A study of angular size-redshift relation for models in which Λ decays as the energy density

R. G. Vishwakarma

IUCAA, Post Bag 4, Ganeshkhind, Pune 411 007, India
E-mail: vishwa@iucaa.ernet.in

Abstract. By modifying the Chen and Wu ansatz, we have investigated some Friedmann models in which Λ varies as ρ. In order to test the consistency of the models with observations, we study the angular size - redshift relation for 256 ultracompact radio sources selected by Jackson and Dodgson. The angular sizes of these sources were determined by using very long-baseline interferometry in order to avoid any evolutionary effects. The models fit the data very well and require an accelerating universe with a positive cosmological constant. Open, flat and closed models are almost equally probable, though the open model provides a comparatively better fit to the data. The models are found to have intermediate density and imply the existence of dark matter, though not as much as in the canonical Einstein-de Sitter model.

PACS numbers: 04.20.Jb, 98.80.Es

1. Introduction

The cosmological constant Λ, which was originally invoked by Einstein to obtain a static solution of his field equations but subsequently rejected by him after the realization that the universe is expanding, has fallen in and out of fashion several times. However recent observations by Perlmutter et al (1999) and Riess et al (1998) strongly favour a significant and positive Λ. Their findings arise from the study of more than 50 type Ia supernovae with redshifts in the range $0.16 \le z \le 0.83$ and suggest Friedmann models with negative pressure-matter such as a cosmological constant, domain walls or cosmic strings (Vilenkin 1985, Garnavich et al 1998). The main conclusion of these works is that the expansion of the universe is accelerating.

Although a cosmic acceleration can also be accounted for by invoking inhomogeneity (though at the cost of the Cosmological Principle) (Pascual-Sanchez 1999, Dabrowski 1999), postulating a Λ-dominated model solves a lot of problems at once. The cosmological constant supplies the "missing matter" required to make $\Omega_{\rm tot} = 1$ (as suggested by the inflationary models, though on the basis of little observational evidence). It modifies CDM by putting more power on large scales, as is compatible with the CMB anisotropy limits. It also removes the inconsistency between the age of the universe and that of the globular clusters for larger values of the Hubble parameter H_0 (the subscript zero denotes the value of the quantity at the present epoch).

However, even with this significant value of Λ, we obviously face the problem that the upper limit of Λ from observations is 120 orders of magnitude below the value for the vacuum energy density predicted by quantum field theory (Weinberg 1989; Carroll, Press and Turner 1992). (It is customary to associate a positive cosmological constant Λ with a vacuum density $\rho_v \equiv \Lambda/8\pi G$.)

Different types of solutions have been proposed to solve this problem and these can be classified into two categories. The first one, advocated by particle physicists, implements some kind of adjustment mechanism, for instance, a counter term in the Lagrangian which goes away with the effective cosmological constant. However, as remarked by Zee (1985), apart from the fine tuning of the parameters required by this method, there is no known symmetry which would guarantee the effective Λ-term being zero. The second approach, advocated by general relativists, considers Λ as a dynamic variable. The link between the two categories is generally considered by studying different classes of scalar fields (Ratra and Peeble 1988).

The second approach, which is essentially phenomenological in nature, has been extensively investigated in the past few years. It argues that, due to the coupling of the dynamic degree of freedom with the matter fields of the universe, Λ relaxes to its present small value through the expansion of the universe and the creation of photons. From this point of view, the cosmological constant is small because the universe is old.

Several ansatzes have been proposed in which the Λ term decays with time (Gasperini 1987, Freese et al 1987, Ozer and Taha 1987, Gasperini 1988, Peebles and Ratra 1988, Chen and Wu 1990, Abdussattar and Vishwakarma 1996, Gariel and Denmat 1999). Of special interest is the ansatz $\Lambda \propto S^{-2}$ (S being the scale factor of the Robertson Walker metric) by Chen and Wu, which has been considered/modified by several authors (Abdel-Rahman 1992; Carvalho, Lima and Waga 1992; Waga 1993; Silveira and Waga 1994). Through dimensional considerations, in the spirit of quantum cosmology, Chen and Wu argued that one can always write the vacuum energy density as $M_{\rm Pl}^4$ times a dimensionless quantity where $M_{\rm Pl} = (\hbar c/G)^{1/2}$ is the Planck mass. They therefore considered

$$\Lambda \propto \frac{1}{l_{\rm Pl}^2}\left[\frac{l_{\rm Pl}}{S}\right]^n, \tag{1}$$

where $l_{\rm Pl} = (G\hbar/c^3)^{1/2}$ is the Planck length. They noted that if one estimates S_0 by ct_0, then $n \leq 1$ or $n \geq 3$ would lead to either too big or too small values of Λ_0 compared to the observed limit. Consequently one needs to put $n = 2$, which gives the right value.

