

U-spin Symmetry in Charmless B Decays

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ABSTRACT

We prove a general theorem about equal CP rate differences within pairs of U-spin related charmless B and B_s decays. Six pairs of decays into two pseudoscalar mesons are identified where such relations hold. Ratios of corresponding rate differences and certain ratios of rates measure U-spin breaking. These processes provide useful information on the weak phase $\gamma = \text{Arg}V_{ub}^*$. Applications of U-spin symmetry to other decays are discussed.

Applying flavor symmetries of strong interactions to B decays into two light pseudoscalar mesons ($B \rightarrow PP$) may provide useful information about phases of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. Using isospin symmetry, the three $B \rightarrow \pi\pi$ decays, $B^0 \rightarrow \pi^+\pi^-$, $\pi^0\pi^0$ and $B^+ \rightarrow \pi^+\pi^0$, were shown [1] to determine quite precisely the weak phase $\alpha = \text{Arg}(-V_{tb}^*V_{td}/V_{ub}^*V_{ud})$ (at least in principle, albeit a potential difficulty in measuring $B^0 \rightarrow \pi^0\pi^0$). Within the approximation of flavor SU(3) symmetry, various aspects of $B \rightarrow K\pi$ decays were studied [2] to learn the weak phase $\gamma = \text{Arg}(-V_{ub}^*V_{ud}/V_{cb}^*V_{cd})$. An accurate determination of γ requires the knowledge of SU(3) breaking and rescattering effects which modify some of the amplitudes. Recently the important role of two B_s decay processes, $B_s \rightarrow K^+K^-$ and $B_s \rightarrow K^-\pi^+$, was demonstrated in this framework [3, 4]. Here one makes explicit use only of a discrete U-spin symmetry transformation interchanging d and s quarks [6].

The purpose of this Letter is to reconsider more generally the implications of U-spin symmetry in charmless B decays [7]. First, we will look at a very general case of charmless B and B_s decays, comparing decays from and into any U-spin related states. We will prove as a general theorem that *pairs of U-spin related processes involve CP rate differences which are equal in magnitude and are opposite in sign*. This property applies not only to two-body and quasi-two-body decays, but in fact to any pair of U-spin related processes. Then, by focusing on $B \rightarrow PP$, we will find altogether twelve

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processes, arranged in six pairs, where in each pair the decay amplitude of one process is related to that of the other by U-spin symmetry, such that the CP rate differences in the two processes are equal in magnitude. Several processes lead to useful information on rescattering effects. A systematic study of U-spin breaking, achieved by comparing some of these processes to others, can lead to an accurate determination of γ .

The implications of U-spin symmetry in all charmless B decays follow from the following general considerations. The low energy effective weak Hamiltonian describing $\Delta S = 1$ B decays is [5]

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left[V_{ub}^* V_{us} \left(\sum_1^2 c_i Q_i^{us} + \sum_3^{10} c_i Q_i^s \right) + V_{cb}^* V_{cs} \left(\sum_1^2 c_i Q_i^{cs} + \sum_3^{10} c_i Q_i^s \right) \right], \quad (1)$$

where c_i are scale-dependent Wilson coefficients and the flavor structure of the various four-quark operators is $Q_{1,2}^{qs} \sim \bar{b}q\bar{q}s$, $Q_{3,\dots,6}^s \sim \bar{b}s \sum \bar{q}'q'$, $Q_{7,\dots,10}^s \sim \bar{b}s \sum e_{q'} \bar{q}'q'$ ($q' = u, d, s, c$). Each of these operators represents an s component (“down”) of a U-spin doublet, so that one can write in short

$$\mathcal{H}_{\text{eff}} = V_{ub}^* V_{us} U^s + V_{cb}^* V_{cs} C^s, \quad (2)$$

where U and C are U-spin doublet operators. Similarly, the effective Hamiltonian responsible for $\Delta S = 0$ decays involves d components (“up” in U-spin) of corresponding operators multiplying CKM factors $V_{ub}^* V_{ud}$ and $V_{cb}^* V_{cd}$,

$$\mathcal{H}_{\text{eff}} = V_{ub}^* V_{ud} U^d + V_{cb}^* V_{cd} C^d. \quad (3)$$

