

Supergravity and “New” Six-Dimensional Gauge Theories

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Abstract

In the first part of this letter, we analyse the supergravity dual descriptions of six-dimensional field theories realized on the worldvolume of (p, q) five-branes (OD5 theory). We show that in order for the low-energy gauge theory description to be valid the θ parameter must be rational. Irrational values of θ require a strongly coupled string description of the system at low-energy. We discuss the phase structure and deduce some properties of these theories. In the second part we construct and study the supergravity description of NS5-branes with two electric RR field, which provides a dual description of six-dimensional theories with several light open D-brane excitations.

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1 Introduction

This letter consists of two parts. In the first part we will consider the $(1, 1)$ supersymmetric six-dimensional gauge theories on the worldvolume of (p, q) five branes in Type IIB string theory introduced in [1]. These theories have been reconsidered in the context of non-commutative deformation of the NS5-brane theory [2, 3] and are named OD5 theories. At low energy the theories reduce to super Yang-Mills (SYM) theories with gauge group $SU(s)$ where s is the greatest common divisor of p and q .

While the low energy interactions dominated by the gauge kinetic term cannot distinguish the different (p, q) theories with the same low-energy gauge group, it was argued in [1] that the θ term related to the π_5 homotopy group of $SU(s)$ is an observable that distinguishes the low-energy gauge theories. One issue which is not settled is what values of θ are possible in the low-energy theories. The construction in [1] considered rational values of the θ parameter. A generalization to irrational θ was suggested in [4].

In order to study this question we will consider the dual string (supergravity) description. We will show that in order for the low-energy gauge theory description to be valid θ must be rational. Irrational values of θ require a strongly coupled Type IIB string description of the system at low-energy. We will discuss the phase structure and deduce some properties of these theories.

In the second part of the letter we will construct and analyse the supergravity description of NS5-branes with two electric RR field. This corresponds to a generalization of the ODp theories to six-dimensional theories with several types of light open D-branes. We will find two interesting cases. The first case corresponds to N Type IIB NS5-branes in the presence of electric RR 2-form and 6-form potentials. The background contains (D(-1),D1,D3,D5)-branes charges, and the six-dimensional worldvolume theory contains the corresponding D-branes. In addition the background contains F-string and NS5-branes charges. This is analogous to Type IIB string theory, namely the six-dimensional theory contains all the possible branes up to dimension six. All together, all the background fields have their $SL(2, Z)$ counterparts. This points to an $SL(2, Z)$ symmetry of the six-dimensional theory, much like Type IIB string theory.

The second case corresponds to N Type IIA NS5-branes in the presence of electric RR 3-form and 5-form potentials. The background contains (D0,D2,D4)-branes charges, and the six-dimensional worldvolume theory contains the corresponding D-branes. In addition the background contains F-string and NS5-branes charges. This is analogous to Type IIA string theory, namely the six-dimensional theory contains all the possible branes up to dimension six.

On the supergravity side these will relate and generalize other constructions in [5]–[17].

The letter is organized as follows. In section 2 we will analyse the (p, q) theories from field theory and supergravity points of view and analyse the possible values of the low-energy θ parameter. In section 3 we will construct the supergravity

description of NS5-branes with two electric RR field and study the corresponding six-dimensional theories with several types of open D-branes.

2 “New” Six-Dimensional Gauge Theories

2.1 Field Theory

We will consider the (1, 1) supersymmetric six-dimensional gauge theories on the worldvolume of (p, q) five branes in Type IIB string theory. At low energy the theories reduce to super Yang-Mills (SYM) theories with gauge group $SU(s)$ where s is the greatest common divisor of p and q . Consider the low-energy effective action

$$S_{eff} = \int d^6x \left(\frac{1}{g_{YM}^2} \text{Tr} F^2 + \theta_{YM} \text{Tr} F \wedge F \wedge F + \dots \right), \quad (1)$$

where the dots correspond to higher dimension operators as well as the supersymmetric completion. Clearly, the low energy interactions dominated by the F^2 term cannot distinguish the different (p, q) theories with the same low-energy gauge group. However, it was argued in [1] that the theta term in (1), which is a consequence of $\pi_5(SU(s)) = \mathbf{Z}$, $s > 2$, is an observable that distinguishes the low-energy gauge theories. The construction in [1] considered rational values of the θ_{YM} parameters. A generalization to irrational values of θ_{YM} was suggested in [4].

