ESTIMATION OF CONFIDENCE INTERVALS IN MEASUREMENTS OF TRILINEAR GAUGE BOSON COUPLINGS

B.P. Kerševan,* *B. Golob, G. Kernel, T. Podobnik*
Faculty for Mathematics and Physics, Ljubljana, Slovenia; Jožef Stefan Institute, Ljubljana,Slovenia

Abstract
Theoretical models, describing expected values of observables used in triple gauge coupling measurements at LEP2, impose different constraints on the values of measured quantities. Due to a presence of model excluded regions of possible measurements, estimation of confidence intervals turns out to be delicate. Instead of widely used classical central confidence intervals, estimation of confidence intervals, based on the likelihood ratio ordering is presented. The advantage of this method is that it always results in non-empty confidence intervals and properly takes into account a possibility of measurements outside the interval of model allowed values, thus giving correct coverage for any possible measurement outcome.

1. INTRODUCTION

Estimation of trilinear gauge boson couplings (TGC's) is one of advantages offered at e^+e^- collisions above the $W^\pm$ pair production threshold at LEP. In the present work attention is focused on the estimation of confidence intervals, resulting from the determination of TGC parameters at the W^+W^-V vertex, with $V \equiv Z^0, \gamma$. TGC values were extracted from the process $e^+e^- \to W^+W^- \to q_1\overline{q_2}q_3\overline{q_4}$, i.e. with each $W^\pm$ producing two jets of hadrons (Ref.[1]).

To avoid various specifics pertaining to diverse analyses, the sample analysis presented in this paper is done on the generator level, using the angular distributions and cross-sections as predicted by the EXCALIBUR event generator (Ref.[2]). In this analysis, the number of W^+W^- events corresponds to an integrated luminosity of $L = 200\ pb^{-1}$ taken at the center-of-mass energy of 189 GeV, roughly corresponding to the situation of the four LEP experiments in 1998. A 100 % selection and reconstruction efficiency with no background contamination is assumed.

The actual analyses of data, collected by DELPHI spectrometer at LEP in 1998 at an average center-of-mass energy of 189 GeV, are described in Ref.[3], while description of measurements at lower energies can be found in Refs.[4],[5]. In next sections only the common features relevant to the subject under study will be mentioned.

Results in TGC measurements are usually given in terms of the parameters Δg_1^Z, the difference between the value of the overall $W^+W^-Z^0$ coupling strength and its Standard Model prediction, $\Delta\kappa_\gamma$, the difference between the dipole coupling κ_γ and its Standard Model value, and λ_γ, the $W^+W^-\gamma$ quadrupole coupling parameter, corresponding to the Baur parameterization (Ref.[1]). The parameters involved are chosen so that the Standard model prediction gives a null value for all the three quantities. Whenever variation of one of the three TGC parameters is considered, the values of the other two are fixed at the values, predicted by the Standard Model.

Inferences about the models, describing the processes involving TGC's, are usually made in terms of point or interval estimates of unknown parameters. Most widely used method of confidence interval estimation is based on maximum likelihood approach, quoting the classical central confidence intervals. Predicted values of observables, mapping an outcome of a measurement to the parameter of interest, may be subject to constraints from theoretical models. In this case it can happen that a result of the

* Corresponding author

measurement yields an empty confidence interval at a given value of confidence level (CL). In general, several prescriptions are applied in order to solve this problem: from shifting the point estimate to the nearest theoretically allowed value, to *ad hoc* scalings of the confidence intervals. Lately, Feldman and Cousins (Ref.[6]) offered several plausible arguments for the use of the unified approach, based on the ordering of confidence intervals according to the likelihood ratio. The approach was originally developed to deal with small signals but its application to extraction of all kinds of bound parameters is straightforward. In the present paper this approach is applied in determination of confidence intervals for TGC parameter measurements.

In the next section a brief description of the likelihood ratio ordering for the determination of confidence interval is given. Section 3 describes evaluation of the measurement uncertainty using information form the total and differential $e^+e^- \to W^+W^- \to q_1\overline{q_2}q_3\overline{q_4}$ cross-section, followed by results from the combined information. Differences to a commonly used classical central interval estimation are exposed on the way. Conclusions are drawn in the last section.

2. CONFIDENCE INTERVALS OF PHYSICALLY BOUND PARAMETERS

A common prescription for determination of confidence intervals using the likelihood function $\ln \mathcal{L}(\vec{X}|\vec{\alpha})$ is given by the following condition (Ref.[7]): At a certain confidence level CL, the confidence interval for the estimation of a set of k unknown parameters $\vec{\alpha}_{true}$ given a measurement $\vec{X}^0$, is a union of all values of $\vec{\alpha}$ which satisfy the condition:

$$-2 \ln R(\vec{X}^0|\vec{\alpha}) \leq \chi^2_{CL}(k), \tag{1}$$

where $\chi^2_{CL}(k)$ is the CL point of the chi-square distribution with k degrees of freedom, i.e. the probability content of the $\chi^2(k)$ in the limits $[0, \chi^2_{CL}(k)]$ is CL. The likelihood ratio $\ln R$ term is defined as:

$$\ln R(\vec{X}^0|\vec{\alpha}) = \ln \mathcal{L}(\vec{X}^0|\vec{\alpha}) - \ln \mathcal{L}(\vec{X}^0|\vec{\alpha}_{best}) = \ln \frac{\mathcal{L}(\vec{X}^0|\vec{\alpha})}{\mathcal{L}(\vec{X}^0|\vec{\alpha}_{best})}, \tag{2}$$

where $\vec{\alpha}_{best}$ is the maximum likelihood point estimate. According to classical frequentist definition the confidence interval obtained by using criterion Eq.(1) should contain the true unknown value $\vec{\alpha}_{true}$ in CL of cases, thus satisfying the required coverage condition:

$$P(\vec{\alpha}_{true} \in CI = \{\cup\, \vec{\alpha}_i\}) = CL, \tag{3}$$

where the confidence interval $CI = \{\cup\, \vec{\alpha}_i\}$ is a defined as a union of all the values $\vec{\alpha}_i$ that satisfy Eq.(1).

In case of one unknown parameter the condition Eq.(1) translates into the well known *one-half rule*: the confidence interval is a union of all values of the parameter for which the value of the log likelihood function is less than 0.5 below maximum. The criterion Eq.(1) is exact only in the asymptotic limit (i.e. large enough statistics), nevertheless it is shown to be valid in a wide range of analyses (Ref.[7]). A need for special care has, however, been demonstrated in presence of physical boundaries (Ref.[6]), which can trivially be extended to all cases where certain regions of parameter space are excluded by an assumed physical model.

To show this explicitly the following example should be considered: the binned extended maximum likelihood (EML - Ref.[8]) method assumes the p.d.f. to be of the form:

$$P(\vec{n}|\vec{\mu}) = \prod_i^k \frac{\mu_i^{n_i}}{n_i!} e^{-\mu_i}, \tag{4}$$

where k is the number of bins and $\vec{\mu}$ represent the array of unknown parameters. Given a specific measurement $\vec{n}^0$ the corresponding log likelihood function is:

$$\ln \mathcal{L}(\vec{n}^0|\vec{\mu}) = \sum_i^k n_i^0 \ln \mu_i - \mu_i - \ln n_i^0!. \tag{5}$$

Assuming μ to be the true value, repeating the experiments would yield the values of $\vec{n}^0$ distributed according to the p.d.f. given in Eq.(4).The $\ln R$ function, given by Eq.(2) is in case of EML:

$$\ln R(\vec{n}^0|\vec{\mu}) = \sum_i^k \Delta\mu_i - n_i^0 \ln(1 + \frac{\Delta\mu_i}{\mu_i}), \tag{6}$$

with $\Delta\mu_i$ defined as $\Delta\mu_i = \mu_i^{best} - \mu_i$.

The values n_i^0 can be substituted by $n_i^0 = \mu_i + \delta_i$ where μ_i is the expected value of n_i^0 and δ_i is a (small) deviation in the asymptotic limit. A short calculation in case of unbound values of μ_i also gives $\mu_i^{best} = n_i^0$. The latter expression is however not correct for every possible value of n_i^0 in case the values of μ_i are bound by a model. In this case a modification should be made by adding an additional function of the measured $\vec{n}^0$ that incorporates the model boundaries on $\vec{\mu}$. The corrected expression is thus $\mu_i^{best} = n_i^0 + f_i(n_i^0)$ and subsequently the term $\Delta\mu_i$ can be expressed as $\Delta\mu_i = \delta_i + f_i(n_i^0)$. Assuming the terms δ_i and $\Delta\mu_i$ to be small the logarithmic term in equation Eq.(6) is expanded into a Taylor series and only terms up to a second order are kept; the expression becomes:

$$\ln R(\vec{n}^0|\vec{\mu}) = \sum_i^k -\frac{\delta_i^2}{2\mu_i} + \sum_i^k \frac{f_i(n_i^0)}{2\mu_i} = -\frac{\chi^2(k)}{2} + g(\vec{n}^0). \tag{7}$$

The above expression shows that in the absence of the model boundary on parameters $\vec{\mu}$ and thus the term $g(\vec{n}^0)$, the $\ln R$ is indeed distributed as a $\chi^2(k)$ and the condition in Eq.(1) holds. However, in case the parameters are bound, the additional term spoils this dependence and the coverage condition given by Eq.(3) is no longer satisfied. An additional point is that in derivation of Eq.(7) it was surmised the term $\Delta\mu_i$ to be small, which might in this case not hold even in the asymptotic limit. In this eventuality a more general condition should be applied instead of Eq.(1):