Carvalho et al (1992) realized that (1) is not the only possible dynamic law for Λ given by the Chen and Wu ansatz. One may, for example, also write

$$\Lambda \propto \frac{1}{l_{\rm Pl}^2}\left[\frac{t_{\rm Pl}}{t_H}\right]^n, \tag{2}$$

where $t_{\rm Pl}$ and $t_{\rm H} \equiv H^{-1}$ are Planck and Hubble times respectively. With the same argument as stated above, one finds $\Lambda \propto H^2$.

Thus the Chen and Wu ansatz was generalized by Carvalho et al by considering $\Lambda = \alpha S^{-2} + \beta H^2$ and later on by Waga (1993) by considering $\Lambda = \alpha S^{-2} + \beta H^2 + \gamma$, where α, β and γ are adjustable dimensionless parameters.

In this paper, we modify the Chen and Wu ansatz further. Since a non-zero Λ might be thought of as producing significant non-gravitational long range forces in the evolution of the universe and consequently influencing the large-scale dynamics, it is natural to hope that the sought dynamical law for Λ might also depend upon some cosmological parameter related to the material content in the universe. Moreover, there is still room for any dimensionless parameter in the Chen and Wu ansatz. One parameter of interest is the density parameter Ω, which is the density of the matter in units of the critical density, i.e., $\Omega \equiv \rho/\rho_c = 8\pi G\rho/3H^2$. We thus consider the ansatz

$$\Lambda = n\ \Omega\ H^2, \tag{3}$$

where n is a new dimensionless cosmological parameter to be determined from the observations. We here do not consider the S^{-2}-dependence because, in view of the present estimate of Λ being of order of H_0^2, an H^2-dependence seems more natural if Λ is to relax to its present estimate due to the expansion of the universe.

Equation (3), which can alternatively be written as $\Lambda = (8n\pi G/3)\rho$ or $\Omega_\Lambda = (n/3)\Omega$, puts Λ on the same footing as the energy density ρ and this corroborates the Machian view. (Here $\Omega_\Lambda \equiv \rho_v/\rho_c = \Lambda/3H^2$ denotes the energy density of vacuum in units of the critical density.)

Incidentally we note that if one estimates the present "radius" of the universe by $S_0 \approx ct_0 \approx cH_0^{-1}$, then the present gravitational attraction $4\pi cG\rho_0/3H_0$ roughly balances the cosmic repulsion $\Lambda_0 cH_0^{-1}$ provided $\Lambda_0 \approx \Omega_0 H_0^2$.

In this paper we investigate cosmological models based on the dynamical law (3), in the framework of general relativity, and study their observational implications. The paper is organized as follows: The basic equations describing the models are presented in Section 2. In Section 3, we study the angular size - redshift relation in the models, this providing one of the classical tests of the observational cosmology. For this purpose, we consider the data set of 256 ultra compact radio sources (with redshifts in 16 bins in the range 0.5 - 3.8) from Jackson and Dodgson (1997). Our results are discussed and compared with previous works in Section 4.

2. The models and the field equations

In order to introduce the ansatz (3) and compare our conclusions with earlier works, we here consider the Friedmann-Lemaitre-Robertson-Walker models. For a perfect fluid, the models are fully specified by the scale factor $S(t)$ and the curvature index $k \in \{-1, 0, 1\}$ of the spatial hypersurfaces $t =$ constant. Denoting the derivative with respect to the cosmic time t by a dot and using units with $c = 1$, the models are governed by
the Raychaudhuri equation:

$$-\frac{\ddot{S}}{S} = \frac{4\pi G}{3}(\rho + 3p) - \frac{\Lambda}{3} \tag{4}$$

and the Friedmann equation:

$$\frac{\dot{S}^2}{S^2} + \frac{k}{S^2} = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3}. \tag{5}$$

We also assume that $p = w\rho$ is the equation of state of the perfect fluid matter. Causality then requires $-1 \leq w \leq 1$. With the help of (3), equation (4) reduces to

$$-\frac{\ddot{S}}{S} = \frac{4\pi G}{3}\left(1 + 3w - \frac{2n}{3}\right)\rho, \tag{6}$$

implying that inflationary solutions (i.e., solutions with negative deceleration parameter $q \equiv -S\ddot{S}/\dot{S}^2$) are possible for $n > 3(1+3w)/2$. We also note that the models are open ($k < 0$), flat ($k = 0$) or closed ($k > 0$) according as $\Omega \lesseqgtr 3/(n+3)$ respectively, provided $n > -3$. For $n \leq -3$, the non-static model is always open.