In the following we will construct a dual string (supergravity) description of these theories and analyse the possible values of θ_{YM} . We will then use the dual description to study some properties of the theories.

2.2 The Dual String (Supergravity) Description

Consider a system of Type IIB NS5-branes in the presence of an electric RR 6-form potential. This branes system can be considered as a coinciding (D5, NS5) branes system. Recently, this theory has been reconsidered in the context of non-commutative deformation of the NS5-brane theory [2, 3] and was called OD5 theory. We denote the worldvolume coordinates by x_0, \dots, x_5 .

The supergravity background of the q NS5-branes in the presence of an electric RR 6-form reads

$$\begin{aligned} ds^2 &= h^{-1/2} \left[-dx_0^2 + \sum_{i=1}^5 dx_i^2 + f(dr^2 + r^2 d\Omega_3^2) \right], \\ f &= 1 + \frac{ql_s^2}{\cos \gamma r^2}, & h^{-1} &= \sin^2 \gamma f^{-1} + \cos^2 \gamma, \\ \chi &= \frac{\tan \gamma}{g_s} f^{-1} h + \chi_0, & l_s^{-2} dA_{\alpha\beta} &= -2q\epsilon_3 \chi, \\ e^{2\phi} &= g_s^2 f h^{-2}, & l_s^{-2} dB_{\alpha\beta} &= 2q\epsilon_3. \end{aligned} \quad (2)$$

Here χ is the RR scalar, $A_{\alpha\beta}$ is the 2-form dual to RR 6-form, where α, β are two coordinates of the 3-sphere, and ϵ_3 denotes its volume 3-form. $B_{\alpha\beta}$ is the NS 2-form potential. Note that in comparison to the solution presented in [5] we added a constant χ_0 . This constant will be relevant later for the discussion of the θ parameter and its possible values. The Type IIB complexified coupling τ combines the dilaton and the RR scalar $\tau = ie^{-\phi} + \chi$.

The requirement of having p D5-branes charge implies the condition

$$\chi_0 + \frac{\tan \gamma}{g_s} = \frac{p}{q}. \quad (3)$$

We will now consider the limit in which this system of (p, q) five branes decouples from the bulk [2, 3, 5]. In this limit we send $l_s \rightarrow 0$ and keep

$$l_{\text{eff}}^2 \equiv \frac{l_s^2}{\cos \gamma}, \quad u \equiv \frac{r}{l_s^2}, \quad \tilde{g} l_{\text{eff}}^2 \equiv g_s l_s^2, \quad (4)$$

fixed. This requires $g_s \rightarrow \infty$.

In this limit the supergravity solution (2) reads

$$\begin{aligned} l_s^{-2} ds^2 &= (1 + a_{\text{eff}}^2 u^2)^{1/2} \left[-d\tilde{x}_0^2 + \sum_{i=1}^5 d\tilde{x}_i^2 + \frac{q}{u^2} (du^2 + u^2 d\Omega_3^2) \right], \\ \chi &= \frac{\chi_0 + \left(\frac{p}{q}\right) a_{\text{eff}}^2 u^2}{1 + a_{\text{eff}}^2 u^2}, & l_s^{-2} dA_{\alpha\beta} &= -2q\epsilon_3 \chi, \\ e^{2\phi} &= \tilde{g}^2 \frac{(1 + a_{\text{eff}}^2 u^2)^2}{a_{\text{eff}}^2 u^2}, & l_s^{-2} dB_{\alpha\beta} &= 2q\epsilon_3, \end{aligned} \quad (5)$$

where we have rescaled the worldvolume coordinates $\tilde{x}_{0,\dots,5} = \frac{1}{l_{\text{eff}}} x_{0,\dots,5}$, and

