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We construct a smooth gauge for the adjoint field which is free of ambiguities on the lattice. In this Laplacian Center Gauge, center vortices and monopoles appear together as local gauge defects. A numerical study of center vortices in $SU(2)$ and $SU(3)$ shows equality of the Z_N and $SU(N)$ string tensions in the continuum limit, and only then.

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I. INTRODUCTION

Center vortices [1,2] are emerging as a strong candidate for the effective degrees of freedom responsible for the non-perturbative features of QCD. Evidence for the essential role of center vortices essentially comes from numerical lattice simulations. On the lattice, one can separate the Z_N center degrees of freedom (d.o.f.) and the $SU(N)/Z_N$ coset d.o.f. after gauge fixing, then study the properties of each set separately. On one hand, the center-projected theory, made of the Z_N d.o.f., has excitations when the Z_N plaquettes are different from the identity. These excitations form closed co-dimension 2 surfaces in dual space, which disorder the Z_N theory. Numerical evidence has been accumulating, which shows that this disorder generates a Z_N string tension close in value to the original $SU(N)$ one [3]. A physical density of vortices per fm^2 can be extracted [4]. $SU(N)$ chiral symmetry breaking induces a quark condensate and Dirac zeromodes in the Z_N theory also [5]. On the other hand, the coset theory shows no confinement, no chiral symmetry breaking, and no topological charge [6]. It is therefore tempting to adhere to the idea of center dominance, according to which center vortices, and nothing else, are responsible for the non-perturbative Yang-Mills features. One implication of center dominance is that the string tension produced by center vortices should match the Yang-Mills string tension exactly.

However, this construction rests on shaky ground. The connection between Z_N excitations which are located on a plaquette and their $SU(N)$ ancestors, believed to be of macroscopic size - smooth examples of which have been explicitly constructed [7] - proceeds via gauge fixing. As explained in more detail in Section III, the goal of this gauge-fixing step is to bring each gauge link as close as possible to a center element, hence the name Maximal Center Gauge. This goal is equivalent to bringing all adjoint links close to the identity, that is, making the adjoint field smooth. When this is achieved, the assignment of a Z_N element to each link, called center projection, can be done with the most confidence. However, the technical difficulty of fixing a smooth gauge on the lattice is well known. Gauge fixing proceeds by iterative local maximization of a gauge functional, and this procedure terminates when a

local maximum is reached, without any guarantee, or in fact any reasonable hope, of reaching the global maximum. Which local maximum is reached depends on the starting point along the gauge orbit. The various gauges reached correspond to "lattice Gribov copies". This technical problem, which plagues the Landau and Coulomb gauges on the lattice, is commonly believed to be rather harmless there. In Maximal Center Gauge, its harmful effects have been exhibited [8]: the copy obtained when starting the gauge-fixing procedure from Landau gauge corresponds to a very high value of the gauge-fixing functional, higher than a typical local maximum, but gives, after center projection, Z_N excitations which do not confine. Evidence has been produced that, the higher the value of the local maximum one reaches, the smaller the Z_N string tension [9]. Although this evidence is currently under dispute [10], the technical problem of lattice Gribov copies prevents a reasonable degree of confidence in the idea of center dominance.

The present paper, we believe, should restore full confidence in this idea. Here, we address and solve the problem of the gauge ambiguity. This problem was already solved for the Landau gauge in Ref. [11]. There, a different gauge was proposed, the Laplacian gauge, which produces a smooth gauge field like the Landau gauge, but has no lattice Gribov copies. The advantages of such an unambiguous gauge have barely been explored [12,5,13]. In the present paper, we generalize the construction of [11] to $SU(N)$ in the adjoint representation, since we want the adjoint field to be smooth. In our "Laplacian Center Gauge", the adjoint $SU(N)/Z_N$ field is uniquely gauge-fixed –except for exceptional configurations which are genuine Gribov copies–, and a remaining local Z_N gauge freedom subsists. Now, even though the adjoint field is uniquely gauge-fixed in general, there exists a sub-manifold of points where the gauge remains ill-defined. Moreover, these local gauge ambiguities must be related to center vortices as follows. If a Z_N center vortex is found at point x, superimposed with a small, smooth adjoint field, then the fundamental Wilson loop will be $\sim e^{i\frac{2\pi}{N}k}, k \neq 0$. This implies that the adjoint Wilson loop will be $\sim e^{i2\pi k}$. In other words, a center vortex corresponds to a non-trivial mapping of the adjoint Wilson loop to the adjoint group $SU(N)/Z_N$. This non-trivial mapping must generate a gauge singularity for the adjoint field at x. At such points, the gauge is ill-defined and the local Z_N gauge freedom is enlarged. Indeed, by studying the details of the gauge condition, we show that the gauge freedom can be locally enlarged from Z_N to $U(1)$ or $SU(2)$, defining two types of local ambiguities which can be readily identified with center vortices and Abelian monopoles, the latter being embedded in the former. These topological objects appear naturally together as local gauge defects. This unified description reconciles with center dominance the large body of numerical evidence in favor of monopole dominance: Abelian monopoles do not represent an alternative to center vortices as effective degrees of freedom; they are their undissociable partners. In particular, the condensation of monopoles in the confined phase of the $SU(N)$ theory [14,15] implies the percolation of center vortex surfaces, by the following simple argument. A non-zero monopole condensate means that a single monopole can be created at point (x,t), or equivalently that monopoles are free, not confined. Since monopole world–lines are closed loops, the monopole at (x,t) must have a partner at (x',t), and a "single" monopole simply has a partner at infinity. This in turn means that monopole world–lines percolate. Since we show that monopole world–lines are

embedded in center vortex surfaces, percolation of one implies percolation of the other.

We complement the qualitative study of local gauge defects outlined above with a quantitative study, for $SU(2)$ and $SU(3)$. We show that, in Laplacian Center Gauge, the center-projected Z_2 and Z_3 theories confine, while the coset theory does not. This confirms earlier $SU(2)$ results [3,6], and firms up the sketchy $SU(3)$ evidence [16], all obtained in the ambiguous Maximal Center Gauge. But the scenario of center dominance demands *exact* equality of the Z_N and $SU(N)$ string tensions. We test this scenario by considering different Laplacian Center Gauges, obtained from different lattice discretizations of the Laplacian. At finite lattice spacing a, different gauges give widely different Z_N string tensions, which at first sight looks like a terrible blow to center dominance. Nevertheless, we show that, in all three gauges we consider and presumably in any –unambiguous– gauge, equality of the Z_N and $SU(N)$ string tensions is approached as $a \to 0$. Therefore, center dominance is restored in the continuum limit, and only then.