The Einstein field equations, via the Bianchi identities, imply

$$\left(R^{ij} - \frac{1}{2}Rg^{ij}\right)_{;j} = 0 = -8\pi G\left(T^{ij} - \frac{\Lambda}{8\pi G}g^{ij}\right)_{;j} \tag{7}$$

leading to the energy conservation equation

$$\dot{\rho} + 3(1+w)\rho\frac{\dot{S}}{S} + \frac{\dot{\Lambda}}{8\pi G} = 0. \tag{8}$$

By using equation (3), this leads to

$$\rho = CS^{-9(1+w)/(n+3)}, \quad n \neq -3, \quad C = \text{constant} > 0. \tag{9}$$

For $n = -3$, the dynamic model yields $(1 + w)\rho = 0$ with $S = t$ and $k = -1$. This leads to the following two cases. (i) Milne's model: $\rho = p = \Lambda = 0$, (ii) $w = -1$ with the only constraint on ρ and Λ coming from equation (3) leaving their functional form otherwise free. However, $w = -1$ just corresponds to a second cosmological constant and will not be considered further. With the help of (9), the Friedmann equation (5) can be integrated to give

$$t = \int_{S_i}^{S} \left[(n+3)\frac{8\pi GC}{9} y^{(2n-3-9w)/(n+3)} - k\right]^{-1/2} dy, \quad n \neq -3, \tag{10}$$

where we have chosen the origin of time at $S = S_i$ ($\neq 0$) with $\dot{S} = 0$ (occurring at the first time) in the models without big bang and at $S_i = 0$ in those with big bang. Equation (10) implies that $n > -3$ for $k = 0$ or 1. In the case $k = 1$, S is restricted by

$$S^{(2n-3-9w)/(n+3)} \geq 9/8\pi(n+3)GC. \tag{11}$$

The integral in equation (10) can easily be evaluated for a general value of n if $k = 0$. For $k \neq 0$, it can still be evaluated for particular values of n and we describe one such solution below. However numerical integration is possible for any particular value of n.

For $k = 0$, equation (10) yields

$$S = \left[3(1+w)\sqrt{\frac{2\pi GC}{(n+3)}}\; t\right]^{(2/9)(n+3)/(1+w)}. \tag{12}$$

Thus the scale factor S evolves as $S \propto t^{(n+3)/6}$ in the radiation-dominated (RD) era and $S \propto t^{2(n+3)/9}$ in the matter-dominated (MD) era, which is a simple generalisation of the canonical Einstein-de Sitter solution. The modified expression for the energy density,

$$\rho = \frac{n+3}{18(1+w)^2\pi G} t^{-2} \tag{13}$$

in the present case, has the same time-variation as

$$\rho = \frac{1}{6\pi G(1+w)^2} t^{-2}$$

in the Einsten-de Sitter model. The expression for Λ in the present case becomes

$$\Lambda = \frac{4n(n+3)}{27(1+w)^2} t^{-2}, \tag{14}$$

which matches with the natural dimensions of Λ.

Solutions for $k \neq 0$: The solutions with $n = 1$ are as follows.

$\underline{k = 1,\ w = 1/3}$:

$$t = \sqrt{aS - S^2} + \frac{a}{2}\sin^{-1}(2aS - 1) - \frac{3\pi a}{4}, \tag{15}$$

where $a = (32\pi GC)/9$.

