, we find that scaling is very well satisfied for the Laplacian gauge, with both sets of data falling on a universal curve [7].

We fit to our data the same models as considered by Leinweber *et al.* [6] in Landau gauge. Since we have observed scaling, we use our results at the
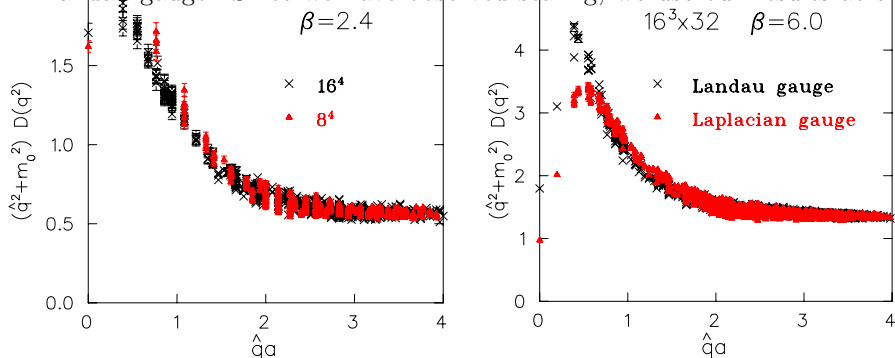

Figure 1: The SU(2) gluon propagator in two different volumes.

Figure 2: The SU(3) gluon propagator in Laplacian and Landau gauges.

2

Model	Z	m	λ or α	A	D(0)	χ^2/d.o.f
Gribov	2.63(2)	0.203(7)			0	5.7
Stingl	2.63(2)	0.203(13)	0.002 (1.1)		0	5.7
Marenzoni	2.47(3)	0.199(6)	0.237(5)		62	4
Cornwall	7.08(9)	0.281(4)	0.265(8)		59	2.5
Model A	1.96(1)	0.654(17)	2.181(67)	8.91(41)	43	1.2

Table 1: best fit of parameter values to our $\beta = 6.0$ data on the $16^3 \times 32$ lattice.

finer lattice spacing ($\beta = 6.0$) for the fits. Table 1 and Fig. 3 summarize the results of the fits to the various models. We find that Gribov–type models are excluded, whereas Cornwall's model is clearly favored among all analytically motivated models. Model "A" [6], which gives a better fit, is phenomenological, contains one more parameter, and misses $D(0)$ by 25%. One can then use the fit to Cornwall's model to analytically continue to negative q^2 and determine the gluon pole mass. This is carried out in Ref. [7].

In conclusion, we see significant modifications from Landau gauge in the infrared. In particular, we find that $D(0)$ obeys scaling, is finite, and volume independent for large enough volumes. We find support for Cornwall's model which fits the momentum dependence of the propagator rather well, whereas models with infrared enhancement of the type $1/(q^2)^2$ or Gribov–type suppression are excluded.
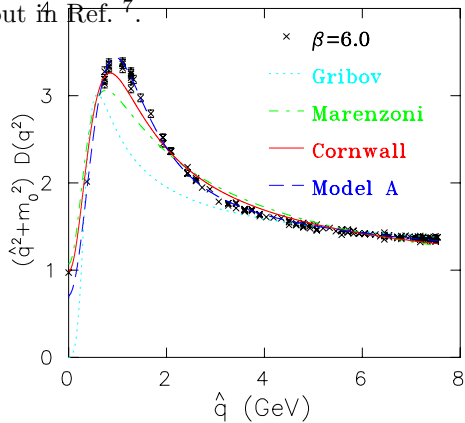

Figure 3: Fits to various models

References

1. J. E. Mandula, Phys. Rep. 315 (1999) 273.
2. J. M. Cornwall, *Phys. Rev.* D**26** (1982) 1453.
3. J. C. Vink and U. Wiese, *Phys. Lett.* B **289** (1992) 122.
4. J. E. Mandula and M. Ogilvie, *Phys. Lett.* B**185** (1987) 127;R. Gupta *et al.*, *Phys. Rev.* D**36** (1987) 2813.
5. P. Marenzoni, G. Martinelli and N. Stella, *Nucl. Phys.* B**455** (1995) 339.
6. D. Leinweber *et al.*, *Phys. Rev.* D**60** (1998) 094507; T. G. Williams, this volume.
7. C. Alexandrou, Ph. de Forcrand and E. Follana, hep-lat/0008012.

8. D. Zwanziger, *Phys. Lett.* B**257** (1991) 168.
9. A. Cucchieri, hep-lat/9908050.