Section II of our paper gives an explicit construction of the Laplacian Center Gauge for $SU(N)$. Section III discusses the local gauge ambiguities and their identification as monopoles and center vortices. Section IV presents our numerical results for $SU(2)$ and $SU(3)$. Section V shows the effect of alternative gauges and discusses center dominance. Section VI summarizes our findings.

II. LAPLACIAN CENTER GAUGE FIXING

The Laplacian gauge was originally proposed by Vink and Wiese [11], who suggested to use the eigenvectors of the covariant Laplacian operator to fix the gauge in a non–Abelian gauge theory. This method takes advantage of a non–local procedure to fix a covariant smooth gauge in an unambiguous way. There are exceptional cases, genuine Gribov copies, in which the gauge fixed configuration is not unique: however such cases can be easily detected by the presence of degenerate eigenvalues and – from the viewpoint of numerical simulations – never occur. The perturbative formulation of the Laplacian gauge in $SU(2)$ has been studied in [17], while the problem of its renormalizability is still open. Since we are interested in reducing the symmetry of the gauge group from $SU(N)$ to its center Z_N, it is useful to consider the Laplacian operator in the adjoint representation. In fact, since the adjoint representation is invariant under gauge transformations in Z_N, the adjoint Laplacian procedure fixes unambiguously the gauge up to the center symmetry, and the Laplacian Center Gauge is just another name for the adjoint Laplacian gauge. The first step towards this construction was already considered, in $SU(2)$, by A. van der Sijs [18].

Consider 4–dimensional lattice $SU(N)$ gauge theory. The adjoint Laplacian operator $\Delta^{ab}_{xy}(\dot{U})$ is given by

$$-\Delta^{ab}_{xy}(\dot{U}) = \sum_{\mu} \Big(2\,\delta_{x,y}\,\delta^{ab} - \dot{U}^{ab}_{\mu}(y-\hat{\mu})\,\delta_{x,y-\hat{\mu}} - \dot{U}^{ba}_{\mu}(y)\,\delta_{x,y+\hat{\mu}} \Big) \tag{2.1}$$

where a,$b = 1, \ldots (N^2-1)$ are color indices and x,y are space–time lattice coordinates. The dotted $\dot{U}_\mu(x)$ are the link variables in the adjoint representation and are related to the links $U_\mu(x)$ in the fundamental by

$$\dot{U}^{ab}_{\mu}(x) = \frac{1}{2}\mathrm{Tr}\left(\lambda_a U_\mu(x)\lambda_b U^\dagger_\mu(x)\right) \tag{2.2}$$

λ_i, $i = 1, \ldots, (N^2-1)$ being the generators of $SU(N)$ with the normalization $\mathrm{Tr}(\lambda_a\lambda_b) = 2\delta_{ab}$. If V is the volume of the lattice, $\Delta(\dot{U})$ is a $[(N^2-1)V] \times [(N^2-1)V]$ real symmetric matrix which depends on the gauge field. The eigenvalues μ_j of Δ are real and the eigenvector equation is

$$\Delta^{ab}_{xy}(\dot{U})\phi^{(j)}_b(y) = \mu_j\, \phi^{(j)}_a(x) \tag{2.3}$$

where $\phi^{(j)}$, $j = 1, \ldots, [(N^2-1)V]$ are the $[(N^2-1)V]$–dimensional (real) eigenvectors. So we can associate (N^2-1)–dimensional real vectors $\phi^{(j)}(x)$ to every lattice site.

Let us consider a gauge transformation $U'_\mu(x) = \Omega(x)U_\mu(x)\Omega^\dagger(x+\hat{\mu})$ on the fundamental links. Then, making use of the property $(\dot{AB}) = \dot{A}\dot{B}$, the eigenvector equation (2.3) becomes

$$\dot{\Omega}^{\dagger\, ai}(x)\Delta^{ik}_{xy}(\dot{U}')\dot{\Omega}^{kb}(y)\phi^{(j)}_b(y) = \mu_j\, \phi^{(j)}_a(x) \tag{2.4}$$

This relation shows that the eigenvalues are gauge invariant and the eigenvectors transform according to $\dot{\Omega}^{ab}(x)\phi^{(j)}_b(x) = \phi^{(j)\,\prime}_a(x)$. This transformation law can be rewritten as follows

$$\Omega(x)\Phi^{(j)}(x)\Omega^\dagger(x) = \Phi^{(j)\,\prime}(x) \tag{2.5}$$

where we have defined the $su(N)$ matrices (i.e. in the $SU(N)$ algebra) $\Phi^{(j)}(x) = \sum_{a=1}^{N^2-1}\phi^{(j)}_a(x)\lambda_a$ and $\Phi^{(j)\,\prime}(x) = \sum_{a=1}^{N^2-1}\phi^{(j)\,\prime}_a(x)\lambda_a$. Gauge transformations rotate the vectors $\phi^{(j)}(x)$ in color space and so we can fix the gauge by requiring a conventional arbitrary orientation for the $\phi^{(j)}(x)$. This orientation may depend on the lattice site and it is fixed once and for all; the simplest choice is to make it space–time independent. We will see below that to perform the reduction of the gauge symmetry from $SU(N)$ to Z_N we only need to fix the orientation of two eigenvectors of the Laplacian operator. Since we are interested in fixing a smooth gauge, we consider the two lowest eigenmodes $\phi^{(1)}$ and $\phi^{(2)}$.

The gauge fixing procedure can be split in two steps. In the first, one rotates $\Phi^{(1)}(x)$ at every x so that $\Phi^{(1)\,\prime}(x)$ is in the Cartan subalgebra of $su(N)$. This leaves a residual symmetry corresponding to gauge transformations belonging to the Cartan subgroup $U(1)^{N-1}$. Therefore, at that stage we already have obtained an unambiguous Abelian gauge, called Laplacian Abelian Gauge in [18]. Nothing more can be done with only one eigenvector. To further reduce the gauge freedom we must consider a second step where the second eigenvector $\phi^{(2)}$ is taken into account. The gauge transformation that has rotated $\Phi^{(1)}(x)$ to the Cartan subalgebra, maps $\Phi^{(2)}(x)$ to $\Phi^{(2)\,\prime}(x)$. While $\Phi^{(1)\,\prime}(x)$

is invariant under gauge transformations in $U(1)^{N-1}$, this is in general not the case for $\Phi^{(2)\,\prime}(x)$ (special cases are discussed later). The symmetry can now be fixed to Z_N by considering gauge transformations in $U(1)^{N-1}$ which make some conventionally chosen color components of the twice rotated matrix $\Phi^{(2)\,\prime\prime}(x)$ vanish. The remnant center gauge freedom, consistently with the construction, can not be fixed within the described procedure in the adjoint representation. In fact, as follows from equation (2.5), the matrices $\Phi^{(j)}(x)$ are invariant under gauge transformations in the center group Z_N (these gauge transformations are the identity in the adjoint representation). Now we describe explicitly how to perform the presented two–step program.

