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A METHOD TO MEASURE THE BETA-BEATING IN A 90 DEGREES PHASE ADVANCE LATTICE

G. Morpurgo, J. Klem

Abstract

A method to compute the beta-beating in an accelerator, using as input the measurement of the phase advance of a betatron oscillation between three BPMs (beam position monitors), had been developed in the past. The beauty of this method is that the result does not depend on the BPM relative errors on the measurement of the oscillation amplitude. Unfortunately, this method is not applicable when the phase advance between two of the three BPMs is (close to) 180 degrees. In this latter case the measurement of the amplitude of the beam oscillation should be combined with the phase advance measurement to get a complete picture of the beta-beating. Detecting and filtering BPMs with large gain errors requires some care. A method dealing with these aspects has been developed. Examples obtained by applying the method to real data from LEP and SPS are shown.

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A method to measure the beta-beating in a 90 degrees phase advance lattice

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Abstract

A method to compute the beta-beating in an accelerator, using as input the measurement of the phase advance of a betatron oscillation between three BPMs (beam position monitors), had been developed in the past[1]. The beauty of this method is that the result does not depend on the the BPM relative errors on the measurement of the oscillation amplitude. Unfortunately this method is not applicable when the phase advance between two of the three BPMs is (close to) 180 degrees. In this latter case the measurement of the amplitude of the beam oscillation should be combined with the phase advance measurement to get a complete picture of the beta-beating. Detecting and filtering BPMs with large gain errors requires some care. A method dealing with these aspects has been developed. Examples obtained by applying the method to real data from LEP and SPS are shown.

1 MEASURING BETA-BEATING IN A 90 DEGREES PHASE ADVANCE LATTICE

In the presence of a quadrupole field (gradient) error at s_0, the betatron beta function is modified accordingly to the well known formula

$$\frac{\Delta\beta}{\beta}(s) = \frac{\Delta k\, L\beta_0 \cos(2\pi Q - 2|\mu(s) - \mu_0|)}{2\sin(2\pi Q)},$$

where Δk is the gradient error, L is magnet length, $\mu(s)$ is phase function and Q is the tune. The difference between the observed and predicted betatron phase μ_s also modulates in a similar way, shifted by 90 degrees with respect to the beta modulation.

The modulation of the beta function propagates around the ring with twice the betatron frequency. In an accelerator, we can measure its effect at each Beam Position Monitor (BPM), by measuring the amplitude and the phase of coherent betatron oscillations[2]. By comparing the oscillation amplitudes at the different BPMs, and the relative phases, with the same quantities predicted by the model, we can quantify the amplitude and the phase of the beating wave. But if the phase advance between consecutive BPMs is 90 degrees, the sampling rate of the beating wave becomes 180 degrees, and we are not anymore in the position of assessing the amplitude of the beating wave just by looking at the BPM amplitudes. In fact, we might or might not see the beating effect depending on the phase of the beating wave at which the BPMs are located. This effect is illustrated in Fig. 1. We can, however, get another set of sampling points, shifted by 90 degrees with respect to the first one, if we also consider the beating effect on the phase advance between the BPMs. If we combine the two sets, we get a point every 90 degrees, and we can reconstruct a complete picture of the beating.

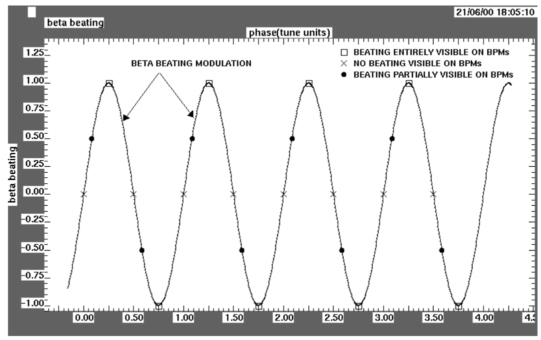

Figure 1: Depending on the phase of the beating wave, oscillation amplitudes detected at BPMs located at 90 degrees betatron phase advance intervals may or may not show the beta beating.