$$a_{\text{eff}}^2 \equiv \frac{l_{\text{eff}}^2}{q}. \quad (6)$$

In this limit the condition (3) reads

$$\chi_0 + \frac{1}{\tilde{g}} = \frac{p}{q}. \quad (7)$$

At the energy range $a_{\text{eff}} u \gg 1$ the RR scalar $\chi = \frac{p}{q}$ and one gets the natural Type IIB θ angle

$$\theta_B \equiv 2\pi \text{Re} \tau = \frac{2\pi p}{q}. \quad (8)$$

At the energy range $a_{\text{eff}} u \ll 1$ the RR scalar is $\chi = \chi_0$, and the theta angle is $\theta = 2\pi\chi_0$. Note, however that in the extreme UV and extreme IR the effective string coupling is large and we have to perform a duality transformation in order

to obtain a weakly coupled description. This is the reason why we cannot identify the low energy Yang-Mills theta angle (1) with $2\pi\chi_0$. Here, the question of what values of theta angles are possible in the low energy SYM theory is formulated as what values of χ are possible in a weakly coupled gauge field theory description at small u . To answer this question we will have to discuss the phase structure of the theories and in particular the regions of validity of the different descriptions.

The curvature of the metric (5) reads

$$l_s^2 \mathcal{R} = \frac{6}{q} \frac{1 - 2a_{\text{eff}}^2 u^2}{(1 + a_{\text{eff}}^2 u^2)^{3/2}}, \quad (9)$$

which is small for a large number of NS5-branes q .

Consider the energy regime $a_{\text{eff}} u \ll 1$. The background (5) simplifies

$$\begin{aligned} l_s^{-2} ds^2 &= \left[-d\tilde{x}_0^2 + \sum_{i=1}^5 d\tilde{x}_i^2 + \frac{q}{u^2} (du^2 + u^2 d\Omega_3^2) \right], \\ \chi &= \chi_0, \quad l_s^{-2} dA_{\alpha\beta} = -2\epsilon_3 q \chi_0, \\ e^\phi &= \frac{\tilde{g}}{a_{\text{eff}} u}, \quad l_s^{-2} dB_{\alpha\beta} = 2\epsilon q. \end{aligned} \quad (10)$$

In order for the supergravity description (10) to be valid we require the effective string coupling $e^\phi \ll 1$. This condition is satisfied when $a_{\text{eff}} u \gg \tilde{g}$, which in this energy range implies that $\tilde{g} \ll 1$. This condition is not satisfied, in particular, in the extreme IR regime where we expect a perturbative SYM field theory description of the system. Since the effective string coupling is large, we have to find an appropriate weakly coupled dual description. In our case, an S-duality transformation $\tau \rightarrow -\frac{1}{\tau}$ does not make the effective string coupling small because of the presence of the RR scalar $\chi = \chi_0$.

We distinguish two cases. In the first case χ_0 is rational and in the second χ_0 is irrational. Let us start with the first case. We can make the effective string coupling small by the use of a more general $SL(2, Z)$ transformation

$$\tau \rightarrow \frac{a\tau + b}{c\tau + d}, \quad B \rightarrow dB - cA, \quad A \rightarrow -bB + aA \quad (11)$$

where $ad - bc = 1$ and $c\chi_0 + d = 0$. The last requirement can be satisfied since χ_0 is rational. Note that with this transformation the NS 2-form potential is mapped to zero. Thus, the five branes charges are mapped $(p, q) \rightarrow (s, 0)$ where s is the greatest common divisor of p and q ¹, and the RR scalar is mapped to the rational number $\chi = \frac{a}{c}$.

¹More precisely, $s = \frac{a}{c}$ which is the greatest common divisor of p and q upon comparing the θ angles in IR and UV.