$$-\ln R(\vec{X}^0|\vec{\alpha}) \leq -\ln R_{CL}(k, \vec{\alpha}), \tag{8}$$

where a dependence of $\ln R_{CL}$ on $\vec{\alpha}$ should also be assumed. The boundary term $\ln R_{CL}(k, \vec{\alpha})$ may in many cases not be calculable analytically and thus Monte-Carlo simulation has to be employed. The Monte-Carlo method consists of generating MC events according to a given p.d.f. at a certain value of $\vec{\alpha}$, each time calculating the $\ln R$ value by using Eq.(2) and ordering the events according to this value, i.e. obtaining a $dN/d\ln R$ distribution. The $\ln R_{CL}(k, \vec{\alpha})$ limit is then obtained by requiring CL fraction of events with the lowest $\ln R$ to be contained in the interval $[\ln R_{CL}(k, \vec{\alpha}), 0]$. Subsequently the procedure should be repeated for every possible value of $\vec{\alpha}$ to obtain a full confidence belt. This method is by a short inspection completely analogous to the confidence belt construction using the unified approach as described in Ref.[6].

3. ESTIMATION OF TRILINEAR GAUGE BOSON COUPLINGS

Trilinear gauge couplings affect the differential cross-section $d\sigma/d\Omega$, where Ω represents the phase space of five independent kinematic quantities derived from the four-momenta of the four-fermion final state. In the given analysis however, only $d\sigma/d\cos\theta_{W-}$ is considered, i.e. differential cross-section as a function of the cosine of W^- production angle $\cos\theta_{W^-}$, which is defined as the angle between the direction of W^- boson and incoming e^-. Other kinematic quantities are extremely difficult to reconstruct due to reconstruction difficulties and ambiguities in the fully hadronic $e^+e^- \to W^+W^- \to q_1\overline{q_2}q_3\overline{q_4}$ decay channel and are also much less sensitive to the TGC values.

The sample analysis determines the point estimates and confidence intervals of the three parameters Δg_1^Z,$\Delta\kappa_\gamma$ and λ_γ using the maximum likelihood method. Procedure consists of three steps:

- Determination of TGC parameters using the total cross-section dependence on TGC-s and maximizing a Poisson log likelihood function:

$$\ln \mathcal{L}(N|\alpha) = N \ln \mu(\alpha) - \mu(\alpha) - \ln N!, \tag{9}$$

where N represents the measured number of events and $\mu(\alpha)$ the expected number of events as a function of the TGC parameters α.

- Determination of TGC parameters using angular distribution $\frac{1}{\sigma}\frac{d\sigma}{d\cos\theta_{W-}}$ dependence on the TGC parameters maximizing a binned multinomial log likelihood function:

$$\ln\mathcal{L}(N;\vec{n}|\alpha) = \ln N! + \sum_{i}^{k}\{n^{(i)}\ln p_i(\alpha) - \ln n^{(i)}!\}, \tag{10}$$

  where $\vec{n} = \{n^{(1)}, n^{(2)}, \ldots, n^{(k)}\}$ denotes an array of measured number of events in k bins and $p_i(\alpha)$ the corresponding probabilities.

- Combining the information by using extended maximum likelihood and maximizing a binned Poisson log likelihood function:

$$\ln\mathcal{L}(N;\vec{n}|\alpha) = \sum_{i}^{k}\{n^{(i)}\ln\mu_i(\alpha) - \mu_i(\alpha) - \ln n^{(i)}!\}, \tag{11}$$

  with $\mu_i(\alpha)$ representing the expected number of events in each bin.

As mentioned in the introduction in this analysis only one of the TGC parameters is left free while the other two are kept at SM values during maximization. The functional dependence of the quantities $\mu(\alpha), p_i(\alpha)$ and $\mu_i(\alpha)$ are determined from Monte-Carlo studies.

