Corrections to the Higgs Boson Masses and Mixings from Chargino, W and Charged Higgs Exchange Loops and Large CP Phases

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Abstract

One loop contributions to the Higgs boson masses and mixings from the chargino sector consisting of the chargino, the W, and the charged Higgs boson ($\chi^+ - W - H^+$) exchanges and including the effects of large CP violating phases are computed. It is found that the chargino sector makes a large contribution to the mixings of the CP even and the CP odd Higgs sectors through the induced one loop effects and may even dominate the mixing generated by the stop and the sbottom sectors. Effects of the chargino sector contribution to the Higgs boson masses are also computed. It is found that the sum of the $\chi^+ - W - H^+$ exchange contribution lowers the lightest Higgs boson mass and worsens the fine tuning problem implied by the LEP data. Phenomenological implications of these results are discussed.

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1 Introduction

In the Supersymmetric Standard Model [MSSM][1] the loop corrections to the effective potential[2, 3] make very important contributions to the Higgs masses[4]. Thus in the absence of the loop corrections the lightest Higgs mass satisfies the inequality $m_h < M_Z$ already in contradiction with the current lower limits from LEP. However, with the inclusion of radiative corrections the lightest Higgs mass can be lifted above M_Z. The major correction to the lightest Higgs mass comes from the stop exchange contribution and analyses have been extended to include sbottom exchange, the leading two loop contributions and several other refinements[4]. These refined analyses are then expected to yield Higgs masses accurate to a level of 1-2 GeV. In this paper we give an analysis of the effect of CP violating phases on the Higgs masses including the effects from the exchange of charginos, the W boson, and the charged Higgs. These contributions modify very significantly the mixings between the CP even and the CP odd Higgs sectors from the CP violation effects arising from the stop and sbottom exchange loops. Below we first describe the motivation that leads us to the analysis of the effects of large CP phases in this context.

As is well known in SUSY models with softly broken supersymmetry new sources of CP violation arise from the soft SUSY breaking parameters which are in general complex. The natural size of these phases is large, typically O(1), and an order of magnitude estimate shows that they can lead to the electric dipole moment (edm) of the electron and of the neutron in excess of the current experiment which for the electron is $|d_e| < 4.3 \times 10^{-27} ecm$[5] and for the neutron is $|d_n| < 6.3 \times 10^{-26} ecm$[6]. There are several solutions suggested in the literature to overcome this problem. Thus one possibility suggested early on was that the phases could be small[7] while another possility is that the emds are suppressed because of the heaviness of the sparticle spectrum that enters in the loops that contribute to the edms[8]. Each of these possibilities is not very attractive. Thus the assumption of small phases constitutes a fine tuning while the assumption that the sparticle spectrum is heavy may put the sparticles even beyond the reach of the Large Hadron Collider (LHC). Another possibility suggested recently seems more encouraging, i.e., that the CP phases have their natural sizes O(1) and the compatibility with the experimental edm constraints occurs because of internal cancellations among the various contributions to the edms[9]. In this scenario one can have a light sparticle spectrum which would be accessible at colliders. This suggestion has been investigated in considerable further detail[10]. We note in passing that in several recent works an assumption has been made in order to overcome the edm problem that the entire CP violating effect in the SUSY sector arises from the phase of the trlinear coupling in the third generation sector. In the absence of a symmetry that would guarantee the vanishing of the CP violating phase of μ, the phases of the gauginos, and the phases in the first two generations such an assumption is equivalent to fine tuning all these phases to zero.

In the presence of large CP phases the effects of such phases on low energy phenomenon can be very significant and analyses have been carried out to investigate their effects on dark matter, on $g_\mu - 2$ and on other low energy processes. One area of special interest to us here where the presence of large CP violating phases have been investigated is the Higgs sector[11]. It was pointed out in Ref.[11] that the presence of CP violating phases in the soft SUSY breaking sector will induce CP violating effects in the Higgs sector allowing a mixing of the CP even and CP odd Higgs sectors. One consequence of this mixing is that the Higgs mass matrix no longer factors into a 2×2 CP even Higgs mass matrix times a CP odd Higgs sector. Consequently the diagonalization of the neutral Higgs mass matrix involves the diagonalization of a 3×3 matrix in MSSM reflecting the mixing between the two CP even and one CP odd Higgs fields. Effects of CP violating phases arising from the exchange of the stops and sbottoms were computed in Ref.[11, 12, 13, 14, 15]. In this work we include also the corrections due to the exchange of the charginos, the W boson, and the charged Higgs. The chargino exchange brings in an additional CP violating phase which is the phase of the SU(2) gaugino mass $\tilde{m}_2$.

To define notation we recall that in mSUGRA[16, 1] the low energy parameters are given by m_0, $m_{\frac{1}{2}}$, A_0, $\tan\beta$ and θ_{μ_0} where m_0 is the universal scalar mass, $m_{\frac{1}{2}}$ is the universal gaugino mass, A_0 is the universal trilinear coupling, $\tan\beta = \frac{v_2}{v_1}$ is the ratio of the Higgs VEVs, where the VEV of H_2 gives mass to the up quarks and the VEV of H_1 gives mass to the down quarks and the leptons, and θ_{μ_0} is the phase of the Higgs mixing parameter μ_0. In mSUGRA there are only two independent CP phases which can be taken to be θ_{μ_0}, the phase of μ_0 and α_{A_0}, the phase of A_0. In this paper, however, we shall carry out the analysis for the more general case of the MSSM. In this case we shall treat the phases α_{A_t}, α_{A_b}, θ_μ, and the phases ξ_i (i=1,2,3) of the $SU(3)\times SU(2)\times U(1)$ gaugino masses $\tilde{m}_i$ (i=1,2,3) all taken at the electro-weak scale to be independent. In MSSM the Higgs sector at the one loop level is described by the scalar potential[1]

$$V(H_1, H_2) = V_0 + \Delta V \tag{1}$$

In our analysis we use the renormalization group improved effective potential where

$$\begin{aligned} V_0 = m_1^2|H_1|^2 + m_2^2|H_2|^2 + (m_3^2 H_1.H_2 + H.C.) \\ + \frac{(g_2^2+g_1^2)}{8}|H_1|^4 + \frac{(g_2^2+g_1^2)}{8}|H_2|^4 - \frac{g_2^2}{2}|H_1.H_2|^2 + \frac{(g_2^2-g_1^2)}{4}|H_1|^2|H_2|^2 \end{aligned} \tag{2}$$

where $m_1^2 = m_{H_1}^2 + |\mu|^2$, $m_2^2 = m_{H_2}^2 + |\mu|^2$, $m_3^2 = |\mu B|$ and $m_{H_{1,2}}$ and B are the soft SUSY breaking parameters, and ΔV is the one loop correction to the effective potential and is given by[2]

$$\Delta V = \frac{1}{64\pi^2} Str(M^4(H_1, H_2)(log\frac{M^2(H_1, H_2)}{Q^2} - \frac{3}{2}) \tag{3}$$

where $Str = \sum_i C_i(2J_i+1)(-1)^{2J_i}$ where the sum runs over all particles with spin J_i and $C_i(2J_i+1)$ counts the degrees of the particle i, and Q is the running scale. In the evaluation of ΔV one should include the contributions of all of the fields that enter in MSSM. This includes the Standard Model fields and their superpartners, the sfermions, the higgsinos and the gauginos[3]. The one loop corrections to the effective potential make significant contributions to the minimization conditions[3].

As observed in Ref.[11] as a consequence of the CP violating effects in the one loop effective potential the Higgs VEVs develop an induced CP violating phase through the minimization of the effective potential. One can parametrize this effect by the CP phase θ_H where

$$\begin{aligned} (H_1) = \begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} v_1 + \phi_1 + i\psi_1 \\ H_1^- \end{pmatrix} \\ (H_2) = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix} = \frac{e^{i\theta_H}}{\sqrt{2}} \begin{pmatrix} H_2^+ \\ v_2 + \phi_2 + i\psi_2 \end{pmatrix} \end{aligned} \tag{4}$$

The non-vanishing of the phase θ_H can be seen by looking at the minimization of the effective potential. For the present case with the inclusion of CP violating effects the variations with respect to the fields $\phi_1, \phi_2, \psi_1, \psi_2$ give the following

$$-\frac{1}{v_1}(\frac{\partial \Delta V}{\partial \phi_1})_0 = m_1^2 + \frac{g_2^2+g_1^2}{8}(v_1^2 - v_2^2) + m_3^2 \tan\beta\cos\theta_H \tag{5}$$

$$-\frac{1}{v_2}(\frac{\partial \Delta V}{\partial \phi_2})_0 = m_2^2 - \frac{g_2^2+g_1^2}{8}(v_1^2 - v_2^2) + m_3^2 cot\beta\cos\theta_H \tag{6}$$

$$\frac{1}{v_1}(\frac{\partial \Delta V}{\partial \psi_2})_0 = m_3^2 \sin\theta_H = \frac{1}{v_2}(\frac{\partial \Delta V}{\partial \psi_1})_0 \tag{7}$$

where the subscript 0 means that the quantities are evaluated at the point $\phi_1 = \phi_2 = \psi_1 = \psi_2 = 0$. As noted in Ref.[13] only one of the two equations in Eq.(7) is independent.