Under this $SL(2, Z)$ transformation we have

$$g'_s = \frac{c^2}{g_s}, \quad l'^2_s = \frac{g_s l_s^2}{c}, \quad (12)$$

and therefore

$$\tilde{g}' = \frac{c^2}{\tilde{g}}, \quad l'^2_{\text{eff}} = \frac{\tilde{g} l_{\text{eff}}^2}{c}. \quad (13)$$

The dilaton maps to

$$e^{2\phi'} = \frac{\tilde{g}'^3 l'^2_{\text{eff}} \tilde{u}^2}{s}. \quad (14)$$

The metric and the dilaton take the familiar form of the D5-branes background

$$\begin{aligned} ds^2 &= l'^2_s \left(\frac{u}{(g_{YM}^2 s)^{1/2}} (-dx_0^2 + \sum_{i=1}^5 dx_i^2) + \frac{(g_{YM}^2 s)^{1/2}}{u} (du^2 + u^2 d\Omega_3^2) \right), \\ e^{\phi'} &= \frac{(g_{YM}^2 s u^2)^{1/2}}{s}, \end{aligned} \quad (15)$$

where

$$g_{YM}^2 \equiv \tilde{g}'^3 l'^2_{\text{eff}}, \quad (16)$$

and we have rescaled $\tilde{u} \rightarrow \frac{u}{c}$. The coordinates x_i are the same as the ones in (2).

Defining the dimensionless effective Yang-Mills coupling by $g_{\text{eff}}^2 \equiv g_{YM}^2 s u^2$, we recast the curvature and dilaton in the form [18]

$$\begin{aligned} l'^2_s \mathcal{R} &\sim \frac{1}{g_{\text{eff}}}, \\ e^{\phi'} &\sim \frac{g_{\text{eff}}}{s}. \end{aligned} \quad (17)$$

When $g_{\text{eff}} \ll 1$ we have a good description of the system as a perturbative $SU(s)$ SYM theory with a rational θ_{YM} term. This is valid at low energies $u \ll \frac{1}{(g_{YM}^2 s)^{1/2}}$.

Consider now the cases when χ_0 is irrational. Since the equation $c\chi_0 + d = 0$ is not satisfied for integers c and d , we cannot use $SL(2, Z)$ transformations in order to find a background with a small dilaton². Thus, we must remain at low-energy with a description where the effective string coupling is large and the curvature small (for large q). In particular, we do not have at low-energy a perturbative $SU(s)$ SYM theory with irrational θ_{YM} term.

²Since at the supergravity level the symmetry group is $SL(2, R)$, one could try to satisfy the condition $c\chi_0 + d = 0$ by taking non-integer c, d . However, with such a transformation we will end up with non-integer D5-branes charge.

Let us turn to the UV regime $a_{\text{eff}}u \gg 1$. Here the effects of nonzero RR fields are important. Again, in order for the supergravity description to be valid we require $e^\phi \ll 1$. This condition is satisfied when $a_{\text{eff}}u \ll \tilde{g}^{-1}$, which in this energy range implies that $\tilde{g} \ll 1$. As before, when this condition is not satisfied we can make the effective string coupling small by the use of an $SL(2, Z)$ transformation, with the condition $c_q^2 + d = 0$. By making use of this transformation, the NS 2-form potential is mapped to zero while the RR fields and dilaton map to

$$\chi' = \frac{a}{c}, \quad l_s'^{-2} dA'_{\alpha\beta} = -s\epsilon_3, \quad e^{2\phi'} = \frac{s\tilde{g}'}{(l_{\text{eff}}'u)^2}. \quad (18)$$

The metric reads

$$ds^2 = \left[-dx_0^2 + \sum_{i=1}^5 dx_i^2 + \frac{R^2}{u^2} (du^2 + u^2 d\Omega_3^2) \right], \quad (19)$$

where $R^2 = s\tilde{g}'l_{\text{eff}}'^2$.

Consider now a graviton scattering in the background (18),(19). Let the graviton be polarized along the worldvolume of the five branes $\Psi = e^{i\omega t}\psi(u)$ ³. $\psi(u)$ satisfies the differential equation

$$\frac{\partial^2 \psi}{\partial u^2} + \frac{3}{u} \frac{\partial \psi}{\partial u} + \frac{\omega^2 R^2}{u^2} \psi = 0. \quad (20)$$

The solutions to equation (20) take the form u^α , where

$$\alpha = -1 \pm \sqrt{1 - \omega^2 R^2}. \quad (21)$$

When $\omega R > 1$, α becomes imaginary and we get a wave-like solution. This means that there is a nonzero absorption cross section (in the decoupling limit) for a graviton scattered on the five branes with the energy $E > \frac{1}{R}$.