Step 1

The starting point is equation (2.5) for the first eigenvector $\phi^{(1)}$. $\Phi^{(1)}(x)$ is an $N \times N$ (traceless) hermitian matrix, and so there exist $N \times N$ unitary matrices $\Omega(x)$ which diagonalize it (we recall that the diagonal elements are the eigenvalues of $\Phi^{(1)}(x)$). This is equivalent to requiring that $\Phi^{(1)\,\prime}(x)$ be in the Cartan subalgebra of $su(N)$. Since we want $\Omega(x)$ to be a gauge transformation in $SU(N)$, we must require that $\det(\Omega(x)) = 1$. The condition that $\Phi^{(1)\,\prime}(x)$ is diagonal does not fix $\Omega(x)$ – e.g. the gauge symmetry – up to $U(1)^{N-1}$ transformations yet. In fact there are non-diagonal $SU(N)$ gauge transformations which, when applied to $\Phi^{(1)\,\prime}(x)$, have the only effect of exchanging the position of the eigenvalues along the diagonal. In order to eliminate this permutation arbitrariness, it is necessary to impose an ordering of the eigenvalues. This is simple to do since the eigenvalues of a hermitian matrix are real. Once some conventional ordering has been chosen, $\Omega(x)$ is really defined up to $U(1)^{N-1}$ transformations. A last remark concerns the explicit evaluation of $\Omega(x)$, which, in view of (2.5), is the problem of diagonalizing a hermitian matrix. For the $SU(2)$ and $SU(3)$ gauge groups, this evaluation can be performed analytically while for $SU(N \geq 4)$ it is necessary to make use of numerical methods.

Step 2

In **Step 1** we have found $\Omega(x)$ defined up to gauge transformations $V(x) \in U(1)^{N-1}$. A second step is necessary to reduce the symmetry from $U(1)^{N-1}$ to Z_N; this means that we must introduce a criterion which fixes $V(x)$ up to Z_N. Consider the second eigenvector $\phi^{(2)}$ and apply the gauge transformation $\Omega(x)$ to $\Phi^{(2)}(x)$ so as to obtain $\Phi^{(2)\,\prime}(x) = \Omega(x)\Phi^{(2)}(x)\Omega^{\dagger}(x)$. Now we look for $V(x)$ such that the vector $\phi^{(2)\,\prime\prime}(x)$, obtained from $\Phi^{(2)\,\prime\prime}(x) = V(x)\Phi^{(2)\,\prime}(x)V^{\dagger}(x)$, has a conventional orientation in color space. Also in this case there is no need that this orientation be the same at all x but, however, this is the simplest choice. In order to make the discussion more transparent, we explicitly consider the $SU(3)$ case; the generalization to $SU(N)$ is straightforward.

A generic matrix $V(x) \in U(1)^2$ can be written in the form $V(x) = \mathrm{diag}\left(\mathrm{e}^{i(2\alpha(x)+\beta(x))}, \mathrm{e}^{i(-\alpha(x)+\beta(x))}, \mathrm{e}^{-i(\alpha(x)+2\beta(x))}\right)$ with $\alpha(x)$, $\beta(x) \in [0, 2\pi)$. We can fix $V(x)$ up to gauge transformations in Z_3 by imposing the following requirements:

$$
\begin{aligned}
&1)\ \mathrm{Tr}\left(\Phi^{(2)\,\prime\prime}(x)\,\lambda_2\right) = 0 \\
&2)\ \mathrm{Tr}\left(\Phi^{(2)\,\prime\prime}(x)\,\lambda_7\right) = 0 \\
&3)\ \mathrm{Tr}\left(\Phi^{(2)\,\prime\prime}(x)\,\lambda_1\right) > 0, \mathrm{Tr}\left(\Phi^{(2)\,\prime\prime}(x)\,\lambda_6\right) > 0
\end{aligned}
\tag{2.6}
$$

The meaning of these three conditions becomes clearer when one writes explicitly $\Phi^{(2)\,\prime\prime}(x)$

$$
\Phi^{(2)\,\prime\prime}(x) = \begin{pmatrix}
\phi_3^{(2)\,\prime} + \frac{1}{\sqrt{3}}\phi_8^{(2)\,\prime} & \left(\phi_1^{(2)\,\prime} - i\phi_2^{(2)\,\prime}\right)\mathrm{e}^{3i\alpha} & \left(\phi_4^{(2)\,\prime} - i\phi_5^{(2)\,\prime}\right)\mathrm{e}^{3i(\alpha+\beta)} \\
\left(\phi_1^{(2)\,\prime} + i\phi_2^{(2)\,\prime}\right)\mathrm{e}^{-3i\alpha} & -\phi_3^{(2)\,\prime} + \frac{1}{\sqrt{3}}\phi_8^{(2)\,\prime} & \left(\phi_6^{(2)\,\prime} - i\phi_7^{(2)\,\prime}\right)\mathrm{e}^{3i\beta} \\
\left(\phi_4^{(2)\,\prime} + i\phi_5^{(2)\,\prime}\right)\mathrm{e}^{-3i(\alpha+\beta)} & \left(\phi_6^{(2)\,\prime} + i\phi_7^{(2)\,\prime}\right)\mathrm{e}^{-3i\beta} & -\frac{2}{\sqrt{3}}\phi_8^{(2)\,\prime}
\end{pmatrix}
\tag{2.7}
$$

For example, $\alpha(x)$ and $\beta(x)$ can be fixed requiring that

$$
\mathrm{e}^{3i\alpha(x)} = \frac{\phi_1^{(2)\,\prime}(x) + i\phi_2^{(2)\,\prime}(x)}{\sqrt{\phi_1^{(2)\,\prime 2}(x) + \phi_2^{(2)\,\prime 2}(x)}}
\tag{2.8}
$$

$$
\mathrm{e}^{3i\beta(x)} = \frac{\phi_6^{(2)\,\prime}(x) + i\phi_7^{(2)\,\prime}(x)}{\sqrt{\phi_6^{(2)\,\prime 2}(x) + \phi_7^{(2)\,\prime 2}(x)}}
\tag{2.9}
$$

making the elements $(1,2)$ and $(2,3)$ of $\Phi^{(2)\,\prime\prime}(x)$ real, i.e. satisfying conditions 1) and 2). The condition 3) in (2.6) is introduced to eliminate the sign ambiguities $\pm\left(\phi_1^{(2)\,\prime}(x) + i\phi_2^{(2)\,\prime}(x)\right)$ and $\pm\left(\phi_6^{(2)\,\prime}(x) + i\phi_7^{(2)\,\prime}(x)\right)$. In conclusion, after this second step, we have obtained the transformation $W(x) = V(x)\Omega(x)$ which fixes uniquely the gauge up to the center symmetry Z_3. In the $SU(N)$ case, the symmetry must be reduced from $U(1)^{N-1}$ to Z_N by fixing $(N-1)$ phases. This can be accomplished, in a simple and elegant way, by requiring that the $(N-1)$ sub–diagonal elements of $\Phi^{(2)\,\prime\prime}(x)$ be real positive. Moreover, with this choice of constraints, the conditions defining the monopoles turn out to be particularly simple.