Now, assume for simplicity [8] that one compares two decay processes, $\Delta S = 1$ and $\Delta S = 0$, in which the initial and final states are obtained from each other by a U-spin transformation, $U : d \leftrightarrow s$. Eqs. (2) and (3) then imply that if the $\Delta S = 1$ amplitude (for the process $B \rightarrow f$) is written as

$$A(B \rightarrow f, \Delta S = 1) = V_{ub}^* V_{us} A_u + V_{cb}^* V_{cs} A_c, \quad (4)$$

where A_u and A_c are complex amplitudes (involving CP-conserving phases), then the corresponding $\Delta S = 0$ amplitude (for $UB \rightarrow Uf$) is given by

$$A(UB \rightarrow Uf, \Delta S = 0) = V_{ub}^* V_{ud} A_u + V_{cb}^* V_{cd} A_c. \quad (5)$$

The amplitudes of the corresponding charge-conjugate processes are

$$A(\bar{B} \rightarrow \bar{f}, \Delta S = -1) = V_{ub} V_{us}^* A_u + V_{cb} V_{cs}^* A_c, \quad (6)$$

and

$$A(U\bar{B} \rightarrow U\bar{f}, \Delta S = 0) = V_{ub} V_{ud}^* A_u + V_{cb} V_{cd}^* A_c. \quad (7)$$

To appreciate the powerful implication of U-spin symmetry in B and B_s decays, we note the following. The rates of the four processes (4)–(7) depend on four quantities, $|V_{ub}^* V_{us} A_u|$, $|V_{cb}^* V_{cs} A_c|$, $\delta \equiv \text{Arg}(A_u A_c^*)$ and $\gamma \equiv \text{Arg}(-V_{ub}^* V_{ud} V_{cb} V_{cd}^*)$. Naively it may seem possible to use the corresponding four rates for a determination of these four

quantities including the weak phase γ . However, there exists a general U-spin relation between corresponding CP rate differences

$$|A(B \rightarrow f)|^2 - |A(\bar{B} \rightarrow \bar{f})|^2 = -[|A(UB \rightarrow Uf)|^2 - |A(U\bar{B} \rightarrow U\bar{f})|^2] . \quad (8)$$

This equality, which follows from the CKM unitarity relation [9]

$$\text{Im}(V_{ub}^* V_{us} V_{cb} V_{cs}^*) = -\text{Im}(V_{ub}^* V_{ud} V_{cb} V_{cd}^*) , \quad (9)$$

prohibits a determination of γ from the four rates alone. Towards the end of this Letter we will return to the question of determining γ by using another input measurement.

Eq. (8), our general theorem for equal CP rate asymmetries, can be used to test the validity and accuracy of U-spin symmetry in *all types of B decays*. These include decays into two pseudoscalars (such as the pair $B^0 \rightarrow K^+\pi^-$ and $B_s \rightarrow \pi^+K^-$), decays into a pseudoscalar and a vector meson (e.g. $B^0 \rightarrow K^{*+}\pi^-$ and $B_s \rightarrow \rho^+K^-$), decays into two vector mesons (e.g. $B^0 \rightarrow K^{*+}\rho^-$ and $B_s \rightarrow \rho^+K^{*-}$) and decays into multibody states (e.g. $B^+ \rightarrow K^+\pi^+\pi^-$ and $B^+ \rightarrow \pi^+K^+K^-$). U-spin symmetry implies equal and opposite sign CP rate differences within every pair of U-spin related processes. Consequently, in each pair the process with the smaller rate is expected to have a larger CP asymmetry. For instance, the CP asymmetry in $B^+ \rightarrow \bar{K}^0K^+$ is expected to be much larger than in $B^+ \rightarrow K^0\pi^+$ (see discussion below). Of course, U-spin is only an approximate symmetry. The magnitudes of U-spin breaking are expected to be different in these various decays, as can be demonstrated by the measured rates.

