Tau neutrino magnetic moments from the Super-Kamiokande and ν e-scattering data

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Abstract

Combined results on $\nu_\mu \to \nu_\tau$ oscillations and νe-scattering from the Super-Kamiokande and LAMPF experiments, respectively, limit the Dirac ν_τ diagonal magnetic moment to $\mu_{\nu_\tau} < 1.9 \times 10^{-9} \mu_B$. For the scheme with 3 Majorana neutrinos the LAMPF results allow the limitation of effective ν_τ magnetic moment to $\mu_{\nu_\tau} < 7.6 \times 10^{-10} \mu_B$. The moments in the scheme with additional Majorana light sterile neutrinos as well as experiments on stimulated radiative neutrino conversion are also discussed.

1 Introduction

In the Standard Model(SM) neutrinos are massless particles with zero electric charge and magnetic moments. Nonzero electromagnetic properties of neutrinos have been discussed in many extensions of the SM [1]. In its simplest extension, neutrinos acquire magnetic moments through radiative corrections [2, 3, 4]

$$\mu_{ij} = \frac{3eG_F m_{ij}}{8\sqrt{2}\pi^2} = 3.2 \times 10^{-19} (m_{ij}/1\ eV)\mu_B\,, \quad (1)$$

where G_F is the Fermi constant, $\mu_{ij}(i, j = e, \mu, \tau)$ is the neutrino magnetic moment matrix, m_{ij} is the Dirac neutrino mass matrix, and $\mu_B = e/2m_e$ is the Bohr magneton. So the magnetic moments are too small to give any observable effect. However, there are less trivial extensions of the SM [5] predicting magnetic moments at the level of $(10^{-10} - 10^{-11})\mu_B$, that are large enough to be observed in neutrino electromagnetic interactions.

If neutrinos are Majorana particles, their diagonal magnetic moments must be zero from the CPT consideration, but can have transition moments that couple one neutrino mass eigenstate to another. Dirac neutrinos can have both diagonal and transition moments.

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In the case of vacuum neutrino mixing, an evolution of the effective magnetic moment of the initial flavour states depends on whether the neutrinos are Dirac or Majorana particles, and whether the transition magnetic moments are involved. Consider the following example (see also Section 2,3). In the case of two neutrino mixing:

$$\begin{aligned} \nu_\mu &= cos\theta \cdot \nu_1 + sin\theta \cdot \nu_2 \\ \nu_\tau &= -sin\theta \cdot \nu_1 + cos\theta \cdot \nu_2 \end{aligned} \tag{2}$$

where ν_1, ν_2 denote mass eigenstates and θ is a mixing angle. Let ν_1, ν_2 be the Dirac neutrinos with only diagonal magnetic moments. This is the scheme used by the Particle Data Group [6].

The effective magnetic moments of μ_{ν_μ} and μ_{ν_τ} neutrino are:

$$\begin{aligned} \mu^2_{\nu_\mu} &= cos^2\theta \cdot \mu^2_{\nu_1} + sin^2\theta \cdot \mu^2_{\nu_2} \\ \mu^2_{\nu_\tau} &= sin^2\theta \cdot \mu^2_{\nu_1} + cos^2\theta \cdot \mu^2_{\nu_2} \end{aligned} \tag{3}$$

As can be seen here, there is no dependence of μ_{ν_μ} or μ_{ν_τ} on distance L or neutrino energy E_ν [7], i.e. once Dirac ν_μ is produced it will propagate through space with the value of the effective magnetic moment remaining constant, while the ν_τ component in the beam will depend on L and E_ν, as described by the well-known formula for two neutrino mixing [8].