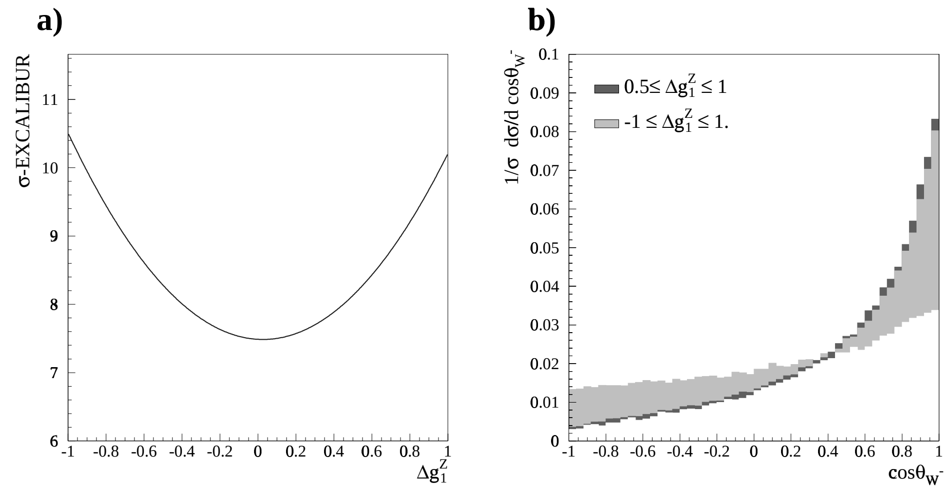

Fig. 1: a) The total $\sigma(e^+e^- \to W^+W^- \to 4q)$ with respect to Δg_1^Z parameter as predicted by the EXCALIBUR event generator. Quantitatively dependence on $\Delta\kappa_\gamma$ and λ_γ is similar. b) The normalised differential cross-section $\frac{1}{\sigma}\frac{d\sigma}{d(\cos\theta_{W-})}$ as a function of Δg_1^Z parameter. The total shaded region represents the change of distribution in a range $[-1, 1]$ and the dark shaded one the change in the range $[0.5, 1]$ of the given parameter. Quantitatively dependence on $\Delta\kappa_\gamma$ and λ_γ is similar, albeit somewhat weaker.

3.1 Total cross-section analysis

Theoretical dependence of the $\sigma(e^+e^- \to W^+W^- \to q_1\overline{q_2}q_3\overline{q_4})$ cross-section on the TGC parameters was obtained using the EXCALIBUR generator (Ref.[2]) and has a well-known parabolic shape (Fig.1a). Minimum of the cross-section dependence on Δg_1^Z is around 7.5 pb, which at a given integrated luminosity yields a number of expected signal events $\mu \gtrsim 1490$. The parabolic dependence of the cross-section on the TGC parameters and the corresponding expected number of events $\mu(\alpha) = L\cdot\sigma(\alpha)$ clearly shows that there is a model excluded region given by the interval $[0, \mu_0]$, where $\mu_0 = L \cdot \sigma_0$ is the lowest value on the parabola. Consequently any measured total number of $e^+e^- \to W^+W^- \to q_1\overline{q_2}q_3\overline{q_4}$ events sufficiently lower than μ_0 excludes the model at a certain CL, or in other words, for such a measurement we obtain an empty classical confidence interval at a given CL as shown in Fig. 2. Examples of possible measurement results are shown on the same plots for $N_0 = 1497$, corresponding to the Standard Model expectation of $\alpha = 0$, as well as for $N_0 = 1530$ and $N_0 = 1450$.

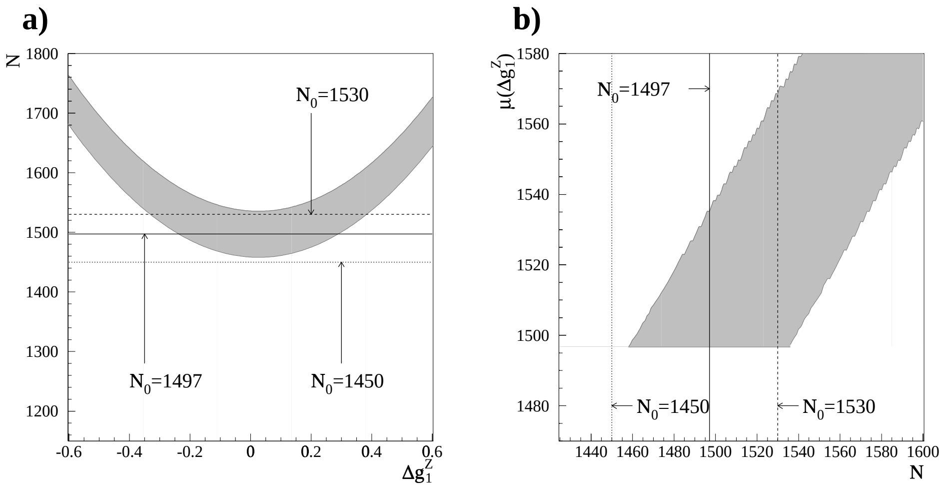

Fig. 2: a) Confidence belt constructed using central intervals at $CL = 68.3\%$, plotted as Δg_1^Z vs. N. In case of $N_0 = 1450$ it is evident that the confidence interval is an empty set. b) The same confidence belt plotted in the more conventional form N vs. $\mu(\Delta g_1^Z)$; in case of TGC parameters this form involves a two to one mapping due to the parabolic dependence of μ on Δg_1^Z.

If one wants, on the other hand, to extract the TGC parameter *within* the presumed model, while preserving the correct coverage, the Feldman and Cousins unified approach (Ref.[6]) using likelihood ratio ordering should be applied. Using this procedure a confidence belt yielding a non-empty confidence interval for every measured value is indeed obtained as shown in Fig. 3.