2 RECOMBINATION BY LOCAL FIT

In the hypothesis of no local gradient error between two consecutive BPMs, and of no error on the BPM calibration gain factors, we can solve (numerically) the system of 4 equations

$$\beta_1^{th}\beta_2^{th} sin^2\phi_{12}^{th} = \beta_1^{me}\beta_2^{me} sin^2\phi_{12}^{me} \quad (1)$$

$$\frac{\beta_1^{me}}{\beta_2^{me}} = \frac{ampli_{BPM1}^2}{ampli_{BPM2}^2} \quad (2)$$

$$\beta_1^{me} = \beta_1^{th}(1 + A * cos(\theta)) \quad (3)$$

$$\beta_2^{me} = \beta_2^{th}(1 + A * cos(\theta + 2\phi_{12}^{me})) \quad (4)$$

where the superscript *th* refers to the quantities predicted by the theoretical model, and *me* refers to the measured quantities. The four unknowns are the real values of beta (β_1^{me} and β_2^{me}), and the amplitude A and the phase θ of the beating wave (which we assume to be the same at the two BPMs because of the hypothesis on no local errors).

Figure 2 shows, for every LEP BPM, the amplitude of a coherent betatron oscillation and the difference between the measured and the predicted phase advance. By applying our method to each consecutive pair of BPMs, we get the amplitude and the phase of the beating wave at each BPM, and the beta values at the BPMs themselves. For every BPM actually we get two results, one by using the BPM and the previous one, the second by using the BPM and the next one. If the two results differ significantly, either one of the three BPMs measured badly, or there is a local gradient error. In this latter case, the beating in the machine on both sides of the BPM will be different, whereas

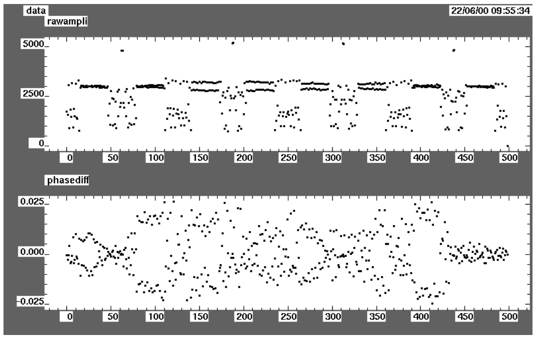

Figure 2: Measurement of vertical coherent betatron oscillation at LEP : a)Amplitude at each BPM and b) difference between the theoretical and measured phases. The 8 arcs with 90 degrees phase advance between consecutive BPMs are clearly visible.

in the former case it will be roughly the same. Figure 3 shows the computed amplitude of the beating along LEP, and Fig. 4 the phase of the beating wave at each BPM. By using the computed values of the beta function at each BPM we can renormalize the measured amplitude of the oscillations (Fig. 5) and get a first idea on the precision of the BPM calibration gain factors (which at LEP is very good). Finally, Fig. 6 shows the beating smoothed by

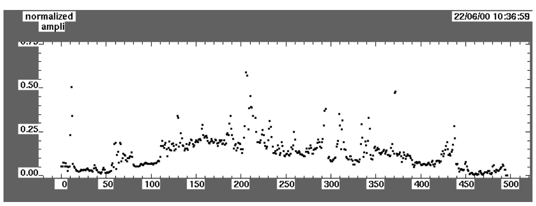

Figure 3: Amplitude of the beta beating along LEP.