2 Chargino,W and charged Higgs contributions

The contribution of the stop and of the sbottom exchange contributions have been discussed at great length in the literature[4]. More recently these analyses have been extended to take account of the CP violating effects arising from the soft SUSY breaking parameters in these sectors[11, 12, 13, 14, 15]. We have reanalysed the stop and sbottom contributions with CP violating effects and these results are listed in Appendix A where we also compare our results with the previous analyses. The main focus of this work, however, is to compute the contributions of the chargino loops to the Higgs masses. The charginos, the W and the charged Higgs boson form a sub-sector as it is the splittings among these particles that leads to a non-vanishing contribution to the one loop effective potential. The one loop correction from this sector is given by

$$\Delta V(\chi^+, W, H^+) = \frac{1}{64\pi^2}(\sum_{a=1,2}(-4)M^4_{\chi_a^+}(log\frac{M_{\chi_a^+}}{Q^2} - \frac{3}{2}) + 6M_W^4(log\frac{M_W^2}{Q^2} - \frac{3}{2}) + 2M^4_{H^+}log(\frac{M_{H^+}}{Q^2} - \frac{3}{2})) \tag{8}$$

The chargino mass matrix is given by

$$M_C = \begin{pmatrix} \tilde{m}_2 & g_2 H_2^0 \\ g_2 H_1^0 & \mu \end{pmatrix} \tag{9}$$

where $\mu = |\mu|e^{i\theta_\mu}$ and $\tilde{m}_2 = |\tilde{m}_2|e^{i\xi_2}$. For the purposes of the analysis it is more convenient to deal with the matrix $M_C M_C^\dagger$ where

$$M_C M_C^\dagger = \begin{pmatrix} |\tilde{m}_2|^2 + g_2^2|H_2^0|^2 & g_2(\tilde{m}_2 H_1^{0^*} + \mu^* H_2^0) \\ g_2(\tilde{m}_2^* H_1^0 + \mu H_2^{0^*}) & |\mu|^2 + g_2^2|H_1^0|^2 \end{pmatrix} \tag{10}$$

The chargino eigen values are given by

$$M^2_{\chi^+_{1,2}} = \frac{1}{2}[|\tilde{m}_2|^2 + |\mu|^2 + g_2^2(|H_2^0|^2 + |H_1^0|^2)]$$
$$\pm\frac{1}{2}[(|\tilde{m}_2|^2 - |\mu|^2 + g_2^2(|H_2^0|^2 - |H_1^0|^2))^2 + 4g_2^2|\tilde{m}_2 H_1^{0*} + \mu^* H_2^0|^2]^{\frac{1}{2}} \tag{11}$$

We note that in the supersymmetric limit $M_{\chi^+_{1,2}} = M_{H^+} = M_W$ and the loop correction Eq.(8) vanishes. Further, as we will discuss later the inclusion of the W and the H^+ exchange along with the chargino exchange is also needed to achieve an approximate Q independence of the corrections to the Higgs masses and mixings from this sector. In this sense $M_{\chi^+_a}$, H^+ and W form a sub-sector and that is the reason for considering this set in Eq.(8). With the inclusion of the stop and the sbottom contributions (see Appendix A) and of the chargino contributions one finds that θ_H is determined by the equation

$$m_3^2 \sin\theta_H = \frac{1}{2}\beta_{h_t}|\mu||A_t| \sin\gamma_t f_1(m^2_{\tilde{t}_1}, m^2_{\tilde{t}_2}) + \frac{1}{2}\beta_{h_b}|\mu||A_b| \sin\gamma_b f_1(m^2_{\tilde{b}_1}, m^2_{\tilde{b}_2}) - \frac{g_2^2}{16\pi^2}|\mu||\tilde{m}_2| \sin\gamma_2 f_1(m^2_{\tilde{\chi}_1}, m^2_{\tilde{\chi}_2}) \quad (12)$$

where

$$\beta_{h_t} = \frac{3h_t^2}{16\pi^2}, \quad \beta_{h_b} = \frac{3h_b^2}{16\pi^2}; \quad \gamma_t = \alpha_{A_t} + \theta_\mu, \quad \gamma_b = \alpha_{A_b} + \theta_\mu, \quad \gamma_2 = \xi_2 + \theta_\mu \quad (13)$$

and $f_1(x,y)$ is defined by

$$f_1(x,y) = -2 + log\frac{xy}{Q^4} + \frac{y+x}{y-x}log\frac{y}{x} \quad (14)$$

To construct the mass squared matrix of the Higgs scalars we need to compute the quantities

$$M^2_{ab} = (\frac{\partial^2 V}{\partial\Phi_a \partial\Phi_b})_0 \quad (15)$$

where Φ_a (a=1-4) are defined by

$$\{\Phi_a\} = \{\phi_1, \phi_2, \psi_1, \psi_2\} \quad (16)$$

and as already specified the subscript 0 means that we set $\phi_1 = \phi_2 = \psi_1 = \psi_2 = 0$ after the evaluation of the mass matrix. The tree and loop contributions to M^2_{ab} are given by

$$M^2_{ab} = M^{2(0)}_{ab} + \Delta M^2_{ab} \quad (17)$$

where $M^{2(0)}_{ab}$ are the contributions at the tree level and ΔM^2_{ab} are the loop contributions where

$$\Delta M^2_{ab} = \frac{1}{32\pi^2} Str(\frac{\partial M^2}{\partial\Phi_a}\frac{\partial M^2}{\partial\Phi_b} log\frac{M^2}{Q^2} + M^2 \frac{\partial^2 M^2}{\partial\Phi_a \partial\Phi_b} log\frac{M^2}{eQ^2})_0 \quad (18)$$

Computation of the 4×4 Higgs mass matrix in the basis of Eq.(16) gives

$$\begin{pmatrix} M_Z^2 c_\beta^2 + M_A^2 s_\beta^2 + \Delta_{11} & -(M_Z^2 + M_A^2) s_\beta c_\beta + \Delta_{12} & \Delta_{13} s_\beta & \Delta_{13} c_\beta \\ -(M_Z^2 + M_A^2) s_\beta c_\beta + \Delta_{12} & M_Z^2 s_\beta^2 + M_A^2 c_\beta^2 + \Delta_{22} & \Delta_{23} s_\beta & \Delta_{23} c_\beta \\ \Delta 13 s_\beta & \Delta_{23} s_\beta & (M_A^2 + \Delta_{33}) s_\beta^2 & (M_A^2 + \Delta_{33}) s_\beta c_\beta \\ \Delta_{13} c_\beta & \Delta_{23} c_\beta & (M_A^2 + \Delta_{33}) s_\beta c_\beta & (M_A^2 + \Delta_{33}) c_\beta^2 \end{pmatrix} \tag{19}$$

where $(c_\beta, s_\beta) = (\cos\beta, \sin\beta)$. In the above the explicit Q dependence has been absorbed in m_A^2 which is given by

$$m_A^2 = (\sin\beta\cos\beta)^{-1}(-m_3^2\cos\theta + \frac{1}{2}\beta_{h_t}|A_t||\mu|\cos\gamma_t f_1(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2)$$
$$+\frac{1}{2}\beta_{h_b}|A_b||\mu|\cos\gamma_b f_1(m_{\tilde{b}_1}^2, m_{\tilde{b}_2}^2) + \frac{g_2^2}{16\pi^2}|\tilde{m}_2||\mu|\cos\gamma_2 f_1(m_{\chi_1^+}^2, m_{\chi_2^+}^2)) \tag{20}$$