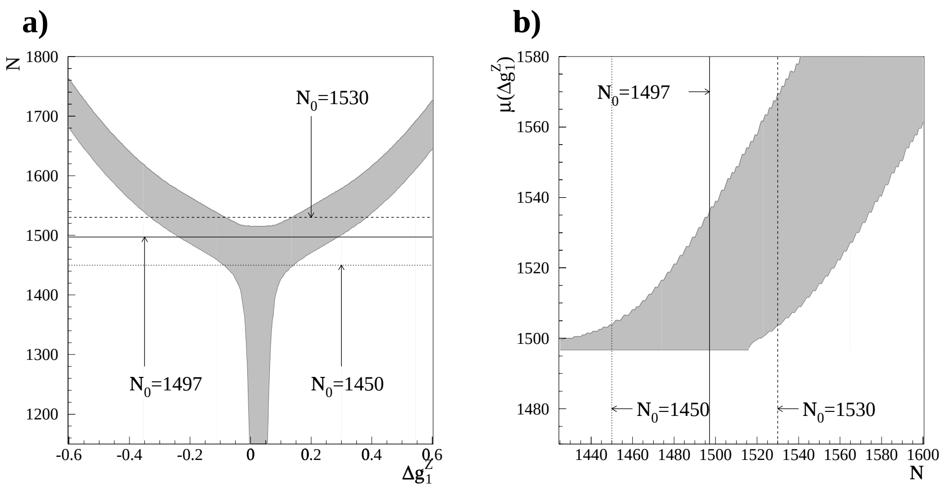

Fig. 3: a) Confidence belt constructed using likelihood ratio ordering, plotted as Δg_1^Z vs. N. It is evident that whatever the value N_0 the confidence interval is never an empty set. b) The same confidence belt plotted in the form N vs. $\mu(\Delta g_1^Z)$; note that the shape is equivalent to the case (Ref.[6]) with $\mu_0 = 0$.

As shown in section 2 the presence of the model boundary affects the confidence interval estimation in maximum likelihood method from simple one-half rule to a more complex prescription given in Eq.(8), where the limits of the new confidence belt $[\ln R_{CL}(k,\vec{\alpha}),0]$ depend on the TGC parameter. In this case the Monte Carlo technique is applied by generating measurement results N according to p.d.f. Eq.(9) and by that obtaining the distribution of $-\ln R$ for different assumed values of α. Due to the model excluded region of possible N, estimated value α^{best} is given by solving:

$$\left\{ \begin{array}{lll} \mu(\alpha^{\mathrm{best}}) & = N \; ; & \text{if } \; N \geq \mu_0 \\ \alpha^{\mathrm{best}} & = \; \alpha_0 \; ; & \text{if } \; N < \mu_0 \end{array} \right\} \tag{12}$$

where the value of the TGC parameter which yields a minimal cross-section is denoted by α_0.

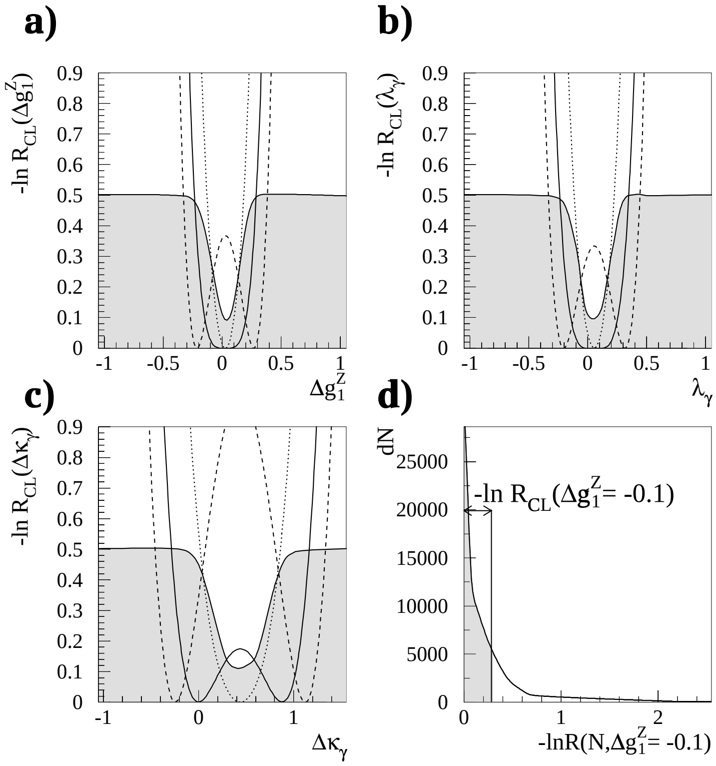

Fig. 4: $-\ln R_{CL}(\alpha)$ for a) $\alpha = \Delta g_1^Z$, b) $\alpha = \lambda_\gamma$ and c) $\alpha = \Delta\kappa_\gamma$ using the total cross-section information. Shadowed regions represent calculations for $CL = 68.3\%$. Lines show examples of measurement results as $-\ln R(N_0,\alpha)$ for $N_0 = 1497$ (full line), $N_0 = 1530$ (dashed line) and $N_0 = 1450$ (dotted line). d) $dN/d(-\ln R)$ distribution obtained by MC simulation in case of $\Delta g_1^Z = -0.1$.