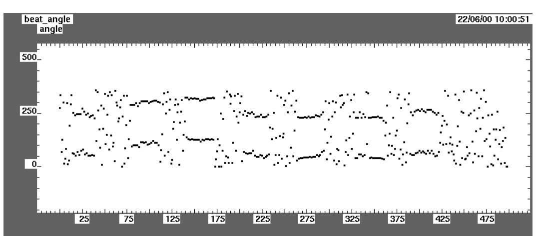

Figure 4: Phase (in degrees) of the beating wave at the LEP BPMs.

performing a long (histogram) and a short (line) sliding average, and its standard deviation. The method produces acceptable results, and it could be refined by further analysis (trying to discard or correct BPMs which are obviously wrong, trying to combine the data of more BPMs to improve the fit).

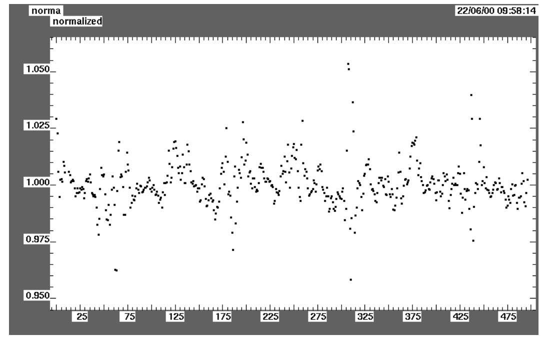

Figure 5: Renormalized amplitude at each BPM. It should be proportional to the BPM calibration gain factor.

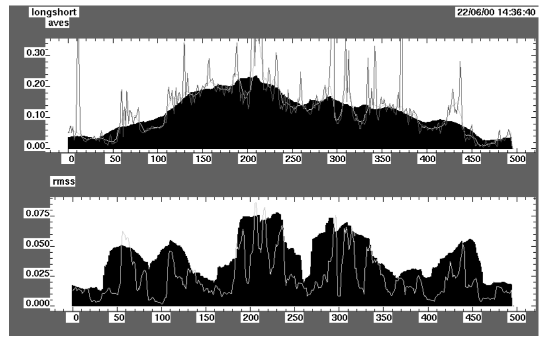

Figure 6: Above : the beating smoothed by performing a sliding average (histogram, sliding over 50 BPMs; line, sliding over 10). Below, the standard deviations for the above distributions.

3 MEASURING BETA-BEATING IN AN ALMOST 90 DEGREES PHASE ADVANCE LATTICE

Another method could be used, complementary to the first one, in a machine like the CERN SPS, where the optics is very regular all around the ring, and the phase advance is just less than 90 degrees (88.7 degrees). In this case, if the natural sources of beating in the unperturbed machine are small, a large beating perturbation generated ad-hoc by modifying the strength of a single quadrupole will dominate, and it will be visible at some BPMs mainly as amplitude beating, and at other BPMs mainly as phase beating (Fig. 7). As we can measure very accurately the phase, we can then fit the beating visible as phase difference between the theoretical and the measured phase, using a sinusoid with a frequency twice as large as the betatron phase advance. The result of the fit, shifted by 90 degrees, can be transferred to the amplitude plane, where the measurement depends critically on the BPM calibration gain factors. By comparing the measured amplitudes with the result of the fit, this method can help us in determining these factors.We have performed similar measurements at the SPS[3]. In Fig. 8 we show the amplitude and the phase modulation measured at the BPMs. As one can easily see, where the (visible) amplitude of the phase modulation has a maximum, the (visible) amplitude of the amplitude modulation

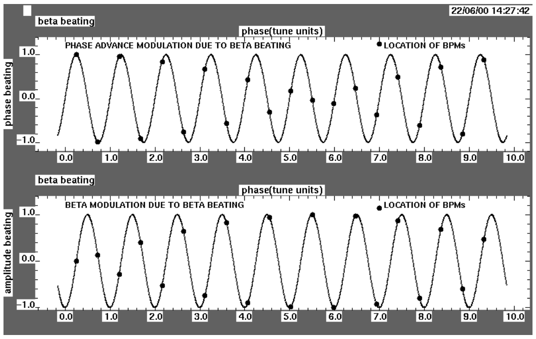

Figure 7: If the phase advance between consecutive BPMs is just less than 90 degrees (86 in this plot), the beta beating will be visible mainly in phase in some parts of the machine, and mainly in amplitude in other parts.