Taking into account recent results from the Super-Kamiokande detector on evidence for $\nu_\mu \rightarrow \nu_\tau$ oscillations with close to maximal mixing angle $sin^2 2\theta = 0.8 - 1$ [9], one gets

$$\mu_{\nu_\mu} \simeq \mu_{\nu_\tau} \tag{4}$$

Taking into account the upper limit to the magnetic moment of muon neutrinos from the $\nu - e$ scattering experiment at LAMPF [10]:

$$\mu_{\nu_\mu} < 7.4 \times 10^{-10} \mu_B \tag{5}$$

a conservative upper bound on the diagonal magnetic moment of tau neutrino can easily be derived:

$$\mu_{\nu_\tau} < 1.9 \times 10^{-9} \mu_B \tag{6}$$

as well as on the magnetic moments of the mass eigenstates :

$$\begin{aligned} \mu_{\nu_1} &< 1.9 \times 10^{-9} \mu_B \\ \mu_{\nu_2} &< 1.9 \times 10^{-9} \mu_B \end{aligned} \tag{7}$$

Thus, measurements of components of the neutrino mixing matrix would allow extraction of the fundamental magnetic moments μ_{ij}. Note that the limit of Eq.(6) is more then two orders of magnitude better then the limit extracted from the νe-scattering by BEBC, $\mu_{\nu_\tau} < 5.4 \times 10^{-7}\mu_B$ [11] or, limits extracted from the e^+e^- experiments, see e.g. [12] and its references.
In experiments the neutrino originates in weak decays and is always a flavour eigenstate. Therefore, one information on neutrino magnetic moments is obtained in flavour bases, while the fundamental magnetic moments are associated with the mass eigenstates. In this paper we consider the combined oscillation and magnetic moment effects for both Dirac and Majorana neutrino cases in both mass and flavour eigenstate bases. The basic formulae are given in Section 2. In section 3 their application to the experimental bounds on neutrino magnetic moments and to the experiments on stimulated radiative neutrino conversion is considered.

2 Basic formulae

The Lagrangian describing the neutrino interaction with the electromagnetic field due to non-zero anomalous transition magnetic and dipole moments in the mass eigenstates basis takes the form

$$L_\mu = \frac{1}{2}\sum_{i,j=1,N} \bar{\nu}_j \sigma_{\mu\nu}(\mu_{ji} + d_{ji}\gamma_5)\nu_i F_{\mu\nu} + h.c. \tag{8}$$

Here ν_i and ν_j are neutrinos with masses m_i and m_j respectively. The Lagrangian (8) can be rewritten in the form

$$L_\mu = \frac{1}{2}\sum_{i,j=1,N} \bar{\nu}_{jR}\mu_{jiRL}\sigma_{\mu\nu}\nu_{iL}F_{\mu\nu} + h.c. \equiv \frac{1}{2}\bar{\nu}_R \hat{\mu}_{RL}\sigma_{\mu\nu}\nu_L F_{\mu\nu} + h.c.\,, \tag{9}$$

where $\nu_{iL,R} = \frac{1}{2}(1 \mp \gamma_5)\nu_i$ and

$$\mu_{jiRL} = (\mu - d)_{ji} + (\mu + d)^*_{ij} \tag{10}$$

For the Dirac neutrino (for the Majorana neutrino we have to substitute $\mu_{ji} \to 2Im(\mu_{ji})$, $d_{ji} \to 2Im(d_{ji})$) the decay width $\nu_i \to \nu_j + \gamma$ is determined by the formula

$$\Gamma(\nu_i \to \nu_j + \gamma) = \frac{m_i^3(|\mu_{ji}|^2 + |d_{ji}|^2)}{8\pi}(1 - \frac{m_j^2}{m_i^2})^3 \quad , \tag{11}$$

Or in terms of inverse radiative lifetime formula (11) takes the form

$$\tau_\gamma^{-1} = 5.3 s^{-1}(\frac{\mu_{ji,eff}}{\mu_B})^2(\frac{m_i^2 - m_j^2}{m_i^2})^3(\frac{m_i}{eV})^3, \tag{12}$$

where $\mu^2_{ji,eff} = |\mu^2_{ji}| + |d^2_{ji}|$. In general, there are other electromagnetic form-factors besides magnetic and dipole moments but due to gauge invariance only magnetic and dipole form-factors contribute to radiative decays [1]. Nonzero magnetic and dipole neutrino moments lead in particular to additional contributions to weak neutrino-electron scattering, namely for $E_e \gg m_i, m_j$

$$\frac{d\sigma}{dE_e} = \frac{d\sigma_{st}}{dE_e} + \frac{d\sigma_\mu}{dE_e}\,, \tag{13}$$