The first term in the second brace on the right hand side of Eq.(20) is the tree term, while the second, the third and the fourth terms come from the stop,sbottom and chargino exchange contributions. We give now our computation of the Δ's. For Δ_{ij} one has

$$\Delta_{ij} = \Delta_{ij\tilde{t}} + \Delta_{ij\tilde{b}} + \Delta_{ij\chi^+} \tag{21}$$

where $\Delta_{ij\tilde{t}}$ is the contribution from the stop exchange in the loops, $\Delta_{ij\tilde{b}}$ is the contribution from the sbottom exchange in the loops and $\Delta_{ij\chi^+}$ is the contribution from the chargino sector in the loops. $\Delta_{ij\tilde{t}}$ and $\Delta_{ij\tilde{b}}$ are listed in Appendix A. In the analysis of the chargino exchange we shall approximate the chargino eigen values given by Eq.(11) by

$$M_{\chi_{1,2}^+}^2 \simeq M_W^2 + \frac{1}{2}(|\tilde{m}_2|^2 + |\mu|^2)$$
$$\pm[\frac{1}{4}(|\tilde{m}_2|^2 - |\mu|^2)^2 - M_W^2\cos 2\beta(|\tilde{m}_2|^2 - |\mu|^2) + 2M_W^2|\tilde{m}_2\cos\beta + \mu^*\sin\beta|^2]^{\frac{1}{2}} \tag{22}$$

where we have ignored the term of O(M_W^4) inside the square root. This approximation leads us to achieve an independence on Q of the chargino-W-H^+ exchange correction and is similar to the approximation of dropping the D terms in the squark masses in the stop exchange correction (see Appendix A). Below we list the result of our analysis of the $\Delta_{ij\chi^+}$ chargino exchange contributions. They are given by

$$\Delta_{11\chi^+} = \frac{g_2^2}{8\pi^2}M_W^2\frac{((|\tilde{m}_2|^2 + |\mu|^2)\cos\beta + 2|\tilde{m}_2||\mu|\sin\beta\cos\gamma_2)^2}{(m_{\chi_1^+}^2 - m_{\chi_2^+}^2)^2} f_2(m_{\chi_1^+}^2, m_{\chi_2^+}^2)$$

$$-\frac{g_2^2}{4\pi^2}M_W^2\frac{((|\tilde{m}_2|^2+|\mu|^2)\cos^2\beta+|\tilde{m}_2||\mu|\sin 2\beta\cos\gamma_2)}{(m^2_{\chi_1^+}-m^2_{\chi_2^+})}ln(\frac{m^2_{\chi_1^+}}{m^2_{\chi_2^+}})$$
$$-\frac{g_2^2}{16\pi^2}M_W^2\cos^2\beta ln(\frac{m^4_{\chi_1^+}m^4_{\chi_2^+}}{M_W^6M_{H^+}^2}) \quad (23)$$

where

$$f_2(x,y)=-2+\frac{y+x}{y-x}ln\frac{y}{x} \quad (24)$$

$$\Delta_{22\chi^+}=\frac{g_2^2}{8\pi^2}M_W^2\frac{((|\tilde{m}_2|^2+|\mu|^2)\sin\beta+2|\tilde{m}_2||\mu|\cos\beta\cos\gamma_2)^2}{(m^2_{\chi_1^+}-m^2_{\chi_2^+})^2}f_2(m^2_{\chi_1^+},m^2_{\chi_2^+})$$
$$-\frac{g_2^2}{4\pi^2}M_W^2\frac{((|\tilde{m}_2|^2+|\mu|^2)\sin^2\beta+|\tilde{m}_2||\mu|\sin 2\beta\cos\gamma_2)}{(m^2_{\chi_1^+}-m^2_{\chi_2^+})}ln(\frac{m^2_{\chi_1^+}}{m^2_{\chi_2^+}})$$
$$-\frac{g_2^2}{16\pi^2}M_W^2\sin^2\beta ln(\frac{m^4_{\chi_1^+}m^4_{\chi_2^+}}{M_W^6M_{H^+}^2}) \quad (25)$$

$$\Delta_{12\chi^+}=\frac{g_2^2}{8\pi^2}M_W^2[((|\tilde{m}_2|^2+|\mu|^2)\cos\beta+2|\tilde{m}_2||\mu|\sin\beta\cos\gamma_2)]$$
$$\frac{[((|\tilde{m}_2|^2+|\mu|^2)\sin\beta+2|\tilde{m}_2||\mu|\cos\beta\cos\gamma_2)]}{(m^2_{\chi_1^+}-m^2_{\chi_2^+})^2}f_2(m^2_{\chi_1^+},m^2_{\chi_2^+})$$
$$-\frac{g_2^2}{8\pi^2}M_W^2\frac{((|\tilde{m}_2|^2+|\mu|^2)\sin 2\beta+2|\tilde{m}_2||\mu|\cos\gamma_2)}{(m^2_{\chi_1^+}-m^2_{\chi_2^+})}ln(\frac{m^2_{\chi_1^+}}{m^2_{\chi_2^+}})$$
$$-\frac{g_2^2}{32\pi^2}M_W^2\sin 2\beta ln(\frac{m^4_{\chi_1^+}m^4_{\chi_2^+}}{M_W^6M_{H^+}^2}) \quad (26)$$

$$\Delta_{13\chi^+}=\frac{g_2^2}{4\pi^2}M_W^2[((|\tilde{m}_2|^2+|\mu|^2)\cos\beta+2|\tilde{m}_2||\mu|\sin\beta\cos\gamma_2)]$$
$$\frac{|\tilde{m}_2||\mu|\sin\gamma_2}{(m^2_{\chi_1^+}-m^2_{\chi_2^+})^2}f_2(m^2_{\chi_1^+},m^2_{\chi_2^+})-\frac{g_2^2}{4\pi^2}M_W^2\frac{|\tilde{m}_2||\mu|\sin\gamma_2\cos\beta}{(m^2_{\chi_1^+}-m^2_{\chi_2^+})}ln(\frac{m^2_{\chi_1^+}}{m^2_{\chi_2^+}}) \quad (27)$$

$$\Delta_{23\chi^+}=\frac{g_2^2}{4\pi^2}M_W^2|\tilde{m}_2||\mu|\sin\gamma_2$$
$$\frac{[((|\tilde{m}_2|^2+|\mu|^2)\sin\beta+2|\tilde{m}_2||\mu|\cos\beta\cos\gamma_2]}{(m^2_{\chi_1^+}-m^2_{\chi_2^+})^2}f_2(m^2_{\chi_1^+},m^2_{\chi_2^+})$$
$$-\frac{g_2^2}{4\pi^2}M_W^2\frac{|\tilde{m}_2||\mu|\sin\gamma_2\sin\beta}{(m^2_{\chi_1^+}-m^2_{\chi_2^+})}ln(\frac{m^2_{\chi_1^+}}{m^2_{\chi_2^+}}) \quad (28)$$

$$\Delta_{33\chi^+} = \frac{g_2^2}{2\pi^2} \frac{M_W^2 |\tilde{m}_2|^2 |\mu|^2 \sin^2 \gamma_2}{(m^2_{\chi_1^+} - m^2_{\chi_2^+})^2} f_2(m^2_{\chi_1^+}, m^2_{\chi_2^+}) \tag{29}$$

We note that all the $\Delta_{ij\chi^+}$ have no explicit Q dependence. Inclusion of the W and the H^+ exchange along with the chargino exchange was necessary to achieve the Q independence. We further note that unlike the third generation contributions where one needs to worry about the possibility of significant QCD corrections, the chargino exchange is purely electro-weak in nature and thus largely free of such corrections. Eqs.(20-29) constitute the main new theoretical computations in this paper. Using these equations and the results of the analysis of Appendix A one finds Δ_{ij} of Eq.(21) and thus computes the matrix of Eq.(19). One may reduce the 4×4 matrix of Eq.(19) by introducing a new basis $\{\phi_1, \phi_2, \psi_{1D}, \psi_{2D}\}$ where ψ_{1D}, ψ_{2D} are defined by

$$\begin{aligned} \psi_{1D} &= \sin\beta\psi_1 + \cos\beta\psi_2 \\ \psi_{2D} &= -\cos\beta\psi_1 + \sin\beta\psi_2 \end{aligned} \tag{30}$$

In this basis the field ψ_{2D} decouples from the other three fields. ψ_{2D} is a zero mass state and is the Goldstone field. The Higgs $(mass)^2$ matrix M^2_{Higgs} of the remaining three fields is given by

$$M^2_{Higgs} = \begin{pmatrix} M_Z^2 c_\beta^2 + M_A^2 s_\beta^2 + \Delta_{11} & -(M_Z^2 + M_A^2) s_\beta c_\beta + \Delta_{12} & \Delta_{13} \\ -(M_Z^2 + M_A^2) s_\beta c_\beta + \Delta_{12} & M_Z^2 s_\beta^2 + M_A^2 c_\beta^2 + \Delta_{22} & \Delta_{23} \\ \Delta_{13} & \Delta_{23} & (M_A^2 + \Delta_{33}) \end{pmatrix} \tag{31}$$

We note that in principle it is possible that due to cancellations between the stop and the chargino contributions to Eq.(12) that θ_H vanishes or becomes very small. However, even in this case the mixings between the CP even Higgs sector and the CP odd Higgs sector can still occur because the parameters that determine this mixings are γ_t, γ_b and γ_2 and not θ_H. γ_t gets directly related to θ_H when contributions to θ_H other than the stop exhanges are ignored.