We can recast equation (20) as

$$-\frac{\partial^2}{\partial u^2} g(u) + v(u)g(u) = 0, \quad (22)$$

where $g(u) = u^{3/2}\psi(u)$ and

$$V(u) = \left(\frac{3}{4} - \omega^2 R^2\right) \frac{1}{u^2}. \quad (23)$$

The shape of the potential (23) changes at $\omega R = \frac{\sqrt{3}}{2}$.

As in [21], here the theory has a mass gap

$$M_{\text{gap}} \sim \frac{1}{(s\tilde{g}'l_{\text{eff}}'^2)^{1/2}}. \quad (24)$$

³This analysis is similar to that done for NS5-branes in the presence of RR 2-form or 3-form [19, 20].

In comparison with the ordinary NS5-branes [21] where we have $M_{\text{gap}} \sim 1/\sqrt{s l_s^2}$ which corresponds to a string excitation, here the mass gap is of the order of the D1-branes tension. Note that this is also the case for D5-branes in the presence of a rank four B field [6].

3 NS5-branes with two electric RR-fields

The theories of Type II NS5-branes in the presence of an electric RR $(p+1)$ -form (ODp) are six dimensional theories with open Dp-branes. The supergravity description of these has been recently considered in [5]. The background of NS5-branes in the presence of an electric RR $(p+1)$ -form has also a magnetic RR $(5-p)$ -form. In this section we will generalize this and study Type II NS5-branes in the presence of two electric RR fields, which corresponds to six-dimensional theories with several types of open D-branes. This can be done, for instance, by starting with the Type IIB background of D5-branes in the presence of a rank four B field and using various T, S dualities ⁴. There are four classes of solutions that we get:

- (A) Two electric RR $(p+1)$ -forms potentials, $p = 1, 2, 3, 4$.
- (B) Electric RR $(p+1)$ and $(p+3)$ -forms, $p = 0, 1, 2, 3$, with a magnetic B field.
- (C) Electric RR $(p+1)$ and $(5-p)$ -forms, $p = 0, 1, 2$.
- (D) Electric RR $(p+1)$ and $(7-p)$ -forms, $p = 1, 2, 3$, with an electric B field.

Note that for all these backgrounds, for each q -form potential there is also a $(6-q)$ -form potential. After taking the decoupling limit, the backgrounds (A), (B), (C) are dual to theories which are already known. Cases (A) and (B) correspond to the ODp theories (with extra fields). Case (C) corresponds to the theory of NS5-branes in the presence of a light-like RR field [5]. Case (D) is new and it is the aim of this section to analyse it.

The supergravity background of N NS5-branes in the presence of electric RR $(p+1)$ and $(7-p)$ -forms, $p = 1, 2, 3$, reads ⁵

$$ds^2 = (h_1 h_2)^{-1/2} \left(-dx_0^2 + dx_1^2 + h_1 \sum_{i=2}^{6-p} dx_i^2 + h_2 \sum_{j=7-p}^5 dx_j^2 + f (dr^2 + r^2 d\Omega_3^2) \right) \quad (25)$$

where the functions f and h_i are given by

$$f = 1 + \frac{N l_s^2}{\cos \theta_1 \cos \theta_2 r^2}, \quad h_i^{-1} = \sin^2 \theta_i f^{-1} + \cos^2 \theta_i. \quad (26)$$

⁴Starting with the Type IIB background of D5-branes in the presence of a rank six B field does not lead to new solutions.

⁵The supergravity solution when $p = 1$ has already been constructed in [12].