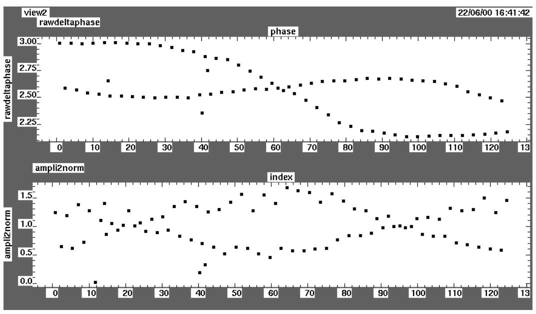

Figure 8: Coherent betatron oscillations amplitude and phase modulation induced by a quadrupole error at the SPS. The location of the quadrupole error is not contained in the plot.

has a mimimum, and viceversa. For sake of simplicity we have not shown the entire SPS; the location of the modified quadrupole is out of the right border of the picture. The phase modulation has a overall slope, due to the difference between the tune of the SPS when the data were taken, and the theoretical tune. If we remove this slope and recenter the modulation around 0, as mentioned before we can fit it by a sinus with a frequency twice as large as the betatron phase advance (first order fit). We could then try to fit the amplitude modulation with the corresponding cosinus function.

We realize, however, that after fitting the data with the best sinusoid, we are left with a first residual. Interesting enough, we took many measurements in different conditions (different strengths of the quadrupole error, different quadrupole, different tune), but this residual was essentially unmodified. This residual can be fitted very well by a sinusoid advancing with 4 times the betatron phase advance (second order fit), leaving an almost negligible second residual. The data originally used for the fit, together with the two residuals, are shown in Fig. 9, where Fig. 10 shows the difference between the data used for the fit and the first residual. This piece of data is amazingly clean, and it will encorage us in trying to understand how to combine in the amplitude plane the effect of the first and of the second order fits.

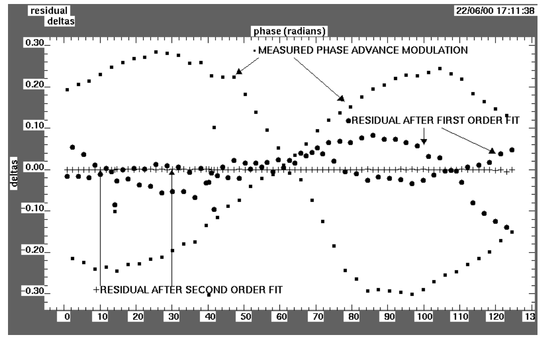

Figure 9: The beating induced phase modulation to be fitted, together with the residual of the fit, and the residual of the second order fit.

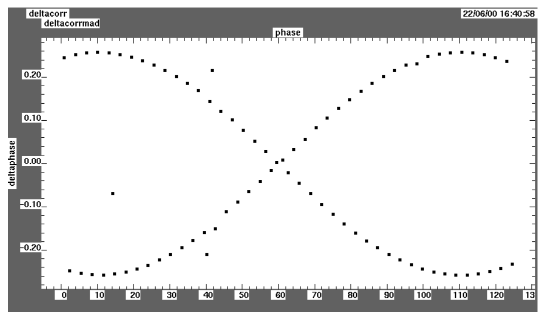

Figure 10: The beating induced phase modulation to be fitted, to which the fit of the first residual had been subtracted. Notice the cleanliness of the data.

4 CONCLUSION

We have presented ideas on how to evaluate the beta beating in machines where the phase advance between BPMs is equal or close to 90 degrees. These methods can be helpful in getting a better understanding of the machines, and in detecting and correcting problems with the BPM calibration gain factors.

5 REFERENCES

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