Laplacian Center Vortices[a]

Ph. de Forcrand
Inst. für Theoretische Physik, ETH Hönggerberg, CH-8093 Zürich, Switzerland
and
CERN, Theory Division, CH-1211 Genève 23, Switzerland
E-mail: forcrand@itp.phys.ethz.ch

M. Pepe
Inst. für Theoretische Physik, ETH Hönggerberg, CH-8093 Zürich, Switzerland
E-mail: pepe@itp.phys.ethz.ch

I present a unified picture of center vortices and Abelian monopoles. Both appear as local gauge ambiguities in the Laplacian Center Gauge. This gauge is constructed for a general $SU(N)$ theory. Numerical evidence is presented, for $SU(2)$ and $SU(3)$, that the projected Z_N theory confines with a string tension similar to the non-Abelian one.

1 Motivation and technical problem

The road traveled by physicists in their efforts to identify the effective, InfraRed degrees of freedom of QCD is far from straight. It bifurcates in many directions, most of which are still under exploration. Currently, the two most popular effective descriptions of confinement are in terms of Abelian monopoles [1,2] and center vortices [3,4]. I want to show that these two descriptions can be unified: these two branches of the road merge together, perhaps indicating that we are traveling towards a piece of Truth.

The study of center vortices, first proposed by Mack [4] and by 't Hooft [3], has been revived by Greensite and collaborators. They project the $SU(2)$ lattice gauge theory to a Z_2 gauge theory, by partially fixing the gauge to Maximal Center Gauge, defined as the gauge in which

$$\sum_{x,\mu} |Tr\ U_\mu(x)|^2 \quad \text{maximum} \quad . \tag{1}$$

In this gauge, the center-projected links are $z_\mu(x) \equiv \text{sign}(Tr\ U_\mu(x))$. This Z_2 theory has defects corresponding to plaquettes taking value -1. Greensite et al.[5] showed that the string tension σ given by these defects closely matches that of the original $SU(2)$ theory. Being skeptical about this, I investigated with M. D'Elia the coset theory, made of positive-trace links $U'_\mu(x) \equiv z_\mu(x)\ U_\mu(x)$. This theory has more short-range disorder than the original one, but carries

[a] presented by Ph. de Forcrand.

no center vortices. Could it be that $\{U'_\mu(x)\}$ would show long-range order, and thus not confine? To my surprise, we found [6] that in $\{U'_\mu(x)\}$: (i) $\sigma = 0$; (ii) $\langle\bar{\psi}\psi\rangle = 0$; (iii) $Q_{top} = 0$. Removal of center vortices causes deconfinement, chiral symmetry restoration and suppression of topological excitations. All non-perturbative properties disappear. Therefore, center vortices must carry the non-perturbative degrees of freedom.

There are two difficulties with this conclusion. First, a great deal of numerical evidence has been accumulated which ties confinement with Abelian monopoles instead of center vortices. Secondly, the above findings may depend on the choice of local maximum in (1). It is the purpose of this talk to resolve these two difficulties, as outlined already in [7].

The second problem is shared by the Abelian projection, which also proceeds by gauge fixing via the iterative, local maximization of a gauge functional. But in the Maximal Center Gauge, this problem can be acute. As shown in [8], the local maximum reached after starting from Landau gauge leads to a very small density of center vortices, which actually do not confine. Following this severe warning, some studies try to obtain the center-vortex properties of the global maximum of (1) by taking the highest among m local maxima, and extrapolating to $m \to \infty$ [9]. One feature underlines the difficulty of this approach: the extrapolated value for the global maximum of (1) falls *below* the measured value obtained by the procedure of [8].

To illustrate why this technical problem is so hard to resolve, let us consider a toy example. Take a 1-dimensional ring of $U(1)$ links $\{e^{i\theta_i}, i = 1,..,N\}$ such that the gauge invariant loop which they form is -1: $\prod_N e^{i\theta_i} = -1$. For such a system, the global maximum of (1), corresponding to Maximal Center Gauge, is obtained when $\theta_{i_0} = \pi$ for one link i_0, and $\theta_i = 0$ $\forall i \neq i_0$. The "kink" $\theta_{i_0} = \pi$ can be placed anywhere, giving rise to an $N-$fold degeneracy. The gauge-fixing functional (1) takes value N. Let us now fix this system to Landau gauge, defined as the gauge which maximizes $\sum_N ReTr\ U_i$. This is achieved when $\theta_i = \pi/N$. All link angles are small and there is no sign of a kink. The Center Gauge functional (1) then takes the value

$$\sum_N |Tr\ U_i|^2 = N cos\frac{2\pi}{N} \approx N - \frac{2\pi^2}{N} \tag{2}$$

One can check that Landau gauge represents a local maximum of (1); we see that the difference between this local maximum and the global one is vanishingly small as $N \to \infty$.

This case is not just a toy example. An Abelian loop having a phase of π is precisely the signature of an $SU(2)$ center vortex, as illustrated in Fig.1. It