where $\frac{d\sigma_{st}}{dE_e}$ is the standard weak scattering cross section and

$$\frac{d\sigma_\mu}{dE_e} = \frac{\pi\alpha^2}{m_e^2}\frac{(|\mu_{\nu,eff}|^2}{\mu_B^2}(\frac{1}{E_e - m_e} - \frac{1}{E_\nu}) \tag{14}$$

is the additional contribution to the weak cross section from nonzero magnetic and dipole neutrino moments. In formula (14) $\mu_{\nu,eff}$ is determined by nonzero magnetic and dipole moments. These two contributions are incoherent within the limits of vanishing neutrino mass since weak scattering preserves the neutrino helicity whereas magnetic scattering changes it helicity. Eqs.(8,9) are written in the neutrino eigenstates mass basis.

Now, consider the case of Majorana neutrinos. In the Weyl basis the interaction (9) can be rewritten in the form

$$L_\mu = \frac{1}{4}\sum_{i,j=1,N} \bar{\nu}^c_{L,i}\sigma_{\mu\nu}\tilde{\mu}^M_{ij}\nu_{L,j}F_{\mu\nu} + h.c. \tag{15}$$

The magnetic moment matrix $\tilde{\mu}^M_{ij}$ is antisymmetic $\tilde{\mu}^M_{ij} = -\tilde{\mu}^M_{ji}$ for Majorana neutrinos [1] and $\nu^c_L \equiv (\nu_L)^c = \frac{1+\gamma_5}{2}\nu^c$, $\nu^c = C\gamma^0\nu^*$, $C = i\gamma^2\gamma^0$, $\bar{\nu}^c = \nu C$. Here ν_i are the neutrinos in the eigenstates mass basis with masses m_i. The realistic case corresponds to N=3 (electron, muon and tau neutrinos). However, recent results from a solar neutrino search, Super-Kamiokande and LSND results favour the very intriguing possibilty of the existence of a 4-th light neutrino (sterile neutrino) [19], so maybe $N > 3$.

For the scattering of the neutrino ν_i with definite mass on the target, the contribution to the total cross section from a nonzero neutrino magnetic moment matrix is

$$\sigma_\mu(\nu_i e \to ...) = \sum_j \sigma_\mu(\nu_i e \to \nu_j e) \sim \sum_j |\mu^M_{ji}|^2 = ((\hat{\mu}^M)^+\hat{\mu}^M)\,, \tag{16}$$

where $\hat{\mu}^M_{ij} = \tilde{\mu}^M_{ij}$.

Both here and elsewhere in this paper we ignore spins and phase space factors. The total cross section is the sum of different mass eigenstate transitions combined incoherently, since in principle their contributions are distinguishable in the final state.

The relation among neutrino states in flavour and mass eigenstates bases is determined by the neutrino mixing matrix, namely [8]

$$|\nu^F_{iL} >= \sum_{j=1,N} U_{ij} |\nu_{jL} > \tag{17}$$

Here $|\nu^F_{iL} >$ are neutrino states in flavour basis ($|\nu^F_{1L} >= |\nu_{eL} >$, $|\nu^F_{2L} >= |\nu_{\mu L} >$, $|\nu^F_{3L} >= |\nu_{\tau L} >$, ...). Note that we have written the expression (17) at $t = 0$, at arbitrary time t the matrix U_{ij} depends on time or on the distance l between the source of neutrinos and detector (neutrino oscillations phenomenon), namely [8]

$$U_{ij}(l) = U_{ij} \exp(-iE_j l) \approx \exp(-iEl) \times U_{ij} \exp[-i(m_j^2/2E)l] \tag{18}$$

Here U_{ij} and $U_{ij}(l)$ are unitary matrices.