Two last important remarks concern the accidental degeneracy of μ_1 or μ_2 in (2.3) and the sign arbitrariness in the eigenvectors $\phi^{(j)}$. In the described procedure we have assumed that the two lowest eigenvalues μ_1 and μ_2 of the Laplacian operator Δ are both not degenerate. If either one is degenerate, the gauge fixing can not be carried out in an unambiguous way; however these cases are really exceptional and in the numerical simulations never occur. Moreover this degeneracy is easy to check and so these events can be detected. The second point is about the arbitrariness in the eigenvectors $\phi^{(j)}$, since they are defined up to a global scale factor and a global sign. The two–step method to fix the gauge makes use of the orientation of the vectors $\phi^{(1)}(x)$ and $\phi^{(2)}(x)$ in color space. So, while the rescaling can not give rise to any ambiguity in the procedure, the freedom in the choice of the global sign can. This freedom can

be eliminated with a conventional choice on $\phi^{(j)}$.

III. LOCAL GAUGE AMBIGUITIES

Local defects may occur in the Laplacian Center Gauge fixing procedure. In this section we discuss how they show up and how they can be associated with monopoles and center vortices. This association is consistent with the initial proposal by 't Hooft of identifying gauge fixing defects and topological features of the gauge fixed theory. Moreover, in this approach, we will make apparent the close relation which exists between monopoles and center vortices in the Laplacian Center Gauge. Let us start by showing the conditions that give rise to defects at each step of the gauge fixing procedure.

Step 2 ill–defined

In **Step 2** we fix the $U(1)^{N-1}$ symmetry by looking for gauge transformations $V(x)$ in the Cartan subgroup that make some color components of $\phi^{(2)\,\prime\prime}(x)$ vanish. This is equivalent to requiring that particular entries in the complex matrix $\Phi^{(2)\,\prime\prime}(x)$ be real positive. If it happens that, at some point x, any of these entries is zero, the second step can not be carried out completely. Let us illustrate the argument by considering $SU(3)$. The conditions (2.6) fix $V(x)$ via the phases $\alpha(x)$ and $\beta(x)$ given by (2.8) and (2.9). If, at some point x, one of the following cases takes place

$$\begin{aligned} &1)\ \phi_1^{(2)\,\prime}(x) - i\phi_2^{(2)\,\prime}(x) = 0 \\ &2)\ \phi_6^{(2)\,\prime}(x) - i\phi_7^{(2)\,\prime}(x) = 0 \end{aligned} \tag{3.1}$$

the corresponding phase – $\alpha(x)$ for 1) and $\beta(x)$ for 2) – is not defined. Thus, at point x the gauge symmetry is not reduced to Z_3 and one of the two $U(1)$ subgroups is left unfixed. The conditions 1) or 2), and more generally the corresponding ones for $SU(N)$, set two constraints, and so the points x where the gauge symmetry is promoted from Z_N to $U(1)$ form 2–dimensional surfaces in the 4–dimensional space–time.

Step 1 ill–defined

In **Step 1** we reduce the gauge symmetry from $SU(N)$ to $U(1)^{N-1}$ by diagonalizing $\Phi^{(1)}(x)$ with a conventional ordering for the eigenvalues along the diagonal. If, at some point x, two eigenvalues happen to be equal, the conventional ordering is no longer unique and the procedure becomes ill-defined at that point. For instance, let us consider $SU(3)$. $\Phi^{(1)\,\prime}(x)$ can be written in the form

$$\Phi^{(1)\,\prime}(x) = a(x)\lambda_3' + b(x)\lambda_8 \tag{3.2}$$

where $a(x)$ and $b(x)$ are non-negative and $\lambda_3' = \frac{1}{2}(\sqrt{3}\lambda_3 + \lambda_8)$ (λ_3 and λ_8 are the diagonal Gell–Mann matrices). In writing (3.2) we have assumed a conventional decreasing ordering along the diagonal for the eigenvalues. $\Phi^{(1)\,\prime}(x)$ has

two equal eigenvalues in one of the following two cases:

$$\begin{aligned} &1)\ a(x) = 0 \\ &2)\ b(x) = 0 \end{aligned} \tag{3.3}$$

When one of these occurs, the residual gauge symmetry is $SU(2) \times U(1)$ instead of $U^2(1)$. Suppose that case 1) takes place, then ${\Phi^{(1)}}'(x)$ is invariant under gauge transformations generated by λ_8, λ_1, λ_2 and λ_3 and the last three matrices generate an $SU(2)$ subgroup. For the generic $SU(N)$ group, the symmetry is $SU(2) \times U(1)^{N-2}$. Let us now show that the points x where the first step can be performed only partially and the gauge symmetry is reduced to $SU(2) \times U(1)^{N-2}$, form 1–dimensional strings in 4–dimensional space–time. Imagine that the matrix $\Omega(x)$ which diagonalizes $\Phi^{(1)}(x)$ is computed in two stages. In the first stage, we partially diagonalize $\Phi^{(1)}(x)$ so as to leave only two off–diagonal elements different from zero. In the second stage, an $SU(2)$ transformation is considered which makes these last two off–diagonal elements vanish. This second stage is equivalent to the diagonalization of an $su(2)$ matrix $s(x) = \sum_1^3 s_k(x)\sigma_k$. Without loss of generality, we can always suppose that the $(N-2)$ diagonal elements found after the first stage and not touched by the $SU(2)$ transformation of the second stage, are all different, and that the equality of two eigenvalues of $\Phi^{(1)}(x)$ can show up only in the second stage (this means that, in general, the position of the non-zero off–diagonal elements in the partially diagonalized matrix depends on x and on the conventional ordering). Then the equality of two eigenvalues takes place at the points x where $s_1(x) = s_2(x) = s_3(x) = 0$: since three constraints must be satisfied, the points x form 1–dimensional strings in 4–dimensional space–time.

Now we present an argument showing that the gauge fixing defects of **Step 2 ill–defined** can be associated with center vortices. Again we consider $SU(3)$ and the extension to $SU(N)$ is straightforward.