We can obtain an approximation to the chargino corrections to the Higgs masses using a perturbation expansion. We order the eigen values so that in the limit of no mixing between the CP even and the CP odd states one has $(m_{H_1}, m_{H_2}, m_{H_3})$ $\to (m_H, m_h, m_A)$. Defining $m_h = m_h^0 + (\Delta m_h)_{\chi^+}$ where m_h^0 is the lightest Higgs mass without the chargino loop contribution, and $(\Delta m_h)_{\chi^+}$ is the correction due to the chargino exchange loops, and with $(\Delta m_H)_{\chi^+}$ and $(\Delta m_A)_{\chi^+}$ similarly defined, one finds

$$
\begin{aligned}
(\Delta m_H)_{\chi^+} &= (2m_H^0)^{-1}(\Delta_{11\chi^+}\cos^2\alpha + \Delta_{22\chi^+}\sin^2\alpha + \Delta_{12}\sin 2\alpha) \\
(\Delta m_h)_{\chi^+} &= (2m_h^0)^{-1}(\Delta_{11\chi^+}\sin^2\alpha + \Delta_{22\chi^+}\cos^2\alpha - \Delta_{12\chi^+}\sin 2\alpha) \\
(\Delta m_A)_{\chi^+} &= (2m_A^0)^{-1}\Delta_{33\chi^+}
\end{aligned}
\tag{32}
$$

where

$$
\begin{aligned}
\cos 2\alpha &\simeq \frac{M_{11}^2 - M_{22}^2}{\sqrt{(trM^2)^2 - 4(detM^2)}} \\
\sin 2\alpha &\simeq \frac{2M_{12}^2}{\sqrt{(trM^2)^2 - 4(detM^2)}}
\end{aligned}
\tag{33}
$$

where the matrix M^2 is the 2×2 matrix in the upper left hand corner of Eq.(31), i.e.,

$$
(M^2) = \begin{pmatrix} M_{11}^2 & M_{12}^2 \\ M_{21}^2 & M_{22}^2 \end{pmatrix} = \begin{pmatrix} M_Z^2 c_\beta^2 + M_A^2 s_\beta^2 + \Delta_{11} & -(M_Z^2 + M_A^2)s_\beta c_\beta + \Delta_{12} \\ -(M_Z^2 + M_A^2)s_\beta c_\beta + \Delta_{12} & M_Z^2 s_\beta^2 + M_A^2 c_\beta^2 + \Delta_{22} \end{pmatrix}
\tag{34}
$$

Numerically the approximation of Eq.(32) turns out to be accurate to within a few percent compared to the exact results obtained from diagonalization of the 3×3 matrix of Eq.(31).

3 Size of chargino sector loop contributions

We discuss now the numerical size of the chargino sector exchange contributions. The current lower limits on the light Higgs masses correspond to $m_h > 88.3$ GeV for the light CP even Higgs and $m_A > 88.4$ GeV for the CP odd Higgs[17]. In our analysis we shall examine the part of the MSSM parameter space where these limits are obeyed although it should be kept in mind that the analysis leading to these limits included no CP violating effects.[15]. Since the general parameter space of MSSM is rather large, we shall limit ourselves to a more constrained set for the purpose of this numerical study. We shall use for our parameter space the set $m_0, m_{\frac{1}{2}}$, m_A, $|A_0|$, $\tan\beta$, θ_μ, α_{A_0}, ξ_1, ξ_2 and ξ_3. The other sparticle masses are obtained from this set using the renormalization group equations evolving the GUT parameters from the GUT scale down to the electro-weak scale. However, this is only a convenience and in general one can use the general MSSM parameter space to test the size of the corrections computed here. As discussed in Ref.[9, 10, 18] there exists a very significant part in the MSSM parameter space where the EMD constraints are satisfied with large phases. For the purposes of this analysis we shall assume that this is the case and not revisit the problem of imposing the edms constraints.

Using the constrained parameter space described above we plot in Fig.1 the quantity Δ_{13} as a function of the CP phase ξ_2. The Δ_{13} plots exhibited in Fig.1 contain the stop, the sbottom and the chargino sector contributions while the horizontal lines exhibit Δ_{13} without the inclusion of the chargino sector contribution. The analysis shows that the chargino sector contribution to Δ_{13} is comparable to the stop exchange contribution. Further one finds that the chargino sector contribution can be either positive or negative relative to the stop and sbottom sector contribution. Thus the chargino sector contribution can constructively interfere with the stop and sbottom sector contribution enhancing the CP even and CP odd mixing by as much as a factor of two. However, in other regions of the parameter space it can produce a negative interference reducing significantly the mixing of the CP even and CP odd Higgs sectors. A similar analysis holds for Δ_{23} and in Fig.2 we give a plot of Δ_{23}, with and without the contribution from the chargino sector, as a function of ξ_2. One finds again that the chargino sector makes a large contributions to Δ_{23}. Further, as for the case of Δ_{13}, the chargino sector contributions can either constructively or destructively interfere with the stop and sbottom sector contribution and thus the chargino sector contribution can either enhance or reduce the size of Δ_{23}.

In Fig.3 a plot of the percentage of the CP even component ϕ_1 of H_1 (upper sets) and the CP odd component ψ_{1D} of H_1 (lower sets) including the stop, the sbottom and the chargino sector contributions is given as a function of ξ_2 while the horizontal lines give the plots when the chargino sector contribution is omitted. (The ϕ_2 component is negligible and is not exhibited.) A comparison of the plots with and without the chargino sector contribution shows that the chargino sector makes a large relative contribution to the ϕ_1 and the ψ_{1D} components and further that this contribution can either constructively or destructively interfere with the contribution coming from the stop and sbottom sector contribution. We note that for the inputs of Fig.3 the mixings between the CP even and the CP odd components are essentially maximal. This phenomenon is a consequence of large $\tan\beta$ and we will study this in greater depth when we discuss the analysis of Fig.4. A similar analysis for H_2 yields a much smaller effect, i.e., less than a percent or so for this case where H_2 is the eigen state which limits to the lightest CP even

Higgs state in the case when one ignores the mixing between the CP even the CP odd states. Thus one concludes that the lightest Higgs state develops a negligible CP odd component as a consequence of mixing and remains essentially a CP even state. A similar conclusion was arrived at in previous analyses[13] without the inclusion of the chargino sector contribution. The analysis of H_3 parallels the analysis of H_1 except that the CP even and the odd components reverse their roles. Thus one can easily obtain the percentages of ϕ_1 and ψ_{1D} components in H_3 from Fig.3 by interchanging ϕ_1 and ψ_{1D}. Again one finds that the ϕ_2 component of H_3 is small for the input of Fig.3.

We discuss now the $\tan\beta$ dependence of the mixing between between the CP even and the CP odd sectors. We illustrate this dependence in Fig.4 where the CP odd component ψ_{1D} of H_1 is plotted as a function of ξ_2 for values of $\tan\beta$ ranging from 5 to 40. One finds that for $\tan\beta \leq 10$ the CP odd component of H_1 is less than fraction of a percent. The fraction of the CP odd component grows to the level of a few percent for values of $\tan\beta$ in the range 15-20. This trend continues and one finds large mixings as $\tan\beta$ gets large, ie., in the neighborhood of 25 or larger. The theoretical reason for this strong dependence of the mixings on $\tan\beta$ can be easily understood. Thus as $\tan\beta$ becomes large $\cos\beta$ becomes vanishingly small, and from Eq.(31) one finds that the two heavier Higgs eigen masses become essentially degenerate. This degeneracy of masses implies that the mixings are no longer suppressed by the factor Δ_{ij}/M_A^2 etc but rather it is the ratio of the $\Delta's$ themselves that determines the mixings. Consequently in the region of large $\tan\beta$ the mixings between the CP even and the CP odd sectors become large. In the analysis presented so far we have investigated the dependence of the mixings of the CP even and the CP odd sector on ξ_2 which is the phase of the $SU(2)$ gaugino mass $\tilde{m}_2$. One also expects a significant dependence of the mixings on the other phases. As an illustration in Fig.5 we give an analysis of the mixings as a function of θ_μ. Here, as in Fig.3, we plot the CP even and the CP odd components of H_1, i.e., of ϕ_1 and of ψ_{1D} but now as a function of θ_μ for the inputs given in the figure caption. The dashed curves are for the case without the chargino sector contribution while the solid curves are with the chargino sector contribution. Again one finds that the chargino sector makes a significant contribution relative to the stop and sbottom sector contribution.

Finally, we discuss the contribution of the chargino sector to the lightest Higgs mass. One finds that the chargino sector contribution is typically of order 1-2 GeV and is negative. Some typical examples of the sizes of the $\chi^+ - W - H^+$ contribution are given in Table 1. The effect of CP phases on the $\chi^+ - W - H^+$ correction to the Higgs masses is typically small, i.e., the variation in the corrections is a few percent at best. The precision analyses of the Higgs masses including radiative corrections from the stop and sbottom sector exchanges and including leading order corrections from two loop corrections and other refinements purport to achieve an accuracy of 1-2 GeV in the prediction of the lightest Higgs boson mass. Since the chargino sector contribution with or without CP violating effects lies in this range it appears reasonable to include this correction in the precision prediction of the lightest Higg boson mass. The chargino sector corrections to the mass eigen values of the other two (H_1, H_3) Higgs bosons is significantly smaller and can be safely neglected.