For the scattering of the neutrino ν^F_i with definite flavour according to the principle of superposition, the amplitude $A(\nu^F_i e \to \nu_j e)$ is proportional to

$$A(\nu^F_i e \to \nu_j e) \sim \sum_k \mu^M_{jk} U_{ik} \tag{19}$$

The total cross section is proportional to

$$\sigma_\mu(\nu^F_i e \to ...) = \sum_j \sigma_\mu(\nu^F_i e \to \nu_j e) \sim \sum_j |\sum_k \mu^M_{jk} U_{ik}|^2 \equiv |\mu^F_{eff,i}|^2 \tag{20}$$

One can rewrite $|\mu^F_{eff,i}|^2$ in the form

$$|\mu^F_{eff,i}|^2 = (\hat{U}^{T+} \hat{\mu}^{M+} \hat{\mu}^M \hat{U}^T)_{ii} = ((\hat{\mu}^F)^+ \hat{\mu}^F)_{ii}, \tag{21}$$

where

$$\hat{\mu}^F = (\hat{U^T})^+ \hat{\mu}^M \hat{U}^T\,, \tag{22}$$

$(\hat{U})_{ij} = U_{ij}$, $(\hat{U}^T)_{ij} = (\hat{U})_{ji}$, $(\hat{U}^+)_{ij} = (\hat{U})^*_{ji}$, $(\hat{\mu}^M)_{ij} = \tilde{\mu}^M_{ij}$

Experimentally, neutrinos are produced in flavour states (e.g., ν_{eL} or $\nu_{\mu L}$ neutrinos). If the distance between the source and detectror is not small they will oscillate into other flavour states. In matrix notations we have the following equation relating neutrino wave function at $l = 0$ and at $l = ct$

$$|\nu^F_{Li}(l) >= \sum_j (\hat{U}'(l))_{ij} |\nu^F_{Lj}(l = 0) >\,, \tag{23}$$

where

$$\hat{U}'(l) = \hat{U}(l) \hat{U}^{-1}(l = 0)$$

For the scattering of the neutrino state $|\nu^F_{Li}(l)>$, which is a mixture of the flavour neutrino states, the formulae (21-22) take place with the substitution of $\hat{U} \to \hat{U}(l)$.

For instance, formula (20) takes the form

$$\sigma_\mu(\nu^F_{Li}(l)\, e \to ...) = \sum_j \sigma_\mu(\nu^F_{Li}(l)\, e \to \nu_j\, e) \sim \sum_j |\mu^F_{ji}(l)|^2 \tag{24}$$
$$= ((\hat{\mu}^F(l))^+ \hat{\mu}^F(l))_{ii} \equiv |\mu^F_{eff,i}(l)|^2,$$

where

$$\hat{\mu}^F(l) = (\hat{U}(l)^T)^+ \hat{\mu}^M \hat{U}(l)^T\,, \tag{25}$$

$$\hat{\mu}^F(l=0) = \hat{\mu}^F \tag{26}$$

Thus, for example, in the Majorana case, the total cross section for the magnetic scattering can be interpreted as a sum of cross sections corresponding to the *initial* to *final* flavor transitions combined incoherently, since in principle their contributions are distinguishable in the final state. In general, there will be dependence of the effective magnetic moment on the distance l in flavour bases [3].

For the case $N = 2$, the neutrino magnetic mass matrix has the form $\tilde{\mu}^F_{ij} = \epsilon_{ij}\mu$ ($\epsilon_{12} = -\epsilon_{21} = 1, \epsilon_{11} = \epsilon_{22} = 0$) and $\mu^F \mu^{F+} = |\mu^2| \times I$, where I is unit matrix. Therefore the magnetic transition moment $|\mu|$ in flavour and mass eigenstates bases coincides and does not depend on distance l. For the most realistic case $N = 3$ neutrino magnetic matrix $\tilde{\mu}^F_{ij}$ has three nonzero matrix elements $\tilde{\mu}^F_{12} \equiv \mu_{e\mu}$, $\tilde{\mu}^F_{13} \equiv \mu_{e\tau}$ and $\tilde{\mu}^F_{23} \equiv \mu_{\mu\tau}$ and for ν_e-, $\nu_\mu-$, $\nu_\tau-$ electron scattering in accordance with formulae (24-26), where the corresponding effective moments are

$$\mu^2_{\nu_e,eff} = |\mu_{e\mu}|^2 + |\mu_{e\tau}|^2, \tag{27}$$

$$\mu^2_{\nu_\mu,eff} = |\mu_{e\mu}|^2 + |\mu_{\mu\tau}|^2, \tag{28}$$

$$\mu^2_{\nu_\tau,eff} = |\mu_{e\tau}|^2 + |\mu_{\mu\tau}|^2 \tag{29}$$

Trivial inequality takes place

$$\mu^2_{\nu_\tau,eff} \leq \mu^2_{\nu_e,eff} + \mu^2_{\nu_\mu,eff} \tag{30}$$

[3]This fact is also mentioned in ref.[7]