Suppose that, at some point x_0, case 1) of (3.1) takes place. Let us define ${\phi^{(2)}_{1,2}}'(x) = {\phi^{(2)}_1}'(x) + i{\phi^{(2)}_2}'(x)$ and consider its Taylor expansion around x_0:

$${\phi^{(2)}_{1,2}}'(x) = (x - x_0) \cdot \nabla {\phi^{(2)}_{1,2}}'(x_0) + \mathcal{O}(x - x_0)^2 \tag{3.4}$$

obtained keeping in mind that ${\phi^{(2)}_{1,2}}'(x_0) = 0$. Consider then a plane passing through x_0. Moving on this plane in the neighborhood of x_0, ${\phi^{(2)}_{1,2}}'(x)$ has, up to some complex rescaling factor, a hedgehog shape that we can parametrize in polar coordinates

$${\phi^{(2)}_{1,2}}'(r,\theta) \simeq f(r,\theta)\, \mathrm{e}^{i\theta} \tag{3.5}$$

with $f(r,\theta)$ a real positive function. If we now impose the gauge fixing conditions (2.6) and we take into account

(2.7), we obtain that, in the neighborhood of x_0 and on the considered plane, the **Step 2** gauge transformation is given by $V(x) = \mathrm{diag}(\mathrm{e}^{-i\frac{2\theta}{3}}, \mathrm{e}^{i\frac{\theta}{3}}, \mathrm{e}^{i\frac{\theta}{3}})$. In this argument we do not consider the $U(1)$ subgroup parametrized by $\beta(x)$, since its contribution can be factored out. Applying the gauge transformation V

$$A_\mu \longrightarrow V(A_\mu + \frac{i}{g}\partial_\mu)V^\dagger \tag{3.6}$$

we obtain that, moving in the plane in the neighborhood of x_0, the gauge field gains the term

$$\frac{i}{g}V\partial_\mu V^\dagger = -\frac{1}{3gr}\,\lambda'_3\,\vec{\mathrm{e}}_\theta \tag{3.7}$$

If we now integrate over a closed path C around x_0 we have

$$\oint_C \frac{1}{3gr}\,r\,dr\,d\theta = \frac{2\pi}{3} \tag{3.8}$$

This means that a Wilson loop belonging to the considered plane and encircling x_0 gets a non-trivial factor $\mathrm{e}^{i\frac{2\pi}{3}}\mathbb{I}$ with respect to the center group Z_3. A similar discussion can be carried out for case 2) of (3.1). According to the presented argument, we establish a connection between center vortices and gauge fixing defects arising from **Step 2**. As for those arising from **Step 1**, 't Hooft already has shown how such gauge fixing defects can be identified with monopole world–lines [14]. Here we discuss the feature that, in the Laplacian Center Gauge, center vortices and monopoles turn out to be closely related in a unified description. Take the $SU(3)$ theory and consider the 2–dimensional surface Σ of the center vortices defined by $\phi_1^{(2)\,\prime}(x) = 0$, $\phi_2^{(2)\,\prime}(x) = 0$. Similarly to equation (3.2), we can write the diagonal elements of $\Phi^{(2)\,\prime\prime}(x)$ in the form $c(x)\lambda'_3 + d(x)\lambda_8$ where now $c(x)$ and $d(x)$ can be also negative. At every point x, $a(x)$ and $c(x)$ may have identical or opposite sign. Let us restrict ourselves to $x \in \Sigma$, and suppose that $a(x)$ and $c(x)$ have identical sign in some parts of Σ, opposite sign in others. By continuity, this implies that these patches must be separated by 1–dimensional closed strings where $a(x) = 0$ or $c(x) = 0$, the former of which defines a monopole world–line. Thus the 2–dimensional surface of center vortices contains embedded monopole world–lines. Moreover, all monopole world–lines are embedded in center vortex surfaces. To see this, consider an $SU(2)$ static monopole at some point x_0 in 3–dimensional space. $\phi^{(1)}$ vanishes at the monopole core x_0 and in its neighborhood has a hedgehog shape in color space. So there will necessarily exist some direction along which $\phi^{(1)}(x)$ is parallel to $\phi^{(2)}(x)$ – which is the condition identifying center vortices in $SU(2)$ –. Thus the core of a monopole is nested in the 1–dimensional closed string of a center vortex, which appears as two half-Dirac strings. Considering now the time evolution also, we see that a monopole world–line is always embedded in a 2–dimensional surface of center vortices. Since, in general, the pattern of the gauge symmetry promotion at the monopole core is $U(1)^{N-1} \longrightarrow SU(2)\times U(1)^{N-2}$

– i.e. one of the $U(1)$ subgroups is promoted to $SU(2)$ –, the argument given here for $SU(2)$ can be extended to a generic $SU(N)$. Fig.1 sketches the embedding of monopole world–lines in center vortex surfaces just discussed.

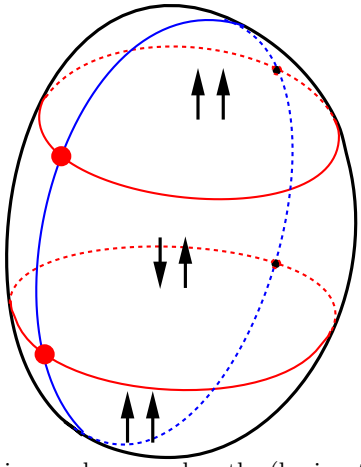

FIG. 1. The connection between center vortices and monopoles: the (horizontal) monopole world–lines separate two patches of center vortex surface with opposite eigenvector orientations; each monopole is attached to two center vortex strings.

This close connection between monopoles and center vortices has actually been observed on the lattice, and presented as a puzzle [19]: on a cooled $SU(2)$ lattice configuration, almost all Abelian monopoles, identified as a $3d$ cube through which one measures a 2π magnetic flux, are attached to two center vortex plaquettes. This is precisely the situation we describe.

IV. NUMERICAL RESULTS

We have performed numerical simulations to investigate the role of the center degrees of freedom in the $SU(2)$ and $SU(3)$ lattice gauge theories. According to the discussion of the two previous sections, we have made use of the Laplacian Center Gauge fixing to reduce the symmetry to the center subgroup. In this approach – considering for instance $SU(3)$ – we have described how center vortices can be detected by looking for the points x where, for example, $\phi^{(2)}_{1,2}{}'(x) = 0$ or, equivalently, $\phi^{(2)}(x)$ describes a 2π rotation in color space around the λ'_3 axis, moving along a closed contour encircling x. The method is the same for a generic group $SU(N)$. As a by-product, monopole world–lines are identified by the points x where two eigenvalues of $\Phi^{(1)}{}'(x)$ are equal; but our aim here is to study the center degrees of freedom and not the Abelian ones. Numerical simulations are performed on a lattice and so the eigenvectors of the adjoint Laplacian operator can be computed only at discrete points. This implies that an interpolation procedure must be defined to detect center vortices and, if desired, monopoles. We have tried several interpolation methods in order to investigate the robustness of the results with respect to different choices. Unfortunately, the arbitrariness introduced by this necessary step seems large at the lattice spacings we considered. Furthermore, we have not found an interpolation scheme which guarantees that center vortex surfaces are closed. Therefore, we have modified our procedure to identify center vortices on the lattice, and adopt as an intermediate step the center projection proposed