$\underline{k = 1,\ w = 0}$:

$$t = a^4\{-\cos x \sin^7 x - \frac{7}{6}\cos x \sin^5 x - \frac{35}{24}\cos x \sin^3 x$$

$$-\frac{35}{16}\cos x \sin x + \frac{35}{16}x\}, \tag{16}$$

with

$$a \sin^2 x = S^{1/4}. \tag{17}$$

$\underline{k = -1,\ w = 1/3}$:

$$t = \sqrt{aS + S^2} - \frac{a}{2}\cosh^{-1}(2aS + 1). \tag{18}$$

$\underline{k = -1,\ w = 0}$:

$$t = a^4\{\cos^{-8} x \sin^7 x - \frac{7}{6}\cos^{-6} x \sin^5 x + \frac{35}{24}\cos^{-4} x \sin^3 x - \frac{35}{16}\cos^{-2} x \sin x$$

$$+\frac{35}{16}\ln(\sec x + \tan x)\}, \tag{19}$$

with

$$a\tan^2 x = S^{1/4}. \tag{20}$$

One can also approximate the solutions for a general n if there is no significant variation in the value of Ω compared to that in H during the expansion of the universe. (This assumption is trivially satisfied in models with $n = 3(1+3w)/2$ and obviously in flat models also.) By using equation (3), the Raychaudhuri equation (4) can be written as

$$\dot{H} + \left(\{3 + 9w - 2n\}\frac{\Omega}{6} + 1\right) H^2 = 0. \tag{21}$$

If the variation in Ω during a time interval is not significant compared to that in H, one can approximate the solution of equation (21) as

$$t \approx mH^{-1}, \tag{22}$$

where $m = 6/[6 - (2n - 3 - 9w)\Omega]$. This gives an approximate age of the universe as $t_0 \approx mH_0^{-1}$. Hence $t_0 > H_0^{-1}$ for $n > 1.5$. The corresponding approximate evolution of S turns out to be

$$S \propto t^m. \tag{23}$$

Although we shall not use these approximate solutions given by equations (22) and (23) in further analysis, we shall see in section 4 that the estimates of t_0 calculated from these equations are very close to those obtained from equation (10).

In order to compare the models with observations and determine the relative contributions of the different cosmological parameters at the present epoch, it is necessary to recast equations (4) and (5) in the following forms:

$$2[q_0 + \Omega_{\Lambda 0}] = \Omega_0, \tag{24}$$

$$1 + \Omega_{k0} = \Omega_0 + \Omega_{\Lambda 0}. \tag{25}$$

Here $\Omega_k \equiv k/S^2H^2$ is defined as the curvature parameter. Now the age of the universe follows from equation (10) as

$$H_0 t_0 = \int_{\phi_i}^{1} \left[\left(1 + \frac{n}{3}\right)\Omega_0 \phi^{(2n-3)/(n+3)} - \Omega_{k0}\right]^{-1/2} d\phi, \tag{26}$$

where $\phi = S/S_0$ and $\phi_i = S_i/S_0$.

3. Comparison of the models with observations

We now want to study the consistency of our models with observations. We know that as we look back from our position at $r = 0$ and $t = t_0$ to some object at a radial coordinate distance r_1, we are also looking back to some time $t_1 < t_0$ and some expansion factor $S_1 = S(t_1) < S_0$. Note, however, that neither r_1, t_1, nor S_1 are directly measurable; what are measured are physical properties like redshift, proper or apparent angular sizes, velocities, luminosities etc. Since the various cosmic distance measures depend sensitively on the parameters of the models, the physical properties of the distant objects are also influenced by these parameters. In particular, the dependence of the angular size Θ of a standard measuring rod upon redshift z depends upon these parameters. For this reason, the Θ - z relation was proposed as a potential test for cosmological models by Hoyle (1959). The original expectation that the test would be able to distinguish between the geometries of the various cosmological models, was not fulfilled because of the intrinsic scatter and evolution of the sources. However, Kellermann (1993) argued that the evolutionary effects could be controlled by choosing a sample of ultra compact radio sources, with angular sizes of the order of a few milliarcseconds measured by very long-base line interferometry (VLBI). These sources, being short-lived and deeply embedded inside the galactic nuclei, are expected to be free from evolution on a cosmological time scale and thus comprise a set of standard objects, at least in a statistical sense. He used a sample of 79 such sources and showed that a credible Θ - z relation emerged. His results were, however, limited to showing that the corresponding Θ - z diagram favoured an Einstein-de Sitter model (with $\Omega_0 = 1$), thus implying the existence of a large amount of cold dark matter. Jackson and Dodgson (1996) extended this work by including a Λ term and showed that Kellermann's data could also fit a low density, highly decelerating model with large negative Λ.