The trace of $\hat{\mu}^{F+}\hat{\mu}^F$ is invariant under unitary transformations linking flavour and mass eigenstates bases. For the pattern involving 3 Majorana neutrinos, therefore the following combination does not depend on the distance l and coincides for eigenstates mass and flavour bases:

$$I_3 \equiv \frac{1}{2} Tr(\hat{\mu}^{F+}\hat{\mu}^F) = |\mu_{e\mu}|^2 + |\mu_{e\tau}|^2 + |\mu_{\mu\tau}|^2 \qquad (31)$$

Note that the schemes with an additional light sterile neutrino [13] are now rather popular due to the fact that 3 neutrinos scheme cannot explain simultaneously atmospheric, solar and LSND results. In the scheme with Majorana neutrinos it corresponds to the case $N > 3$. Consider the case $N = 4$ (with an additional light sterile neutrino). For such a case, in addition to nonzero matrix elements for 3×3 matrix we have nonzero matrix elements μ_{es}, $\mu_{\mu s}$, $\mu_{\tau s}$ describing transitions of ν_e, ν_μ, ν_τ neutrinos to the sterile neutrino ν_s, and in the formulae (27-29) we have to add $|\mu_{es}^2|$, $|\mu_{\mu s}^2|$ and $|\mu_{\tau s}^2|$ on the right-hand sides of the equations (27), (28) and (29), respectively. The corresponding invariant (analogue of (31)) is

$$I_4 = I_3 + |\mu_{es}|^2 + |\mu_{\mu s}|^2 + |\mu_{\tau s}|^2 \qquad (32)$$

Consider briefly the case of Dirac neutrinos. The derivation of the main formulae is similar to the Majorana neutrino case except for two differences. The first is that the neutrino magnetic matrix $\hat{\mu}_{RL}$ in formula (9) is not asymmetric and in general is arbitrary. The second difference is that diagonalization of neutrino mass matrix is performed as an independent unitary rotation of left-handed and right-handed neutrinos by two independent unitary matrices $\hat{U}_L$ and $\hat{U}_R$. The relation among neutrino magnetic moments in mass eigenstates and in flavour bases has the form

$$\hat{\mu}_{RL}^F = \hat{U}_R^{T+} \hat{\mu}_{RL} \hat{U}_L^T \qquad (33)$$

Note that in general the flavour basis for the right-handed neutrino is not well defined and can be arbitrary. So we can work in a basis where $\hat{U}_R = 1$, but the main formulae for effective magnetic moments in the flavour basis do not depend on $\hat{U}_R$. The effective magnetic moments for flavour neutrinos are determined by formulae analogous to (20-26), namely

$$\mu_{\nu_i,eff}^2 = (\hat{\mu}_{RL}^{F+}\hat{\mu}_{RL}^F)_{ii} = (\hat{U}_L^{T+}\hat{\mu}_{RL}^+\hat{\mu}_{RL}\hat{U}_T)_{ii} \qquad (34)$$

For instance, if in massive eigenstates basis Dirac neutrinos are diagonal (this scenario is used by the Particle Data group [6]), i.e.

$$\mu_{ij,RL} = \mu_j \delta_{ij}, \qquad (35)$$

then in the flavour basis we have

$$\mu^2_{\nu_i,eff} = \sum_j |U_{ij}|^2 |\mu_j|^2 \tag{36}$$

and there is no dependence on the distance l or neutrino energy for effective magnetic moments (36) [7]. In the general case , the effective magnetic moments for ν_e, ν_μ and ν_τ electon scattering are determined by the flavour magnetic moment matrix, namely

$$\mu^2_{\nu_e,eff} = |\mu^F_{ee,RL}|^2 + |\mu^F_{\mu e,RL}|^2 + |\mu^F_{\tau e,RL}|^2, \tag{37}$$

$$\mu^2_{\nu_\mu,eff} = |\mu^F_{\mu\mu,RL}|^2 + |\mu^F_{e\mu,RL}|^2 + |\mu^F_{\tau\mu,RL}|^2, \tag{38}$$

$$\mu^2_{\nu_\tau,eff} = |\mu^F_{\tau\tau,RL}|^2 + |\mu^F_{e\tau,RL}|^2 + |\mu^F_{\mu\tau,RL}|^2 \tag{39}$$