Table 1:

$\tan\beta$	m_h without $\chi^+ - W - H^+$	m_h with $\chi^+ - W - H^+$
5	116.82	115.42
10	121.76	120.45
15	122.70	121.41
20	123.0	121.71
25	123.11	121.84
30	123.16	121.88

Table caption: Input parameters are $m_0 = 500$, $m_{\frac{1}{2}} = 400$, $m_A = 200$, $|A_0| = 1000$, $\theta_\mu = 0.5$, $\alpha_{A_0} = 0.5$, $\xi_1 = 0.4$, $\xi_2 = 0.5$, $\xi_3 = 0.6$. All masses are in GeV and all angles are in radians.

There are several consequences of the CP even and CP odd Higgs mixings implied by large phases. Some of these have been discussed in Refs.[13, 15]. One consequence is the effect on the quark and the lepton couplings with the Higgs, i.e., the couplings $\bar{q}qH_i$ and $\bar{l}lH_i$ (i=1,2,3). These modifications affect the phenomenology for Higgs searches at colliders. We point out here that the vertices involving the couplings of the Higgs with the charginos (χ_a^+, a=1,2) and the neutralinos ($\chi_n, n = 1-4$) are also affected, i.e., the chargino Higgs couplings $\bar{\chi}_a^+\chi_b^+H_i$ (a,b=1,2), and the neutralino Higgs couplings $\bar{\chi}_n\chi_m H_i$ (n,m=1-4). Specifically, the couplings of the lightest neutralino (χ_1) to the Higgs will depend on the parameters which mix the CP even and the CP odd Higgs sector and will affect dark matter analyses. Thus the neutralino relic density analysis which involves the process $\chi_1 + \chi_1 \to \bar{f}f$ etc, with the Higgs poles apprearing in the direct channel, will be affected. We expect these effects to arise from the couplings of the H_1 and H_3 and expect them to give significant effects only for large values of $\tan\beta$, i.e., $\tan\beta > 20$ where the mixing effects become significant. Similarly the analysis of the direct detection of dark matter which involves the scattering process $\chi_1 + q \to \chi_1 + q$, with the Higgs poles entering in the cross channel, will be affected. A detailed discussion of these phenomena is outside the scope of this paper.

4 Conclusions

In this paper we have analysed the effects of the χ^+, the W and the H^+ exchange contributions to the Higgs boson masses and mixings in the presence of large CP violationg effects. We find that this sector makes a large contribution to the mixing between the CP even and the CP odd Higgs states and in certain parts of the parameter space the mixings generated by the chargino sector may dominate the mixings generated by the stop and sbottom sector exchanges. We also find that in terms of sizes the chargino sector contributions are significantly larger than the sbottom exchange corrections which have been included in previous analyses. The size of the mixing effects are seen to depend sharply on the value of $\tan\beta$ with the mixing effects becoming large as $\tan\beta$ gets large and for values of $\tan\beta$ larger than 30 the mixings between the CP even and the CP odd sector become maximal. These mixings have important implication for Higgs phenomenology at colliders. We have also analysed the effects of the chargino sector contribution on the lightest Higgs mass. We find that the chargino sector contribution to the lightest Higgs boson mass lies in the range of 1-2 GeV. This effect is relevant in the precision predictions of the lightest Higgs boson mass. Further, we find that typically the chargino sector contribution is negative and lowers the lightest Higgs boson mass and leads to a slight worsening of the fine tuning problem already implied by the non observation of the Higgs boson at LEP thus far[19]. A similar analysis can be carried out for the neutralino sector contribution to the Higgs boson masses and mixings. This analysis is underway and will be reported in a separate communication.

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5 Appendix A: Stop and sbottom contributions

For completeness we give here an analysis of the one loop contributions from the stop and sbottom sectors with inclusion of CP violating effects. The stop $(mass)^2$ matrix is given by

$$M_{\tilde{t}}^2 = \begin{pmatrix} M_Q^2 + h_t^2|H_2^0|^2 + \frac{(g_2^2-g_1^2/3)}{4}(|H_1^0|^2 - |H_2^0|^2) & h_t(A_t^* H_2^{0*} - \mu H_1^0) \\ h_t(A_t H_2^0 - \mu^* H_1^{0*}) & M_U^2 + h_t^2|H_2^0|^2 + \frac{g_1^2}{3}(|H_1^0|^2 - |H_2^0|^2) \end{pmatrix} \tag{35}$$

where $A_t = |A_t|e^{i\alpha_{A_t}}$. The contribution to the one loop effective potential from the stop and top exchanges is given by

$$\Delta V(\tilde{t}, t) = \frac{1}{64\pi^2}(\sum_{a=1,2} 6M_{\tilde{t}_a}^4(log\frac{M_{\tilde{t}_a}^2}{Q^2} - \frac{3}{2}) - 12m_t^4(log\frac{m_t^2}{Q^2} - \frac{3}{2})) \tag{36}$$

Using the above potential our analysis for $\Delta_{ij\tilde{t}}$ gives

$$\Delta_{11\tilde{t}} = -2\beta_{h_t} m_t^2|\mu|^2 \frac{(|A_t|\cos\gamma_t - |\mu|cot\beta)^2}{(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2)^2} f_2(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2) \tag{37}$$

$$\Delta_{22\tilde{t}} = -2\beta_{h_t} m_t^2 \frac{|A_t|^2[|A_t| - |\mu|cot\beta\cos\gamma_t]^2}{(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2)^2} f_2(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2)$$
$$+2\beta_{h_t} m_t^2 ln(\frac{m_{\tilde{t}_1}^2 m_{\tilde{t}_2}^2}{m_t^4}) + 4\beta_{h_t} m_t^2 \frac{|A_t|[|A_t| - |\mu|cot\beta\cos\gamma_t]}{(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2)} ln(\frac{m_{\tilde{t}_1}^2}{m_{\tilde{t}_2}^2}) \tag{38}$$

$$\Delta_{12\tilde{t}} = -2\beta_{h_t} m_t^2 \frac{|\mu|[|A_t|\cos\gamma_t - |\mu|cot\beta]}{(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2)} ln(\frac{m_{\tilde{t}_1}^2}{m_{\tilde{t}_2}^2})$$
$$+2\beta_{h_t} m_t^2 \frac{|\mu||A_t|[|A_t|\cos\gamma_t - |\mu|cot\beta][|A_t| - |\mu|cot\beta\cos\gamma_t]}{(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2)^2} f_2(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2) \tag{39}$$

$$\Delta_{13\tilde{t}} = -2\beta_{h_t} m_t^2 \frac{|\mu|^2|A_t|\sin\gamma_t[|\mu|cot\beta - |A_t|\cos\gamma_t]}{\sin\beta(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2)^2} f_2(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2) \tag{40}$$

$$\Delta_{23\tilde{t}} = -2\beta_{h_t} m_t^2|\mu||A_t|^2 \frac{\sin\gamma_t(|A_t| - |\mu|cot\beta\cos\gamma_t)}{\sin\beta(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2)^2} f_2(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2)$$
$$+2\beta_{h_t} \frac{m_t^2|\mu||A_t|\sin\gamma_t}{\sin\beta(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2)} ln(\frac{m_{\tilde{t}_1}^2}{m_{\tilde{t}_2}^2}) \tag{41}$$

$$\Delta_{33\tilde{t}}=-2\beta_{h_t}\frac{m_t^2|\mu|^2|A_t|^2\sin^2\gamma_t}{\sin^2\beta(m^2_{\tilde{t}_1}-m^2_{\tilde{t}_2})^2}f_2(m^2_{\tilde{t}_1},m^2_{\tilde{t}_2}) \tag{42}$$

The expressions of our Δ_{11}, Δ_{22}, Δ_{13}, Δ_{23} and Δ_{33} agree with those of previous authors. However, there is a difference between our Δ_{12} and the Δ_{12} of Ref.[13] in the presence of phases although the two expressions agree in the limit when there are no phases.