Later on, more extensive exercise was carried out on a bigger sample of 256 ultra compact sources by Jackson and Dodgson (1997). They then concluded that the canonical inflationary cold dark matter model ($\Omega_0 = 1,\ \Omega_\Lambda = 0$) was ruled out at the 98.5 percent confidence level and that a low-density Friedmann model with either sign of Λ was favoured.

The original data set for the ultra compact sources used by Jackson and Dodgson was compiled by Gurvits (1994). The full sample included 337 sources, out of which Jackson and Dodgson selected the sources with z in the range 0.5 to 3.8 for reasons discussed in their paper. These sources, 256 in number, were binned into 16 redshift bins, each bin containing 16 sources. Recently the compilation of Jackson and Dodgson has been used by Banerjee and Narlikar (1999) in the quasi-steady state cosmology. We also use this data set to study the Θ - z relation in the present models and check its consistency with observations. For this purpose, we now derive the Θ - z relation in the models.

If the observer at $r = 0$ and $t = t_0$ receives the light from a source at a radial distance r_1 with redshift z, the Hoyle's formula gives the apparent angular size Θ of the source as

$$\Theta = \frac{d(1+z)}{r_1 S_0}, \tag{27}$$

where d is the known (or assumed) proper size of the source and the coordinate radius r_1 is given by

$$\eta(r_1) = \int_{S_0/(1+z)}^{S_0} \frac{dS}{S\dot{S}} \tag{28}$$

with

$$\begin{aligned} \eta(r_1) &= \sin^{-1} r_1, \quad && k = 1 \\ &= r_1, && k = 0 \\ &= \sinh^{-1} r_1, && k = -1. \end{aligned} \tag{29}$$

Using (3), (5) and (9), equation (28) reduces to

$$\eta(r_1) = \frac{1}{S_0 H_0} \int_1^{1+z} \left[\left(1 + \frac{n}{3}\right) \Omega_0 \psi^{9/(n+3)} - \Omega_{k0}\psi^2 \right]^{-1/2} d\psi, \tag{30}$$

where $\psi = S_0/S$. We also note that

$$S_0 = \frac{1}{H_0}\sqrt{\frac{k}{\Omega_{k0}}}, \qquad k \neq 0 \tag{31}$$

and equations (3) and (25) give

$$n = \frac{3}{\Omega_0}(1 + \Omega_{k0}) - 3. \tag{32}$$

It is clear from equations (27) and (29) - (32) that, once we fix Ω_{k0} and Ω_0, the theoretical Θ - z relation can be completely worked out for given d and H_0. We assume $H_0 = 65$ km/s/Mpc and $d = 10$ pc (this does not have any consequences as there is still quite uncertainty in the value of H_0) and calculate the theoretical $\Theta(z)$ at the mean bin redshifts for a range of parameters Ω_0 and Ω_{k0}. Using the observed values of Θ_i and the same standard errors σ_i of the ith redshift bin as used by Jackson and Dodgson, we compute χ^2 according to

$$\chi^2 = \sum_{i=1}^{16} \left[\frac{\Theta_i - \Theta(z_i)}{\sigma_i}\right]^2 . \tag{33}$$

This has 16 degrees of freedom as there are no constraints on the parameter space. We have performed an extensive investigation in order to find best-fit parameters. For this purpose, we have tried to minimise χ^2 with respect to Ω_{k0} and Ω_0 and find the best-fitting solution as $\Omega_{k0} = -0.22$ and $\Omega_0 = 0.49$, which is an open model. However, flat and closed models also fit the data quite satisfactorily. This has been shown through some examples and the results have been plotted in Figures 1 and 2.

4. Interpretation and conclusion

Figure 1 illustrates four typical cases of Ω_{k0}: for each Ω_{k0}, we have plotted the values of χ^2 against Ω_0. The 95 percent line (with 14 degrees of freedom) intersects each curve and leaves sufficient portions of the curves below it, which shows a reasonably good fit. The minimum value of χ^2 decreases for lower density universes. Moreover, in accordance with the standard model, we have lower density for the open model. However, we also note that, contrary to the claims of Jackson and Dodgson (1997), the closing of the universe with $\Omega_{\Lambda 0}$ does not abolish the need to postulate nonbaryonic matter, though the density is not as high as in Kellermann's model. Although the test is not conclusive in determining the curvature signature k, leaving all three signs almost equally probable, the fit of the data with the open model is excellent, with a probability exceeding 50 percent! The corresponding probability for the other optimum case ($\Omega_{k0} = 0.3$) is about 30 percent which also represents a good fit. The other two cases lie in between. The predicted age limit in these optimum cases is about $(1-1.4)H_0^{-1}$. The formal best-fitting values are shown in Table 1.