In order to illustrate the consequences of this very general feature in particular cases, we proceed to study in detail the overall implications of U-spin symmetry in decays to two light pseudoscalar mesons. Among all sixteen measurable B meson decays [10] of the form $B, B_s \rightarrow \pi\pi, K\pi, K\bar{K}$ one can identify a dozen processes which can be arranged in six U-spin related pairs:

1. $B^0 \rightarrow K^+\pi^-$ vs. $B_s \rightarrow \pi^+K^-$,
2. $B_s \rightarrow K^+K^-$ vs. $B^0 \rightarrow \pi^+\pi^-$,
3. $B^0 \rightarrow K^0\pi^0$ vs. $B_s \rightarrow \bar{K}^0\pi^0$,
4. $B^+ \rightarrow K^0\pi^+$ vs. $B^+ \rightarrow \bar{K}^0K^+$,
5. $B_s \rightarrow K^0\bar{K}^0$ vs. $B^0 \rightarrow \bar{K}^0K^0$,
6. $B_s \rightarrow \pi^+\pi^-$ vs. $B^0 \rightarrow K^+K^-$.

In case 3 the $\Delta U = 1/2$ transition operators lead to $U = 1$ states to which only the $U = 0$ component of the π^0 contributes. (The two pseudoscalar mesons are in an S-wave implying that they are in a symmetric U-spin state).

Including $B_s \rightarrow \pi^0\pi^0$, which is related by isospin to $B_s \rightarrow \pi^+\pi^-$, this consists of all but the following three $B \rightarrow PP$ ($P = \pi, K$) decays: $B^+ \rightarrow K^+\pi^0$, $B^+ \rightarrow \pi^+\pi^0$ and $B^0 \rightarrow \pi^0\pi^0$. As our theorem Eq. (8) states, in the U-spin symmetry limit the two CP

rate differences within each of the above six pairs of processes are equal in magnitude and have opposite signs. Deviations from equal asymmetries measure U-spin breaking.

For a more detailed study of these processes it is convenient to apply a diagrammatic approach to SU(3), describing amplitudes in terms of SU(3) flavor flow topologies [10]. In this description, the two amplitudes of a pair of U-spin related processes have similar expressions, differing only by being strangeness changing in one case (denoted by primes) and strangeness conserving in the other (involving no primes) *which proves the equality of corresponding CP rate differences*. For instance,

$$\begin{aligned} A(B^0 \rightarrow K^+\pi^-) &= -P' - T' - \frac{2}{3}P'_{EW}{}^c, \\ A(B_s \rightarrow \pi^+K^-) &= -P - T - \frac{2}{3}P_{EW}{}^c. \end{aligned} \quad (10)$$

Thus, we write only expressions for $\Delta S = 1$ processes [10]:

$$\begin{aligned} A(B^0 \rightarrow K^+\pi^-) &= -P' - T' - \frac{2}{3}P'_{EW}{}^c, \\ A(B_s \rightarrow K^+K^-) &= -P' - T' - \frac{2}{3}P'_{EW}{}^c - PA' - E', \\ \sqrt{2}A(B^0 \rightarrow K^0\pi^0) &= P' - \frac{1}{3}P'_{EW}{}^c - P'_{EW} - C', \\ A(B^+ \rightarrow K^0\pi^+) &= P' - \frac{1}{3}P'_{EW}{}^c + A', \\ A(B_s \rightarrow K^0\bar{K}^0) &= P' - \frac{1}{3}P'_{EW}{}^c + PA', \\ A(B_s \rightarrow \pi^+\pi^-) &= -PA' - E'. \end{aligned} \quad (11)$$