by Greensite and collaborators. Following this prescription, every gauge fixed link $U_\mu(x)$ is decomposed as the product of two parts

$$U_\mu(x) = Z_\mu(x) \cdot U'_\mu(x) \tag{4.1}$$

where $Z_\mu(x)$ is the center projected link living in Z_N and $U'_\mu(x)$ is the coset link taking values in $SU(N)/Z_N$. For instance, this splitting can be carried out by requiring that $|\arg(\mathrm{Tr}(U'_\mu(x)))| \leq \pi/N$. Thus, starting from an $SU(N)$ gauge fixed configuration, one builds up two projected configurations, made of the $Z_\mu(x)$ and the $U'_\mu(x)$ links. Then, if $W(C)$ is a Wilson loop along the closed contour C, making use use of the decomposition (4.1), one can write

$$W(C) = \sigma(C)\, W'(C) = \left[\prod_{p\in\Sigma} \sigma(p)\right] W'(C) \tag{4.2}$$

where $\sigma(p)$ is the plaquette in the center projected variables, and $W'(C)$ and $\sigma(C) \equiv \prod_{p\in\Sigma} \sigma(p)$ are the Wilson loops evaluated with the coset links $U'_\mu(x)$ and with the center links $Z_\mu(x)$ respectively. $\prod_{p\in\Sigma}$ is the product over all the plaquettes p belonging to a surface Σ supported on C; the value of $\sigma(C)$ does not depend on the choice of Σ and, for simplicity, we can suppose that it is the planar surface bounded by C. Since we have fixed a gauge where U'_μ is smooth, we expect that, to a reasonable approximation, $W(C)$ has a non-trivial value with respect to Z_N if and only if $\sigma(C)$ does. In this approach, a non-trivial value for $\sigma(p)$ is the "signal" for a center vortex.

We have collected 1000 $SU(2)$ configurations at three different values of the coupling constant $\beta = 2.3$, 2.4 and 2.5 on a 16^4 lattice; for $SU(3)$ we have generated 500 configurations on a 16^4 lattice at $\beta = 6.0$.

The following figure shows our measurement of the Creutz ratios $\chi(R) = -\ln(\langle W(R,R)\rangle\langle W(R-1,R-1)\rangle/\langle W(R,R-1)\rangle^2)$ for $SU(2)$ at $\beta = 2.4$ ($W(R,T)$ is the Wilson loop $R \times T$).

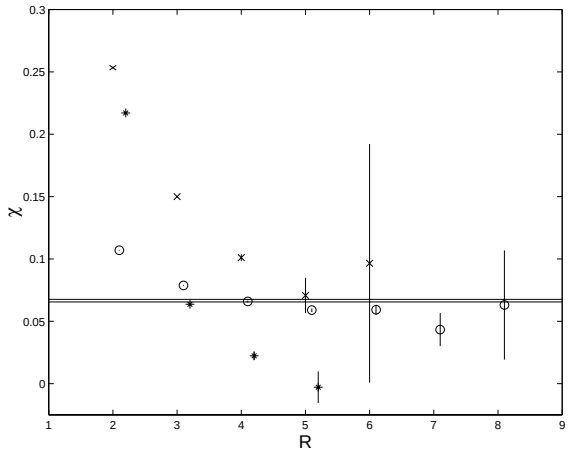

FIG. 2. Creutz ratios at $\beta = 2.4$. Crosses refer to $SU(2)$, circles to Z_2 and stars to the coset group $SU(2)/Z_2$. The continuous strip is the value of the string tension in the literature [20].

Crosses refer to $SU(2)$, circles to center projection after Laplacian Center Gauge fixing, and stars to the coset part. The continuous strip is the value in the literature [20,21] of the $SU(2)$ string tension for the chosen set of parameters. These numerical results show, on one hand, the flattening of the Creutz ratios in the Z_2 sector and, on the other hand, the vanishing of the Creutz ratios computed with the coset links. We have obtained analog behaviours for the other two values of β: 2.3 and 2.5. A similar study has been carried out for the $SU(3)$ lattice gauge theory. The next figure shows our results for the Creutz ratios in the Z_3 sector after Laplacian Center Gauge fixing of 500 configurations.

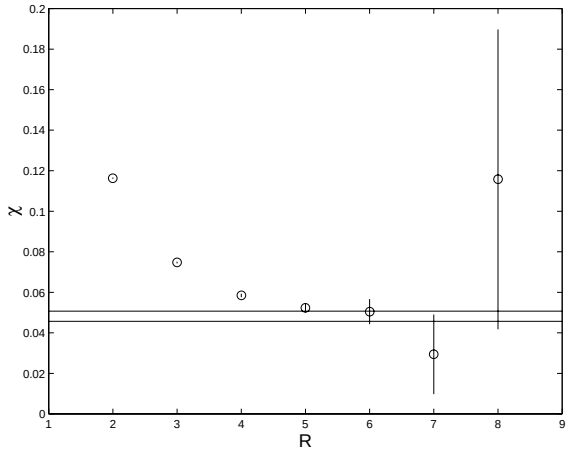

FIG. 3. Creutz ratios at $\beta = 6.0$ in the center sector Z_3 of $SU(3)$. The continuous strip is the value of the string tension in the literature [22].

The continuous strip is the value in the literature [22] of the $SU(3)$ string tension at the considered set of parameters. Also in this case, one clearly sees a nice flattening to a non-vanishing value of the Creutz ratios evaluated with the center projected links.

Therefore, our $SU(2)$ and $SU(3)$ results are qualitatively similar to those previously obtained in Direct Maximal Center gauge in $SU(2)$. They confirm the finding that the center-projected theory confines, with a string tension close to that of the original non-Abelian theory, whereas the coset theory does not. One difference however is apparent: the Creutz ratios tend to an asymptotic value much more slowly in Laplacian gauge than in DMC gauge. This is caused by the presence of many more close pairs of center vortices. Indeed, the vortex density is much higher, by a factor 3 to 5, than that measured in DMC gauge. A similar increase in the density of Abelian monopoles was observed previously in the Laplacian Abelian gauge [18]. One may consider this a practical nuisance, since these additional vortex pairs make the extraction of the projected string tension more noisy. One may instead consider that the center-projected theory represents the original one more closely, since it partially matches the short-distance increase of the force.

Finally, one can compare DMC and Laplacian gauges from the point of view of the computer effort. Because some eigenvectors of the adjoint Laplacian must be computed iteratively, one may get the impression that the Laplacian gauge is computationally expensive. That impression is misleading. Using the public-domain package ARPACK [23] to solve the eigenvalue problem, the computer time needed to gauge fix one 16^4 configuration is about the same as

for 50 Monte Carlo sweeps in the case of $SU(2)$, and 300 to 500 for $SU(3)$. This is far less than required to fix to DMC gauge iteratively.