Ω_{k0}	Ω_0	$\Omega_{\Lambda 0}$	n	χ^2	$H_0 t_0$
-0.22	0.49	0.29	1.79	13.34	1.05
0	0.58	0.42	2.17	13.61	1.15
0.1	0.62	0.48	2.32	14.00	1.22
0.3	0.70	0.60	2.61	15.96	1.41

It is, therefore, clear from Table 1 that the data favour an accelerating universe with a positive Λ which is interesting in view of the results from the supernovae data. However, it may be noted that a decelerating universe also cannot be ruled out completely. We mention, for example, a model $\Omega_0 = 0.42$ and $\Omega_{\Lambda 0} = -0.02$ with $d = 8.9$ pc which has a fairly good fit ($\chi^2 = 15.39$) to the data. However, this model is not interesting on the grounds of age considerations. It may be noted that if we use the approximate solution given by equations (22) and (23) to calculate $H_0 t_0$, the estimates never differ by more than 4.5 percent from those shown in Table 1.

In Figure 2, we have plotted the theoretical Θ - z curves for the four models discussed above and compared them with the Jackson and Dodgson' data points. The figure shows that the models fit the data very well.

Acknowledgements
The author thanks Professor J V Narlikar for fruitful and inspiring discussions. The author also gratefully acknowledges IUCAA and MRI for hospitality where this work was done. Thanks are also due to the referees for useful comments which helped in improving the paper.

References:

Abdel-Rahaman A-M M 1992 Phys. Rev. D **45** 3492

Abdussattar and Vishwakarma R G 1996 Pramana - J. Phys **47** 41

Banerjee S K and Narlikar J V 1999 MNRAS **307** 73

Carroll S M Press W H and Turner E L 1992 Ann. Rev. Astron. Astrophys. **30** 499

Carvalho J C Lima J A S and Waga I 1992 Phys. Rev. D **46** 2404

Chen W and Wu Y S 1990 Phys. Rev. D **41** 695

Dabrowski M P 1999 Preprint gr-qc/9905083

Freese K Adams F C Friemann J A and Mottolla E 1987 Nucl. Phys. B **287** 797

Gariel J and Le Denmat G 1999 Class. Quant. Gravit. **16** 149

Garnavich et al 1998 Astrophys. J. **509** 74
Gasperini M 1987 Phys. Lett. B **194** 347
______ 1988 Class. Quant. Gravit. **5** 521
Gurvits L I 1994 Astrophys. J. **425** 442
Hoyle F 1959 in Bracewell R N ed. IAU Symp. No. 9 Paris Sympo. Radio Astronomy, Stanford Univ. Press, Stanford
Jackson J C and Dodgson M 1996 MNRAS **278** 603
______ 1997 MNRAS **285** 806
Kellermann K I 1993 Nat. **361** 134
Ozer M and Taha M O 1987 Nucl. Phys. B **287** 776
Pascual-Sanchez J F 1999 Preprint gr-qc/9905063
Peebles P J E and Ratra B 1988 Astrophys. J. **325** L17
Perlmutter S et al 1999 Astrophys. J. **517** 565
Ratra B and Peebles P J E 1988 Phys. Rev. D **37** 3406
Riess A G et al 1998 Astron. J. **116** 1009
Silveira V and Waga I 1994 Phys. Rev. D **50** 4890
Vilenkin A 1985 Phys. Rep. **121** 265
Waga I 1993 Astrophys. J. **414** 436
Weinberg S 1989 Rev. Mod. Phys. **61** 1
Zee A 1985 in High Energy Physics, Proceedings of 20th Annual Orbis Scientiae

FIGURE CAPTIONS:

Figure 1. χ^2 plotted against Ω_0 for 4 typical models. The minimum value of χ^2 decreases for lower density models.

Figure 2. Best-fitting $\log \Theta / \log z$ curves for 4 models, to be compared with Jackson and Dodgson' data points.

TABLE CAPTION:

Table 1: Best-fitting values of the different parameters for four typical cases of Ω_{k0}.