In the convention of Eqs. (1)–(7) the above amplitudes are decomposed into two sets of terms containing $V_{cb}^*V_{cs}$ and $V_{ub}^*V_{us}$. (Correspondingly, amplitudes in the second Eq. (10) involve $V_{cb}^*V_{cd}$ and $V_{ub}^*V_{ud}$). The first set consists of a QCD penguin P' , an electroweak penguin P'_{EW} , a color-suppressed electroweak penguin $P'_{EW}{}^c$ and a penguin annihilation term PA' , while the second set contains a tree T' , color-suppressed C' , annihilation A' and exchange E' amplitude. Amplitudes in the first set carry each a weak phase $\text{Arg}(V_{cb}^*V_{cs}) = 0$, while the other four terms have a phase γ .

Let us discuss briefly the magnitudes of various terms. The amplitudes obey the following hierarchy relations

$$|P'| \gg |T'| \sim |P'_{EW}| \gg |C'| \sim |P'_{EW}{}^c|, \quad (12)$$

where a hierarchy factor of about 0.2 or 0.3 describes the ratio of sequential amplitudes. This hierarchy was anticipated [10, 11] from the corresponding CKM factors, a color factor, QCD and electroweak loop factors. It is supported both by relating $B \rightarrow K\pi$ and $B \rightarrow \pi\pi$ data [12] using flavor SU(3) [13] and by recent QCD calculations [14, 15]. The other three amplitudes, PA' , A' and E' , in which the spectator quark participates in the interaction, are usually assumed to be small [10] (see also [14, 15]), unless strongly amplified by rescattering [16].

The dominant term in the amplitudes (11) is P' [13], occurring in the first five processes. These decays are expected to have comparable branching ratios of order 10^{-5} , as

measured for $B^0 \rightarrow K^+\pi^-$, $B^0 \rightarrow K^0\pi^0$ and $B^+ \rightarrow K^0\pi^+$ [12]. The decay $B_s \rightarrow \pi^+\pi^-$ and the U-spin related process $B^0 \rightarrow K^+K^-$ involve only the much smaller combination $PA' + E'$, and are anticipated to have much lower rates. Neglecting rescattering, one estimates for $B^0 \rightarrow K^+K^-$ a branching ratio of order 10^{-8} [14]. The present experimental upper limit [12] is two orders of magnitude higher. Lowering this limit by one or two orders of magnitude would settle the question of little rescattering. A way of bounding the rescattering amplitude A' in $B^+ \rightarrow K^0\pi^+$ (or, similarly, of bounding PA' in $B_s \rightarrow K^0\bar{K}^0$) was discussed in [17].

Assuming in the following that $PA' + E'$ can indeed be neglected, one has

$$\begin{aligned} A(B^0 \rightarrow K^+\pi^-) &\approx A(B_s \rightarrow K^+K^-) \\ A(B_s \rightarrow \pi^+K^-) &\approx A(B^0 \rightarrow \pi^+\pi^-). \end{aligned} \quad (13)$$

Once one obtains stringent limits on $PA' + E'$ through $B^0 \rightarrow K^+K^-$ and $B_s \rightarrow \pi^+\pi^-$, these two relations provide tests for U-spin symmetry acting on the spectator quarks. They imply, of course, that the CP rate differences of all four processes are equal. Equal CP rate differences in $B^0 \rightarrow \pi^+\pi^-$ and $B^0 \rightarrow K^+\pi^-$ (assuming $PA' + E' = 0$) were discussed in [18].