V. INTERPRETATION OF THE NUMERICAL RESULTS

The figures presented in the previous section show good agreement between the Creutz ratios evaluated from the center projected links and the value of the string tension in the literature for $SU(2)$ and $SU(3)$ at the same β. However, the importance of this numerical agreement should not be overestimated. Numerical simulations are performed at a finite value of the lattice spacing, and we see no reason to believe that, if lattice artifacts are negligible for the $SU(N)$ gauge theory, their effect is equally small on the results obtained from the center projected model. Nevertheless, our conjecture is that, even if the value of the string tension in the center sector is appreciably modified by lattice artifacts at finite lattice spacing, this dependence has to vanish in the continuum limit. The observation of such a behaviour would be a robust confirmation of the relevance of the center degrees of freedom in the confinement mechanism. In order to investigate this issue, we have considered three different lattice Laplacian operators to fix the gauge. They differ by term which vanish in the continuum limit, i.e. higher derivatives or irrelevant operators. In practice, these new Laplacians have been obtained very simply, by smearing the links U_μ and substituting smeared links in the construction of the Laplacian (2.1). Specifically, we have considered 0, 1 and 5 smearing steps (weight = 0.5) on the U_μ links. We stress that the smeared links are used only to obtain different gauge fixing operators; the gauge transformations are always applied to the non-smeared link configurations, as well as the center projection and measurements.

The same sample of configurations has been fixed in each one of the three gauges, and the string tension has been measured in the center sector after center projection each time. In order to estimate more accurately the string tension from the Z_μ links, we have constructed smeared Wilson loops, where the spatial sides are made of links recursively smeared at a fixed time coordinate. This procedure is identical to that commonly used in the measurement of the $SU(N)$ static potential. It reduces the contribution of excited states, thus improving the statistical accuracy on the string tension. Although in the center projected theory, the existence of a transfer matrix is doubtful, we verified that spatial smearing did not introduce a measurable bias. From the Wilson loop data, we have extracted the Z_N string tension using the same fitting procedure as employed for the $SU(N)$ case, with an ansatz of the form $V_0 + \sigma R - \frac{e}{R}$ for the static potential $V(R)$ [1].

To illustrate our results, we show in the following figures the Z_2 and Z_3 Creutz ratios measured in the three gauges. The gauge dependence of the projected string tension is dramatic.

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[1]We gratefully acknowledge G. Bali for providing us with a data analysis program.

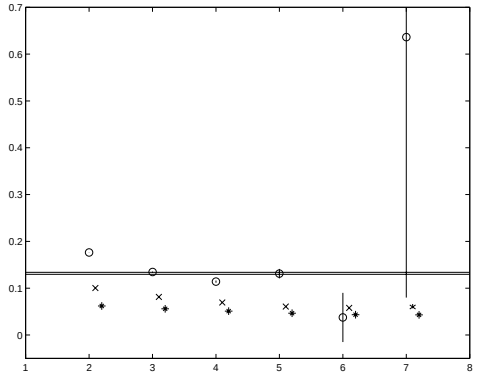

FIG. 4. Z_2 Creutz ratios at $\beta = 2.3$. The continuous strip is the value of the string tension in the literature [20].

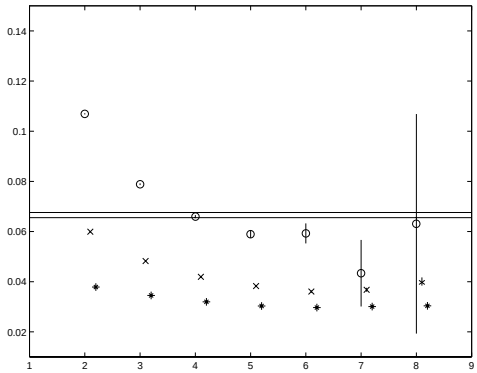

FIG. 5. Z_2 Creutz ratios at $\beta = 2.4$. The continuous strip is the value of the string tension in the literature [20,21].

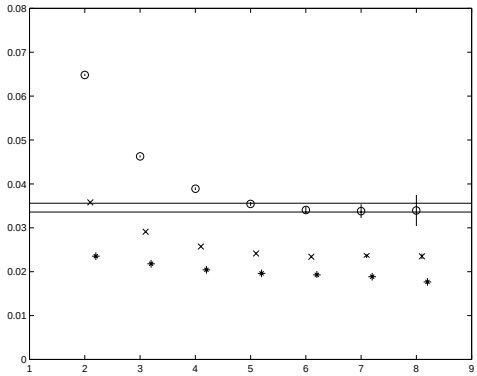

FIG. 6. Z_2 Creutz ratios at $\beta = 2.5$. The continuous strip is the value of the string tension in the literature [20,21].

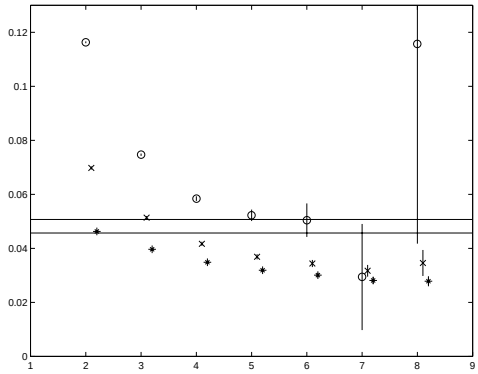

FIG. 7. Z_3 Creutz ratios at $\beta = 6.0$. The continuous strip is the value of the string tension in the literature [22].

The string tensions σ_i ($i = 0, 1, 5$ is the number of smearing steps performed on the links U_μ entering in the Laplacian operator eq.(2.1)) obtained from our 1000 $SU(2)$ configurations at $\beta = 2.3$, 2.4, 2.5 in the three different Laplacian Center Gauges are then compared with the $SU(2)$ string tension $\sigma_{SU(2)}$ reported in the literature. We denote by R_i the ratio $R_i = \sqrt{\sigma_i/\sigma_{SU(2)}}$. The following table summarizes our results:

	$\beta = 2.3$	$\beta = 2.4$	$\beta = 2.5$
R_0	0.813(23)	0.860(20)	0.978(18)
R_1	0.592(12)	0.720(11)	0.804(12)
R_5	0.547(8)	0.653(7)	0.739(11)

As the number of smearing steps i of the links entering the Laplacian increases, the projected string tension σ_i quickly decreases. This qualitative effect is easy to understand. The Laplacian made of smeared links becomes blind to short-range fluctuations, so that, after gauge fixing, the factorization of the Wilson loop eq.(4.2) into a smooth non-Abelian part $W'(C)$ and a disordered center part $\sigma(C)$ is spoiled. The smooth part $W'(C)$ is less smooth and carries some disorder, while the center part $\sigma(C)$ is less disordered, showing a reduced string tension. However, as β increases, the physical smearing radius ρ shrinks to zero, and the Table shows strong indications that the full string tension is recovered after center projection, in the continuum limit $\beta \to \infty$, for any fixed number i of smearing steps.