We discuss next our computation for the sbottom sector. The sbottom mass $(mass)^2$ matrix is given by

$$M^2_{\tilde{b}}=\begin{pmatrix} M_Q^2+h_b^2|H_1^0|^2-\frac{(g_2^2+g_1^2/3)}{4}(|H_1^0|^2-|H_2^0|^2) & h_b(A_b^*H_1^{0*}-\mu H_2^0) \\ h_b(A_bH_1^0-\mu^*H_2^{0*}) & M_D^2+h_b^2|H_1^0|^2-\frac{g_1^2}{6}(|H_1^0|^2-|H_2^0|^2)\end{pmatrix} \tag{43}$$

where $A_b=|A_b|e^{i\alpha_{A_b}}$. The contribution to the one loop effective potential from the sbottom and b exchanges is given by

$$\Delta V(\tilde{b},b)=\frac{1}{64\pi^2}(\sum_{a=1,2}6M^4_{\tilde{b}_a}(log\frac{M^2_{\tilde{b}_a}}{Q^2}-\frac{3}{2})-12m_b^4(log\frac{m_b^2}{Q^2}-\frac{3}{2})) \tag{44}$$

Our computation of $\Delta_{ij\tilde{b}}$ yields

$$\Delta_{11\tilde{b}}=-2\beta_{h_b}m_b^2\frac{|A_b|^2[|A_b|-|\mu|\tan\beta\cos\gamma_b]^2}{(m^2_{\tilde{b}_1}-m^2_{\tilde{b}_2})^2}f_2(m^2_{\tilde{b}_1},m^2_{\tilde{b}_2})$$
$$+2\beta_{h_b}m_b^2ln(\frac{m^2_{\tilde{b}_1}m^2_{\tilde{b}_2}}{m_b^4})+4\beta_{h_b}m_b^2\frac{|A_b|[|A_b|-|\mu|\tan\beta\cos\gamma_b]}{(m^2_{\tilde{b}_1}-m^2_{\tilde{b}_2})}ln(\frac{m^2_{\tilde{b}_1}}{m^2_{\tilde{b}_2}}) \tag{45}$$

$$\Delta_{22\tilde{b}}=-2\beta_{h_b}m_b^2|\mu|^2\frac{(|\mu|\tan\beta-|A_b|\cos\gamma_b)^2}{(m^2_{\tilde{b}_1}-m^2_{\tilde{b}_2})^2}f_2(m^2_{\tilde{b}_1},m^2_{\tilde{b}_2}) \tag{46}$$

$$\Delta_{12\tilde{b}}=-2\beta_{h_b}m_b^2|\mu|\frac{(|A_b|\cos\gamma_b-|\mu|\tan\beta)}{(m^2_{\tilde{b}_1}-m^2_{\tilde{b}_2})}ln(\frac{m^2_{\tilde{b}_1}}{m^2_{\tilde{b}_2}})$$
$$+2\beta_{h_b}m_b^2\frac{|\mu||A_b|[|A_b|\cos\gamma_b-|\mu|\tan\beta][|A_b|-|\mu|\tan\beta\cos\gamma_b]}{(m^2_{\tilde{b}_1}-m^2_{\tilde{b}_2})^2}f_2(m^2_{\tilde{b}_1},m^2_{\tilde{b}_2}) \tag{47}$$

$$\Delta_{13\tilde{b}}=-2\beta_{h_b}m_b^2|\mu||A_b|^2\sin\gamma_b\frac{(|A_b|-|\mu|\tan\beta\cos\gamma_b)}{\cos\beta(m^2_{\tilde{b}_1}-m^2_{\tilde{b}_2})^2}f_2(m^2_{\tilde{b}_1},m^2_{\tilde{b}_2})$$
$$+2\beta_{h_b}m_b^2\frac{|\mu||A_b|\sin\gamma_b}{\cos\beta(m^2_{\tilde{b}_1}-m^2_{\tilde{b}_2})}ln(\frac{m^2_{\tilde{b}_1}}{m^2_{\tilde{b}_2}}) \tag{48}$$

$$\Delta_{23\tilde{b}} = -2\beta_{h_b} m_b^2 |\mu|^2 |A_b| \sin\gamma_b \frac{(|\mu|\tan\beta - |A_b|\cos\gamma_b)}{\cos\beta(m_{\tilde{b}_1}^2 - m_{\tilde{b}_2}^2)^2} f_2(m_{\tilde{b}_1}^2, m_{\tilde{b}_2}^2) \tag{49}$$

$$\Delta_{33\tilde{b}} = -2\beta_{h_b} \frac{m_b^2 |\mu|^2 |A_b|^2 \sin^2\gamma_b}{\cos^2\beta(m_{\tilde{b}_1}^2 - m_{\tilde{b}_2}^2)^2} f_2(m_{\tilde{b}_1}^2, m_{\tilde{b}_2}^2) \tag{50}$$

In the above analysis we have ignored the D terms of the squark (mass)^2 matrices to gain approximate independence of the renormalization scale Q as in the analysis of Ref.[13, 15].

Figure Captions

Fig.1: Plot of Δ_{13} including the stop, sbottom and chargino sector contributions vs ξ_2. The common input for both the top and the bottom curves is $m_0 = 500$, $m_{\frac{1}{2}} = 400$, $M_A = 200$, $|A_0| = 1000$, $\alpha_0 = 0.5$, $\xi_1 = 0.4$ and $\xi_3 = 0.6$ where all masses are in GeV and all angles are in radians. The dashed curve is the case $\tan\beta = 30$, $\theta_\mu = 1$ and the dashed horizontal line is without the inclusion of the chargino sector contribution. The corresponding solid curve and the solid horizontal line are for the same input except that $\tan\beta = 40$ and $\theta_\mu = 0.5$.

Fig.2: Plot of Δ_{23} including the stop, sbottom and chargino sector contributions vs ξ_2 for the same input as in Fig.1. The dashed and solid curves have the same meaning as in Fig.1 and the horizontal lines are the plots without inclusion of the chargino sector contribution also as in Fig.1.

Fig.3: Plot of the CP even component ϕ_1 of H_1 (upper curves) and the CP odd component ψ_{1D} of H_1 (lower curves) including the stop, sbottom and chargino sector contributions as a function of ξ_2 for the same inputs as in Fig.1. The dashed curves are for the case $\tan\beta = 30$, $\theta_\mu = 1$, and the solid curves are for the case $\tan\beta = 40$ $\theta_\mu = 0.5$, and the corresponding horizontal lines are for the cases when the chargino sector contributions are neglected.

Fig.4: Plot of the modulus square of the CP odd component in H_1 as a function of ξ_2 for various values of $\tan\beta$ with all the other parameters being the same as in Fig.3 with $\theta_\mu = 0.5$. The values of $\tan\beta$ from bottom up are 5,10,15,20,30,40.

Fig.5: Plot of the CP even component ϕ_1 of H_1 (upper curves) and the CP odd component ψ_{1D} of H_1 (lower curves) as a function of θ_μ including the stop, sbot-

tom and chargino sector contributions (solid) and without inclusion of chargino contributions (dashed) for the following input: $m_0 = 500$, $m_{\frac{1}{2}} = 400$, $m_A = 300$, $A_0 = 1000$, $\tan\beta = 30$, $\alpha_{A_0} = -0.4$, $\xi_1 = 0.4$, $\xi_2 = 0.5$, $\xi_3 = 0.6$, where all masses are in GeV and all angles are in radians.