In addition we have RR fields A and an electric B field

$$\begin{aligned}
A_{2\dots(6-p)}^{(5-p)} &= \frac{\tan \theta_1}{g_s} f^{-1} h_1, & A_{(7-p)\dots 5}^{(p-1)} &= \frac{\tan \theta_2}{g_s} f^{-1} h_2, \\
-A_{01(7-p)\dots 5}^{(p+1)} &= \frac{\sin \theta_1 \cos \theta_2}{g_s} f^{-1} h_2, & A_{01\dots(6-p)}^{(7-p)} &= -\frac{\sin \theta_2 \cos \theta_1}{g_s} f^{-1} h_1, \\
B_{01} &= \sin \theta_1 \sin \theta_2 f^{-1}, & e^{2\phi} &= g_s^2 f h_1^{-(p-1)/2} h_2^{(p-5)/2}. \quad (27)
\end{aligned}$$

In the decoupling limit we take $l_s \rightarrow 0$ and keep

$$u \equiv \frac{r}{l_s}, \quad b_i \equiv \frac{l_s}{\cos \theta_i}, \quad g_s l_s \equiv \tilde{g} (b_1 b_2)^{1/2}, \quad (28)$$

fixed. In this limit the metric (25) reads

$$l_s^{-2} ds^2 = (\hat{h}_1 \hat{h}_2)^{-1/2} \left(-d\tilde{x}_0^2 + d\tilde{x}_1^2 + \hat{h}_1 \sum_{i=2}^{6-p} d\tilde{x}_i^2 + \hat{h}_2 \sum_{j=7-p}^5 d\tilde{x}_j^2 + \frac{N}{u^2} (du^2 + u^2 d\Omega_3^2) \right), \quad (29)$$

and the fields (27) are

$$\begin{aligned}
A_{2\dots(6-p)}^{(5-p)} &= \frac{l_s^{(5-p)}}{\tilde{g}} \left(\frac{b_1}{b_2} \right)^{(p-4)/2} a_1^2 u^2 \hat{h}_1, & A_{(7-p)\dots 5}^{(p-1)} &= \frac{l_s^{p-1}}{\tilde{g}} \left(\frac{b_1}{b_2} \right)^{(p-2)/2} a_2^2 u^2 \hat{h}_2, \\
-A_{01(7-p)\dots 5}^{(p+1)} &= \frac{l_s^{(p+1)}}{\tilde{g}} \left(\frac{b_1}{b_2} \right)^{p/2} a_2^2 u^2 \hat{h}_2, & A_{01\dots(6-p)}^{(7-p)} &= -\frac{l_s^{(7-p)}}{\tilde{g}} \left(\frac{b_1}{b_2} \right)^{(p-6)/2} a_1^2 u^2 \hat{h}_1, \\
e^{2\phi} &= \tilde{g}^2 \frac{(b_1/b_2)^{3-p}}{a_1 a_2 u^2} \hat{h}_1^{-(p-1)/2} \hat{h}_2^{(p-5)/2}, & B_{01} &= l_s^2 a_1 a_2 u^2. \quad (30)
\end{aligned}$$

Here $a_1 = \frac{b_1}{N b_2}$, $a_2 = \frac{b_2}{N b_1}$, $\hat{h}_i^{-1} = 1 + a_i^2 u^2$, and we have rescaled the coordinates

$$\tilde{x}_{0,1} = \frac{1}{(b_1 b_2)^{1/2}} x_{0,1}, \quad \tilde{x}_{2,\dots,(6-p)} = \frac{b_1^{1/2}}{b_2^{1/2} l_s} x_{2,\dots,(6-p)}, \quad \tilde{x}_{(7-p),\dots,5} = \frac{b_2^{1/2}}{b_1^{1/2} l_s} x_{(7-p),\dots,5}. \quad (31)$$

In the following we will consider the different cases $p = 1, 2, 3$.

$p = 1$

This case corresponds to N Type IIB NS5-branes in the presence of electric RR 2-form and 6-form potential. The background contains (D(-1), D1, D3, D5)-branes charges, and the six-dimensional worldvolume theory contains the corresponding D-branes. In addition the background contains F-string and NS5-branes charges. This is analogous to Type IIB string theory, namely the six-dimensional theory contains all the possible branes up to dimension six. All together, all the background fields have their $SL(2, Z)$ counterparts. This points to an $SL(2, Z)$ symmetry of the six-dimensional theory, much like Type IIB string theory.