If, on the other hand, the amount of smearing was adjusted as a function of the lattice spacing so that the physical smearing radius ρ would remain constant, then one would achieve a continuum limit where, presumably, the center-projected string tension would be a fraction of the non-Abelian one. This fraction should decrease as the smearing radius ρ grows. When ρ reaches the physical size of a center vortex, the smeared Laplacian becomes blind to center vortices and the decomposition (4.2) becomes completely ineffective. This simple reasoning shows that the covariant operator used for the gauge fixing must be local for the center dominance scenario to be correct.

A similar analysis has been carried out for $SU(3)$ and the following table shows the corresponding results:

	$\beta = 6.0$
R_0	0.93(6)
R_1	0.818(32)
R_5	0.739(24)

The trend is similar to $SU(2)$. Furthermore, the ratios R_i are very close to those measured in $SU(2)$ at $\beta = 2.5$. Following the argument presented above, this indicates that the typical vortex size, in lattice units, is the same in both cases. With $a(SU(3), \beta = 6.0) \approx 0.1$ fm and $a(SU(2), \beta = 2.5) \approx 0.085$ fm, we obtain that the ratios of $SU(3)$ over $SU(2)$ center vortex sizes is about $0.1/0.085 \sim 1.18$. Such a slight increase is consistent with the expected increase of the adjoint string-breaking distance [24], which is connected to the center vortex size [25].

VI. CONCLUSION

The standard iterative local maximization methods used to fix the gauge on the lattice are ambiguous: they stop when *any* local maximum has been reached. Because of this ambiguity, one tends to mistrust measurements performed in such ill-defined gauges, as well as the physical models based on these measurements. This applies in particular to the scenario of vortex dominance, according to which the string tension of the non-Abelian theory can be exactly reproduced by considering only its center degrees of freedom, identified after gauge-fixing. Evidence for this scenario comes exclusively from lattice studies using ambiguous gauges. Some amount of counter-evidence [8,9] has also been reported, again in similar ambiguous gauges.

Motivated by our skepticism, we have constructed, by generalizing the approach of [11], a gauge which smooths the adjoint $SU(N)/Z_N$ field like the usual Maximal Center Gauge (MCG), but has no ambiguity. After gauge fixing to this Laplacian Center Gauge, a remaining local Z_N gauge freedom subsists. The local gauge defects which appear in this gauge are of two types: co-dimension 2, where the remaining gauge freedom is enlarged from Z_N to $U(1)$; and co-dimension 3, where it is further enlarged to $SU(2)$. These two types of defects can be identified with center vortices and Abelian monopoles respectively, with the latter embedded in the former. Thus we provide a natural, unified description of these two objects which had been considered as alternative choices of effective degrees of freedom.

We have numerically implemented the Laplacian Center Gauge for $SU(2)$ and $SU(3)$. In addition to being free of the ambiguities which plague the Maximal Center Gauge, our gauge is also computationally cheaper. A measurement of Creutz ratios in the center-projected Z_2 and Z_3 theories superficially confirms earlier observations made in MCG, albeit with a higher density of center vortices: the center-projected theory confines like the original one, while the coset $SU(N)/Z_N$ theory does not. Upon closer scrutiny however, the center-projected string tension is smaller than the original one. The difference between the two can be varied by arbitrary amounts, by adding higher derivative terms to the Laplacian used for the gauge fixing. Nevertheless, we have shown strong evidence that this difference

vanishes as the continuum limit of the lattice theory is taken. The gauge dependence of the center-projected string tension, clearly visible at finite lattice spacing a, goes away as $a \to 0$. Therefore, our study calls attention to lattice artifacts in the center projection, but confirms the center dominance scenario.

Since the center degrees of freedom (d.o.f.) account for the non-Abelian string tension, one may wonder if the center-projected theory can reproduce other non-perturbative features of the non-Abelian one, like chiral symmetry breaking or topological susceptibility. Indeed, one would expect this to be the case, since the coset $SU(N)/Z_N$ theory appears deprived of all these non-perturbative features [6]. However, one encounters two difficulties when trying to measure the appropriate observables in the Z_N theory: lattice artifacts and non-positivity of the transfer matrix.

We have already emphasized how lattice artifacts, in the choice of the discretized Laplacian or in the center projection step, can spoil the matching of the Z_N string tension with the $SU(N)$ one. Lattice artifacts affect the center vortex density so strongly that we cannot really quote a value for this important quantity. They also strongly influence the measurement of the chiral condensate in the Z_N theory [5]. Naturally, they also make it conceptually difficult to define a topological charge on the lattice, especially for a discrete Z_N theory.

Another, fundamental, difficulty comes from the non-locality of the gauge condition. After gauge fixing, all gauge links are correlated with each other, and this correlation persists after center projection. Furthermore, it may not die out exponentially at large distances in the center-projected theory, making it impossible to define a local effective Hamiltonian and a positive transfer matrix. Indeed, we have observed symptoms of this disease when attempting to measure glueball masses in the Z_N theory: in several cases, correlations at large distances would become significantly negative, and impossible to interpret as the Euclidean-time propagator of a superposition of eigenstates of a Hamiltonian. Difficulties have also been reported for the Abelian projection [26,27]. Symptoms may even be present for the string tension, since in Ref. [18], Creutz ratios measured after Abelian projection in the Maximal Abelian Gauge appear to *increase* with distance; a proper, normal decrease was restored when using instead the Laplacian Abelian Gauge. We also observed a similar increasing behaviour for the Z_N Creutz ratios when we applied Direct Maximal Center Gauge to our $SU(2)$ configurations [28]. One may wonder if the center d.o.f. are responsible for some non-perturbative features of the Yang-Mills theory, like the string tension, and not for others, like the glueball mass. Or it may be that a local effective Hamiltonian can truly be defined after center projection, but that an imperfect identification of the center d.o.f. spoils this construction: this would be one way to explain the restoration of the proper, decreasing behaviour of the Creutz ratios in the Laplacian gauge. One bold attitude towards this problem consists of making an ansatz for this effective Hamiltonian and test its consequences [29]. Much work remains to be done in studying the role and the interaction of the center d.o.f.

Here we have tried to put on firm numerical ground the first step in this ambitious program: Z_N center degrees of freedom are responsible for the full string tension of the $SU(N)$ theory. In the process of establishing this, we have

clarified the meaning of center vortices and Abelian monopoles as local gauge defects.

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