An example of a MC generated $-\ln R$ distribution for $\Delta g_1^Z = -0.1$ is shown in Fig. 4d). 68.3% of $-\ln R(N,\Delta g_1^Z = -0.1)$ values lie in the interval $[0, 0.282]$ and hence $-\ln R_{CL}(\Delta g_1^Z = -0.1) = 0.282$. Repeating the random generation and calculation of $-\ln R_{CL}$ for different TGC parameters α results in a $[0, -\ln R_{CL}(\alpha)]$ confidence belt, which is shown in Fig. 4a-c). For large absolute values of TGC parameters, corresponding to measurements N far away from the model excluded region, the $-\ln R_{CL}$ value agrees with $\frac{1}{2}$ as expected. Boundaries of the excluded region manifest themselves as the deeps in $-\ln R_{CL}$, centered at values of TGC parameters for which the value of cross-section is minimal ($\Delta g_1^Z = 0.024$, $\Delta\kappa_\gamma = 0.438$, $\lambda_\gamma = 0.056$). Examples of possible measurement results are again shown on the same plots in the form of $-\ln R(N_0,\alpha)$, for $N_0 = 1497$, $N_0 = 1530$ and $N_0 = 1450$.

3.2 Angular distribution analysis

The normalised differential cross-section $\frac{1}{\sigma}\frac{d\sigma}{d\cos\theta_{W^-}}$ has a nonlinear dependence on the TGC parameters; its dependence on Δg_1^Z, as obtained by EXCALIBUR generator (Ref.[2]), is shown in Fig.1b). The angular distribution changes rapidly in the vicinity of the SM value, while at larger positive values the distribution change decreases and eventually the shape starts turning back towards the standard model, indicating a presence of a 'turning point'. Therefore, as in the case of the total cross-section measurement, the number of expected events within bins of $\cos\theta_{W^-}$ for different values of TGC parameters is limited by the model. Hence again deviations from the one-half rule are expected. As noted in section 3 a binned likelihood method corresponding to the multinomial p.d.f., is applied (c.f. Eq.(10)). Due to multidimensional nature a simple representation of the confidence belt construction is impossible. Instead, generation of likelihood ratio distribution as described in previous subsection should be applied.

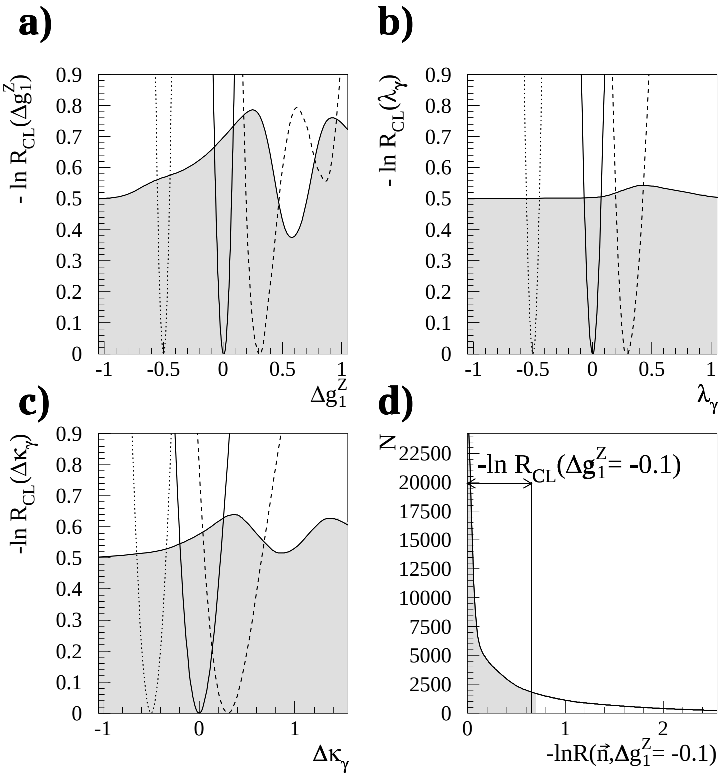

Fig. 5: a)-c) $-\ln R_{CL}(\alpha)$ for the three TGC parameters using the angular distribution information. Results of possible measurements are shown as $-\ln R(\vec{n},\alpha)$ for SM prediction $\alpha = 0$ obtained by EXCALIBUR generator (full line), $\alpha = -0.5$ (dotted line) and $\alpha = 0.3$ (dashed line). Deviations from the one half rule are most pronounced for Δg_1^Z and $\Delta\kappa_\gamma$. d) Example of $-\ln R_{CL}$ estimation at 68.3% CL in the case of $\Delta g_1^Z = -0.1$.

In the analysis 50 bins in $\cos\theta_{W^-}$ were used. For each value of the TGC parameter from -2.0 to 2.0 in steps of 0.1, numbers of events n_i in angular bins were randomly generated according to multinomial p.d.f. (Ref.[7]) using the standard routine of CERNLIB package (Ref.[9]). Probabilities were calculated from the EXCALIBUR predicted number of events in the i-th bin as $p_i(\alpha) = \mu_i(\alpha)/\sum_i \mu_i(\alpha)$. Point estimate α^{best} for each random generation of $\vec{n}$ was given by the value of α maximising Eq.(10). An example of the $-\ln R(\vec{n},\Delta g_1^Z)$ distribution for $\Delta g_1^Z = -0.1$ is shown in Fig. 5d), together with the interval $[0, -\ln R_{CL}(\Delta g_1^Z = -0.1)]$ for $CL = 68.3\%$.