Deviations from equalities in (13) (once $PA' + E'$ is sufficiently bounded) measure U-spin symmetry breaking. In the approximation of factorized hadronic amplitudes [14, 15], U-spin breaking is introduced through the ratio of products of corresponding form factors and decay constants, $f = F_{B_s K}(m_K^2)/F_{B\pi}(m_K^2) \approx F_{B_s K}(m_\pi^2)/F_{B\pi}(m_\pi^2)$,

$$\begin{aligned} A(B_s \rightarrow K^+K^-) &= fA(B^0 \rightarrow K^+\pi^-), \\ A(B_s \rightarrow K^-\pi^+) &= fA(B^0 \rightarrow \pi^+\pi^-). \end{aligned} \quad (14)$$

The rates of these four processes can be used not only to determine the U-spin breaking factor f , but also to check the factorization assumption by finding equal ratios of amplitudes in the two cases.

A similar U-spin equality in the absence of large rescattering effects (i.e., neglecting $PA' - A'$) holds between the amplitudes of $B^+ \rightarrow K^0\pi^+$ and $B_s \rightarrow K^0\bar{K}^0$, and between the corresponding U-spin related processes, $B^+ \rightarrow \bar{K}^0K^+$ and $B^0 \rightarrow \bar{K}^0K^0$. Again, the rates of these four processes can be used to test U-spin symmetry and to measure U-spin symmetry breaking.

A method of determining the weak phase γ , using $B^+ \rightarrow K^0\pi^+$ and the U-spin related processes $B^0 \rightarrow K^+\pi^-$ and $B_s \rightarrow \pi^+K^-$, was discussed in [4]. Here we reiterate the most important features of this suggestion, which can be explained simply in terms of our general U-spin considerations leading to Eqs. (4)–(7). As noted, the rates of the four processes, in our case $B^0 \rightarrow K^+\pi^-$, $B_s \rightarrow \pi^+K^-$ and their charge-conjugates, depend on the magnitudes of two hadronic amplitudes, $P' + (2/3)P'_{EW^c}$ and T' , and on their relative strong phase and weak phase γ . (Note that $|P + (2/3)P'_{EW^c}| = \tan\theta_c|P' + (2/3)P'_{EW^c}|$ while $|T| = \tan^{-1}\theta_c|T'|$). The four rates obey a U-spin equality (8) between the two rate asymmetries in $B^0 \rightarrow K^\pm\pi^\mp$ and $B_s \rightarrow \pi^\pm K^\mp$. Thus, to solve for γ one needs one more input. This input is provided by the charge-averaged rate of $B^\pm \rightarrow K^0\pi^\pm$, where neglecting rescattering can be justified by improving bounds on $B^0 \rightarrow K^+K^-$ and $B_s \rightarrow \pi^+\pi^-$.

This method can be generalized to other pairs of U-spin related decays. For instance, one may use $B^0 \rightarrow K^{*+}\pi^-$, $B_s \rightarrow \rho^+K^-$ and their charge-conjugates, complemented by information on the charge-averaged rate of $B^\pm \rightarrow K^{*0}\pi^\pm$. As we have shown, U-spin breaking effects can be measured by comparing rates and asymmetries of corresponding processes. Including such effects in the analysis would result in a more precise determination of γ .

Another suggested method [3], based on comparing time-dependent CP asymmetries in $B^0 \rightarrow \pi^+\pi^-$ and in $B_s \rightarrow K^+K^-$ (for which one must tag the flavor of the neutral mesons at time of production), can also be generalized to other U-spin related decays. This includes comparing $B^0 \rightarrow K_S\pi^0$ with $B_s \rightarrow K_S\pi^0$ and comparing $B^0 \rightarrow K_S K_S$ with $B_s \rightarrow K_S K_S$. One measures for both channels a $\cos \Delta mt$ and a $\sin \Delta mt$ term, or alternatively an oscillation amplitude and an oscillation phase. These four quantities determine four unknowns: the ratio of two hadronic amplitudes involving $V_{ub}^*V_{us}$ and $V_{cb}^*V_{cs}$, a strong final-state phase between the two amplitudes, and two weak phases β and γ . U-spin breaking effects, represented in the factorization approximation by certain ratios of form factors and decay constants, cancel one another in the asymmetries.

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