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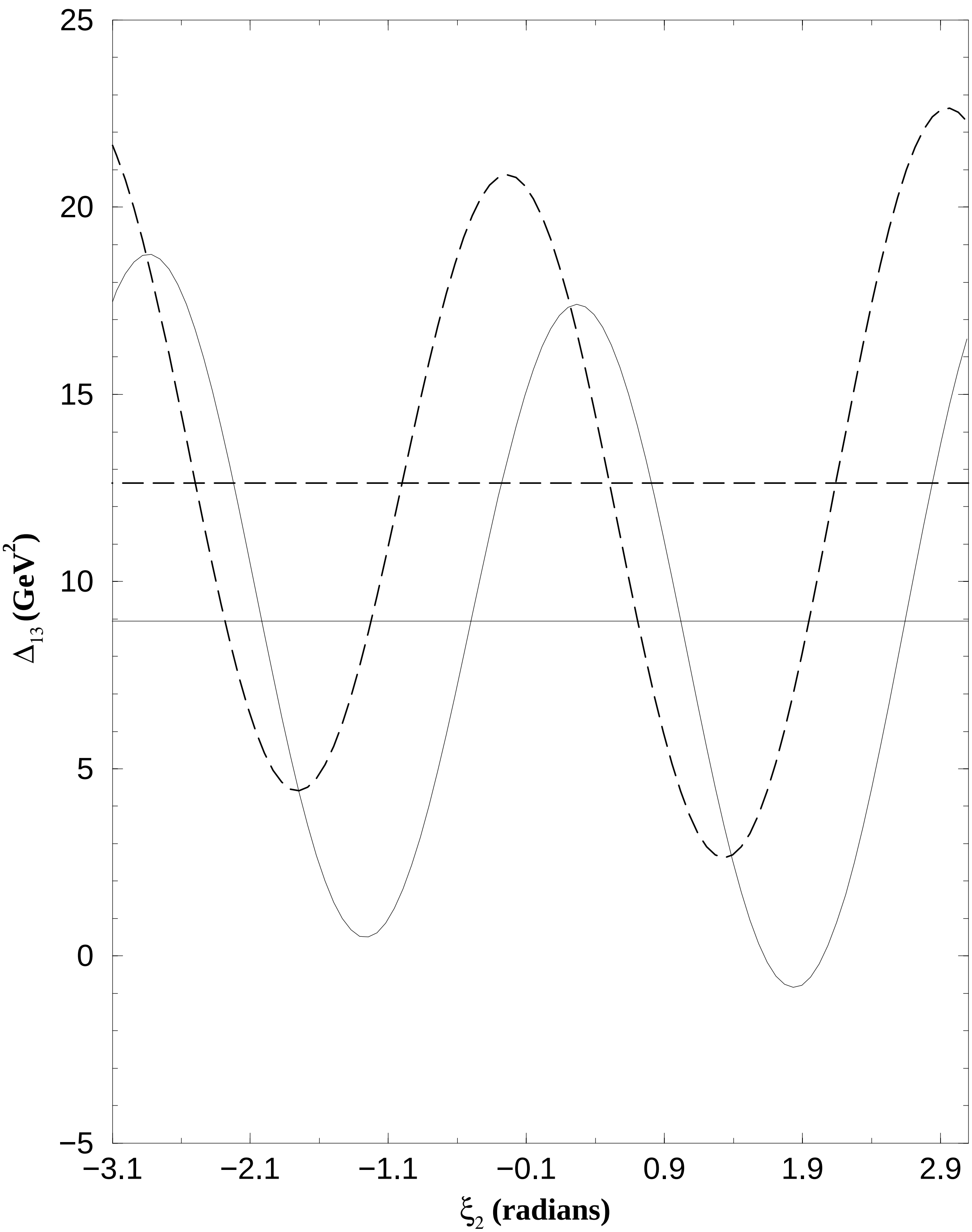
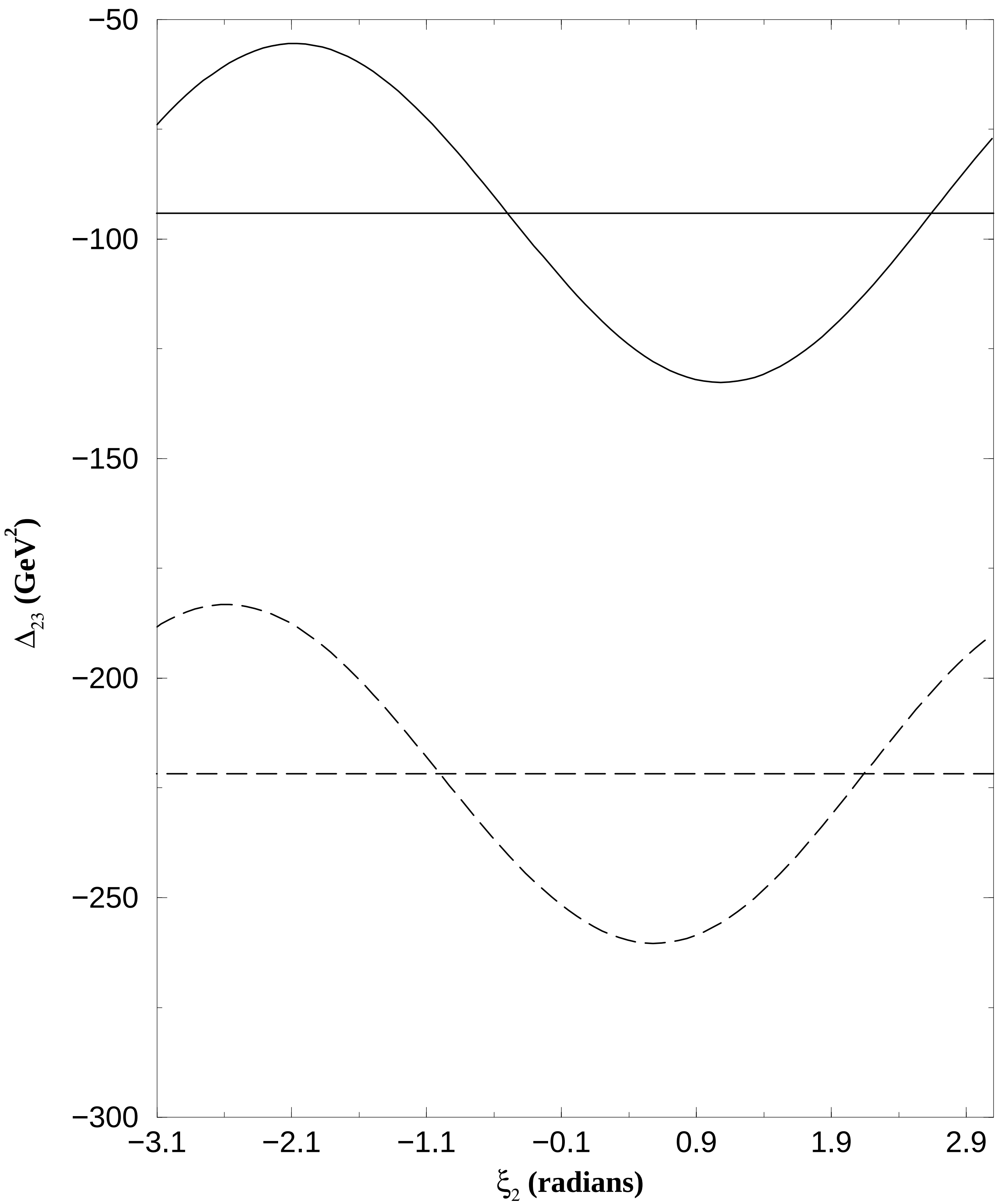
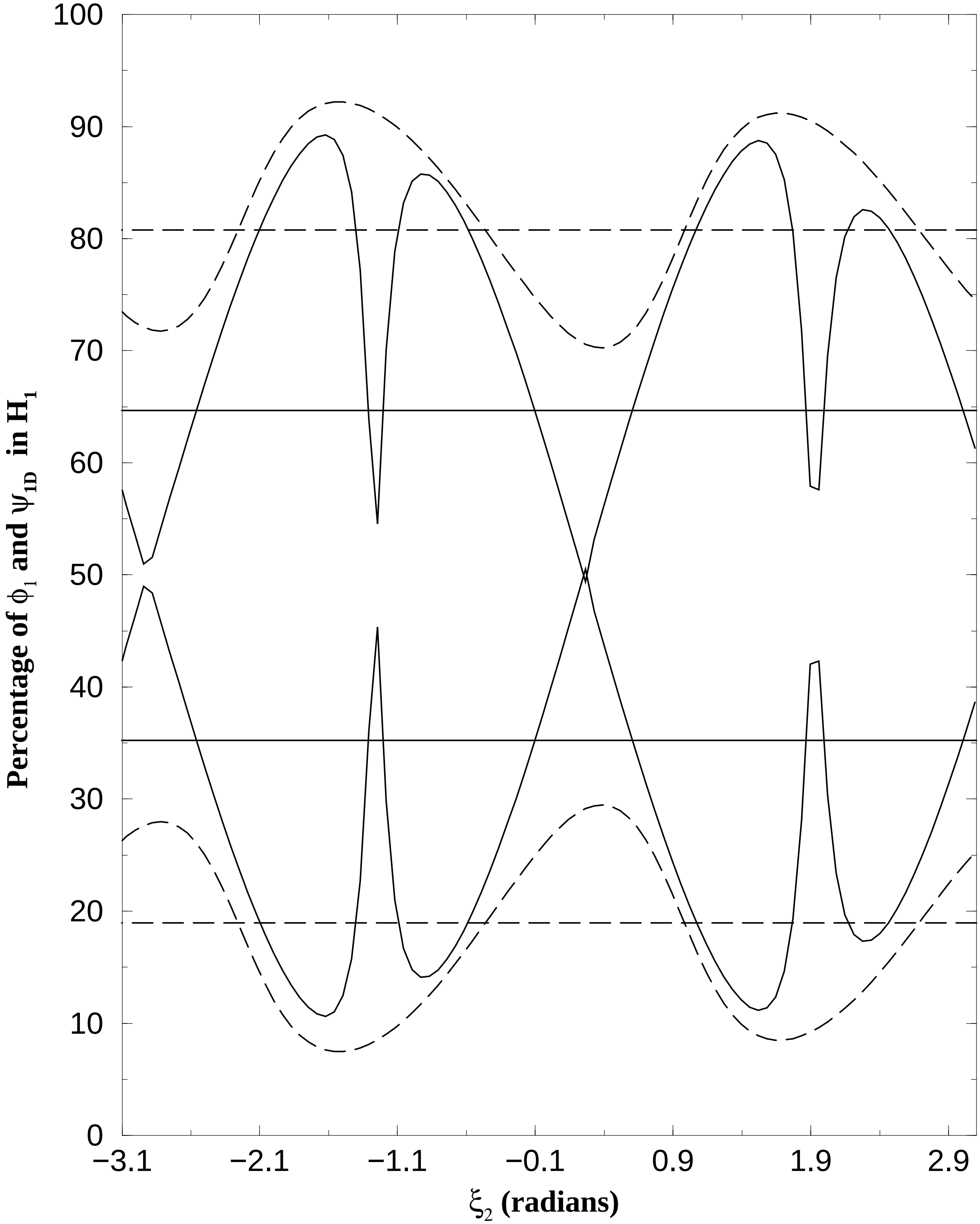