Note that in general $\mu_{e\mu,RL} \neq \mu_{\mu e,RL}$. For the case of Dirac neutrinos the corresponding invariant (31) is

$$Tr(\hat{\mu}^{F+}\hat{\mu}^F) = \mu^2_{\nu_e,eff} + \mu^2_{\nu_\mu,eff} + \mu^2_{\nu_\tau,eff} \tag{40}$$

3 Experimental bounds on magnetic moments

In this section we consider the following bounds on neutrino magnetic moment matrix elements resulting from reactor and accelerator experiments: [4]

$$\mu_{\nu_e} < 1.8 \times 10^{-10} \mu_B \tag{41}$$

$$\mu^2_{\nu_e} + 2.1 \mu^2_{\nu_\mu} < 1.16 \times 10^{-18} \mu^2_B \tag{42}$$

$$\mu_{\nu_\tau} < 5.4 \times 10^{-7} \mu_B \tag{43}$$

from the reactor [15], νe scattering at LAMPF [10] and BEBC [11] data, respectively. The analysis of Super-Kamiokande results allows to obtain similar or more stringent bounds [7], [17], [16]: [5]

[4]The astrophysical bounds on neutrino magnetic moments can be found in ref.[14].

[5]It should be noted that the bound of ref.[16] is obtained in the flavour bases.

$$\mu_{\nu_e} < (1.6 - 2) \times 10^{-10} \mu_B \qquad (44)$$
$$\mu_{\nu_\tau} < 1.6 \times 10^{-7} \mu_B$$

In the derivation of the bounds (41-43) the distance between the source and the detector was small compared to possible neutrino oscillation length so the possible influence of the neutrino oscillation on the extraction of bounds on neutrino magnetic moments (the dependence of the extracted magnetic moments on the distance between neutrino source and detector) is negligible. In experiments we measure inclusive cross sections of the flavour neutrinos scattering on target, so the experimental bounds (41-43) on diagonal neutrino magnetic moments are also valid for the corresponding effective flavour magnetic moments which are generalizations of the diagonal magnetic moments for the case of nondiagonal magnetic moments.

Consider at first the case of the 3 Majorana neutrinos. Using reactor bound (41) and formulae of Eqs.(27,28) we find that

$$|\mu_{e\mu}|, |\mu_{e\tau}| < 1.8 \times 10^{-10} \mu_B \qquad (45)$$

LAMPF bound (42) leads to

$$|\mu_{e\mu}| < 6.1 \times 10^{-10} \mu_B \, , \qquad (46)$$

$$|\mu_{e\tau}| < 1.1 \times 10^{-9} \mu_B \, , \qquad (47)$$

$$|\mu_{\mu\tau}| < 7.4 \times 10^{-10} \mu_B \, , \qquad (48)$$

Bounds (45-48) have been obtained for magnetic moments in the flavour bases. Note that from the inequalities (45-48) we find that

$$I_3 = |\mu_{e\mu}|^2 + |\mu_{e\tau}|^2 + |\mu_{\mu\tau}|^2 < 0.61 \times 10^{-18} \mu_B^2 \qquad (49)$$

