ON THE PROBLEM OF LOW COUNTS IN A SIGNAL PLUS NOISE MODEL

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Abstract
Suppose that an observed count n, say, is composed of a signal plus a background variable, where the expected value of the background is known but that of the signal is not. What special techniques, if any, are appropriate if the observed count is smaller than the expected background? We argue that it is appropriate to base inferences on the conditional distributions of the count given that background variable is at most n. This proposal is supported by the ancillary nature of the background and a connection with admissibility

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Discussion after talk of Michael Woodroofe. Chairman: Jim Linnemann.

Gary Feldman

As Bob showed you in his talk, in the Nomad experiment we used the unified approach to combine different bins, and some of the bins obviously had signal greater than background, some had signal less than background, and so forth. Now if you just have a simple Poisson experiment, it's well-known that if you divide it into many bins, and combine them in this way, you get identical answers as if you just throw them all into the same bin. The question is: what happens if you try to condition - if you divide a Poisson into many bins, condition each of those bins and then combine? Does the system still work, and if not, what's the implication for the type of thing we did in Nomad?

M. Woodroofe

No, you can't combine. Once you've conditioned, you've destroyed this property of adding things up. As far as trying to apply this method when you're getting data from several different sources, I would try to do the combination to the maximum extent possible, and then condition. Now that might end up having two or three conditions. If two groups of experiments were similar to each other within groups, but not between groups, I might combine within groups, condition within groups, and then multiply the likelihood functions together.

Tom Junk

Just a question of symmetry. If you're looking for a signal that's negative (neutrino disappearance, or something that interferes destructively with the background), can some similar kind of conditioning be applied when you have too much background?

M. Woodroofe

I don't think so. What we have done would not work if the μ could be negative. Somehow then having observed the count of N, you don't really know that the background was less than or equal to N.

Carlo Giunti

So in your method you don't have correct coverage. I would like to know how you interpret the limit.

M. Woodroofe

We do not have unconditional coverage. I tried to argue that the conditional ensemble was better than the unconditional one. We should try to match the experiment that was actually done in the ensemble. So that means that we have conditional coverage: The conditional probability of coverage is 90%, and it does not mean that the unconditional probability of coverage is 90%, and in fact it is not. It can be quite a bit less, as in the example that Bob showed me. Now it is in principle, and I think probably in practice, possible to have the best of both worlds, I mean, to have both types of coverage, at the expense of having some over-coverage.

Peter Clifford

Would you like to say something about the very special way in which you are using ancillarity? It's not the classical. The classical definition is that you have a number of sufficient statistics, and you look at a function of the sufficient statistics which has a distribution which doesn't depend on the parameter, and you condition on that. You are conditioning on something which you don't actually observe, and it's certainly not a function of sufficient statistics. It's a very clever idea, but it's quite unusual. I wonder if you could say something about how you see that fitting into the classical definition.

M. Woodroofe

What we're doing goes beyond anything that Fisher actually said, or anything I can find that his followers have actually said. There is a paper called "The functional model" by David and Stone, I think, in which they mention this possibility. They say 'this doesn't quite fit into our general scheme'. So the answer to the question you asked is: Yes, what we are doing is different, and is suspect for precisely that reason. We think it makes sense, and we need to explore it now in more cases. We have really only worked out the one Poisson case, which does have the special feature that $N = 0$ implies $B = 0$.