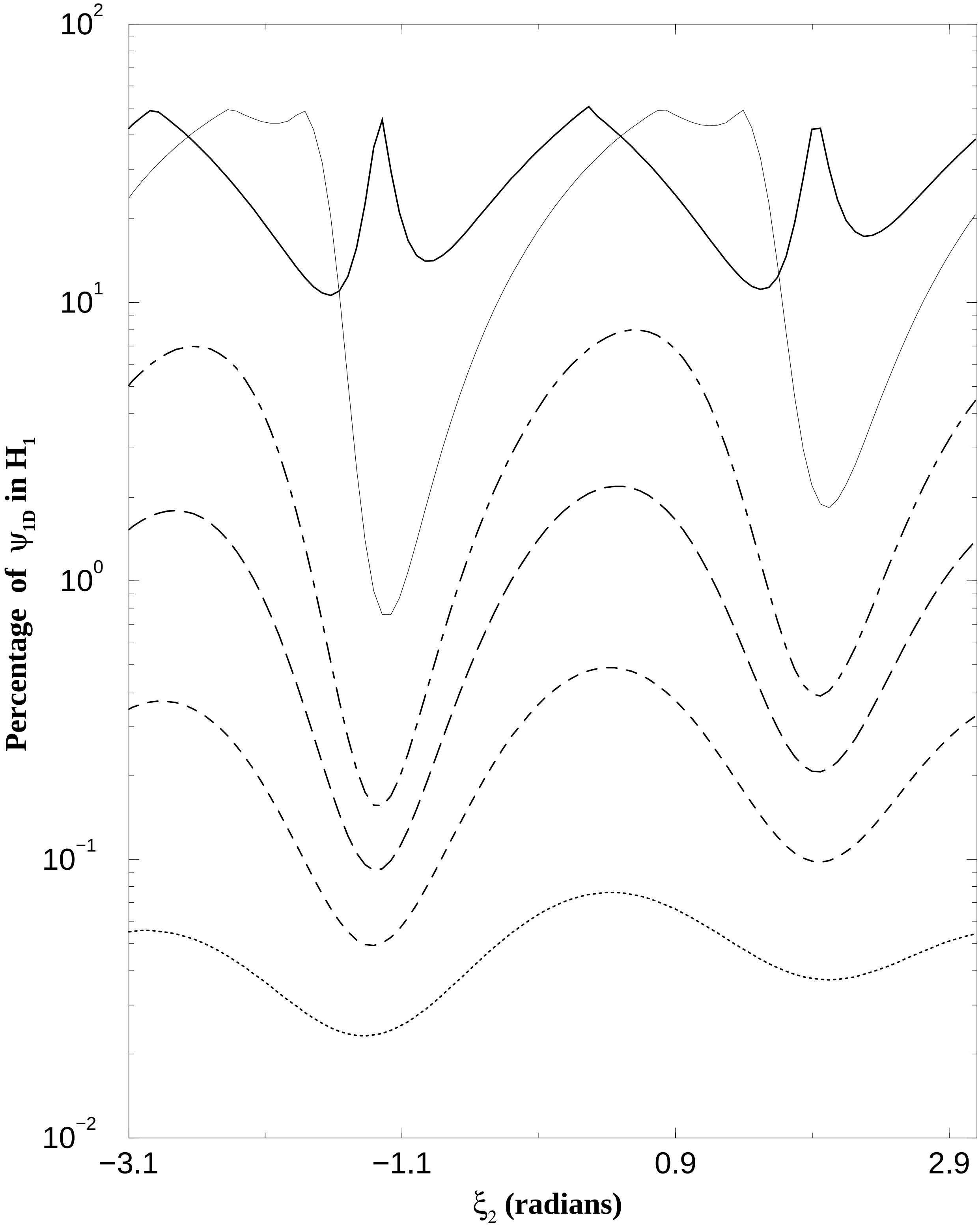
25
20
15
10
5
0
−5
−3.1
−2.1
−1.1
−0.1
0.9
1.9
2.9
Δ13 (GeV2)
ξ2 (radians)



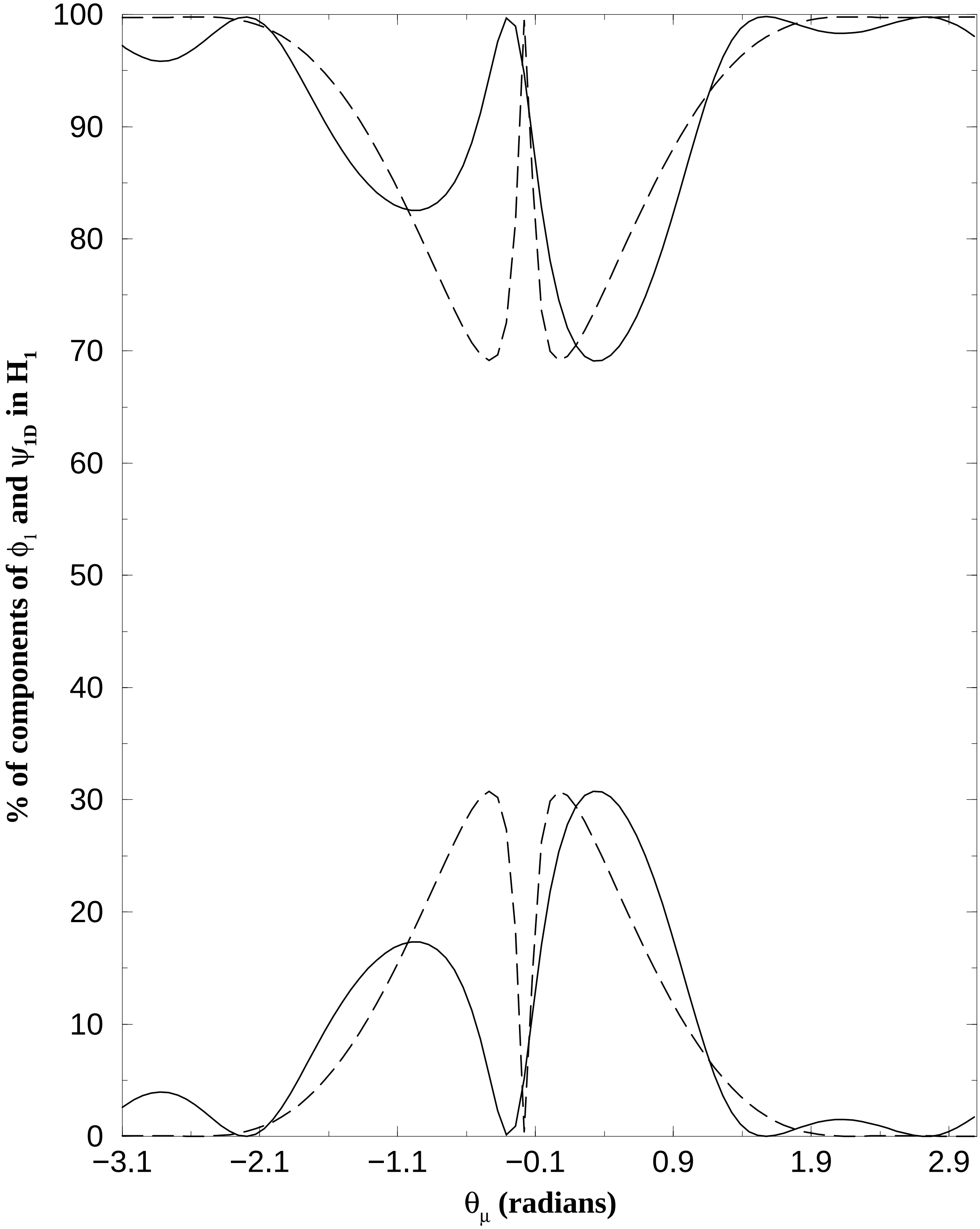
−50
−100
−150
−200
−250
−300
−3.1
−2.1
−1.1
−0.1
0.9
1.9
2.9
Δ_{23} (GeV^2)
ξ_2 (radians)



Percentage of ϕ_1 and ψ_{1D} in H_1
ξ_2 (radians)
100
90
80
70
60
50
40
30
20
10
0
−3.1
−2.1
−1.1
−0.1
0.9
1.9
2.9



10^2
10^1
10^0
10^{-1}
10^{-2}
Percentage of ψ_{1D} in H_1
−3.1
−1.1
0.9
2.9
ξ_2 (radians)



100
90
80
70
60
50
40
30
20
10
0
−3.1
−2.1
−1.1
−0.1
0.9
1.9
2.9
θμ (radians)
% of components of φ1 and ψ1D in H1