Such an $((F, D1), (NS5, D5))$ system has been also considered in [12]. If we denote the F-string, D1-branes, NS5-branes and D5-branes charges by the integers N', M', N, M respectively, there is a relation [12]

$$(N, M) = s(c, d), \quad (N', M') = s'(-d, c), \quad (32)$$

where the (c, d) and (s, s') pairs are relatively prime.

Consider the supergravity background (29), (30). The dimensionless deformation parameters are of the form $a_i u, i = 1, 2$. Assume for simplicity that $a_1 = a_2 = a_{eff}$. The supergravity background reads

$$\begin{aligned} l_s^{-2} ds^2 &= (1 + a_{eff}^2 u^2) \left[-d\tilde{x}_0^2 + d\tilde{x}_1^2 + \frac{\sum_{i=1}^5 d\tilde{x}_i^2}{1 + a_{eff}^2 u^2} + \frac{N}{u^2} (du^2 + u^2 d\Omega_3^2) \right] \\ A_{2345} &= \frac{l_s^4}{\tilde{g}} A, & \chi &= \frac{1}{\tilde{g}} A, \\ A_{01} &= -\frac{l_s^2}{\tilde{g}} A, & A_{012345} &= -\frac{l_s^6}{\tilde{g}} A, \\ B_{01} &= l_s^2 a_{eff}^2 u^2, & e^{2\phi} &= \tilde{g}^2 a_{eff}^2 u^2 A^{-2} \end{aligned} \quad (33)$$

where

$$A = a_{eff}^2 u^2 / (1 + a_{eff}^2 u^2). \quad (34)$$

Similarly to the OD5 theory considered in the previous section, the background contains a RR scalar χ . As done there, we can add to χ a constant χ_0 , which will also modify the RR 2-form and 6-form. Note also that the dilaton in (33) is exactly the same as that of the OD5 theory, and is large in the IR and UV regimes. Again due to the RR scalar, we have to make use of the $SL(2, Z)$ symmetry in order to pass to a weakly coupled dual description.

In the IR, the theory is similar to the OD5. We can neglect the RR potentials. However, the RR scalar $\chi = \chi_0$ implies a specific form of the $SL(2, Z)$ symmetry and the analysis is identical to that performed in the previous section leading to the same conclusions regarding the low-energy θ parameter.

In the UV regime the theory differs from the OD5 theory. Now we have other RR fields as well as an electric B field. The dilaton is large. Since the RR scalar is rational we can use the $SL(2, Z)$ symmetry to map the solution to a weakly coupled one, with RR 6-form charge as well as an electric B field. This structure is very reminiscent of the NCOS theory, now in six-dimensions.

Finally, we note that the conjecture of the self-duality of OD1 theory [2] is a special case of the $SL(2, Z)$ symmetry of this theory in the energy range $a_1 u \ll 1$.

$p = 2$

This case corresponds to N Type IIA NS5-branes in the presence of electric RR 3-form and 5-form potentials. The background contains (D0,D2,D4)-branes charges, and the six-dimensional worldvolume theory contains the corresponding D-branes. In addition the background contains F-string and NS5-branes charges. This is analogous to Type IIA string theory, namely the six-dimensional theory contains all the possible branes up to dimension six.

Consider the supergravity background (29), (30). The dimensionless deformation parameters are of the form $a_i u$, $i, j = 1, 2$. Assume for simplicity that $a_1 = a_2 = a_{eff}$. The supergravity background reads

$$\begin{aligned}
l_s^{-2} ds^2 &= (1 + a_{\text{eff}}^2 u^2) \left[-d\tilde{x}_0^2 + d\tilde{x}_1^2 + \frac{\sum_{i=1}^5 d\tilde{x}_i^2}{1 + a_{\text{eff}}^2 u^2} + \frac{N}{u^2} (du^2 + u^2 d\Omega_3^2) \right] , \\
A_{234} &= \frac{l_s^3}{\tilde{g}} A , & A_5 &= \frac{l_s}{\tilde{g}} A , \\
A_{015} &= -\frac{l_s^3}{\tilde{g}} A , & A_{01234} &= -\frac{l_s^5}{\tilde{g}} A , \\
B_{01} &= l_s^2 a_{\text{eff}}^2 u^2 , & e^{2\phi} &= \tilde{g}^2 a_{\text{eff}}^2 u^2 A^{-2} ,
\end{aligned} \tag{35}$$

where A is given by (34).