As shown in section (2) this invariant is independent on distance l between the neutrino source and detector and is the same for mass eigenstates and flavour eigenstates bases. So we find that in any basis, nondiagonal neutrino transition moments in scheme with 3 Majorana neutrinos have to be less than

$$|\mu_{ij}| < 7.8 \times 10^{-10} \mu_B \qquad (50)$$

In particular, from the inequality (49) and formula of Eq.(29) we find that in a scheme with 3 Majorana neutrinos the effective magnetic moment of $\tau-$neutrino has to be less than

$$\mu_{\nu_\tau,eff} < 7.6 \times 10^{-10}\mu_B \tag{51}$$

For the scheme with additional sterile Majorana neutrinos, bounds (45-48) are also valid. From the reactor bound (41) we find that $|\mu_{es}| < 1.8 \times 10^{-10}\mu_B$. From the LAMPF bound Eq.(42) we find that $|\mu_{es}| < 1.1 \times 10^{-9}\mu_B$ and $|\mu_{\mu s}| < 7.4 \times 10^{-10}\mu_B$. From BEBC bound (43) we find that $|\mu_{\tau s}| < 5.4 \times 10^{-7}\mu_B$. Using the formula (32) one can find that in any basis (mass eigenvalue basis for instance) nonzero transition magnetic moments in the scheme with additional light Majorana neutrinos have to be less than

$$|\mu_{ij}| < 5.4 \times 10^{-7}\mu_B \tag{52}$$

For 3 Dirac neutrinos, in addition to bounds on diagonal magnetic moments similar bounds on the transition magnetic moments can be deduced. Namely, in all bases, nondiagonal neutrino magnetic moments have to be less than $5.4 \times 10^{-7}\mu_B$. It should be noted that bound (44) on a diagonal magnetic moment of Dirac τ-neutrino obtained from Super-Kamiokande data in the assumption of the nearly maximal $\nu_\tau - \nu_\mu$ mixing leads to the same bound on $|\mu_{\tau s}|$ in the model with 4 Majorana neutrino (ν_e, ν_μ, ν_τ and ν_s). For the model with 3 Dirac neutrino the bound of (44) gives a similar bound on transition magnetic moments $\mu_{e\tau}$, $\mu_{\mu\tau}$. For the interpretation of Super-Kamiokande data as $\nu_\mu - \nu_s$ oscillation using the results of ref.[11] it can be found that in 4-Majorana neutrino model bound (47) also takes place. Note, that bounds on heavy sterile neutrino transition magnetic moments were obtained from the NOMAD data in ref. [20].

For 3 Dirac neutrinos in the assumption of the diagonal magnetic moments in the mass eigenstates basis and with maximal $\nu_\mu - \nu_\tau$ mixing [6] we find that $|U_{1j}|^2 \approx \delta_{1j}$, $|U_{22}|^2 \approx |U_{23}|^2 \approx |U_{32}|^2 \approx |U_{33}|^2 \approx 0.5$, $|U_{21}|^2 \approx |U_{31}|^2 \approx 0$ and as a consequence

$$\mu^2_{\nu_\tau,eff} \approx \mu^2_{\nu_\mu,eff} \approx \frac{1}{2}(\mu_2^2 + \mu_3^2) \tag{53}$$

Using the limit (42) we obtain the bounds of Eq.(6,7) discussed above.

There are interesting proposals on the search for radiative neutrino transitions through neutrino flavour conversion in a superconducting RF cavity installed in a neutrino beam [21]-[23]. Consider now the application of the obtained bounds on the neutrino radiative decays taking the example of the model with 3 Majorana neutrinos. It follows from Eqs.(12,50) that

[6]The assumption of nearly maximal $\nu_\mu - \nu_\tau$ mixing is the simplest interpretation of Super-Kamiokande data

$$\tau(\nu_i \to \nu_j + \gamma) > 3 \times 10^{17} s \times (\frac{m_i^2}{|m_j^2 - m_i^2|})^3 (\frac{eV}{m_i})^3 \tag{54}$$

If neutrino masess m_i and m_j are degenerate, suppression factors for the neutrino radiative lifetime arises dramatically. For example, for $(\frac{|m_j^2 - m_i^2|}{m_i^2}) \simeq 10^{-3}$ and $m_i^2 \simeq 1\ eV^2$ the radiative neutrino lifetime is $\tau(\nu_i \to \nu_j + \gamma) \gtrsim 10^{26}$ sec. This estimate, compared with expected sensitivity of $\lesssim 10^{20}$ sec for the same mass values in the proposed experiments, makes quite difficult for them to compete with the bound of Eq.(50).

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