The $-\ln R_{CL}(\alpha)$ dependence is shown for all three TGC parameters in Fig. 5a-c). As in the case of the total cross-section measurement, examples of possible experimental results are shown on the same plots in the form of $-\ln R(\vec{n}_0, \alpha)$ functions. The chosen ones correspond to the exact SM distribution ($\vec{n}_0$ equal to the MC prediction for $\alpha = 0$), and to $\alpha = -0.5, 0.3$ for each of the TGC parameters respectively.

Substantial deviations from the one-half rule can be seen around the 'turning point' of the distribution dependence on the TGC parameter involved (see fig 1b). For example in fig. 5a) in case of $\Delta g_1^Z = 0.3$ the likelihood ratio method gives two disconnected intervals at 68.3% CL while the one-half rule would give only one interval which is not equal to either of the two. The deviations from the one-half rule in case of λ_γ are only slight.

3.3 Combined analysis

As a final step in our analysis the two informations obtained from angular distribution and total cross-section can be combined simply by multiplying the p.d.f.-s which correspond to extended maximum likelihood analysis given by Eq.(11). The $-\ln R_{CL}(\alpha)$ values can again be obtained by MC simulation and the results are shown in Figs. 6a-c). At the assumed luminosity and precision of the measurement (i.e. selection and reconstruction efficiency being ideal) significant deviations from the one-half rule remain evident only in the case of $\Delta\kappa_\gamma$, however this cannot be generalised to a real physical analysis with lower statistics and/or precision.

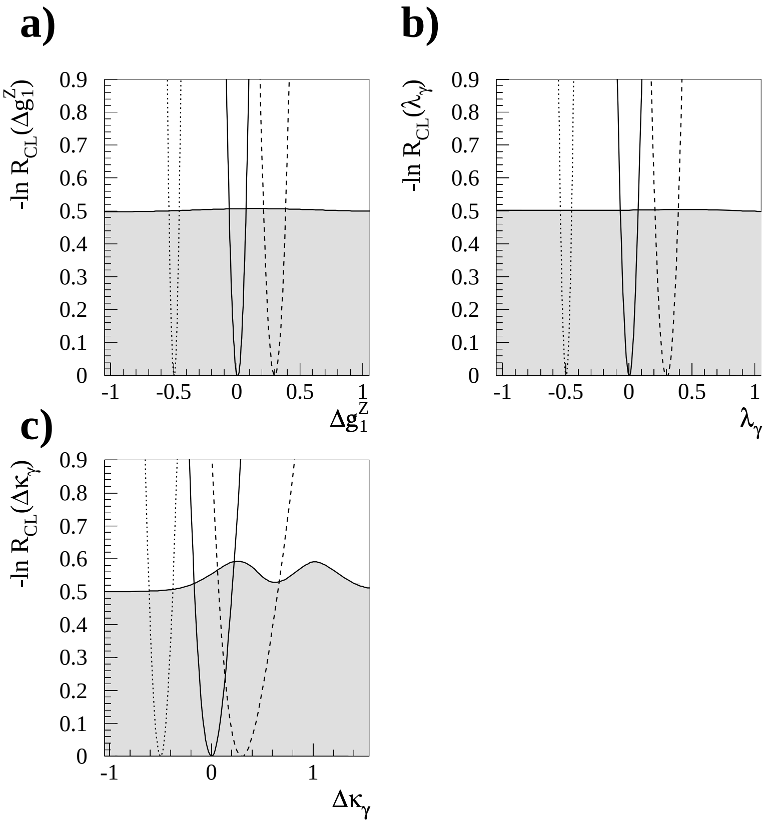

Fig. 6: $-\ln R_{CL}(\alpha)$ for the three TGC parameters in case of using combined total cross-section and angular distribution information. The deviations from one half rule remain evident in case of $\Delta\kappa_\gamma$, whereas in the case of the other two TGC parameters the deviations from the one half rule are negligible at the assumed luminosity, selection and reconstruction efficiency.

4. CONCLUSION

Estimation of confidence intervals for a parameter μ stemming from a measurement of observable x is delicate in the presence of model boundaries for possible measurement outcomes. Feldman and Cousins recently suggested Ref.([6]) a unified approach to the classical statistical analysis, based on the likelihood ratio ordering. Advantage of such an approach is in obtaining confidence intervals *within* a model assumed, taking into account measurements which would yield an empty classical confidence interval, i.e. decoupling goodness-of-fit from CI estimation while preserving the correct coverage.