Apart from the the RR 3-form potentials, this system is the OD4 theory studied in [2] and from the supergravity point of view in [5]. NS5-branes with one type of RR field on their worldvolume have been also considered in [13] and from the supergravity point of view in [14, 15, 16].

In the IR and UV regimes the dilaton is large and the good description is given by an eleven-dimensional supergravity background. Consider the UV regime. The eleven-dimensional supergravity background

$$\begin{aligned}
l_p^{-2} ds^2 &= \hat{h}^{-1/2} \left(\frac{u^{2/3}}{N^{1/3} (b_1^3 b_2)^{1/2}} (-dx_0^2 + dx_1^2 + \hat{h}_1 d\tilde{x}_{2,3,4}^2 + \hat{h}_2 d\tilde{x}_5^2 \right. \\
&\quad \left. + \frac{N^{2/3}}{b_2^2 u^{4/3}} \hat{h}_2^{-1} (dx_6 + a_2^2 u^2 \hat{h}_2 d\tilde{x}_5)^2 + \frac{N^{2/3} b_2}{R u^{4/3}} (du^2 + u^2 d\Omega_3^2) \right) , \\
C_{234} &= \frac{l_p^3}{b_1^3} a_1^2 u^2 \hat{h}_1 , & C_{015} &= \frac{l_p^3}{b_2^3} a_2^2 u^2 \hat{h}_2 , \\
C_{016} &= \frac{l_p^3}{(b_1 b_2)^{3/2}} a_1 a_2 u^2 , & C_{012346} &= \frac{l_p^6}{b_1^6} a_1^2 u^2 \hat{h}_1 ,
\end{aligned} \tag{36}$$

where $\hat{h}_i^{-1} = 1 + a_i^2 u^2$. The rescaled coordinates are defined as

$$\tilde{x}_{2,3,4} = \frac{b_1^{3/2}}{l_p^{3/2}} x_{2,3,4} , \quad \tilde{x}_5 = \frac{b_2^{3/2}}{l_p^{3/2}} x_5 . \tag{37}$$

This background can be considered from another point of view. One can start with M5-branes with worldvolume coordinates x_0, \dots, x_5 and smear in the direction x_6 . One then rotates in x_5, x_6 plane. With the addition of the C fields one has

$$\begin{aligned}
ds^2 &= h_1^{-1/3} \left(f^{-1/3} (-dx_0^2 + dx_1^2 + h_1 dx_{2,3,4}^2 + h_2 dx_5^2) + f^{2/3} h_2^{-1} (dx_6 + A dx_5)^2 \right. \\
&\quad \left. + f^{2/3} (dr^2 + r^2 d\Omega_3^2) \right) , \\
C_{234} &= \tan \theta_1 f^{-1} h_1 , & C_{015} &= -\sin \theta_1 \cos \theta_2 f^{-1} h_2 , \\
C_{016} &= \sin \theta_1 \sin \theta_2 f^{-1} , & C_{012346} &= -\sin \theta_2 \cos \theta_1 f^{-1} h_1 , \\
A &= \tan \theta_2 f^{-1} h_2 , & f &= 1 + \frac{N l_p^3}{\cos \theta_1 \cos \theta_2 R r^2} , \tag{38}
\end{aligned}$$

where R is the radius of x_6 , and $h_i^{-1} = \sin^2 \theta_i f^{-1} + \cos^2 \theta_i$. The decoupling limit is defined such that $l_p \rightarrow 0$ with

$$u = \frac{R^{1/2} r}{l_p^{3/2}} , \quad b_i^{3/2} = \frac{l_p^{3/2}}{\cos \theta_i} , \tag{39}$$

and R fixed. This leads to the background (36), (37). Thus, we can think of (36) OM theory [2] “at an angle”.

$p = 3$

When $p = 3$, the supergravity background is S-dual to the background describing the NCSYN theory in (5+1)-dimensions with four non-commutative directions. The phase structure and some properties of this theory have been studied in [6].

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