Example of measurement in the proximity of the model limits is triple gauge coupling determination at LEP2 collider. Using the total number of observed $e^+e^- \to W^+W^-$ events as an observable for estimation of TGC's of two charged and a neutral gauge boson reveals a discrepancy between the confidence intervals calculated by both methods. The discrepancy reflects the model excluded region of expected number of events below the minimum of the parabola that describes the $\sigma(e^+e^- \to W^+W^-)$ dependence on the TGC parameter. Using the likelihood ratio approach, the confidence intervals can be deduced for each measurement of the total number of events N, even when N is lower than the minimal expected number of events. In case of the classical central intervals such a measurement would lead to an empty confidence interval at a certain confidence level CL.

Another observable, applicable to the TGC measurements at LEP2, is the distribution $\frac{1}{\sigma}\frac{d\sigma}{d\cos\theta_{W^-}}$, where θ_{W^-} represents the angle between the direction of W^- boson and incoming e^-. Like the total cross-section for $W^\pm$ pair production, angular distribution shows a non-linear dependence on the parameters of interest and model excluded region of expected number of events in bins of $\cos\theta_{W^-}$. Since the multidimensional nature of the multinomial probability density function, describing numbers of events in individual angular bins, prevents a classical confidence belt construction, a large number of toy MC experiments has been performed, resulting in the distribution of the likelihood ratio and consequently in construction of the confidence intervals. Again a significant difference is observed with regard to the classical central confidence intervals.

Following the procedure used for the two measurements, the total cross-section and the angular distribution, confidence intervals for the three TGC parameters were evaluated also for the case of combined information. These are found to be in agreement with the intervals obtained from the widely used one-half rule, for the Δg_1^Z and λ_γ parameters, while small differences remain in the case of $\Delta\kappa_\gamma$. It should be noted that the sample analysis was done on the generator level assuming ideal selection and reconstruction; a more realistic analysis, including reconstruction effects in determination of the $W^\pm$ charge and its direction, might give raise to larger deviations from the classical intervals. Hence in the TGC measurements, because of the proximity of the model bounds, one should calculate the confidence intervals based on the likelihood ratio ordering at least in order to check the reliability of the quoted errors.

References

[1] G. Gounaris, J.-L. Kneur, D. Zeppenfeld, in *Physics at LEP2*, eds. G. Altarelli, T. Sjöstrand, F. Zwirner, CERN **96-01** Vol. 1 (1996) 525.

[2] F.A. Berends, R. Kleiss, R. Pitau, in *Physics at LEP2*, eds. G. Altarelli, T. Sjöstrand, F. Zwirner, CERN **96-01** Vol. 2 (1996) 23.

[3] B.P. Kersevan, T. Podobnik, G. Kernel, B. Golob et al., DELPHI Coll., DELPHI internal note, DELPHI **99-57** PHYS 826;
T.J.V. Bowcock, C. DeClercq, G. Fanourakis, D. Fassouliotis, D. Gelé, B. Golob, B. Kerševan, A. Kinvig, V. Kostioukhine, J. Libby, A. Leisos, N. Mastroyiannopoulos, C. Matteuzzi, M. McCubbin, M. Nassiakou, G. Orazi, U. Parzefall, H.T. Phillips, R.L. Sekulin, F. Terranova, S. Tzamarias, A.

Van Lysebetten, O. Yushchenko et al., DELPHI Coll., paper submitted to the HEP’99 Conference, DELPHI **99-63** CONF 250 (1999)

[4] P. Abreu et al., DELPHI Coll., Phys. Lett. **B459** (1999) 382.

[5] G. Abbiendi et al., OPAL Coll., Eur. Phys. J. **C8** (1999) 191.
P. Abreu et al., DELPHI Coll., CERN-EP **99-62** (1999), acc. by Phys. Lett. B
R. Barate et al., ALEPH Coll., Phys. Lett. **B422** (1998) 369.
M. Acciarri et al., L3 Coll., Phys. Lett. **B413** (1997).

[6] G.J .Feldman, R.D. Cousins, Phys.Rev. **D57** (1998) 3873.

[7] W.T. Eadie, D. Drijard, F.E. James, M .Roos and B. Sadoulet, *Statistical Methods in Experimental Physics* North Holland, Amsterdam and London, (1971).

[8] L. Lyons, W. Allison (Oxford U.), *Maximum Likelihood or Extended Maximum Likelihood?* Nucl.Instrum.Meth.**A245:530** (1986)

[9] Description available at http://wwwinfo.cern.ch/asdoc/Welcome.html

Discussion after talk of Borut-Paul Kersevan. Chairman: Wilbur Venus.

Bob Cousins

I have a question about when an interval is split into two intervals. Is it like the case of neutrino oscillations? Would that make sense, or does it not make sense.

Borut-Paul Kersevan

It does make sense. Due to a turning point in the angular distribution dependence on TGC-S, we have two local minima in the minus log-likelihood curve, even at this sensitivity on generator level; the distribution shapes on the two sides of the turning point are not equivalent, but with given statistics we can get a jump (change of global minimum) to the other side of the turning point, so this is also a cause of bias. Actually having the two intervals correctly set means that the likelihood ratio approach gives the correct coverage. This approach can be used in confidence belt computation and maximum likelihood, either of which would take into account the biases or the discrepancies as well, so it's sensible.